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ASSERTION WITHOUT EMPIRICAL BASIS: AN ECONOMETRIC APPRAISAL OF MONETARY TRENDS IN ... THE UNITED KINGDOM BY MILTON FRIEDMAN AND ANNA J. SCHWARTZ

by

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ABSTRACT

This paper critically re-evaluates some of the fundamental empirical claims about monetary behaviour in the United Kingdom made by Milton Friedman and Anna J. Schwartz in their 1982 book Monetary Trends in the United States and the United Kingdom. We focus on six aspects of their analysis: the exogeneity of money; their claims of the constancy and correct specification of their money-demand equation; their interpretation of a dummy variable in that equation as capturing a "shift in liquidity preference" for 1921-55; their treatment of the interdependence of money, income, prices, and interest rates; and their use of phase-average data. They fail to support many of their empirical assertions with valid econometric evidence: in particular, they leave untested many conditions necessary to sustain their inferences. However, those conditions either are in part directly testable from their data or have testable implications: we test many of those hypotheses and reject virtually all of them. We reject basic claims made for their empirical model of money demand, e.g., those of parameter constancy, price homogeneity, and normality of the disturbances. En route, we show that their model of velocity as a constant performs poorly relative to the "will-o'-the-wisp" model of velocity as a random walk. As constructive evidence against their models, we develop a money-demand model superior to either model of velocity, and which has an unexplained residual variance less than one tenth that of their money-demand equation. This paper, however, is not an "anti-monetarist" critique; rather, it is a pro-econometrics tract which highlights the practical dangers of seeking to analyse complex stochastic processes while eschewing modern econometric methods.

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I. Introduction

> "..., the only relevant test of the validity of a hypothesis is comparison of its predictions with experience. The hypothesis is rejected if its predictions are contradicted ..."
>
> Milton Friedman (1953, pp. 8-9)

> "In the social sciences there is no such thing as refutation; only embarrassment."
>
> Converse's Law as formulated by John Converse in 1977 and quoted in Charles Dyke (1981, p. 71).

In their 1982 book Monetary Trends in the United States and the United Kingdom: Their Relation to Income, Prices, and Interest Rates, 1867-1975, Milton Friedman and Anna J. Schwartz present a wealth of empirical statistical material claimed to substantiate a range of economic hypotheses. Their approach involves transforming annual observations to averages over phases of "NBER reference cycles", followed by detailed graphical and regression studies of the resulting numbers. For excellent reviews and useful summaries, see *inter alia* Charles Goodhart (1982), Thomas Mayer (1982), Tim Congdon (1983), Michael J. Artis (1984) and Basil Moore (1983).

Their chapter summarising their "Principal Empirical Findings" is ten pages long and contains many claims based on inferences from their statistical analyses. There are too many claims to investigate in a single paper, but we show below that, despite their painstaking analyses, Friedman and Schwartz (1982) (abbreviated to FS henceforth) have not credibly established a significant number of their main empirical claims about monetary behaviour in the United Kingdom. The claims in question concern the "exogeneity" of money; the constancy over time of the parameters in their money-demand model and the adequacy of its specification; the validity of ignoring the mutual interdependence of money, income, prices, and interest rates when using regression techniques; and the success of their phase-averaging procedure in isolating long-run behaviour. Most of their claims either are in part directly testable from their data or have testable implications. Nevertheless, FS do not actually present any tests of those claims. By itself, that lacuna removes the credibility from their claims although it does not establish that their assertions are incorrect. However, we have tested those assertions, focussing on the specification of their money-demand equation over the historical period they selected. We have found that most of their assertions are in fact rejected by their data, leaving their conclusions stranded as assertions devoid of empirical support. Whether their theoretical models constitute a useful framework for viewing "long-run" monetary history is logically a separate issue (albeit an important one) from that of the credibility of their assertions. Evidence pertinent to the former issue is also adduced below. Nevertheless, our primary concern here is to demonstrate that one cannot take at face value many of the inferences conducted by FS.

After considering the data and data transformations used by FS (Section II), we evaluate their results for velocity and money-demand relationships as fitted to phase-average data for the UK and reject almost all their claims for those relationships (Section III). Our analysis of such claims derives from a coherent statistical framework and theory of testing which we exposit in Sections IV and V. Systematically applying that methodology, we investigate demand-for-money functions using the annual UK data series published by FS and attempt to reconcile our estimates based on annual data with theirs on phase-average data (Section VI and Appendix B). Section VII concludes the paper.

Although we reject many of FS's central empirical claims, this paper is not an "anti-monetarist" critique. Rather, it is a "pro-econometrics" tract which highlights the practical dangers of seeking to analyse complex stochastic processes while eschewing modern econometric methods.

II. The data series and transformations

We consider immediately the annual and phase-average data series to be used in our study, note certain reservations on the measurement and interpretation of those series, and discuss some of the statistical effects of transforming the annual data to phase averages.

In Table 4.9, FS report data series from 1871 to 1975 on the "annual" values in the UK of the broad money stock (M), real net national income (I), the price level (P), short-term and long-term nominal interest rates (RS and Rℓ), population (N), the exchange rate against the US dollar (E, in \$ per £), and high-powered money (H). We also include FS's (Table 4.8) series for the price level in the USA (P*) in our data set.[1] Unless otherwise noted, capital letters denote both the generic name and the level; logs of scalars are in lower case, vectors and matrices in bold face. Relevant series are rescaled proportionately from 1871 to 1920 to remove the break in 1920 when Southern Ireland ceased to be part of the United Kingdom. Otherwise, the data are unaltered. Figures I-III show various time series after the rescaling.

However, FS do not directly analyse this "raw" data, and instead transform the annual series by averaging separately over contraction and expansion phases of data-selected choices of "reference business cycles". Their objective in so doing is to extract the "longer-term movements" in the data (the focus of their study; nb. pp. 13-14). Further, they claim that phase-averaging reduces serial correlation arising from the business cycle (p. 78) and attenuates measurement errors (p. 86). Attaining those effects is important to their statistical analysis. Over some hundred years of annual observations, FS identify 37 such phases for the UK, with

[1] For details of construction and definition of all series, see FS (chapters 4 and 5) and our Appendix A.

Figure I. The log of the velocity of money (v_t) and the yield on 3-month bills (RS_t) (annual data).

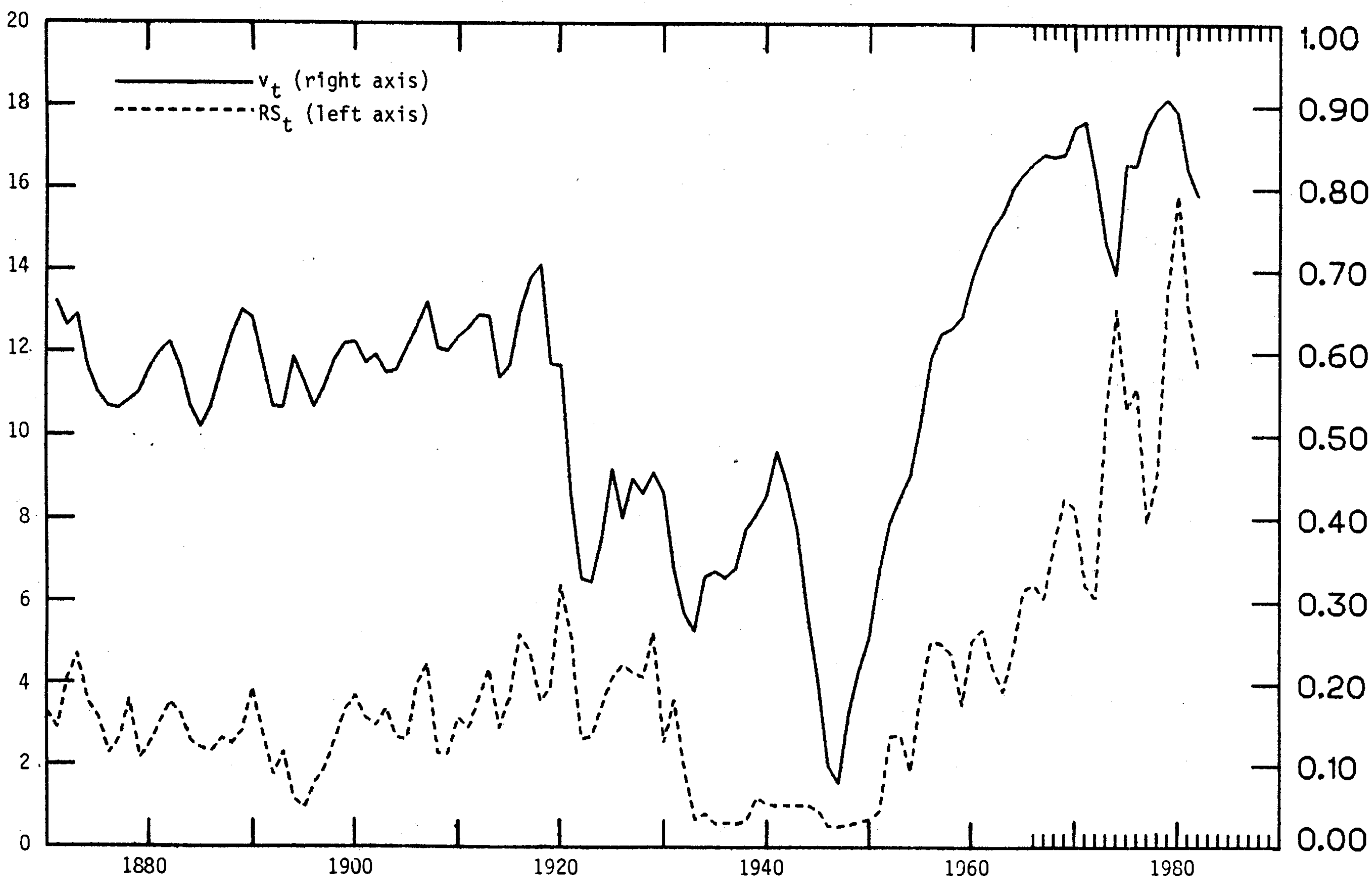


Figure II. The log of the real money stock [$(m-p)_t$] and the log of real income (i_t) (annual data).

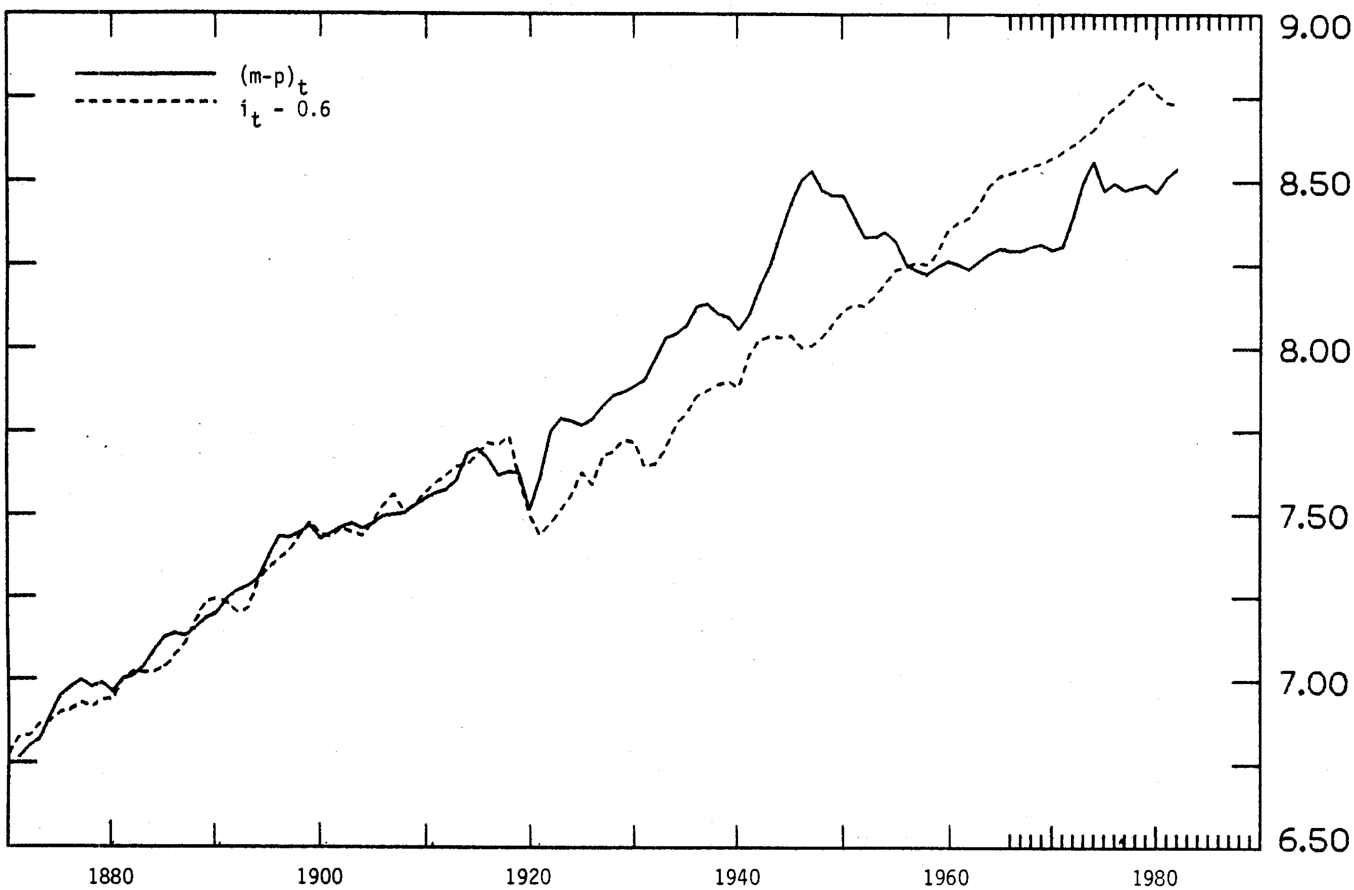
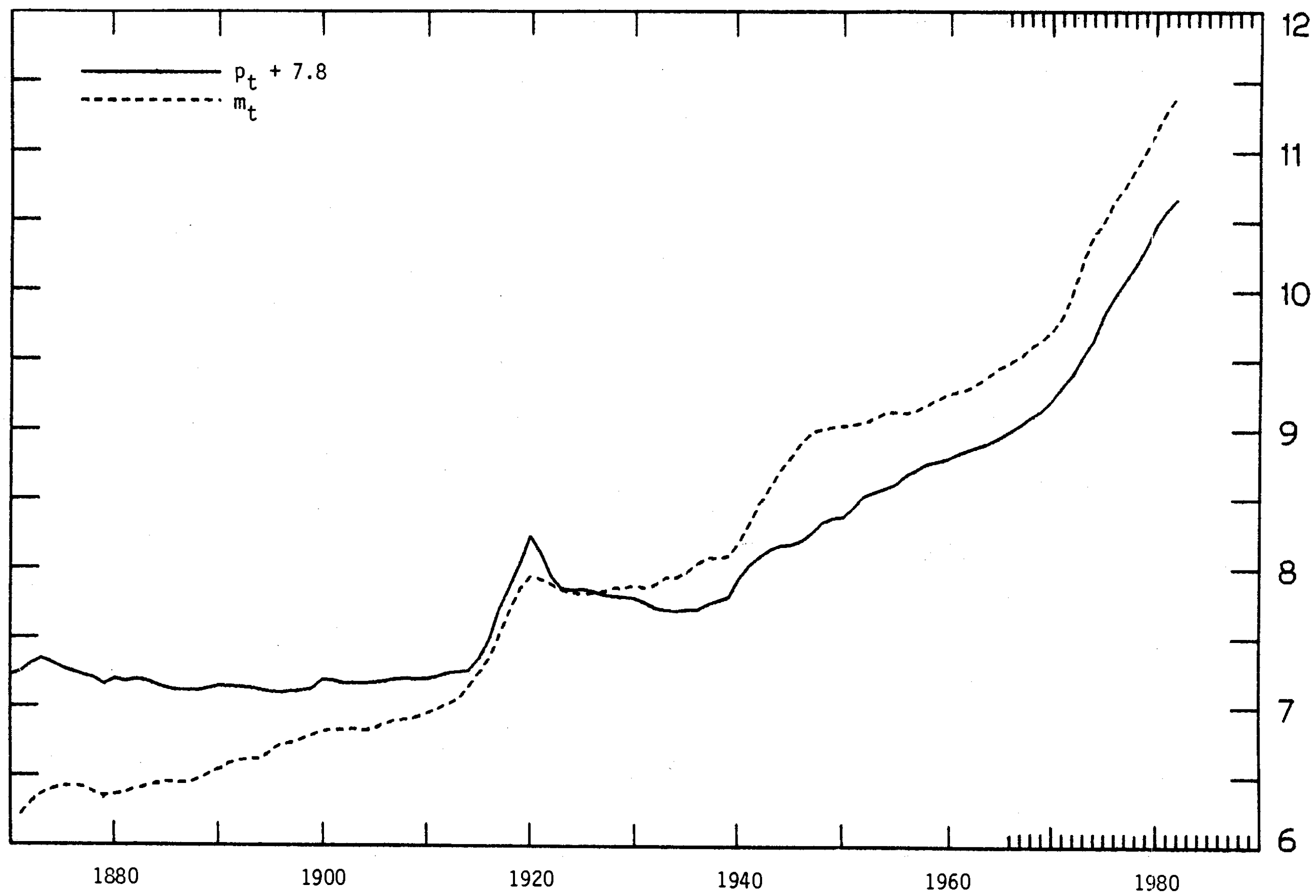


Figure III. The log of the implicit price deflator (p_t) and the log of the nominal money stock (m_t) (annual data).



average lengths of 2.1 and 3.4 years respectively for contraction and expansion. The phase averages, weighted by duration, are taken as the basic units of analysis (pp. 75-79 and Table 5.8). FS analyse their phase-average data both in levels and as "rates of change". The latter are based on "differences" from one expansion (or contraction) to the next, so using triplets of overlapping phases (again weighted appropriately).[2] The data to which these moving-average filters are applied are the logarithms of money, prices, incomes, and population but the original values of interest rates.[3]

Throughout their book, FS emphasise primarily an "errors-in-variables" paradigm. However, precisely what properties FS ascribe to those "errors

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[2]There are typographical errors in the reported values for "rates of change" in Table 5.10 affecting observations 18 through 21. We corrected those errors following the procedures described by FS (Section 3.2): our amendments are reported in Appendix F.

[3]We have several caveats about their data; and while these are not germane to an attempt to re-analyse their numbers, nevertheless we briefly note the most important issues which would need clarification prior to drawing substantive conclusions from the results reported below.

(i) Their choice of monetary measure seems too broad to represent transactions demand and too narrow for an overall index of "liquidity", especially given the rapid growth of the Building Society movement over the last century. M is based on the UK monetary measure M2 (pp. 111-114), and excludes shares and deposits in Building Societies. Note that FS centre their money-stock figures on mid-years by averaging successive end-of-year values.

(ii) The measurement of the price series (P) is "corrected" for rationing and controls (pp. 115-120), with real income then derived by deflating nominal income. It is difficult to understand why they should wish to hold measured nominal income constant when they believe measured prices are incorrect. Furthermore, over a century, one cannot but be concerned about the effects on the measurement of P of the many dramatic changes which have occurred in quality-adjusted real (i.e., relative) prices (such as for computational power).

(iii) The rate of return on physical assets is measured as the rate of change of nominal income $G(\bar{p}+\bar{I})$. Since changes in nominal income cannot be excluded on *a priori* grounds as a determinant of current holdings of money, such an approximation confounds dynamic reactions with substitution effects. In any case, the evidence FS report in favour of their proxy seems to us rather unfavourable (compare FS's discussion on pp. 508, 510 with their Chart 10.12).

Proper treatment of those difficulties is outside our scope of evaluating FS's results on the data they record.

of measurement" we could not ascertain from their discussion. Several hints are provided: for example, FS claim that the importance of errors in variables is reduced by averaging, but enhanced by differencing (p. 86). That suggests the model of the measurement errors being close to white noise since averaging is of little help if errors are persistent (e.g., highly autoregressive; see Appendix B).

All the variables undoubtedly contain substantial measurement errors, especially when interpreted as correspondences to economically meaningful latent constructs. But, it seems unimaginable that such time series of over a century would not contain large systematic errors. Differencing would remove most effects of such errors, so rates of change could well be absolutely more accurate than levels in that their errors were (say) 2% of the level when the level was mismeasured by (say) 10%.[4] While FS use "differencing" as a filter, it is not used that way in our re-evaluation below, but occurs only as a convenient way of imposing parameter transformations (cf. James Davidson, Hendry, Frank Srba, and Stephen Yeo (1978, pp. 673-674)). Thus, the relative accuracy of estimation in levels and differences is not directly relevant to this study, as all our econometric models are effectively in levels. Where any assumption is needed, we will take the measurement errors of the levels of variables to be highly autoregressive with a small innovation variance (see Appendix B).

Use of phase-average data raises three distinct issues requiring examination: the theoretical statistical effects of phase-averaging (qua aggregation), the impact of selecting the intervals over which to average by prior analysis of an interrelated data set, and the observed effects of

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[4]Using the earlier example, consider the enormous falls in the real price of but increases in the volume of computing since World War II and the consequential difference it makes as to whether 1947 or 1982 prices are used in measuring real income. Yet the changes from year to year could be reasonably accurately measured as a percentage of the level.

FS's phase-averaging. We take the first immediately whilst postponing discussion of the latter two until Section III.

Phase-averaging sequentially applies two filters to the annual data: the first averages that data (as with a moving average) and thus implies a re-parameterisation of the data generation process; the second selects the phase-average data from the averaged series (i.e., marginalises the data density with respect to the unwanted intermediate observations), thereby entailing a (statistical) reduction in that re-parameterisation.[5] A loss of information results from such aggregation. That would be unimportant if all the parameters of interest could be recovered from the aggregated series and clearly FS believe such parameters can be (e.g., the long-run elasticity of interest rates in a money-demand equation). Many of the parameters which we consider to be of interest cannot be obtained from FS's phase data, including parameters relevant to tests of Granger (1969) non-causality, short-run variability in the postulated relationships, and the dynamic mechanisms whereby the economy adjusts to "shocks". Further, some of those parameters are fundamental in deriving "long-run" parameters (cf. Davidson et al (1978, p. 681)). Hence, we will analyse primarily the annual data and only briefly investigate the phase-average series. That precipitates the issue of reconciling and comparing results from the two data sets: Appendix B attempts an analysis, based on one possible, if

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[5]To illustrate, consider fixed n-period phase-averaging of the annual series $\{x_t;\ t=..,1,2,...,T\}$ with T a multiple of n. Letting L be the lag operator such that $Lx_t=x_{t-1}$, the first filter is $(1+L+L^2+...+L^{n-1})/n$, so the averaged series is $\{x_t^*;\ t=..,1,2,...,T\}$ where $x_t^* = (x_t+x_{t-1}+x_{t-2}+...+x_{t-n+1})/n$. The second filter selects every n^{th} observation of x_t^*, so the phase-average series is $\{x_{jn}^*;\ j=..,1,2,...,(T/n)\}$, denoted $\{\bar{x}_j;\ j=..,1,2,...,J\}$. In fact, FS include turning points in both preceding and following phases (but weight them by half the normal weight in each), so the first filter is actually $(.5+L+L^2+...+L^{n-1}+.5L^n)/n$ and the second is unchanged. However, the statistical effects of using phase-averages with (rather than without) overlap appear minor in comparison to those of using phase-average (rather than annual) data (cf. pp. 75, 84-85 and our Appendix B).

simple, data process. That analysis implies properties for the phase-average and annual data closely in line with those observed; and, it supports the notion that a substantial loss of information has indeed resulted from phase-averaging.

III. Estimates and tests using the phase-average data

In this section, we address two fundamental claims of FS: the constancy and the correct specification of their money-demand equation. En route to testing those propositions, we find that their model of velocity performs worse than the "will-o'-the-wisp" model of velocity as a random walk. For their money-demand equation, the data reject their claims and assumptions of parameter constancy, price homogeneity, absence of trends, and normality of the disturbances. Further, we find that their models of nominal income and prices appear to add no new inferences since they are approximate re-normalisations of their money-demand equation. Finally, we show that phase-averaging does not achieve its principal claimed benefits. The approach taken leads naturally into Sections IV and V on modelling and testing.

The first empirical issue which we address is the assertion by FS that

> ... a numerically constant velocity does not deserve the sneering condescension that has become the conventional stance of economists. It is an impressive first approximation that by almost any measure accounts for a good deal more than half of the phase-to-phase movements in money or income. Almost certainly, measurement errors aside, it accounts for a far larger part of such movements than the other extreme hypothesis — that velocity is a will-o'-the-wisp reflecting independent changes in money and income. (p. 215)

We begin by testing that proposition.

Consider the data on the so-called velocity of circulation of the money stock:

(1) $\bar{v}_j = -(\bar{m}-\bar{p}-\bar{I})_j \qquad j = 1,\ldots,J$

where a super-script "bar" denotes phase-averaging and there are J phases.[6] Thus, $\bar{m}_j$, $\bar{p}_j$, and $\bar{i}_j$ are the phase-average data on logs of the money stock, price level, and constant price income such that $(\bar{p}+\bar{i})_j$ is the phase average of the log of measured nominal income. Figure IV shows the time series for $\bar{v}_j$ and reveals a highly autoregressive variable, as confirmed by inspecting the Durbin-Watson statistic (dw) when velocity is regressed on a constant:

$$(2) \qquad \hat{\bar{v}}_j = \underset{(.028)}{.53}$$

$$J = 36 \qquad \hat{\sigma} = 16.92\% \qquad dw = .31 \quad .$$

Throughout, estimated standard errors are shown in parentheses (except in Table I, where we report results from FS) and $\hat{\sigma}$ is the standard deviation of the residuals, adjusted for degrees of freedom.[7] One possible statistical equivalent of a will-o'-the-wisp is a random walk. Although the sample size is small due to aggregation over time, one cannot reject the hypothesis that $\bar{v}_j$ is a random walk using the table for the Durbin-Watson statistic prepared by Denis Sargan and Alok Bhargava (1983).[8] Alternatively, using the approach proposed by David Dickey and Wayne Fuller (1979, 1981), we obtain:

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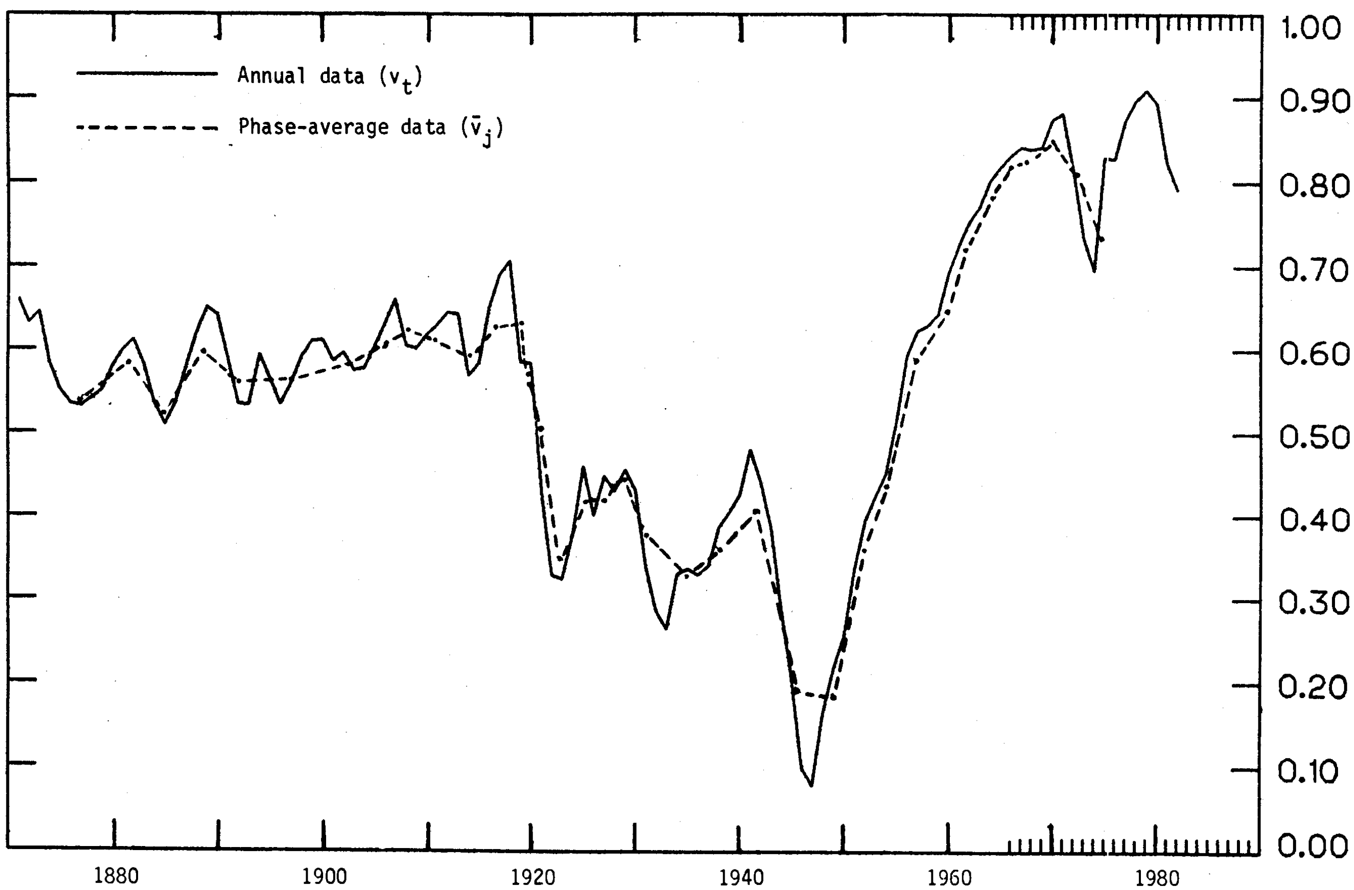
[6]Velocity is $V \equiv (P \cdot I)/M$, so the log of velocity is $v = p+i-m$. Without loss of clarity, we often refer to v as "velocity".

[7]All our estimates using phase-average data are based on weighted least squares, correcting for the different phase lengths. However, experiments revealed that parameter estimates were not very different whether ordinary or weighted least squares were used.

[8]Sargan and Bhargava propose testing that the errors in a regression are a random walk by seeing how close dw is to zero, and give upper and lower bounds for dw under the hypothesis of a random walk. Note that, for a static regression, $dw \approx 2(1-\rho)$ where ρ is the first-order autoregressive coefficient of the disturbance. Adding FS's two dummies $\bar{W}$ and $\bar{S}$ to (2) increases dw to .56, indicating some success by FS in selecting the period over which $\bar{S}$ is non-zero.

Figure IV. The log of the velocity of money: annual and phase-average data.



(3) $$\Delta_1\hat{\bar{v}}_j = \underset{(.042)}{.023} - \underset{(.077)}{.028}\, \bar{v}_{j-1}$$

$J = 35 \qquad \hat{\sigma} = 7.22\% \qquad dw = 1.70$.

The t-ratio on $\bar{v}_{j-1}$ (denoted τ) is only -.37, so again there is no significant evidence against the random walk hypothesis.[9]

We note that John Gould and Charles Nelson (1974) sought to test whether velocity was a random walk, following Friedman and Schwartz's (1963) results for the USA. As Bhargava (1983) shows, the claim that velocity is a constant can be tested as an assertion of existence through testing whether the deviations from that constant are a random walk; that is one interpretation of Gould and Nelson's approach to testing.[10] (For an exposition, see Appendix C.) On that basis, the results in (2) and (3) do not allow us to reject the non-existence of the model that velocity is a constant. Further, since the standard deviation of $\Delta_1\bar{v}_j$ is under half that of $\bar{v}_j$, the will-o'-the-wisp model appears to account for a far larger percentage of the observed variability in velocity than the model that velocity is a constant.

Turning to regression results, Table I reports the "main" estimates gleaned from FS which they record as establishing evidence concerning the

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[9]With the lag operator L defined as $Lx_t=x_{t-1}$, we let the difference operator Δ_1 be $(1-L)$; hence $\Delta_1 x_t=x_t-x_{t-1}$. More generally, $\Delta_r^q x_t=(1-L^r)^q x_t$. If q (or r) is undefined, it is taken to be unity. For phase-average data, differencing is defined in terms of phases rather than years, so that (e.g.) $\Delta_1\bar{x}_j=\bar{x}_j-\bar{x}_{j-1}$.

If $\bar{v}_j$ has a unit root, then the coefficient of $\bar{v}_{j-1}$ in (3) is zero. Dickey and Fuller (1979; 1981, pp. 1065-1066) derive the distribution of the t-ratio on $\bar{v}_{j-1}$ if that is so: Dickey in Fuller (1976, p. 373) gives points on that distribution. Also, note that, while $\bar{v}_j = \bar{v}_{j-1}$ is equivalent to $\bar{v}_j = c$ (a constant) in a deterministic world, there is a massive difference in implications when random errors are added to the two possibilities.

[10]Bhargava also demonstrates the invariance of the Durbin-Watson statistic to the value of the constant and reports significance levels for the null hypothesis that the errors are a random walk.

Table I. Final "levels" estimates with phase-average data for the "main" equations in FS

Source

(I.1)
$$\widehat{(\bar{m}-\bar{p}-\bar{n})} = \underset{(.08)}{.16} + \underset{(18.13)}{.88}(\bar{i}-\bar{n}) - \underset{(3.42)}{11.16}\overline{RN} - \underset{(.74)}{.22}G(\bar{p}+\bar{i})$$
p. 282

$$+ \underset{(2.38)}{1.4}\bar{W} + \underset{(7.56)}{21.}\bar{S}$$

$R^2 = .970 \quad \hat{\sigma} = 5.54\%$

(I.2)
$$\widehat{(\bar{p}+\bar{i})} = -1.02 + .84\bar{m} + .60(\bar{i}-\bar{n}) + 18.80\overline{RN} - .08G(\bar{p}+\bar{i})$$
p. 349

$$- .9\bar{W} - 10.\bar{S}$$

$R^2 = .9990 \quad \hat{\sigma} = 3.94\%$

(I.3)
$$\hat{\bar{p}} = \underset{(31.1)}{-5.99} - \underset{(7.4)}{.015}\bar{t} + \underset{(17.4)}{1.02}\bar{m} + \underset{(.9)}{.94}\overline{RS} - \underset{(2.8)}{1.10}G(\bar{p}+\bar{i})$$
p. 420

$\hat{\sigma} = 6.0\%$

(I.4)
$$\hat{\bar{i}} = \underset{(34.2)}{6.50} + \underset{(8.3)}{.017}\bar{t} - \underset{(.9)}{.05}\bar{m} + \underset{(2.4)}{2.34}\overline{RS} + \underset{(5.7)}{2.20}G(\bar{p}+\bar{i})$$
p. 420

$\hat{\sigma} = 5.9\%$

Notes: 1. Notation is as in Section II, but 't'-values are in parentheses. RN = RS·H/M; G(·) denotes a rate of change; and $\bar{W}$ and $\bar{S}$ are FS's dummies for "post-war adjustment" and "demand shift", rescaled by 1/100 (see Appendix A).

2. Below, all values of $\hat{\sigma}$ for our equations with logarithmic regressands are quoted relative to the level of the regressand in its original units (i.e., if log(Y) is the regressand, $100\hat{\sigma}$ is a percentage of Y).

demand for money, the effects of money on income, and the effects of money on prices. "Levels" and "rates of change" equations are analysed by FS, but they see little to choose between them (e.g., see p. 286). For brevity, we focus on the equations in levels. While full details can be found in FS, we briefly summarise their interpretation of the estimates as follows: on the assumption that M is "exogenous", M does not cause I (equation (I.4)), and M does cause P (I.3) and hence P·I (I.2). Consequently, (I.1) is interpreted as a money-demand equation, but it is determining P, not M (or RN). If FS are insistent that M is "exogenous", we are in effect left with three models of P, all conditional on M and an interest rate, but with different parameter values and standard errors. While we have not yet presented a coherent statistical framework for interpreting their estimates, the evidence seems strongly against the restrictions imposed on (I.2) to generate (I.1) ($\hat{\sigma}$ increases by 40%), and even more against those in (I.3). However, in keeping with FS's own interpretation, we take (I.1) as being the money-demand function for which a number of assertions are testable.

FS (p. 230) discuss the striking relationship between real money balances and real income illustrated by Chart 6.2, even though residual autocorrelation is apparent visually on noting that real income increases over time. Relaxing the restriction of unit income elasticity in (2), we obtain:

$$(4) \qquad \widehat{(\bar{m}-\bar{p}-\bar{n})}_j = \underset{(.38)}{.11} + \underset{(.08)}{.86}\,(\bar{i}-\bar{n})_j$$

$$J = 36 \qquad R^2 = .96 \qquad \hat{\sigma} = 16.49\% \qquad dw = .33 \, .$$

Given the value of dw, that regression should allow the reader to draw his/her own conclusions on the validity of and "insights" that can be gained from "consider[ing] variables one or two at a time" (p. 215;

contrast with J. Johnston (1963, pp. 179ff) on the effects of autocorrelation, and see Christopher Sims (1980) for a forceful advocacy of less restrictive modelling). On adding $\bar{RN}_j$ to (4), dw increases to only .59, but increases to 1.51 for the model of $(\bar{m}-\bar{p}-\bar{n})$ recorded in Table I. We could closely, but not precisely, reproduce those numbers:

$$(5) \qquad \widehat{(\bar{m}-\bar{p}-\bar{n})}_j = \underset{(.19)}{.012} + \underset{(.049)}{.885}\,(\bar{i}-\bar{n})_j - \underset{(3.3)}{11.21}\,\bar{RN}_j - \underset{(.29)}{.22}\,G(\bar{p}+\bar{i})_j + \underset{(.58)}{1.37}\,\bar{W}_j + \underset{(2.7)}{20.6}\,\bar{S}_j$$

$$J = 36 \qquad \hat{\sigma} = 5.66\% \qquad dw = 1.51 \quad .$$

Much of the reduction in variance from (4) to (5) is due to $\bar{S}_j$, their data-based dummy for "[a]n upward demand shift, produced by economic depression and war" (p. 281), equal to a 21% shift during 1921-55.

A major issue in FS is the claimed (parameter) constancy of their demand-for-money equation (e.g., see pp. 283, 624) although many investigators would regard the need for a shift dummy for one-third of the sample as casting severe doubt on the model's constancy. Interestingly, FS nowhere actually test for constancy. They apparently believe that "high" values of R^2 and/or "low" values of $\hat{\sigma}$ are sufficient in themselves to indicate constancy: such a conclusion is false. We tested the constancy of their money-demand equation once only, on a "split half" basis, refitting (5) to observations 1-18 and predicting 19-36, and testing for constancy using Chow's (1960, pp. 594-5) statistic: that yielded $\eta_1(18,12) = 6.3$ which exceeds the 1% point of the F-distribution. Even although the 95% confidence interval based on $\hat{\sigma}$ is ±11%, parameter non-constancy can be detected. The values of the residual standard error for the two corresponding sub-periods are dramatically different: 2.8% and 6.0% respectively. Such evidence is far from supporting any claim to constancy.

We also tested three other aspects of the money-demand model in (5): price homogeneity, the absence of any trends, and the normality of the residuals. For the first of those, we obtained:

$$(6)\quad (\widehat{\bar{m}-\bar{p}-\bar{n}})_j = \underset{(.38)}{1.01} + \underset{(.08)}{.68}\,(\bar{i}-\bar{n})_j + \underset{(.04)}{.13}\,\bar{p}_j - \underset{(3.8)}{18.4}\,\bar{RN}_j - \underset{(.26)}{.04}\,G(\bar{p}+\bar{i})_j$$

$$+ \underset{(.5)}{1.1}\,\bar{W}_j + \underset{(2.8)}{16.3}\,\bar{S}_j$$

$$J = 36 \qquad \hat{\sigma} = 5.03\% \qquad dw = 1.85 \quad .$$

The coefficient on $\bar{p}_j$ is significant at the 99% level. Next, we obtained:

$$(7)\quad (\widehat{\bar{m}-\bar{p}-\bar{n}})_j = \underset{(.52)}{2.32} + \underset{(.16)}{.14}\,(\bar{i}-\bar{n})_j - \underset{(2.7)}{16.0}\,\bar{RN}_j + \underset{(.24)}{.23}\,G(\bar{p}+\bar{i})_j$$

$$+ \underset{(.5)}{1.8}\,\bar{W}_j + \underset{(3.7)}{6.1}\,\bar{S}_j + \underset{(.0019)}{.0090}\,\bar{t}_j$$

$$J = 36 \qquad \hat{\sigma} = 4.35\% \qquad dw = 1.99 \quad .$$

The coefficient on $\bar{t}_j$ is statistically significant at any reasonable level, so we reject the first two hypotheses. Normality of the disturbances is also rejected, since Carlos Jarque and Anil Bera's (1980) statistic for testing against skewness (SK) and excess kurtosis (EK) is $\xi_5(2) = 6.9$ in (5), with SK = 1.1 and EK = .6.[11] However, for the sub-sample estimates, the value of $\xi_5(2)$ is negligible for the first half and 12.7 for the second. (The 1% point of $\chi^2(2)$ is 9.2.) Thus, the distributions of the residuals have "fat" tails and a skewed shape, inconsistent with normality. Further, (7) fits better than (5) prior to either "removing the serial correlation" (on which issue see Sargan (1980b) and Hendry and Grayham Mizon (1978)) or adding the dummy variables $\bar{W}$ and $\bar{S}$. Equation (5) is clearly neither an

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[11]We modify Jarque and Bera's (1980, p. 257) statistic to be

$$\xi_5(2) = [(T-k)/6]\cdot[SK^2 + EK^2/4] \ ,$$

given that the number of regressors fitted is often large relative to the sample size. $\xi_5(2)$ is asymptotically distributed as $\chi^2(2)$ under the null hypothesis of normality, in which case skewness and excess kurtosis are both zero.

adequate characterisation of the data nor consistent with a theory claiming a constant money-demand equation, homogeneous of degree zero in prices. What is perhaps the most important single claim of FS therefore has no empirical basis, despite their assertion (p. 7) that "[t]his parallelism[12] is a manifestation of the stable demand curve for money plus the excellence of the simple quantity theory approximation".

The other "equations" using phase-average data were not further investigated because, as explained in our Appendix D, "re-normalising" regressions (as FS do) does not provide additional inferences.

FS's primary justification for phase-averaging appears to be the claim that it reduces cyclical effects (pp. 13-14, 78), thereby allowing them to focus on their primary concern, monetary <u>trends</u>. By filtering out the cycle, FS argue phase-averaging should reduce serial correlation in the data and lower the data variance. Secondly, FS claim phase-averaging reduces the effects of measurement errors (p. 86): that also would imply a reduction in the data variance. Contradicting those claims, we now show that phase-averaging negligibly reduced the variance and (substantial) serial correlation in their series for velocity, a series from which they claim to draw important inferences. Further, we demonstrate that a simple first-order autoregressive process <u>can</u> explain those features of the data, and we briefly address the issue of selecting the phase periods.

First, we record the following regression using FS's "raw" annual data:

$$(8) \qquad \hat{v}_t = \underset{(.017)}{.53}$$

$$T = 100 \qquad \hat{\sigma} = 16.91\% \qquad dw = .08 \qquad \tau = -1.2$$

for T annual observations. The results in (8) are very close to those in

[12] Of nominal income and the nominal quantity of money, and of the rate of change of nominal income and the rate of change of the nominal quantity of money.

(2).[13] Comparison with (2) immediately reveals that phase-averaging has little impact on the standard deviation of "velocity" or on the significant serial correlation present in the series. In Figure IV, the annual and phase-average data for velocity are plotted, graphically illustrating the effects of phase-averaging: there is only a small reduction in the (high-frequency) variability at the cost of increasing the size of the "jump" between successive observations. Consequently, phase-averaging has little net effect on the variance of velocity (contrast with pp. 73ff).

Next, we present analytical evidence on the effects of phase-averaging. In Appendix B, we derive the variance formulae based on a first-order autoregressive process for "annual" and phase-average series, both in levels and in differences. In Table II, values for a three-period phase average (assuming an autoregressive coefficient of .96) are compared with the observed statistics for velocity.

Table II. Standard deviations of velocity

Series	Observed value	Value for autoregressive model
v	.169	.169[14]
$\bar{v}$	.169	.166
$\Delta_1 v$	.047	.048
$\Delta_1 \bar{v}$	.071	.067

As can be seen, the theoretical and actual standard deviations are quite

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[13]If we partial out war years from (8), we obtain:

$$\hat{v}_t = \underset{(.018)}{.55} + \sum_{i=1}^{14} a_i WD_i$$

$$T = 100 \qquad \hat{\sigma} = 16.6\% \qquad dw = .17$$

where WD_i denotes a dummy variable which is 1 for the i^{th} year of major war and zero otherwise. These results are virtually identical to those in (8).

[14]The values in Appendix B (Table B.IV) have been normalised on the standard deviation of v because the results in Appendix B are for relative, not absolute, variances.

similar. Further, given an annual autoregressive coefficient of .96 (which is consistent with dw = .08 in (8)), the theoretical model in Appendix B predicts an autoregressive coefficient of .92 for the phase-average series, closely in line with dw = .31 in (2). While we do not think velocity is simply a first-order autoregressive process (and give evidence supporting our view below), these results nevertheless demonstrate the ineffectiveness of phase-averaging in attaining its supposed principal benefits.

Before concluding this section, we briefly return to the issue of selecting phases (raised in Section II). Without undertaking a Monte Carlo study, it is difficult to deduce the statistical effects of selecting the phases over which to average when using information from related time series, but a first approximation is given by examining the over-all effects of phase-averaging on the data series and on econometric modelling in light of the results for fixed-length phase-averaging reported in Appendix B. For velocity, the model of phase-averaging with fixed phase lengths explains several salient features of the data with little direct evidence of the effects of selection. In terms of econometric models, the standard error of FS's "money-demand equation" is around 5% (p. 5) (which some reviewers, and FS, apparently regard as "good"); the equivalent $\hat{\sigma}$ from our annual model is under 2% (see (22)). Thus the additional dynamic information leads to a more than tenfold reduction in the residual variance. That is a larger improvement than that predicted by the simple (fixed phase-length) model, despite FS using carefully selected dates over which to average.

FS's own phase-average data reveal the patent mis-specification and parameter non-constancy of their money-demand equation, despite their assertions to the contrary. Their claim that velocity is a near constant is likewise rejected by the data. More fundamentally, we cannot accept the procedures FS adopt in pre-processing the data to obtain "phase averages". These involve a data-based set of filters, but with no account being taken of the statistical effects of such filtering on later inferences. Moreover, the aggregation to phase averages entails a loss of information on dynamics which we believe renders further modelling of such data a waste of time, especially since phase-averaging failed to achieve the objectives stated by FS. Thus we turn to an analysis of the annual observations in an attempt to develop a money-demand equation which encompasses (5) and explains its failures. To do so, we first consider the statistical framework and the theory of testing on which the analysis of FS's claims (above) was based, as both will prove invaluable in modelling the annual data.

IV. A statistical framework for analysing FS's data

In this section and the one following, we discuss the most relevant aspects of our statistical approach for analysing FS's data. An econometric model is viewed as a (reduced) re-parameterisation of the data generation process achieved by marginalising and conditioning, the latter operation being related to the economic notion of contingent plans based on weakly exogenous variables. Such operations entail that the "error" is a derived rather than an autonomous process, suggesting designing the model to satisfy data-based and theory criteria. For a general exposition and bibliographic perspective, see Hendry and Jean-François Richard (1982, 1983), Hendry (1983), and Hendry and Kenneth Wallis (1984).

FS believe in the existence of a number of constant relationships linking the variables under study and implicitly assume log-normality of those variables (e.g., see pp. 75, 223, 236 combined). Thus, we seem to share the hypothesis that there exists a joint data density for the variables in FS's Table 4.9 with a meaningful parameterisation. We denote that density by $F(\underset{\sim}{X}_T^1 | \underset{\sim}{X}_0;\ \underset{\sim}{\theta})$ where $\underset{\sim}{x}_t$ is the vector of observations on those variables at time t (t=1,...,T), $\underset{\sim}{X}_t^1 = (\underset{\sim}{x}_1 \ldots \underset{\sim}{x}_t)$ so that $\underset{\sim}{X}_T^1$ is the complete sample of data, $\underset{\sim}{X}_0$ denotes the matrix of all relevant initial conditions, and $\underset{\sim}{\theta}$ is the vector of associated parameters. However, from our theoretical perspective, the density $F(\cdot)$ over the variates in their Table 4.9 has already been subjected to considerable marginalisation involving variables which we believe to be important in accounting for the behaviour of the reported series (e.g., fiscal variables, investment, etc.). That could jeopardise the constancy of $\underset{\sim}{\theta}$, which is potentially a function of the omitted variables and would vary as they did.

Given the sequential nature of economic behaviour, $F(\cdot)$ is factorised as:

(9) $$F(\underset{\sim}{X}_T^1 | \underset{\sim}{X}_0;\ \underset{\sim}{\theta}) = \prod_{t=1}^{T} F(\underset{\sim}{x}_t | \underset{\sim}{X}_{t-1};\ \underset{\sim}{\lambda}_t)$$

where $\underset{\sim}{X}_{t-1} = (\underset{\sim}{X}_0\ \underset{\sim}{x}_1 \ldots \underset{\sim}{x}_{t-1})$ and $(\underset{\sim}{\lambda}_1' \ldots \underset{\sim}{\lambda}_T')' = \underset{\sim}{\lambda} = \underset{\sim}{f}(\underset{\sim}{\theta})$ is the corresponding re-parameterisation. A critical claim to be tested is the constancy over time of certain components and/or functions of $\{\underset{\sim}{\lambda}_t\}$. Once the data are appropriately transformed to make normality reasonable, (9) entails:

(10) $$\underset{\sim}{x}_t | \underset{\sim}{X}_{t-1} \sim N(\underset{\sim}{\mu}_t,\ \Sigma_t)$$

where $\underset{\sim}{\mu}_t = E(\underset{\sim}{x}_t | \underset{\sim}{X}_{t-1})$ and so $\underset{\sim}{\lambda}_t$ comprises the non-redundant elements of $\underset{\sim}{\mu}_t$ and Σ_t. Note that if $\underset{\sim}{\varepsilon}_t \equiv \underset{\sim}{x}_t - \underset{\sim}{\mu}_t$, then $\{\underset{\sim}{\varepsilon}_t\}$ is a sequence of martingale differences (and so is an innovation with respect to $\underset{\sim}{X}_{t-1}$, and hence is white noise).

We are already in a position to interpret some of the empirical findings in Section III. For example, averaging over phases of business cycles further reduces the density $F(\underset{\sim}{X}_T^1 | \cdot)$ and the associated parameterisation, resulting in $\prod_{j=1}^{J} F^*(\bar{\underset{\sim}{x}}_j | \bar{\underset{\sim}{X}}_{j-1};\ \underset{\sim}{\lambda}_j^*)$ (say). Thus, if FS's claim that phase-averaging removes the serial correlation in the data arising from the business cycle (p. 78) were true, then this would sustain valid inference without needing to condition on $\bar{X}_{j-1}$ in $F^*(\cdot)$.[15] Certainly they proceed as if conditioning is unnecessary by freely interpreting coefficients divided by standard errors according to (possibly non-central) 't'-distributions in models which are not conditional on the past. It is very regrettable that FS present no evidence in their book to support that practice. Evidence in Section III demonstrated substantial residual serial correlation in some of their models. Huge biases in estimated standard errors can result from untreated residual autocorrelation even when all regressors are strictly exogenous (nb. dw in (2) and (4) above). Those biases are generally downwards for positive autocorrelation. Moreover, biases in parameter estimates occur from serial correlation interacting with regressors which are not all strictly exogenous (see below).

Several assumptions additional to those implicit in (10) must be made to allow an empirically testable model to be formulated. It must be assumed that X_{t-1} can be adequately approximated by $X_{t-1}^{t-\ell}$ (where ℓ is the fixed longest lag considered) without invalidating the innovation properties of the $\{\varepsilon_t\}$. Since we have assumed only <u>conditional</u> normality for x_t, we also need to assume that μ_t is well approximated by a linear function of $X_{t-1}^{t-\ell}$. Thus, we postulate:

$$(11)\quad x_t = \sum_{i=1}^{\ell} \pi_i x_{t-i} + \varepsilon_t \qquad\qquad \varepsilon_t \sim IN(0,\Sigma), \quad t=1,\ldots,T.$$

Many of the assumptions entail restrictions on the observables and hence

[15] We interpret phase-averaging as FS's attempt to eliminate X_{t-1} from $F(x_t|X_{t-1};\lambda_t)$. However, the conditions under which $\bar{X}_{j-1}$ does not appear in $F^*(\bar{x}_j|\bar{X}_{j-1};\lambda_j^*)$ <u>and</u> the parameters of interest can be obtained from $\{\lambda_j^*\}$ are quite restrictive. Thus we will incorporate dynamics into our model directly rather than rely on those conditions. Further, the data reject a critical one of those conditions, the strict exogeneity of m, p, and i (see Hendry and Ericsson (1983, Table C.I) and Appendix G below).

have testable implications: where possible, those have been investigated and relevant statistics are reported. For example, if ℓ has been chosen too small, $\underset{\sim}{\varepsilon}_t$ will not be an innovation; and, if the $\{\underset{\sim}{\pi}_i\}$ are not constant, predictive failure should be observable on sub-samples. Additionally, Granger non-causality tests play the vital role of testing assertions about strict exogeneity within the data set reported (i.e., conditional on $\underset{\sim}{X}_T^1$; see Robert Engle, Hendry, and Richard (1983)).[16] In Appendix G, various tests for strict exogeneity are presented, showing that the null of Granger non-causality can be rejected at the 5% level in most cases. Estimation of (11) also provides baseline innovation variances for the various series; corresponding residual standard errors for m, p, and i are 1.9%, 2.6%, and 2.8%, where ℓ=5 and $\underset{\sim}{x}_t' = (m_t\ h_t\ i_t\ p_t\ RS_t\ R\ell_t\ e_t)$.[17]

Given that (11) is not decisively rejected at the outset, one can turn to modelling $\underset{\sim}{\mu}_t = \sum_i \underset{\sim}{\pi}_i \underset{\sim}{x}_{t-i}$. There are many basic approaches to doing so, including causal chains, simultaneous systems, block recursive models, and various simplifications based directly on (11). All of these raise problems in econometric modelling. Surprisingly, apart from their errors-

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[16]The four distinct concepts of exogeneity, namely weak, strong, super and strict, discussed by Engle et al (1983), correspond to different notions of being "determined outside the model under consideration" according to the purposes of the inferences being conducted (i.e., conditional estimation, prediction, policy analysis, and forecasting, respectively). In no case is it legitimate to "make variables exogenous" simply by not modelling them. We can find no necessary connections between the "causality" of one variable y for another z and their respective exogeneity status in that z_t being "exogenous" or endogenous is neither necessary nor sufficient for it to influence y_t (except for the trivial case that by definition strictly exogenous variables cannot be Granger-caused by endogenous variables). For a useful discussion of the concept of causality in econometrics, see Arnold Zellner (1979).

[17]A constant term and the dummy variables D_1, D_2, and D_3 are also included. We have chosen ℓ=5 on the grounds that that implies 39 parameters estimated per equation in Hendry and Ericsson (1983, Appendix C) and Appendix G herein (vs. 93 observations total), in line with guidelines in Sargan (1980b, p. 880). Also, ℓ=5 implies a maximum lag of approximately one cycle (as measured by FS).

in-variables paradigm, FS ignore almost every other econometric issue, and not just parameter non-constancy and autocorrelation and heteroscedasticity in the residuals. There are no allowances for or tests of mis-specification resulting from the simultaneous determination of the $\underset{\sim}{x}_t$, inappropriate functional forms, dynamic mis-specification, and/or omitted variables.

In whatever approach is taken to modelling $\underset{\sim}{\mu}_t$, as shown above, the associated disturbance is a derived process (being the unexplained component of $\underset{\sim}{x}_t$) rather than an autonomous process. Hence it is relevant to ask whether that disturbance (or any other aspect of the model, e.g., a given parameter) has certain properties necessary for the model to be an adequate representation of the underlying data generation process. The next section considers such properties and the corresponding model evaluation criteria.

V. Model evaluation criteria

Statistical inference in multivariate time-series processes is a hazardous and contentious issue. From the time of R. Hooker (1901) and especially Udny Yule (1926) onwards, the enormous difficulties inherent in conducting valid inference in such processes have gradually become documented. Presently, econometricians are much more aware of the pitfalls in analysing economic time series than of methods which ensure, with any reasonable likelihood, that sensible and sustainable conclusions can be reached. An empirical "conclusion" is deemed sustainable only if it satisfies a range of criteria discussed in detail below. Most of those criteria are well-known and widely accepted, we consider all of them to be justifiable, and we contend that satisfying such criteria constitutes a minimal necessary condition for judging an empirical model to be credible.

The main criteria which we have in mind relate to goodness-of-fit, absence of residual autocorrelation and heteroscedasticity, valid exogeneity, predictive ability, parameter constancy, the statistical and economic interpretation of estimated coefficients, and the validity of a priori restrictions. Rather than discuss each of those issues separately and in an ad hoc manner, we propose the following taxonomy in which design criteria are related to particular types of information available to the modeller. Such information may be partitioned into four primary groups:

(A) sample data believed relevant by the given investigators (i.e., $\underset{\sim}{X}_T^1$),

(B) theory information (T_T^1),

(C) the supposed structure of the measurement system (M_T^1), and

(D) sample data not in $\underset{\sim}{X}_T^1$ but believed relevant by other investigators ($\underset{\sim}{W}_T^1$).

Different investigators may use different subsets of T_T^1 as well as different data. In (A)-(D), all the information sets are dated since, a priori, the content of M_T^1 and T_T^1 need not remain unaltered over time (and generally changes as knowledge accrues).

Within (A) (and equivalently, for (D)), a further threefold partition of $\underset{\sim}{X}_T^1$ into $(\underset{\sim}{X}_{t-1}^1, \underset{\sim}{x}_t, \underset{\sim}{X}_T^{t+1})$ is useful as that corresponds to the division of the data into the (relative) past, present, and future, denoted (A_1), (A_2), and (A_3). That the past is immutable and the future is uncertain is among the basic tenets of economics, so it is unsurprising that procedures for model evaluation should focus separately on those various subsets. The evaluation issues which arise under (A) include:

(A_1) goodness-of-fit and residual variance, residual serial independence and homoscedasticity;

(A_2) the legitimacy of conditioning on contemporaneous variables; and

(A_3) parameter constancy and predictive performance.

Corresponding to the resulting sixfold partition of the information [i.e., (A_1), (A_2), (A_3), (B), (C), (D)], we have the following six evaluation criteria: innovation errors, weak exogeneity, parameter constancy, theory consistency, data admissibility, and parameter encompassing.[18] Those comprise a minimal set of criteria, the satisfaction of which is necessary, but not sufficient, to sustain empirical inferences, and hence forecasting and policy analysis. In statistical terms, each criterion yields a testable null hypothesis (subject to minimal identification requirements), and we have essentially stated those criteria in terms of their corresponding nulls. In Section III (above), we investigated how well certain of their equations perform on most of those criteria, and rejected almost all of the claims FS make for their empirical model of money demand.[19] We cannot do better than cite Friedman (1951, p. 107) in support of our approach:

> It is one of our chief defects that we place all too much emphasis on the derivation of hypotheses and all too little on testing their validity. This distortion of emphasis is frequently unavoidable, resulting from the absence of widely accepted and objective criteria for testing the validity of hypotheses in the social sciences. But this is not the whole story. Because we cannot adequately test the validity of many hypotheses, we have fallen into the habit of not trying to test the validity of hypotheses even when we can do so. We examine evidence, reach a conclusion, set it forth, and rest content, neither asking ourselves what evidence might contradict our hypothesis nor seeking to find out whether it does.

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[18] (D) may be partitioned into (D_1), (D_2), and (D_3), rather naturally inducing encompassing on past data, on exogeneity conditions, and on forecasts. E.g., see Mizon and Richard (1983) and Mizon (1984).

[19] It is unclear to which concept of exogeneity their notion corresponds. Hence it is exceedingly difficult to formulate tests of their assertion that money is "exogenous" although, as noted in Section IV, we can reject the null hypothesis that M is strictly exogenous.

While there are only six evaluation criteria described above, departures from the null hypothesis could take many forms. Table III lists the bulk of the test statistics reported below; the convention used is that $\xi_i(q)$ and $\eta_i(q,r)$ denote statistics which have central $\chi^2(q)$ and $F(q,r)$ distributions respectively under a common null and against the i^{th} alternative. Thus, $\xi_2(q)$ and $\eta_2(q,T-k-q)$ both test for q^{th}-order residual autocorrelation. There are T observations and k regressors in the model under the null. Most of these tests are applied in the Lagrange Multiplier spirit, with <u>all</u> the data used in the estimates quoted and where (e.g.) $\eta_1(q,\cdot)$ acts as a post-estimation diagnostic for predictive failure over the last q observations. Joint tests could be constructed as in Jarque and Bera (1980); and, as Jan Kiviet (1982) also notes, many of the statistics are asymptotically independent so that their χ^2-forms could be added together to construct a "portmanteau" mis-specification statistic. Otherwise, care should be taken to control for Type I errors over the set of tests.

Even if FS had tested and had <u>not</u> rejected such necessary conditions for valid inference entailed in (A), (B) and (C), as discussed in Hendry (1983), those criteria are minimal in that they often can be satisfied simply by <u>designing</u> empirical models appropriately. For example, a theory-based model imposed on data and with any residual serial correlation removed usually satisfies (A_1) and (B); and so on. Consequently, we additionally would require evidence on the ability of their models to encompass rival hypotheses, that is, to demonstrate that the information in (D) is irrelevant, conditional on (A) and (B) (here we assume common

Table III. Criteria for evaluating econometric models

Null	Alternative	Statistic	Sources
(A_1)	q^{th}-order residual autocorrelation	$\xi_2(q)$; $\eta_2(q,T-k-q)$	Box and Pierce (1970); Godfrey (1978), Harvey (1981, p. 173)
(A_1)	q invalid parameter restrictions	$\eta_3(q,T-k-q)$	Johnston (1963, p. 126)
(A_1)	first-order ARCH	$\xi_4(1)$	Engle (1982)
(A_1)	skewness (SK) and excess kurtosis (EK)	$\xi_5(2)$	Jarque and Bera (1980)
(A_1)	heteroscedasticity quadratic in regressors (q quadratic terms)	$\eta_6(q,T-k-q)$	White (1980), Nicholls and Pagan (1983)
(A_2)	q instrumental variables not independent of errors	$\xi_7(q-k)$; $\eta_7(q-k,T-q)$	Sargan (1958, 1964); Sargan (1980a, p. 1136)
(A_3)	predictive failure over a subset of q observations	$\eta_1(q,T-k-q)$	Chow (1960, pp. 594-5)

Notes: 1. We have labelled the Chow statistic $\eta_1(q,T-k-q)$ both to highlight the pre-eminence of the issue of constancy in the substantive debate on monetary behaviour and because of its crucial role as an indirect test of weak exogeneity through testing the conjunction of hypotheses embodied in super exogeneity.

2. The value of q may differ across statistics, as may those of k and T across models and samples.

agreement about and satisfaction of (C)).[20] For single equations estimated by least squares, a *necessary* condition for encompassing is *variance dominance* where one equation variance-dominates another if the former has a smaller variance.[21] It seems natural that a poorly fitting equation cannot account for why a well-fitting equation fits well. Below, we often use variance dominance as a criterion for excluding unacceptable models. For comprehensive accounts of tests for encompassing and of related non-nested hypothesis tests, see Mizon and Richard (1983), Mizon (1984), James MacKinnon (1983), and Hashem Pesaran (1982).

We demonstrate below that FS's results on phase-average data are variance-dominated by the simplest of univariate time-series models for money fitted to their annual data and *hence they cannot even encompass that elementary hypothesis*. An immediate implication is that their money-demand equation is mis-specified (in part because of the time aggregation), so all inferences based thereon are of dubious validity. Indeed, we rejected (above) the constancy of their "best representative" money-demand equation on phase data and will reject the constancy of the best that we can obtain on annual data from 1878 to 1970; constancy fails dramatically when testing over the period since competition and credit control was introduced in 1971.

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[20] Encompassing can be understood intuitively from the following example. Suppose Model 1 *predicts* $\tilde{\underline{a}}$ as the value for the parameter(s) $\underset{\sim}{a}$ in Model 2, whilst Model 2 *actually* has estimate $\hat{\underset{\sim}{a}}$. Then we test the closeness of $\tilde{\underline{a}}$ to $\hat{\underset{\sim}{a}}$, taking account of the uncertainty arising in estimation. Model 1 encompasses Model 2 if $\tilde{\underline{a}}$ is "statistically close" to $\hat{\underset{\sim}{a}}$, so that Model 1 explains why Model 2 obtains the results it does.

[21] Formally, variance dominance refers to the underlying (and unknown) error variances. Without loss of clarity, we often will say a model variance-dominates another if the *estimated residual* variance of the former is smaller than that of the latter.

VI. Econometric modelling of "money demand" using the annual data

As a positive critique to complement the analysis of Section III, we model the demand for money in the UK using FS's annual data. Before doing so, we first develop simple time-series models of the main variables, namely, money, prices, and income: such models establish a useful baseline against which to evaluate money-demand equations. In modelling the demand for money itself, we start with a general autoregressive-distributed lag representation of money conditional on incomes, prices, and interest rates, and evaluate a simplification thereon in light of the model design criteria described in Section V (cf. Hendry and Mizon (1978) and Michael McAleer, Adrian Pagan, and Paul Volker (1985) on modelling from general to simple, and Edward Leamer (1978) for an insightful analysis of specification searches). We use series for 1878-1970 (T = 93) since the data from 1971 to 1982 appear to have a different stochastic structure (e.g., see Table VI and equation (24) below).[22]

Starting from a fifth-order autoregression,[23] we obtained the following simplified description of the time-series behaviour of v_t:

$$(12)\quad \widehat{\Delta_1 v_t} = \underset{[.09]}{.40\Delta_1 v_{t-1}} + \underset{[2.11]}{.54D_1} + \underset{[1.16]}{.14D_2} - \underset{[1.9]}{2.8D_3} + \underset{[.0044]}{.0037}$$

$$T = 93 \quad R^2 = .19 \quad \hat{\sigma} = 4.34\% \quad \eta_1(10,78) = .07 \quad \xi_2(11) = 17.5$$

$$\xi_4(1) = .2 \quad \xi_5(2) = 32.3 \quad SK = -1.1 \quad EK = 2.1 \quad \eta_6(5,83) = 1.6\ .$$

(See Table III above for definitions of the statistics.) In (12), D_1, D_2, and D_3 are zero-one dummies being unity for 1914-18, 1921-55, and 1939-45

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[22] Most estimates have been computed with and without data for war years: where interesting differences emerged, they will be noted and a format like that in footnote 13 used (even though we removed the WD_i by "prior regression" which entailed setting the data for war years to zero after all relevant lags, transformations, etc. were completed).

[23] We chose a fifth-order autoregression because ℓ=5 in the unrestricted models estimated in Section IV.

respectively (see Appendix A); [·] denotes heteroscedasticity-consistent estimated standard errors (see Halbert White (1980) and Desmond Nicholls and Pagan (1983)). Note that (12) has approximately white-noise homoscedastic (but non-normal) residuals. Equation (12) has no "long-run" solution for the level of velocity, being in statistical terms a generalisation of a random walk. Nevertheless, this will-o'-the-wisp model of velocity substantially variance-dominates all of the money-demand models reported by FS, so that their formulation cannot encompass even this naive specification.

The notion that velocity is a near constant[24] has a long history in economics, probably because it seems natural to expect that the money stock normalised by nominal income should manifest much less variability than real money (which is not standardised for real income growth) or nominal money (which ignores changes in both the price level and real income). It is a non sequitur that statistical models of those three magnitudes (i.e., v, (m-p), m) have error variances following the same ranking. In fact, equation (12) is a very poor model of the real money stock conditional on real income (noting that $v_t = -(m-i-p)_t$) since it imposes a unit elasticity at all lags. Surprisingly, it is variance-dominated by a corresponding time-series model for real money which excludes real income altogether:

$$(13)\quad \widehat{\Delta_1(m-p)_t} = \underset{[.11]}{.36}\Delta_1^2(m-p)_{t-1} + \underset{[1.64]}{.05}D_1 + \underset{[.83]}{.89}D_2 + \underset{[1.9]}{2.4}D_3 + \underset{[.0037]}{.0090}$$

$$T = 93 \quad R^2 = .22 \quad \hat{\sigma} = 3.42\% \quad \eta_1(10,78) = .12 \quad \xi_2(10) = 16.0$$

$$\xi_4(1) = 12.6 \quad \xi_5(2) = 5.2 \quad SK = -.2 \quad EK = 1.1 \quad \eta_6(5,83) = 1.7\ .$$

The estimated error variance has dropped to just over 60% of that in (12).

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[24]Concerning its "constancy" over time, note that, excluding war years, the largest change in V_t over a two-year period is 26% and that after its trough in 1947, V_t rose by 80% in the next 23 years. (All percentage measures use the log-symmetric form.) Compare FS's Chart 5.5 (in which UK velocity appears virtually constant) with Figure I.

Even retaining price homogeneity at all lags is a poor assumption since, for the nominal money stock (i.e., excluding all information on both prices and incomes), we have:

$$(14) \quad \widehat{\Delta_1^2 m_t} = \underset{[.08]}{-.37}\Delta_1 m_{t-2} + \underset{[1.5]}{4.5}D_1 - \underset{[.54]}{.64}D_2 + \underset{[1.1]}{4.0}D_3 + \underset{[.0031]}{.0105}$$

$T = 93 \quad R^2 = .40 \quad \hat{\sigma} = 2.21\% \quad \eta_1(10,78) = .63 \quad \xi_2(10) = 26.0$

$\xi_4(1) = .2 \quad \xi_5(2) = 2.8 \quad SK = -.05 \quad EK = .9 \quad \eta_6(5,83) = 5.4$.

The estimated error variance is now roughly a quarter of that in (12) and under half that in (13). It is remarkable that $\hat{\sigma}^2$ should fall so dramatically from (12) to (14), precisely opposite to what one might have anticipated from FS's theories. Since the residual sum of squares can only fall from its value in (14) when p_t and i_t and their lags are added unrestrictedly, the implicit constraints imposed to obtain (13) and (12) are rejected at all reasonable levels of significance. Indeed:

$$(15) \quad \widehat{\Delta_1 v_t} = \underset{[.08]}{.54}\Delta_1 v_{t-1} + \underset{[.06]}{.90}\Delta_1 i_t + \underset{[.06]}{.71}\Delta_1 p_t - \underset{[.10]}{.57}\Delta_1 i_{t-1} - \underset{[.07]}{.47}\Delta_1 p_{t-1}$$

$$- \underset{[1.2]}{3.7}D_1 + \underset{[.48]}{.53}D_2 - \underset{[1.1]}{4.0}D_3 - \underset{[.0032]}{.0070}$$

$T = 93 \quad R^2 = .846 \quad \hat{\sigma} = 1.94\% \quad \eta_1(10,74) = .44 \quad \eta_2(6,78) = 3.8$

$\eta_3(4,84) = 89.2 \quad \xi_4(1) = .4 \quad \xi_5(2) = .5 \quad SK = .2 \quad EK = .01.$

Here, $\eta_3(\cdot)$ tests the specialisation of (15) to (12), which is decisively rejected.

This set of findings does not rule out the class of models which imposes price homogeneity and/or unit income elasticities as being valid characterisations of the equilibrium solution. The flaw in univariate analyses of v_t or $(m-p)_t$ of the type just presented is to impose such restrictions at all lags.

Further, the three results recorded in (12)-(14) imply that the time-series descriptions of $\{p_t\}$ and $\{i_t\}$ must be very different from that of $\{m_t\}$: otherwise aggregation over variables could not have caused such a loss of information. That is indeed the case as shown in (16) and (17).

$$(16)\quad \hat{\Delta_1 p_t} = \underset{[.20]}{.64}\Delta_1 p_{t-1} - \underset{[.14]}{.15}\Delta_1 p_{t-2} + \underset{[2.8]}{6.3}D_1 - \underset{[1.2]}{1.3}D_2 + \underset{[2.5]}{3.1}D_3 + \underset{[.0046]}{.0106}$$

$T = 93 \quad R^2 = .52 \quad \hat{\sigma} = 3.96\% \quad \eta_1(10,77) = .24 \quad \xi_2(10) = 7.3$

$\xi_4(1) = 7.1 \quad \xi_5(2) = 379. \quad SK = -1.3 \quad EK = 9.9 \quad \eta_6(8,79) = 5.4$

$$(17)\quad \hat{\Delta_1 i_t} = \underset{[.12]}{.26}\Delta_1 i_{t-1} + \underset{[.99]}{.23}D_1 + \underset{[.82]}{.57}D_2 - \underset{[1.59]}{.18}D_3 + \underset{[.0052]}{.0112}$$

$T = 93 \quad R^2 = .07 \quad \hat{\sigma} = 3.50\% \quad \eta_1(10,78) = .15 \quad \xi_2(11) = 18.6$

$\xi_4(1) = .8 \quad \xi_5(2) = 41. \quad SK = -.8 \quad EK = 2.9 \quad \eta_6(5,83) = .36$

Since the values of $\hat{\sigma}^2$ in these equations are very much larger than that in (14), it is unsurprising that imposing unit coefficients on prices and incomes at all lags should lead to a severe deterioration in fit.[25]

Returning to Figure I, three relatively distinct epochs of behaviour of v suggest themselves visually: a "cyclic around an average" period for 1875-1914, a "lower but more volatile" period during the inter-war years, and a "long upward trend" from 1947 to 1970 (after which floating exchange rates and competition and credit control were introduced). Any claim to the constancy of a model for money demand would need both constant parameters and a similar goodness-of-fit over each of these epochs; more stringently, it would need to stay constant since 1970. A possible objection might be that a money-demand equation's "constancy" need not be precise but only relatively better than (say) the consumption function's constancy. Since FS assert that their model is indeed constant, such an objection is not germane. However, in response to Mayer (1982, p. 1534),

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[25]Note that the dummy variable D_2, which FS identified as corresponding to a "shift in liquidity preference" for 1921-1955, is <u>not significant</u> in any of equations (12)-(17).

Appendix E briefly documents the comparable constancy of the consumption function using a simple error-correction model over some equivalent epochs, and shows it to be remarkably constant both before and after World War II.

As many reviewers of FS noted, it is extremely difficult to accept in the United Kingdom's institutional structure that money is an exogenous variable (see especially Congdon (1983) and Goodhart (1982)). Rather, given prices, income, and interest rates, and subject to adjustment lags, the private sector determines the volume of money outstanding (for more extensive discussions, see R. Hawtrey (1938, ch. 1-3) and Goodhart (1984)). While agents may control the mean (i.e., planned) value of money, money also acts as a "temporary buffer" for shocks, a buffer which agents cannot (or do not find optimal to) control precisely in every period (see David Laidler (1984)). Thus, we design our model of money demand to allow for disequilibria in agents' holdings of money relative to their *ex ante* plans (rather than postulating instantaneous adjustment). Such disequilibria are removed gradually through "error correction"; the reaction lags of money to changes in its various determinants are allowed to differ for every variate, and are determined from the data. From that viewpoint, velocity is a derived variate, which seems consistent with its time-series behaviour in periods when any of money, prices, or income change substantially.

The properties of and intuition behind error-correction models are important for understanding our approach (see Davidson *et al* (1978, pp. 679-682) and Hendry, Pagan, and Sargan (1984)). Suppose that a non-stochastic steady-state theory suggests proportionality between two variables Y and Z (e.g., consumption and income, money and nominal income, or wages and prices) so that $Y=KZ$ where K is constant for a given growth rate of Z (and so of Y). In logs, that theory becomes $y=\kappa+z$ with $\kappa=\ln(K)$. Without a solid economic theory of the dynamic relationship between the corresponding observable variables y_t and z_t, a general autoregressive-distributed lag relationship is postulated, with the parameters satisfying the restriction entailed by the steady-state solution. For expositional simplicity, we consider only current and one-period lags of y_t and z_t entering the dynamic relationship, as in the following equation:

(18) $$y_t = \alpha + \alpha_1 y_{t-1} + \beta_0 z_t + \beta_1 z_{t-1} + \nu_t \qquad \nu_t \sim IN(0, \sigma_\nu^2) .$$

In (18), long-run homogeneity between y and z requires $\alpha_1+\beta_0+\beta_1=1$. Re-writing (18) with that restriction gives:

(19) $$\Delta_1 y_t = \alpha + \beta\Delta_1 z_t + \gamma(y_{t-1}-z_{t-1}) + \nu_t \qquad \gamma\neq 0$$

where α, β, and γ are corresponding unrestricted parameters. That equation is respresentative of a large class of models satisfying both the economic theoretic restrictions and allowing for general dynamic responses. Intuitively, the term $\beta\Delta_1 z_t$ reflects the immediate impact that a change in z_t has on y_t. The term $\gamma(y_{t-1}-z_{t-1})$ (with γ negative for dynamic stability) is statistically equivalent to having $\gamma(y_{t-1}-\kappa-z_{t-1})$ instead in (19), and hence reflects the impact on $\Delta_1 y_t$ of having y_{t-1} out of line with $\kappa+z_{t-1}$. Such discrepancies could arise from errors in agents' past decisions, with the presence of $\gamma(y_{t-1}-z_{t-1})$ reflecting their attempts to correct such errors; so, (19) belongs to the class of error-correction models. For a steady-state growth rate of Z_t equal to g (i.e., $g=\Delta_1 z_t=\Delta_1 y_t$) and $\nu_t=0$, then, solving (19), we have:

(20) $$Y_t = Z_t \exp\{[-\alpha+g(1-\beta)]/\gamma\} ,$$

reproducing the assumption of proportionality between Y_t and Z_t from the non-stochastic steady-state theory. Note also that $K = \exp\{[-\alpha+g(1-\beta)]/\gamma\}$, which is independent of g only if $\beta=1$ or α depends on g. See Teun Kloek (1984) and Mark Salmon (1982).

Estimation of the "long-run parameter" K requires estimating all parameters in (19), including those corresponding to short-run and disequilibrium effects (i.e., β and γ). When using phase-average data, least-squares estimation is inconsistent for long-run parameters of important subclasses of (18) unless z_t is strictly exogenous (see Appendix B). Even if z_t is strictly exogenous, efficient estimation of those parameters and consistent estimation of standard errors (necessary for valid inferences) require properly modelling the dynamics present. Phase-averaging is neither necessary nor sufficient to capture such dynamics or to permit valid inferences from least-squares estimation. By way of contrast, estimation of (19) by least squares allows valid inference on both $(\alpha\ \beta\ \gamma\ \sigma_\nu^2)$ and K.

Error-correction models fall within the statistical framework of Section IV, noting that (18) (and hence (19)) involves a further factorisation of (9), namely:

$$(21)\quad F(\underset{\sim}{x}_t | \underset{\sim}{X}_{t-1};\ \underset{\sim}{\lambda}_t) = F(\underset{\sim}{y}_t | \underset{\sim}{z}_t,\ \underset{\sim}{X}_{t-1};\ \underset{\sim}{\phi}_1) \cdot F(\underset{\sim}{z}_t | \underset{\sim}{X}_{t-1};\ \underset{\sim}{\phi}_2)$$

where $\underset{\sim}{x}_t' = (\underset{\sim}{y}_t'\ \underset{\sim}{z}_t')$, corresponding to the assertion that the variates in $\underset{\sim}{z}_t$ are weakly exogenous for the parameters $\underset{\sim}{\phi}_1$ in the conditional density (for (19), $\underset{\sim}{\phi}_1' = (\alpha\ \beta\ \gamma\ \sigma_\nu)$, $\underset{\sim}{x}_t' = (y_t\ z_t)$, and $\ell=1$). Such a factorisation of the density is legitimate: whether it successfully isolates parameters of interest will be considered later.

In the remainder of this section, we present a general autoregressive-distributed lag representation of money conditional on prices, incomes, and interest rates, estimated to establish the innovation variance. That formulation is simplified to an error-correction model based on previous money-demand models developed recently in the UK. The static-equilibrium solution is presumed to be of a form $v_t=\psi_0+\psi_1 RS_t$, thus taking v_t and RS_t to be co-integrable (see Appendix C on co-integrability and for a discussion of testing for the existence of such a long-run relationship). Table IV reports the estimates of the unrestricted autoregressive-distributed lag model and the associated test statistics. Equation (22) records our presently "preferred" error-correction simplification of that unrestricted model.

$$(22)\quad \Delta_1\widehat{(m-p)}_t = \underset{[.04]}{.37}\Delta_1^2(m-p)_{t-1} - \underset{[.07]}{.06}\Delta_1^2(m-p)_{t-2} - \underset{[.022]}{.20}(m-p-i)_{t-4}$$
$$+ \underset{[.10]}{.66}(\Delta_4 i_t/4) - \underset{[.04]}{.47}\Delta_1^2 p_t - \underset{[.06]}{.14}\Delta_1^2 p_{t-2} - \underset{[.20]}{.78}RS_t$$
$$- \underset{[1.1]}{3.3}(\Delta_2 R\ell_t/2) + \underset{[1.1]}{1.9}D_1 + \underset{[.6]}{3.6}D_2 + \underset{[.80]}{.64}D_3 - \underset{[.010]}{.086}$$

$T = 93 \quad R^2 = .82 \quad \hat{\sigma} = 1.711\% \quad \eta_1(10,71) = 1.44 \quad \eta_2(6,75) = 3.5$

$\xi_4(1) = .2 \quad \xi_5(2) = .03 \quad SK = -.05 \quad EK = -.005$

(Figure V shows plots of the actual and predicted annual changes in real money over the period 1878-1970.) The static equilibrium solution of (22) yields (ignoring the D_i):

$$(23)\quad M/(P\cdot I) = \exp(-.44 - 4.0RS) \approx .64e^{-.040RS^*}$$

where RS* is RS measured in percentage points. Thus, a one percentage point increase in interest rates (e.g., 5% to 6%) reduces M relative to PI by <u>four percent</u> in the "long run". That long-run solution (i.e., hypothetical equilibrium: see Aris Spanos (1981)) is within the framework of the quantity theory.

Within the straitjacket of the information set FS use, equation (22) is not unreasonable. Directly testing the significance of the additional variables implicit in Table IV yields $\eta_3(21,60) = .97$, so that the residual of (22) appears to be an innovation on the information set generated by $(m, p, i, RS, R\ell, D_1, D_2, D_3)$. Note that the unrestricted equation

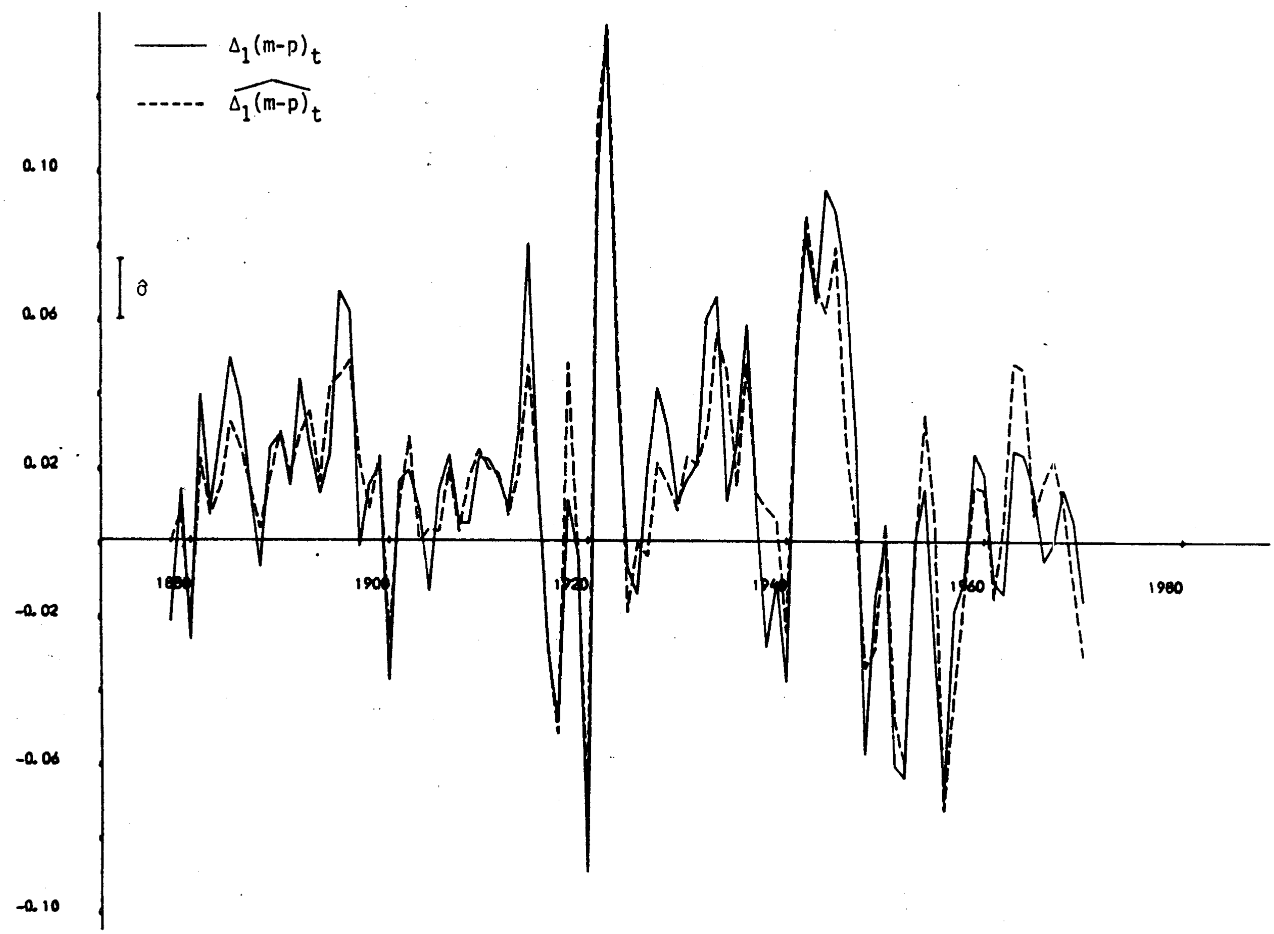
Table IV. A general autoregressive-distributed lag representation for money (m_t), conditional on incomes, prices, and interest rates

Variable	lag i					
	0	1	2	3	4	5
m_{t-i}	-1.0	1.534 [.104]	-.890 [.179]	.500 [.206]	-.147 [.177]	-.059 [.093]
i_{t-i}	.075 [.063]	.019 [.079]	.025 [.066]	-.052 [.076]	.030 [.099]	-.017 [.052]
p_{t-i}	.369 [.061]	-.418 [.086]	.265 [.086]	-.273 [.086]	.058 [.089]	.046 [.076]
RS_{t-i}	-.526 [.287]	.415 [.365]	-.208 [.370]	.148 [.355]	.098 [.390]	-.410 [.311]
$R\ell_{t-i}$	-2.001 [.893]	.027 [1.370]	2.484 [1.498]	-1.433 [1.623]	2.258 [1.515]	-.442 [1.138]
D_1	2.845 [.797]	-	-	-	-	-
D_2	.216 [1.614]	-	-	-	-	-
D_3	2.516 [1.116]	-	-	-	-	-
Constant	-.186 [.105]	-	-	-	-	-

$T = 93$ $R^2 = .99983$ $\hat{\sigma} = 1.718\%$ $\eta_1(10,50) = 1.20$ $\eta_2(6,54) = 3.79$ $\xi_4(1) = .06$

$\xi_5(2) = .07$ $SK = .083$ $EK = -.039$

Figure V. Equation (22): actual and fitted values for $\Delta_1(m-p)_t$.



automatically encompasses the models in equations (12)-(14) as well as (22); that (22) variance-dominates FS's money-demand model in levels by a <u>factor</u> of over <u>tenfold</u>[26]; that the parameters correspond to nearly orthogonal variables (see Table V); that we did not need to run "hundreds of regressions" (p. 266) to establish (22); but that it might be useful to formulate a theory-model of agents' plans and expectations which could yield insight into its dynamics and perhaps produce a more parsimonious parameterisation. The parameters of the non-dummy variables in (22) are precisely estimated; and most of the diagnostics are acceptable, although $\eta_2(\cdot)$ in Table IV and (22) is significant, with the partial sixth-order residual autocorrelation coefficient for the latter being -.4, suggesting that further lags may be necessary in Table IV. The adjustment lags are long in (22), given that the error-correction term (m-p-i) enters at four lags. Further, only <u>changes</u> in the rate of inflation (and not the level of inflation) matter; otherwise, (22) is similar in form to the models developed in Hendry and Mizon (1978), Hendry (1979), John Trundle (1982), and Davidson and Manfred Keil (1981). Compared to (13), the time-series component is retained almost unaltered, so (22) encompasses the univariate model in (13) while highlighting the roles of both the feedback from disequilibria via (m-p-i) and the additional economic variables.

Given that FS postulate a shift in liquidity preference over 1921-55, it seemed sensible to estimate (22) separately for each of the main epochs noted earlier, namely (i) 1878-1913, (ii) 1922-1938, (iii) 1950-1970, and

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[26]By appropriately (statistically) reducing the density $F(\underset{\sim}{y}_t|\underset{\sim}{z}_t,\underset{\sim}{X}_{t-1};\ \underset{\sim}{\phi}_1)$ for our model, one could derive an estimate of the parameters in FS's model corresponding to the density $F^*(\underset{\sim}{\bar{x}}_j|\underset{\sim}{\bar{X}}_{j-1};\ \underset{\sim}{\lambda}_j^*)$. From that constructed estimate and the estimate which FS actually obtain, one could test whether our model using annual data encompasses theirs using phase-average data. Note also that it is easy to construct examples for which the parameters in FS's model would not be constant over time but those in ours would be (e.g., in (21), $\underset{\sim}{\phi}_1$ is constant, $\underset{\sim}{z}_t$ is Granger-caused by $\underset{\sim}{y}_t$, and $\underset{\sim}{\phi}_2$ changes over time).

Table V. Data correlation matrix for the variables in (22)

		$\Delta_1(m-p)_t$	1	2	3	4	5	6	7	8	9	10
1.	$\Delta_1^2(m-p)_{t-1}$	.40 { .63}										
2.	$\Delta_1^2(m-p)_{t-2}$	-.04 {-.09}	-.13									
3.	$(m-p-i)_{t-4}$	-.15 {-.68}	-.05	-.11								
4.	$\Delta_4 i_t/4$	-.11 { .55}	-.12	-.06	.25							
5.	$\Delta_1^2 p_t$	-.45 {-.76}	.16	.20	.04	.26						
6.	$\Delta_1^2 p_{t-2}$	-.02 {-.21}	.06	-.80	.08	.22	-.21					
7.	RS_t	-.28 {-.43}	-.07	.04	-.62	-.06	-.02	.03				
8.	$\Delta_2 R\ell_t/2$	-.53 {-.33}	-.32	-.04	-.21	.08	.01	.13	.62			
9.	D_1	-.06 { .24}	-.07	-.05	-.16	.04	.16	.12	.13	.16		
10.	D_2	.18 { .54}	.01	.00	.71	-.07	-.10	-.08	-.47	-.28	-.19	
11.	D_3	.25 { .09}	.09	.01	.23	.10	-.02	.04	-.33	-.12	-.07	.37

(Figures in curly brackets are partial correlations.)

(iv) 1950-1982 (which thereby adds post-1970 data to (iii)). That not only investigates parameter constancy directly, but also implicitly allows all of the parameters to shift in period (ii) when D_2 is non-zero throughout. Table VI records the results. The coefficients are not so changeable between epochs as to be unrecognizable. Given the simplicity of the specification, these results provide a useful starting point for a more detailed examination of such issues as functional form, omitted variables (e.g., wealth and taxes), and so on, which may produce an improved model with more constant values of $\hat{\sigma}$ (see Imre Lakatos (1970) on progressive research strategies). By way of comparison with (iv), estimating (22) over the 105 observations through 1982 produces:

$$
\begin{aligned}
(24)\quad \Delta_1\widehat{(m-p)}_t = {} & \underset{[.07]}{.41}\Delta_1^2(m-p)_{t-1} + \underset{[.10]}{.10}\Delta_1^2(m-p)_{t-2} - \underset{[.026]}{.20}(m-p-i)_{t-4} \\
& + \underset{[.16]}{.64}(\Delta_4 i_t/4) - \underset{[.07]}{.50}\Delta_1^2 p_t - \underset{[.08]}{.07}\Delta_1^2 p_{t-2} - \underset{[.13]}{.63}RS_t \\
& - \underset{[.88]}{.49}(\Delta_2 R\ell_t/2) + \underset{[1.1]}{1.4}D_1 + \underset{[.8]}{4.2}D_2 + \underset{[.8]}{.7}D_3 - \underset{[.015]}{.097}
\end{aligned}
$$

$T = 105 \quad R^2 = .66 \quad \hat{\sigma} = 2.419\% \quad \eta_1(12,81) = 8.75 \quad \eta_2(6,87) = 1.3$

$\eta_3(21,72) = 2.41 \quad \xi_4(1) = 6.7 \quad \xi_5(2) = 10.2 \quad SK = -.1 \quad EK = 1.6$.

The most noticeable parameter changes from (22) are a fall in the coefficient of $\Delta_2 R\ell_t$ and a large increase in $\hat{\sigma}$, so the orthogonal parameterisation does prove useful for the most part (cf. Table V).[27] Nevertheless, constancy is clearly rejected (see $\eta_1(\cdot)$).

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[27]We (1983, pp. 77-78) experimented with RS replaced by RN, FS's (p. 270) measure of the marginal cost of money (RN ≡ RS·H/M). The resulting estimates are remarkably similar to those found using RS and reveal no improvement in the constancy of the interest-rate coefficient. Following suggestions by Chris Pissarides and recent work by Ross Starr (1983), we also experimented with adding variables which measured interest rate volatility, e.g., using $.2\sum_{i=0}^{4}(RS_{t-i}-RS_t^{+})^2$ where $RS_t^{+} = .2\sum_{i=0}^{4} RS_{t-i}$. The effect was largest for the most recent period but did not produce constant parameters or a constant fit.

Table VI. Sub-sample estimates of (22): $\Delta_1(m-p)_t$ on:

regressor \ period	1878-1970 (0)	1878-1913 (i)	1922-1938 (ii)	1950-1970 (iii)	1950-1982 (iv)
$\Delta_1^2(m-p)_{t-1}$	.37 [.04]	.47 [.07]	.35 [.07]	.39 [.06]	.40 [.14]
$\Delta_1^2(m-p)_{t-2}$	-.06 [.07]	-.11 [.12]	-.29 [.20]	.02 [.11]	.22 [.15]
$(m-p-i)_{t-4}$	-.20 [.022]	-.17 [.04]	.04 [.07]	-.13 [.03]	-.12 [.03]
$\Delta_4 i_t/4$	.66 [.10]	.78 [.15]	-.05 [.34]	.03 [.35]	.25 [.45]
$\Delta_1^2 p_t$	-.47 [.04]	-.68 [.06]	-.05 [.17]	-.60 [.08]	-.85 [.14]
$\Delta_1^2 p_{t-2}$	-.14 [.06]	-.29 [.11]	-.14 [.12]	.01 [.15]	-.10 [.29]
RS_t	-.78 [.20]	-.65 [.34]	.11 [.18]	-.69 [.36]	-.21 [.19]
$\Delta_2 R\ell_t/2$	-3.3 [1.1]	-4.4 [3.0]	-7.4 [2.0]	-.45 [.83]	.15 [.51]
D_1	1.9 [1.1]	-	-	-	-
D_2	3.6 [.60]	-	-	.52 [.68]	1.7 [1.5]
D_3	.64 [.80]	-	-	-	-
Constant	-.086 [.010]	-.075 [.027]	.033 [.032]	-.043 [.013]	-.070 [.021]
T	93	36	17	21	33
R^2	.82	.79	.89	.93	.62
$100\hat{\sigma}$	1.711	1.143	1.824	1.015	2.939
$\eta_1(q,T-k-q),q$	1.4, 10	.2, 5	.9, 2	.8, 5	15.1, 12
$\eta_2(q,T-k-q),q$	3.5, 6	.9, 6	(dw=2.7)	(dw=2.9)	1.2, 6
$\xi_4(1)$	.2	.05	.3	.07	1.9
SK, EK, $\xi_5(2)$	-.05, -.005, .03	-.3, -.4, .6	-.5, -.6, .4	.5, -.4, .5	1.1, 2.8, 12.1

The post-1970 evidence could be interpreted as support for our belief that money is a "buffer" for short-run shocks, so that the error variance is not constant for such equations, but varies with shocks to money supply, e.g., when the authorities attempt to control that variable. However, in models with nearly orthogonal regressors, omission of important explanatory factors whose time-series properties alter also would be reflected primarily in $\hat{\sigma}$ changing.

The large effect of D_2 in (22) is interesting in view of the estimates in the earlier time-series models since its role appears to be to remove an anomaly between v and RS, probably due to the invalidity of unit-elasticity restrictions on p and i. One reason for seeking to explain M without needing D_2 is that we cannot accept that a "shift" of 20% persisting for over 30 years in the relation of M to PI (see equation (22)) is just an "anomaly".[28] Whether one believes that M is exogenous and "causes" P or that it is endogenous and determined by a "stable" money-demand function, the observable discrepancy is massive evidence against either belief.[29] For 1920-1956, (m-p) is grossly out of line with i (even adjusted for RS), a point seen graphically in Figure II. Yet the price level <u>falls</u> in almost

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[28]If D_2 is in fact excluded from the regressor set when simplifying the regression in Table IV, price homogeneity is rejected; or, if price homogeneity is imposed, $\hat{\sigma}$ increases. Thus a radically different approach seems required to sustain even the weaker view that agents plan to hold a certain money stock via a constant behavioural model. Noting that nominal money M_t almost never falls, the class of models discussed by George Akerlof (1979) also merits consideration.

[29]One possible explanation (suggested by Nicholas Dimsdale) for the behaviour of a broad money aggregate like M is via a portfolio model in which the ratio of M to other financial assets is a function of the opportunity costs of holding a non-interest bearing asset and the risks of other assets. Andrew Threadgold has kindly provided his data on Central Government Debt over our period and the similarity of its behaviour to that of M/(PI) is suggestive: we are currently investigating this approach to modelling M without D_2.

every year from 1921 to 1935 and real income experiences its most drastic historical fall in 1920-1922! Thus, the economy seems capable of operating without inflation despite an "excessive" real money stock.

Our econometric model of the demand for money in the UK satisfies a range of statistical criteria as well as incorporates economic theory. Both aspects are essential for a model to characterise adequately the underlying data generation process. However, even with our best effort, our model is not fully satisfactory: estimated adjustment lags are rather long; some parameter non-constancy is evident, mainly through the equation standard error and the coefficients on interest rates; and the dummy variable D_2 seems necessary.[30] That may be less than surprising as the data span a century during which financial institutions altered dramatically: witness the growth of building societies and, after 1970, the introduction of competition and credit control and of floating exchange rates. In spite of manifest deficiencies in our model, the evidence herein suggests that the econometric techniques used to obtain that model are vastly superior in practice to the maze of prior transformations in which FS indulge.

VII. Conclusions

A number of assertions in Friedman and Schwartz (1982) concerning the empirical validity of their money-demand equation have been tested using their data series for the United Kingdom and were found to be without empirical support. Their procedure of averaging data over business-cycle phases did not greatly reduce the serial correlation in the data series, but did lose a great deal of information, thus leading to rather badly fitting equations. We find nothing to recommend in that practice as against analysing the annual data and modelling both "trend" and "cycle".

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[30]For a follow-up to Section 6, see Andrew Longbottom and Sean Holly (1985).

The failure by FS to present statistical evidence pertinent to their main assertions leaves these devoid of credibility. In fact, many of FS's inferences from equations based on the phase-average data are invalidated by residual autocorrelation. Moreover, their "final" money-demand equation is not constant (contrary to their claim); and, on testing obvious assumptions (such as price homogeneity and the absence of trends) implicit in their model, rejection results. Such negative findings are consistent with those reported by Meghnad Desai (1981, especially ch. 4).

Simply "corroborating" a subset of the implications of a theory is not an adequate justification for deeming it useful (see _inter alia_ Friedman (1953), Karl Popper (1959, Section 82) and Lawrence Boland (1982, ch. 1)). That is well illustrated by the contrast between FS's claims to have empirically "corroborated" various aspects of their theories and our evidence that those assertions are actually refutable from the same data. Only _well-tested_ theories which have successfully weathered tests outside the control of their proponents and encompass the gestalt of existing empirical evidence seem likely to provide a useful basis for applied economic analysis and/or policy. Our econometric approach was briefly exposited and illustrated above and it emphasised the crucial role of testing and evaluating models with (at least) the six criteria outlined. The failure to emphasise evaluation and testing is a major lacuna in Leamer's (1983) analysis, and we would view this paper in part as a counter-example to the view that data evidence does not discriminate between alternative hypotheses (cf. McAleer, Pagan, and Volker (1985)). Despite the manifest _insufficiency_ of even "sophisticated falsificationism" as a methodology (e.g., see Alan Chalmers (1976)), rigorous evaluation of empirical claims seems a _necessary_ first step towards taking the con out of economics.

APPENDIX A. Legend

D_1	-	A dummy variable for World War I (= 1 for 1914-18 inclusive, zero elsewhere)
D_2	-	A dummy variable for 1921-55 (= 1 for 1921-55 inclusive, zero elsewhere)
D_3	-	A dummy variable for World War II (= 1 for 1939-45 inclusive, zero elsewhere)
E	-	Exchange rate ($/£)
H	-	High-powered money (million £)
I	-	Real net national product (million 1929 £)
M	-	Money stock (million £)
N	-	Population (millions)
P	-	Deflator of I (1929 = 1.00)
P*	-	Deflator of net national product in the USA (1929 = 1.00)
$R\ell$	-	Long-term interest rate (fraction)
RS	-	Short-term interest rate (fraction)
$\bar{S}$	-	A dummy variable for phase observations 16-28 (1921-1955; = 1 for observations 16-28 inclusive, zero elsewhere)
$\bar{W}$	-	A dummy variable for phase observations 13-15 and 26-28 (1918-1921 and 1946-1955; = -4, -3, -2, 8, 5, and 3 for phase observations 13, 14, 15, 26, 27, and 28 respectively; zero elsewhere)

Coefficients and estimated standard errors of the dummies D_1, D_2, D_3, $\bar{S}$, and $\bar{W}$ are reported times 100 for readability.

These data are as in FS (Tables 4.8 and 4.9), but are rescaled using the figures for 1920 to exclude a proportional effect of Southern Ireland, and have been kindly extended through 1982 by John Trundle at the Bank of England (see Appendix F). Also, P, P*, RS, and $R\ell$ have been divided by 100 so that the values of P and P* in 1929 equal 1.00 (rather than 100) and RS and $R\ell$ are expressed as fractions (rather than as percentages).

APPENDIX B. An analogue model for averaging and differencing

In this appendix, we consider the statistical effects that fixed n-period phase-averaging has on time series generated by a simple dynamic process. We focus on the variance and autocorrelation of the disturbance term for both the levels and difference equations, as FS make claims about what their properties ought to be. We also present certain features of the observed series on velocity and examine how well our analytical model captures them. While it is an extremely simplified characterisation of the phase-averaging adopted by FS, the following model does offer several insights into the likely consequences of their approach.[31]

Consider the time series $\{(y_t : z_t)'; \ t = 1, \ldots, T\}$ such that:

(B1) $y_t = \alpha_0 + \alpha_1 z_t + u_t$

(B2) $u_t = \rho u_{t-1} + \varepsilon_t$

(B3) $z_t = \lambda_1 z_{t-1} + \lambda_2 y_{t-1} + v_t$

$$\begin{bmatrix} \varepsilon_t \\ v_t \end{bmatrix} \sim \mathrm{IN}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \right).$$

Except when otherwise noted, we assume that $(y_t : z_t)'$ is stationary; hence all roots of (B1)-(B3) are stable, i.e., $|\rho| < 1$ and $|\lambda_1 + \alpha_1 \lambda_2| < 1$. For convenience, denote the second root $(\lambda_1 + \alpha_1 \lambda_2)$ by κ. If $\lambda_2 = 0$, z_t is strongly exogenous for $(\alpha_0 : \alpha_1)$; otherwise it is correlated with u_t. Letting the n-period phase average of an arbitrary series $\{x_t\}$ be:

(B4) $\bar{x}_j = n^{-1} \sum_{i=1}^{n} x_{(j-1)n+i} \qquad j = 1, 2, \ldots, (\frac{T}{n})$,

then

(B5) $\bar{y}_j = \alpha_0 + \alpha_1 \bar{z}_j + \bar{u}_j$

from (B1). Average two-(phase-)period differences result in:

(B6) $(.5\Delta_2 \bar{y}_j) = \alpha_1 (.5\Delta_2 \bar{z}_j) + (.5\Delta_2 \bar{u}_j)$.

Our concern is how inference is affected by using (B5) (or (B6)) rather than (B1)-(B3).

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[31] We are indebted to Julia Campos for carefully checking an earlier version of this appendix.

As a baseline, we note that the variance and autocovariances of u_t in (B1) are:

$$\text{(B7)} \quad \tau_i \equiv E(u_t u_{t-i}) = \begin{cases} \sigma_\varepsilon^2/(1-\rho^2) & i = 0 \\ \rho^i \sigma_\varepsilon^2/(1-\rho^2) & i = 1,2,\ldots . \end{cases}$$

Hence the autocorrelation function for u_t is ρ^i ($i>0$). Equivalent formulae for $\bar{u}_j$ in (B5), the levels equation with phase-average data, are:

$$\bar{\tau}_i \equiv E(\bar{u}_j \bar{u}_{j-i}) = \begin{cases} \dfrac{\sigma_\varepsilon^2}{n(1-\rho)^2}\left[1 - \dfrac{2\rho(1-\rho^n)}{n(1-\rho^2)}\right] & i = 0 \quad \text{(B8)} \\ \dfrac{\rho^{n(i-1)}\rho(1-\rho^n)^2\sigma_\varepsilon^2}{n^2(1-\rho^2)(1-\rho)^2} & i = 1,2,\ldots . \quad \text{(B9)} \end{cases}$$

The error variance of (B5) is less than that of (B1) for all $n > 1$, but both equations manifest residual autocorrelation unless $\rho \approx 0$. Some representative values of the variance of $\bar{u}_j$ and its first-order autocorrelation r_1 $\left(= E(\bar{u}_j\bar{u}_{j-1})/E(\bar{u}_j^2)\right)$ are shown in Table B.I.

Table B.I. Variance and first-order autocorrelation of $\bar{u}_j$

	$Var(\bar{u}_j)/\sigma_\varepsilon^2$ (from (B8))				r_1 (from (B8) and (B9))			
ρ \ n	1	2	3	5	1	2	3	5
.0	1.	.5	.33	.2	.0	.0	.0	.0
.5	1.33	1.0	.81	.59	.5	.38	.28	.17
.8	2.8	2.5	2.3	2.0	.8	.72	.64	.50
.95	10.3	10.0	9.8	9.5	.95	.93	.90	.84

The variance falls as n increases, but with an ever smaller proportional effect as ρ increases. As an example, consider that for $\rho = .8$ and $n = 3$, (B8) yields $2.3\sigma_\varepsilon^2$ whereas σ_u^2 from (B7) is $2.8\sigma_\varepsilon^2$, so averaging produces only a small decrease in variance. Moreover, for large ρ, there is only a small reduction in first-order error autocorrelation, whereas autocorrelation at higher orders dies off rapidly.

Least-squares estimation of structural parameters in the presence of autocorrelated disturbances is inefficient. Further, if right-hand side variables are correlated with the disturbance, such estimation is inconsistent:

$$\text{(B10)} \quad E(z_t u_t) = \rho\lambda_2\sigma_\varepsilon^2/[(1-\rho^2)(1-\rho\kappa)] \equiv \sigma_{zu} \text{ (say)}$$

which is zero only if $\rho\lambda_2 = 0$. $E(z_{t-i}\varepsilon_t) = 0$ for all $i \geqq 0$ (and for all ρ and λ_2), so estimation of (B1)-(B2) by <u>autoregressive</u> least squares is consistent. Turning to the phase-average data in (B5):

$$\text{(B1')} \quad E(\bar{z}_j\bar{u}_j) = \lambda_2\sigma_\varepsilon^2(1+\rho)/[4(1-\rho)(1-\rho\kappa)]$$

when n=2 (for illustrative purposes). Thus, even if $\rho = 0$ (and so $\sigma_{zu} = 0$), $E(\bar{z}_j\bar{u}_j) \neq 0$ unless $\lambda_2 = 0$. Conversely, the strict exogeneity of z_t (i.e., $\lambda_2 = 0$) is sufficient for both (B10) and (B11) to be zero.

Next, we consider the effects of two-period differencing of the phase-averages in (B5) to get (B6):

$$\text{(B12)} \quad \mathrm{var}(.5\Delta_2\bar{u}_j) = \frac{\sigma_\varepsilon^2}{2n(1-\rho)^2}\left[1 - \frac{\rho(2-\rho^n)(1-\rho^{2n})}{n(1-\rho^2)}\right]$$

and

$$\text{(B13)} \quad E[(.5\Delta_2\bar{u}_j)(.5\Delta_2\bar{u}_{j-1})] = (1-\rho^{2n})\rho(1-\rho^n)^2\sigma_\varepsilon^2/[4n^2(1-\rho^2)(1-\rho)^2] \;.$$

After rescaling (B12) by 4, the error variances for levels and "rates of change" models will be similar only for quite large values of $|\rho|$ (e.g., $\rho = .9$), in which case the former will still exhibit substantial first-order autocorrelation although the latter will not.

For the differenced data, the equation equivalent to (B11) is:

$$\text{(B14)} \quad E(\Delta_2\bar{z}_j \cdot \Delta_2\bar{u}_j) = [2 - (1+\kappa)^2\kappa^2]\lambda_2\sigma_\varepsilon^2/4$$

when $\rho=0$ and $n=2$. So, in general, different biases will result from fitting (B5) and (B6). Only if z_t is strictly exogenous ($\lambda_2=0$) will both biases be zero.

The variable-length weighting procedures in FS should not alter the substance of this last result; but being data-based to eliminate "cyclical behaviour" (which, e.g., a positively autocorrelated series would appear to manifest), it could seriously alter the nature of the residual autocorrelation. For example, $\{.5\Delta_2\bar{u}_j\}$ should show negative second-order autocorrelation of the form:

$$\text{(B15)} \quad -.5 + \frac{\rho^{n+1}(1-\rho^{2n})(1-\rho^{n})^2}{2n(1-\rho^2)\left[1 - \frac{\rho(2-\rho^n)(1-\rho^{2n})}{n(1-\rho^2)}\right]} \quad .$$

Some representative values of (B15) are given in Table B.II.

Table B.II. Values of second-order autocorrelation for $\Delta_2\bar{u}_j$

ρ \ n	1	2	3	5
.0	-.50	-.50	-.50	-.50
.5	-.38	-.45	-.48	-.50
.8	-.18	-.25	-.32	-.41
.95	-.05	-.03	-.06	-.12

That effect may not carry over for data-selected phase averages.

Descriptively, FS's phase-average data series remain very highly autoregressive (e.g., the Durbin-Watson statistic is .31 for velocity on a constant and remains as low as .59 for real per capita money on a constant, real per capita income, and interest rates). Thus the analysis ought to account for the pure time-series phenomena of the data. Table B.III details such phenomena, using velocity as an example.

Table B.III. Standard deviations of observed velocity

	Annual	Phase-average
Sample size	100	36
Levels	$\hat{\sigma}(v) = .169$	$\hat{\sigma}(\bar{v}) = .169$
Differences	$\hat{\sigma}(\Delta_1 v) = .047$	$\hat{\sigma}(\Delta_1 \bar{v}) = .071$

$\sigma(\cdot)$ denotes the standard deviation of the relevant variable. Other series from FS show analogous results: roughly equal standard deviations in levels, and large reductions in standard deviations by differencing, more so for the annual data. The original series shows first-order autocorrelation close to unity with the resulting differenced series being far less positively autocorrelated than the levels, and close to "white noise" for the phase-average data. The analogous case now is equivalent to $\alpha_1=0$ in (B1) and (in mean deviation form) yields the baseline model:

(B16) $y_t = \rho y_{t-1} + \varepsilon_t \qquad t = 1,\ldots,T$

with

(B17) $\sigma_y^2 = \sigma_\varepsilon^2/(1-\rho^2)$.

For n-period averaging,

(B18) $\bar{y}_j = \rho_p \bar{y}_{j-1} + e_j \qquad j = 1,\ldots,J$

where ρ_p is defined by $[E(\bar{y}_{j-1}^2)]^{-1}E(\bar{y}_j\bar{y}_{j-1})$, i.e., $E(\bar{y}_{j-1}e_j)=0$. Thus,

(B19) $\sigma^2(\bar{y}) = \sigma_e^2/(1-\rho_p^2) = \sigma_y^2[n(1-\rho^2)-2\rho(1-\rho^n)]/[n^2(1-\rho)^2]$,

(B20) $\rho_p = \rho(1-\rho^n)^2/[n(1-\rho^2)-2\rho(1-\rho^n)]$.

Moreover,

(B21) $\Delta_1 y_t = \varepsilon_t + (\rho-1)y_{t-1}$ and

(B22) $\Delta_1 \bar{y}_j = e_j + (\rho_p-1)\bar{y}_{j-1}$ so that

(B23) $\sigma^2(\Delta_1 y) = 2\sigma_\varepsilon^2/(1+\rho)$ and

(B24) $\sigma^2(\Delta_1 \bar{y}) = 2\sigma_e^2/(1+\rho_p)$.

When $n = 3$ and $\rho = .96$, we have $\rho_p = .92$: Table B.IV gives relevant standard deviations in terms of σ_ε.

Table B.IV. Standard deviations from models (B16) and (B18) ($\rho=.96$)

Annual		Phase-average	
σ_y	= 3.57	$\sigma(\bar{y})$	= 3.51
$\sigma(\Delta_1 y)$	= 1.01	$\sigma(\Delta_1 \bar{y})$	= 1.42
σ_ε	= 1.00	σ_e	= 1.39

Those values closely match the corresponding ratios in Table B.III. (See Table II in Section III.)

The final alternative we consider is that the series are random walks: certainly the graphs of several of the variables look like those of the random-walk series reported by Holbrook Working (1934), and $\rho = .96$ is not "far" from unity.[32] However, if $\rho = 1$ in (B16), then $E(y_t^2) = t\sigma_\varepsilon^2$ (for $y_0=0$) and the mean of the standard estimator of the variance of y_t is:

(B25) $$E\left(\frac{1}{T-1} \sum_{t=1}^{T} y_t^2\right) = \sigma_\varepsilon^2 \, T(T+1)/[2(T-1)] \equiv \sigma_T^2(y)$$

(e.g., see Fuller (1976, p. 367)) whereas $\sigma^2(\Delta_1 y) = \sigma_\varepsilon^2$. Working (1960) considers the case of an n-period average of a random walk and shows that, for e_j defined by $\bar{y}_j = \bar{y}_{j-1} + e_j$, its variance is:

(B26) $$\sigma_e^2 = \sigma_\varepsilon^2(2n^2+1)/(3n) .$$

However, the mean of the standard estimator of the variance of $\bar{y}_j$ is:

(B27) $$E\left(\frac{1}{J-1} \sum_{j=1}^{J} \bar{y}_j^2\right) = \sigma_\varepsilon^2 J(3Jn^2-n^2+3n+1)/[6n(J-1)] \equiv \sigma_J^2(\bar{y}) ,$$

assuming $y_0=0$. Table B.V gives various standard deviations for T = 96, J = 35 and n = T/J ($\approx$2.74).

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[32] As noted in the text, it is difficult to reject the null that $\rho=1$; but that could be due to the low power of unit-root tests against alternatives like $\rho=.95$ (e.g., see Bhargava (1983)) and does <u>not</u> entail accepting the null.

Table B.V. Standard deviations of various series (ρ=1.0)

Annual	Phase-average
$\sigma_T(y) = 7.00$	$\sigma_J(\bar{y}) = 7.04$
$\sigma_\varepsilon = 1.00$	$\sigma_e = 1.40$

That table is a much poorer match to Table B.III than is Table B.IV. Like the significant error-correction coefficients established in Section VI, the evidence here favours a highly autoregressive but "just" stationary process for velocity. Conversely, the analysis for that case explains why the models using phase-average data fit much less well than those using annual data.

The effects of measurement errors (denoted by $\{\zeta_t\}$) on the above analysis critically depend on the time-series characteristics of such errors. If the $\{\zeta_t\}$ are white-noise errors in measuring $\Delta_1 y_t$, then they in effect add to $\{\varepsilon_t\}$ in (B16) when ρ is close to unity. However, if the $\{\zeta_t\}$ are white-noise errors in measuring y_t as $y_t^o = y_t + \zeta_t$, then the error in (B16) fitted to $\{y_t^o\}$ becomes $(\varepsilon_t+\zeta_t-\rho\zeta_{t-1})$. Thus, such errors would induce substantial negative residual autocorrelation if σ_ζ^2, the variance of ζ_t, were comparable in magnitude to σ_ε^2, and that would be especially marked for the data $\{\Delta_1 y_t^o\}$. Also, $\sigma^2(y^o) = \sigma_\zeta^2+\sigma_y^2$ and $\sigma^2(\Delta_1 y^o) \approx 2(\sigma_\zeta^2 + \sigma_\varepsilon^2/(1+\rho))$. Conversely, for $\bar{y}_j^o = \bar{y}_j + \bar{\zeta}_j$ (say), $\sigma^2(\bar{y}^o) = \sigma^2(\bar{y}) + \sigma_\zeta^2/n$ and $\sigma^2(\Delta_1\bar{y}^o) = \sigma^2(\Delta_1\bar{y}) + 2\sigma_\zeta^2/n$. Since the effect of measurement errors is much smaller for $\bar{y}$ than for y, given (B16)-(B24) and the values observed in Table B.III, σ_ζ^2 would have to be negligible to match the variance ratios. While that is dependent on assuming (unrealistically) that the process is fully characterised by (B16), nevertheless the evidence is more consonant with white-noise errors in measuring the changes, and relatively small errors generally in comparison to the large standard deviations of the levels of the variables (especially the trending series such as M, P, I, RS, or Rℓ).

APPENDIX C. Testing for the existence of a relationship between variables

In this appendix, we summarise recent results on tests for the existence of a relationship between variables and apply such tests to FS's data for money, prices, incomes, and interest rates. A prior question to what specification to choose in modelling money demand is whether there exists any "long-run", or equilibrium, relationship between money, prices and incomes. If a relationship is defined across evolutionary variables (i.e., ones which need differencing to be stationary), then one characterisation of its existence or the lack thereof is whether or not the variance of deviations from the relationship is bounded over time (e.g., see Clive Granger and Andrew Weiss (1983)). If one cannot reject the hypothesis that the deviations have unbounded variance over time, then the postulated relationship is at best exceedingly tenuous, and at worst meaningless. Since random walks have unbounded variability over time, a natural hypothesis to seek to reject is that the deviations from the relationship constitute a random walk. That comprises the basis of Granger and Engle (1984), drawing on Fuller (1976), Dickey and Fuller (1979, 1981), G.B.A. Evans and N.E. Savin (1981, 1984), and Sargan and Bhargava (1983).

Granger (1981; 1983a, b) defines variables to be integrable of order d (denoted I(d)) if differencing d times is needed to induce stationarity (i.e., I(0)). Variables which are I(d) for $d \geq 1$ have variances which increase rapidly over time whereas I(0) variables do not exhibit that characteristic in large samples. A set of I(d) variables is co-integrable if some linear combination of them is I(0). Thus, we will test the proposition that certain sets of variables which empirically behave as I(1) are co-integrable (i.e., define a long-run relationship) by testing whether the deviations from the relationship are I(1) against the alternative that they are I(0). The exact dynamic specification does not matter for this test of non-existence, since all lags can be re-parameterised as changes relative to the current period, and hence must be of a lower order of integrability than the levels. For example, $\dot{p} \equiv \Delta_1 p_t$ is I(d-1) if p_t is I(d). Thus, omission of lagged variables will not vitiate the test: if the levels are co-integrable, the changes must be also; and if the changes are not co-integrable then the levels cannot be either. Both the Durbin-Watson statistic (dw) used by Bhargava (1983) and the 't'-statistic which Dickey and Fuller (1979) denote by τ_μ (we use τ) provide approximate statistics for investigating the random walk hypothesis in regression residuals, and we calculate both below.

The most general of FS's claims concerning money demand seems to be that there exists a linear relationship linking m, p, i, RS and Rℓ, which we translate to mean that that set of variates must be co-integrable. On the basis of the dw and τ statistics for all of those series individually, the hypothesis that they are I(1) cannot be rejected (although Rℓ had an estimated root larger than unity). Consequently, the putative existence of hypothetical "long-run" relationships is of substantive interest. A narrower version of FS's claim is that there exists a linear relationship connecting v, $\dot{p}$, RS and Rℓ. We have investigated both the general and narrower claims by estimating "staticised" relationships and report tests of the hypothesis that the residuals are a random walk against the alternative of a stationary process, i.e., we seek to reject the null that the residuals from a regression of m_t on p_t, i_t, RS_t, and $R\ell_t$ (or of v_t on RS_t and $R\ell_t$) follow a random walk (i.e., are I(1)). We obtained:

(C1) $\hat{m}_t = 1.17p_t + .83i_t - 8.2RS_t + 1.11$

$T = 93 \quad \hat{\sigma} = 10.3\% \quad dw = .46 \quad \tau = -3.4 \quad \hat{\sigma}^* = 6.15\% \quad dw^* = .77$

(C2) $\hat{v}_t = 7.0RS_t + .308$

$T = 93 \quad \hat{\sigma} = 11.1\% \quad dw = .32 \quad \tau = -2.8 \quad \hat{\sigma}^* = 6.60\% \quad dw^* = .73$

where τ was computed from the estimated residuals, and dw* and $\hat{\sigma}^*$ relate to equivalent regressions including D_1, D_2 and D_3. The large increase between dw and dw* is due almost entirely to FS's shift dummy D_2. Adding $R\ell$ had little effect on $\hat{\sigma}$ or dw.

While the tests are inconclusive, the existence of those relationships at their Waterloo is as much a close-run thing as Wellington's was at his: when the dummies are excluded, the values of dw are close to the lower bounds in Sargan and Bhargava (1983), especially given that an optimal linear combination of stochastic regressors is being estimated. At best, those tests point to an exceedingly tenuous relationship, salvaged only by the inclusion of the shift dummy variable constructed by FS after they noted discrepancies from a linear regression between real money and real income (see FS (pp. 225-238) and our Figure II). Note also that (C1) and (C2) fit much worse than the time-series models reported above, indicating the important role of dynamic reactions in accounting for the data variances.

APPENDIX D. Re-normalisations of regression equations

This appendix briefly illustrates why we view FS's models for $(\bar{p}+\bar{i})$, $\bar{p}$, and $\bar{i}$ (the second through fourth equations reported in Table I) as approximate re-normalisations of their equation for $(\bar{m}-\bar{p}-\bar{n})$ and why such re-normalisations produce no new inferences.

Now, if the joint density $F(\underset{\sim}{x}_t|\underset{\sim}{X}_{t-1};\underset{\sim}{\lambda}_t)$ in (9) involves q different x_{it}'s, it can be factorised in q! different ways. Any given factorisation is statistically legitimate, although most need not yield either constant or interesting parameters. However, in general, it is <u>not</u> legitimate to pick elements from different factorisations and treat them as "independent" summaries of $F(\underset{\sim}{x}_t|\underset{\sim}{X}_{t-1};\underset{\sim}{\lambda}_t)$. For example, if q = 2 with $\underset{\sim}{x}_t' = (x_{1t}\ x_{2t})$, there exist two valid factorisations:

$$\text{(D1)}\quad F(\underset{\sim}{x}_t|\underset{\sim}{X}_{t-1};\underset{\sim}{\lambda}_t) = f_1(x_{1t}|x_{2t},\underset{\sim}{X}_{t-1};\cdot)\cdot f_2(x_{2t}|\underset{\sim}{X}_{t-1};\cdot)$$
$$= g_1(x_{2t}|x_{1t},\underset{\sim}{X}_{t-1};\cdot)\cdot g_2(x_{1t}|\underset{\sim}{X}_{t-1};\cdot)\ .$$

Either $f_1(\cdot)$ and $f_2(\cdot)$ or $g_1(\cdot)$ and $g_2(\cdot)$ may be used; but $f_1(\cdot)$ and $g_1(\cdot)$ do not comprise a legitimate representation, with both sets of "parameters" being of independent interest. For example, when (10) holds (hence $F(\underset{\sim}{x}_t|\cdot)$ is the bivariate normal density) with $\ell=0$ and $\underset{\sim}{\Sigma}_t$ non-singular and constant, $f_1(\cdot)$ and $g_1(\cdot)$ yield:

$$\text{(D2)}\quad x_{1t} = a_0 + a_1x_{2t} + \varepsilon_{1t} \qquad E(\varepsilon_{1t}^2) = \phi_{11}$$
$$\text{(D3)}\quad x_{2t} = b_0 + b_1x_{1t} + \varepsilon_{2t} \qquad E(\varepsilon_{2t}^2) = \phi_{22}$$

with $E(\varepsilon_{1t}\varepsilon_{2t})=0$. Then $a_1/\phi_{11} = b_1/\phi_{22}$ with $a_1b_1 = R^2_{x_1x_2}$. Because the density in (10) is "saturated" by five parameters and the equations in (D2) and (D3) have six, an automatic restriction results. That finding generalises so that for three variables, say:

$$\text{(D4)}\quad x_{it} = \gamma_{i0} + \gamma_{i1}x_{jt} + \gamma_{i2}x_{st} + \varepsilon_{it} \qquad E(\varepsilon_{it}^2) = \phi_{ii}$$

$(i,j,s=1,2,3;\ j,s\neq i;\ s>j)$, then:

$$\text{(D5)}\quad \gamma_{11}/\phi_{11} = \gamma_{21}/\phi_{22}\ ,\quad \gamma_{12}/\phi_{11} = \gamma_{31}/\phi_{33}\ ,\ \text{and}\ \ \gamma_{22}/\phi_{22} = \gamma_{32}/\phi_{33}\ .$$

The regression coefficients are all functions of a smaller set of parameters. Adding an unrestricted set of regressors to every equation does not affect that result either. While the algebra becomes much more complicated if different additional regressors are included in each equation, it remains invalid in principle to mix the choice of factorisations: running regressions with different normalisations cannot establish separate "structural equations" or produce new inferences.

For FS's results reported in Table I, the equations in (D5) do not hold precisely because of the other restrictions imposed and the different sets of auxiliary variables in each regression. Nevertheless, they are near enough satisfied to cast doubt on the notion that three "separate" equations are being reported.

Interestingly, it is not only easy to show that the analysis above is an exact sampling result, but in the process the formulae for any value of q are obtained. The main insight required is simply the identity of inversion and regression. If $\underset{\sim}{y}$ is to be regressed on $\underset{\sim}{X}$, it is well-known that all the relevant ingredients can be obtained by inverting the augmented second-moment matrix $\underset{\sim}{A} = (\underset{\sim}{y}:\underset{\sim}{X})'(\underset{\sim}{y}:\underset{\sim}{X})$. Then:

$$\text{(D6)}\quad \underset{\sim}{A}^{-1} = \begin{bmatrix} (\underset{\sim}{y}'\underset{\sim}{Q}_X\underset{\sim}{y})^{-1} & -(\underset{\sim}{y}'\underset{\sim}{Q}_X\underset{\sim}{y})^{-1}\underset{\sim}{y}'\underset{\sim}{X}(\underset{\sim}{X}'\underset{\sim}{X})^{-1} \\ -(\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'\underset{\sim}{y}(\underset{\sim}{y}'\underset{\sim}{Q}_X\underset{\sim}{y})^{-1} & (\underset{\sim}{X}'\underset{\sim}{X})^{-1}[\underset{\sim}{I}+\underset{\sim}{X}'\underset{\sim}{y}(\underset{\sim}{y}'\underset{\sim}{Q}_X\underset{\sim}{y})^{-1}\underset{\sim}{y}'\underset{\sim}{X}(\underset{\sim}{X}'\underset{\sim}{X})^{-1}] \end{bmatrix}$$

where $\underset{\sim}{Q}_X = \underset{\sim}{I} - \underset{\sim}{X}(\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'$. Thus $(\underset{\sim}{y}'\underset{\sim}{Q}_X\underset{\sim}{y})$ is the residual sum of squares, etc. Normalising on the first diagonal term yields:

$$\text{(D7)}\quad \underset{\sim}{A}^{-1} = (\underset{\sim}{y}'\underset{\sim}{Q}_X\underset{\sim}{y})^{-1}\begin{bmatrix} 1 & -\hat{\underset{\sim}{\beta}}' \\ -\hat{\underset{\sim}{\beta}} & (T-k)\mathrm{Var}(\hat{\underset{\sim}{\beta}})[\underset{\sim}{I}+\underset{\sim}{H}] \end{bmatrix}$$

for k regressors, where $\hat{\underset{\sim}{\beta}} = (\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'\underset{\sim}{y}$ and $\underset{\sim}{H} = \underset{\sim}{X}'\underset{\sim}{y}(\underset{\sim}{y}'\underset{\sim}{Q}_X\underset{\sim}{y})^{-1}\hat{\underset{\sim}{\beta}}'$. Thus, regression is just a normalised and appropriately interpreted inversion of the augmented moment matrix $\underset{\sim}{A}$. But $\underset{\sim}{A}$ involves <u>all</u> of the variables, and the different normalised regressions simply correspond to dividing by different diagonal terms in $\underset{\sim}{A}^{-1}$ and interpreting the associated columns. Consequently, (D5) holds for all T; and the relevant generalisation for <u>any</u> $q = k+1$ follows immediately. In the alternative possibility that the parameters of three <u>interdependent</u> equations linking x_{1t}, x_{2t} and x_{3t} are of interest, then we have a simultaneous equations model and so least-squares estimates are inconsistent.

APPENDIX E. Comparative results on the consumption function

The evidence contrasts interestingly with that on M (see Mayer (1982, p. 1534)); detailed documentation is provided in Hendry (1983), where estimates on annual data for the inter-war and post-war periods separately are reported. The standard error ($\hat{\sigma}$) in <u>both periods</u> is .5% of real non-durable consumers' expenditure ($C^{\dagger}_t$), itself about 90% of total consumers' expenditure during those periods; the sets of estimated parameters are closely similar; and the errors are approximately white noise and homoscedastic in both periods. A "representative" equation for 1922-38 is:

$$\text{(E1)}\quad \Delta_1 \hat{c}^{\dagger}_t = \underset{[.06]}{.50}\Delta_1 i^{\dagger}_t - \underset{[.026]}{.080}(c^{\dagger}-i^{\dagger})_{t-1} - \underset{[.027]}{.053}\Delta_1 p^{\dagger}_t$$

$$T = 17 \quad R^2 = .81 \quad \hat{\sigma} = .53\% \quad \eta_1(4,10) = 1.1 \quad \eta_2(2,12) = .4$$

$$\xi_4(1) = .3 \quad \xi_5(2) = 4.5 \quad SK = -1.0 \quad EK = 1.9 \quad \eta_6(4,10) = .6$$

where $I^{\dagger}_t$ is real personal disposable income in 1938 prices; $P^{\dagger}_t$ is the implicit deflator of $C^{\dagger}_t$; and $i^{\dagger}_t$, $p^{\dagger}_t$, and $c^{\dagger}_t$ are the logs of those variables. That is a remarkably constant relationship, as the equivalent post-war estimates (1954-1980) are:

$$\text{(E2)}\quad \Delta_1 \hat{c}^{\dagger}_t = \underset{[.04]}{.48}\Delta_1 i^{\dagger}_t - \underset{[.02]}{.12}(c^{\dagger}-i^{\dagger})_{t-1} - \underset{[.03]}{.16}\Delta_1 p^{\dagger}_t$$

$$T = 27 \quad R^2 = .90 \quad \hat{\sigma} = .52\% \quad \eta_1(4,20) = 1.0 \quad \eta_2(2,22) = .2$$

$$\xi_4(1) = 3.6 \quad \xi_5(2) = 1.1 \quad SK = .5 \quad EK = -.2 \quad \eta_6(5,19) = .8 \,.$$

See Hendry (1983, equations (13) and (9)).

We consider a "debate" about the relative constancy of these relationships to be sterile: expenditure requires a transactions medium; and as Alfred Marshall (1926, pp. 38-39) remarks, narrow money <u>demand</u> and expenditure profiles are essentially mirror images. We see no immediate implications for policy from the equations for consumers' expenditure and money demand. For example, (E1) is compatible with large changes in the percentage of income consumed, as actually occurred, with $C^{\dagger}_t/I^{\dagger}_t$ falling substantially in both periods. Thus, even if the equation itself does not shift, demand management may be required to stabilise net output.

APPENDIX F. Corrections and extensions to Friedman and Schwartz's data.

Table F.I. Data series from Table 5.10 for phase numbers 16 through 22, corrected

Table 5.10 Rates of Change Computed from Triplets of Reference Phase Averages (Annual Percentage): United Kingdom, 1870-1975

Central Phase of Triplet: Number	Central Phase of Triplet: Midpoint	Per Capita Real Money Balances g_m	Short-Term Interest Rate DR_S	Long-Term Interest Rate DR_L	High-Powered Money g_H	Money Stock per Unit of Output $G_{M/y'}$	Weight g_w
16	1923.0	3.757	-0.296	-0.162	-2.925	-5.462	23.8
17	1925.5	1.028	0.296	0.003	-1.617	-4.702	22.6
18	1927.0	2.255	0.080	0.031	-1.372	-2.155	7.4
19	1928.0	2.477	0.170	-0.010	-1.294	-2.179	4.0
20	1929.0	1.614	-0.407	-0.050	-1.134	0.839	12.8
21	1931.0	3.146	-0.641	-0.256	1.817	0.494	67.1
22	1935.0	2.880	-0.434	-0.189	3.293	0.155	69.3

Table F.II. Data for 1975-1982[33]

Year	Money Stock (£m) M	Nominal Income (£m) I·P	Implicit Price Deflator (1929=100.) P	Population (millions) N	Short-Term Interest Rate (% p.a.) RS	Long-Term Interest Rate (% p.a.) Rℓ	Exchange Rate ($/£) E
1975	36,480.	83,573.	763.7	55.900	10.62	14.66	2.2200
1976	42,572.	97,447.	870.6	55.886	11.19	14.25	1.8049
1977	46,731.	111,915.	975.2	55.852	7.90	12.31	1.7455
1978	52,624.	129,106.	1085.9	55.836	8.97	11.92	1.9197
1979	59,861.	148,566.	1228.0	55.881	13.43	11.38	2.1223
1980	69,329.	169,352.	1454.8	55.945	15.87	11.86	2.3281
1981	80,306.	182,959.	1612.9	56.252	13.16	13.00	2.0254
1982	89,256.	197,003.	1746.3	n/a	11.57	11.91	1.7489

[33] We are indebted to John Trundle of the Bank of England for providing us with these data.

APPENDIX G. Estimation of equation (11)

Table G.I records the estimates pertinent to (11) for $\ell = 5$ for most of the variables in FS's Table 4.9 (i.e., for those other than population). For m, i, p, and e, the null hypothesis of Granger non-causality is rejected at the 5% level. In no case is there significant autoregressive conditional heteroscedasticity (i.e., ARCH; ξ_4), or (with the exception of the long-term interest rate $R\ell$ and high-powered money H) parameter non-constancy over the decade 1961-70 (η_1) or residual autocorrelation (ξ_2). In several cases, however, there is evidence of significant outliers, casting some doubt on the assumption of normality, although dummy variables for each year of the two world wars would remove most such cases (see Table G.III). Overall, therefore, the evidence is consistent with (11).

Table G.II reports related estimates when H is replaced by the US price level P*. Some interesting differences result, unsurprisingly, as P* has rather different time-series behaviour from H. Note that, whether or not war years are included, P* is not Granger-caused by the UK variables and so can be treated as strictly exogenous (cf. Tables G.II and G.IV). Also, M is no longer Granger-caused on this information set, whereas the exchange rate E is the variable most significantly affected by lags of the other variables. Otherwise, results similar to those in Table G.I emerge.

In all four tables, $\eta_3(5,\cdot)$ is the F statistic for testing for the exclusion of $m_{t-1},\ldots,m_{t-5}$ from the reported equation. $\eta_3(30,\cdot)$ is the F statistic for testing for the exclusion of all variables except lags of the dependent variable, dummies, and the constant term from the equation (i.e., testing for Granger non-causality). The 5% significance points for F(5,54), F(30,54), F(5,42), and F(30,42) are 2.38, 1.67, 2.44, and 1.73 respectively. The estimation period is 1878-1970 inclusive (T=93).

Table G.I. Estimates of equation (11) with h: observations for WWI and WWII included

regressor \ dependent variable	m_t	h_t	i_t	p_t	RS_t	$R\ell_t$	e_t
m_{t-1}	1.38 (.19)	.61 (.31)	.42 (.29)	-.19 (.26)	.02 (.08)	-.008 (.024)	.74 (.46)
m_{t-2}	-.73 (.30)	-.36 (.48)	-.55 (.45)	-.22 (.40)	-.22 (.13)	-.035 (.038)	-.31 (.73)
m_{t-3}	.24 (.32)	-.13 (.53)	.48 (.49)	.61 (.44)	.37 (.14)	.095 (.041)	-.87 (.80)
m_{t-4}	.09 (.29)	.53 (.48)	-.98 (.44)	-.15 (.40)	-.25 (.13)	-.071 (.037)	.64 (.72)
m_{t-5}	-.10 (.15)	-.24 (.24)	.63 (.22)	-.04 (.20)	.06 (.06)	.010 (.019)	.07 (.37)
h_{t-1}	.18 (.09)	.73 (.15)	-.21 (.14)	.49 (.12)	.02 (.04)	.006 (.012)	-.36 (.22)
h_{t-2}	.02 (.12)	.05 (.19)	.01 (.18)	-.01 (.16)	.05 (.05)	.003 (.015)	.32 (.29)
h_{t-3}	.08 (.12)	-.12 (.20)	.05 (.18)	-.13 (.16)	-.09 (.05)	-.017 (.015)	.08 (.30)
h_{t-4}	.03 (.12)	-.06 (.20)	.35 (.18)	.01 (.17)	-.01 (.05)	-.010 (.016)	-.07 (.30)
h_{t-5}	-.01 (.10)	-.16 (.16)	-.29 (.14)	.10 (.13)	.03 (.04)	.016 (.012)	-.34 (.24)
i_{t-1}	.02 (.09)	.20 (.14)	.80 (.13)	.04 (.12)	.07 (.04)	.022 (.011)	.24 (.22)
i_{t-2}	.09 (.12)	.04 (.19)	-.06 (.18)	.05 (.16)	-.02 (.05)	-.009 (.015)	-.50 (.29)
i_{t-3}	-.07 (.11)	-.16 (.18)	-.08 (.17)	-.28 (.15)	-.11 (.05)	-.023 (.014)	.15 (.28)
i_{t-4}	-.05 (.12)	-.11 (.19)	.17 (.17)	-.13 (.16)	.10 (.05)	.004 (.015)	.14 (.28)
i_{t-5}	.00 (.09)	-.00 (.15)	.22 (.14)	.12 (.13)	-.02 (.04)	.017 (.012)	-.07 (.23)
p_{t-1}	-.22 (.11)	.09 (.17)	-.14 (.16)	.96 (.15)	.05 (.05)	.024 (.014)	-.06 (.26)
p_{t-2}	.16 (.15)	.29 (.25)	.14 (.23)	-.40 (.21)	.00 (.07)	-.008 (.019)	-.38 (.38)
p_{t-3}	-.30 (.15)	-.22 (.25)	-.34 (.23)	-.12 (.21)	-.06 (.07)	-.004 (.020)	.41 (.38)
p_{t-4}	-.09 (.15)	-.04 (.25)	.40 (.23)	-.26 (.21)	.07 (.07)	-.004 (.020)	.23 (.38)
p_{t-5}	.04 (.10)	.21 (.16)	-.01 (.15)	.02 (.14)	-.02 (.04)	.011 (.013)	-.09 (.25)
RS_{t-1}	.11 (.42)	-.34 (.68)	-.68 (.63)	-.30 (.57)	.61 (.18)	-.050 (.053)	.04 (1.03)
RS_{t-2}	-.25 (.47)	-.42 (.77)	-1.43 (.71)	.24 (.64)	-.19 (.20)	.059 (.060)	3.80 (1.16)
RS_{t-3}	-.14 (.53)	.19 (.86)	.43 (.80)	-.88 (.72)	.37 (.23)	.015 (.068)	-2.70 (1.30)
RS_{t-4}	.09 (.57)	-.03 (.93)	-.69 (.86)	.58 (.78)	-.23 (.25)	.046 (.073)	.72 (1.41)
RS_{t-5}	-.25 (.47)	-.75 (.77)	-.56 (.71)	-1.14 (.64)	-.20 (.20)	-.154 (.060)	.24 (1.16)
$R\ell_{t-1}$	-.94 (1.24)	-3.15 (2.02)	.76 (1.88)	4.61 (1.69)	.62 (.53)	1.255 (.159)	-3.73 (3.06)
$R\ell_{t-2}$	4.17 (1.98)	3.35 (3.23)	3.31 (3.00)	-1.04 (2.70)	-.48 (.85)	-.864 (.253)	-2.46 (4.88)
$R\ell_{t-3}$	-.40 (2.12)	-1.92 (3.46)	-1.64 (3.21)	6.07 (2.89)	-.22 (.91)	.541 (.271)	.27 (5.23)
$R\ell_{t-4}$	2.62 (2.21)	2.65 (3.60)	-.85 (3.35)	1.76 (3.01)	1.43 (.95)	.050 (.283)	-.93 (5.45)
$R\ell_{t-5}$	-.93 (1.51)	-1.10 (2.47)	.84 (2.29)	-3.10 (2.06)	-.92 (.65)	-.055 (.193)	4.87 (3.73)
e_{t-1}	-.04 (.06)	-.09 (.09)	-.07 (.09)	.02 (.08)	.00 (.03)	-.003 (.007)	.77 (.14)
e_{t-2}	-.01 (.07)	.05 (.12)	.02 (.11)	-.08 (.10)	.00 (.03)	.002 (.009)	-.27 (.18)
e_{t-3}	-.02 (.07)	.05 (.12)	-.01 (.11)	.13 (.10)	.04 (.03)	.020 (.009)	.09 (.18)
e_{t-4}	-.10 (.06)	-.06 (.10)	-.08 (.09)	-.13 (.08)	-.01 (.03)	-.004 (.008)	.11 (.15)
e_{t-5}	.05 (.05)	.07 (.08)	-.04 (.07)	-.02 (.07)	-.02 (.02)	-.007 (.006)	-.05 (.12)
D_1	2.34 (1.50)	16.03 (2.45)	5.49 (2.28)	-2.89 (2.05)	-.79 (.65)	.037 (.192)	.66 (3.71)
D_2	-1.03 (2.07)	-4.64 (3.38)	.04 (3.14)	-6.69 (2.82)	-.06 (.89)	-.067 (.265)	3.48 (5.11)
D_3	4.46 (1.42)	7.26 (2.31)	2.16 (2.15)	.70 (1.94)	-1.16 (.61)	-.334 (.182)	.37 (3.50)
Constant	-.97 (.27)	.74 (.44)	.46 (.41)	-1.48 (.37)	.03 (.12)	-.008 (.034)	1.18 (.66)
T	93	93	93	93	93	93	93
R^2	.99982	.99954	.99772	.99910	.88081	.98234	.97765
$100\hat{\sigma}$	1.872	3.052	2.833	2.551	.8066	.2393	4.613
$\eta_1(10,44)$	.34	.73	.34	.94	.53	4.73	1.41
$\xi_2(7)$	5.5	18.0	8.8	14.9	8.2	21.4	13.8
$\xi_4(1)$	.6	2.3	2.9	.4	.3	.7	.9
$\eta_3(5,54)$	16.83	1.43	2.11	1.21	1.44	1.39	1.15
$\eta_3(30,54)$	1.90	1.51	2.42	4.64	1.46	1.57	2.92

Table G.II. Estimates of equation (11) with p*: observations for WWI and WWII included

dependent variable / regressor	m_t	p_t^*	i_t	p_t	RS_t	$R\ell_t$	e_t
m_{t-1}	1.65 (.16)	.50 (.33)	.48 (.23)	.43 (.25)	-.05 (.06)	-.027 (.019)	.51 (.37)
m_{t-2}	-.68 (.29)	-.26 (.60)	-.85 (.41)	-.14 (.44)	-.05 (.11)	-.001 (.034)	-.04 (.67)
m_{t-3}	.28 (.31)	.59 (.63)	.70 (.44)	.21 (.47)	.20 (.12)	.061 (.036)	-.75 (.71)
m_{t-4}	.04 (.28)	-.82 (.58)	-.89 (.40)	-.03 (.43)	-.20 (.11)	-.067 (.033)	.31 (.65)
m_{t-5}	-.06 (.14)	.48 (.29)	.48 (.20)	.04 (.22)	.09 (.05)	.023 (.017)	.11 (.33)
p_{t-1}^*	-.01 (.09)	1.05 (.18)	.14 (.13)	-.03 (.14)	.00 (.03)	.006 (.010)	.41 (.21)
p_{t-2}^*	.10 (.12)	.08 (.25)	-.13 (.17)	.06 (.18)	.05 (.05)	.003 (.014)	-.27 (.28)
p_{t-3}^*	-.01 (.12)	-.09 (.25)	-.41 (.18)	.14 (.19)	.06 (.05)	.017 (.014)	-.44 (.29)
p_{t-4}^*	-.07 (.12)	-.25 (.23)	.20 (.16)	-.20 (.17)	-.10 (.04)	-.022 (.013)	.18 (.26)
p_{t-5}^*	-.09 (.10)	-.10 (.21)	.20 (.15)	-.05 (.16)	-.01 (.04)	-.002 (.012)	.06 (.24)
i_{t-1}	.00 (.11)	-.04 (.22)	.64 (.15)	-.03 (.16)	.08 (.04)	.024 (.012)	-.12 (.24)
i_{t-2}	-.07 (.14)	-.28 (.28)	-.02 (.20)	-.05 (.21)	-.04 (.05)	-.008 (.016)	-.57 (.32)
i_{t-3}	-.08 (.13)	-.39 (.26)	.04 (.18)	-.29 (.20)	-.12 (.05)	-.027 (.015)	.44 (.29)
i_{t-4}	.00 (.13)	.02 (.26)	.18 (.18)	-.08 (.19)	.13 (.05)	.008 (.015)	.31 (.29)
i_{t-5}	-.03 (.10)	.26 (.21)	.23 (.14)	.02 (.15)	-.03 (.04)	.015 (.012)	-.20 (.23)
p_{t-1}	-.10 (.14)	-.14 (.28)	-.37 (.19)	1.11 (.21)	.08 (.05)	.023 (.016)	-.51 (.31)
p_{t-2}	.01 (.19)	-.36 (.40)	.34 (.27)	-.57 (.29)	-.09 (.07)	-.017 (.023)	-.19 (.44)
p_{t-3}	-.28 (.19)	.00 (.40)	.04 (.28)	-.14 (.30)	-.07 (.07)	-.014 (.023)	.69 (.45)
p_{t-4}	.01 (.19)	.10 (.38)	.08 (.26)	-.00 (.28)	.14 (.07)	.016 (.022)	-.02 (.43)
p_{t-5}	.10 (.12)	.04 (.24)	-.02 (.17)	.02 (.18)	-.05 (.04)	.003 (.014)	-.21 (.27)
RS_{t-1}	-.12 (.45)	-.04 (.92)	.13 (.63)	-.93 (.68)	.46 (.17)	-.094 (.052)	1.18 (1.03)
RS_{t-2}	-.16 (.49)	1.21 (1.01)	-1.65 (.70)	.50 (.75)	-.18 (.19)	.060 (.057)	2.58 (1.13)
RS_{t-3}	-.36 (.54)	-1.04 (1.11)	-.05 (.77)	-1.14 (.83)	.47 (.20)	.042 (.063)	-3.17 (1.24)
RS_{t-4}	-.21 (.62)	-.43 (1.28)	-1.12 (.88)	.30 (.95)	-.34 (.24)	.034 (.073)	-.18 (1.43)
RS_{t-5}	-.31 (.51)	-1.08 (1.04)	.15 (.72)	-1.17 (.78)	-.24 (.19)	-.176 (.059)	1.64 (1.17)
$R\ell_{t-1}$	-1.13 (1.34)	1.40 (2.73)	1.14 (1.89)	4.85 (2.03)	.38 (.50)	1.207 (.156)	-5.51 (3.07)
$R\ell_{t-2}$	4.89 (2.17)	-1.47 (4.43)	3.74 (3.07)	-.52 (3.30)	-.47 (.82)	-.872 (.252)	2.11 (4.97)
$R\ell_{t-3}$	.29 (2.27)	8.94 (4.64)	-.89 (3.22)	6.26 (3.45)	-.12 (.86)	.539 (.264)	1.77 (5.21)
$R\ell_{t-4}$	2.36 (2.35)	-3.70 (4.79)	-.70 (3.32)	1.08 (3.56)	1.53 (.89)	.068 (.273)	-.57 (5.38)
$R\ell_{t-5}$	-.73 (1.72)	2.40 (3.51)	-2.60 (2.43)	-2.32 (2.61)	-.09 (.65)	.163 (.200)	3.03 (3.94)
e_{t-1}	-.04 (.06)	-.08 (.13)	-.03 (.09)	.03 (.10)	-.03 (.02)	-.011 (.007)	.73 (.15)
e_{t-2}	.01 (.08)	-.12 (.16)	.11 (.11)	-.06 (.12)	.01 (.03)	.002 (.009)	-.15 (.18)
e_{t-3}	-.05 (.08)	.14 (.16)	.03 (.11)	.05 (.12)	.01 (.03)	.018 (.009)	.06 (.17)
e_{t-4}	-.07 (.07)	-.07 (.13)	-.07 (.09)	-.08 (.10)	.01 (.02)	.000 (.008)	.11 (.15)
e_{t-5}	.09 (.05)	.08 (.11)	-.15 (.08)	.04 (.08)	-.01 (.02)	-.004 (.006)	-.16 (.12)
D_1	4.53 (1.55)	6.56 (3.15)	1.65 (2.19)	2.57 (2.35)	.15 (.58)	.210 (.180)	.46 (3.54)
D_2	-5.97 (2.10)	-12.93 (4.28)	1.71 (2.97)	-14.31 (3.19)	-.60 (.79)	-.139 (.244)	2.24 (4.80)
D_3	4.85 (1.44)	3.29 (2.93)	-.40 (2.03)	2.98 (2.18)	-.55 (.54)	-.223 (.167)	.61 (3.29)
Constant	-.34 (.17)	-.38 (.34)	.30 (.24)	-.59 (.26)	-.00 (.06)	-.019 (.020)	.59 (.39)
T	93	93	93	93	93	93	93
R^2	.99980	.99627	.99773	.99872	.89592	.98339	.97801
$100\hat{\sigma}$	1.997	4.076	2.826	3.034	.7538	.2322	4.576
$\eta_1(10,44)$	.47	.43	.32	1.19	.41	4.25	1.59
$\xi_2(7)$	9.4	8.2	12.4	10.0	7.1	17.5	11.4
$\xi_4(1)$	1.2	.3	1.4	.0	.7	.5	.1
$\eta_3(5,54)$	65.89	2.01	1.92	3.24	1.61	1.96	1.17
$\eta_3(30,54)$	1.45	1.58	2.44	2.76	1.93	1.77	3.00

Table G.III. Estimates of equation (11) with h: observations for WWI and WWII excluded (i.e., 1914-1919, 1939-1946)

regressor \ dependent variable	m_t	h_t	i_t	p_t	RS_t	$R\ell_t$	e_t
m_{t-1}	1.29 (.19)	.53 (.28)	.26 (.33)	-.04 (.29)	.11 (.11)	.031 (.029)	1.52 (.58)
m_{t-2}	-.83 (.30)	-.67 (.44)	-.52 (.51)	-.33 (.45)	-.30 (.16)	-.082 (.045)	-1.79 (.90)
m_{t-3}	.62 (.31)	.54 (.46)	.64 (.54)	.80 (.47)	.37 (.17)	.107 (.047)	.44 (.94)
m_{t-4}	-.15 (.28)	-.30 (.41)	-.92 (.48)	-.42 (.42)	-.23 (.15)	-.070 (.043)	-.30 (.85)
m_{t-5}	-.05 (.14)	.16 (.21)	.54 (.25)	.03 (.22)	.07 (.08)	.012 (.022)	.43 (.43)
h_{t-1}	.16 (.12)	.72 (.18)	-.39 (.21)	.60 (.18)	-.03 (.07)	-.009 (.018)	-.95 (.36)
h_{t-2}	.19 (.17)	.13 (.25)	.33 (.29)	-.03 (.25)	.04 (.09)	.002 (.026)	1.12 (.51)
h_{t-3}	-.17 (.18)	-.28 (.26)	.07 (.30)	-.76 (.26)	-.08 (.10)	-.008 (.026)	-.52 (.52)
h_{t-4}	.18 (.17)	.09 (.25)	.14 (.29)	.56 (.25)	.04 (.09)	-.004 (.025)	.16 (.50)
h_{t-5}	.03 (.12)	-.19 (.17)	-.16 (.20)	-.04 (.18)	-.02 (.07)	.002 (.018)	-.47 (.36)
i_{t-1}	-.04 (.09)	.01 (.13)	.78 (.15)	.08 (.13)	.08 (.05)	.021 (.013)	.22 (.26)
i_{t-2}	.05 (.11)	.16 (.16)	-.06 (.19)	-.08 (.16)	-.03 (.06)	-.003 (.017)	-.48 (.33)
i_{t-3}	-.03 (.10)	-.10 (.15)	-.04 (.18)	-.14 (.15)	-.12 (.06)	-.027 (.016)	.27 (.31)
i_{t-4}	.01 (.11)	-.06 (.15)	.17 (.18)	-.14 (.16)	.11 (.06)	.006 (.016)	.12 (.32)
i_{t-5}	-.04 (.09)	.07 (.13)	.18 (.15)	.09 (.13)	-.02 (.05)	.014 (.013)	-.07 (.26)
p_{t-1}	-.26 (.11)	.08 (.16)	-.27 (.19)	.99 (.16)	.09 (.06)	.027 (.016)	-.24 (.33)
p_{t-2}	.21 (.15)	.22 (.22)	.31 (.26)	-.23 (.23)	-.01 (.08)	-.004 (.023)	.16 (.46)
p_{t-3}	-.43 (.15)	-.07 (.22)	-.42 (.26)	-.35 (.23)	-.09 (.08)	-.001 (.023)	.18 (.45)
p_{t-4}	-.03 (.15)	.07 (.21)	.28 (.25)	.05 (.22)	.08 (.08)	-.005 (.022)	.56 (.44)
p_{t-5}	-.05 (.09)	.16 (.13)	.01 (.15)	-.06 (.13)	-.02 (.05)	.018 (.014)	-.09 (.27)
RS_{t-1}	.07 (.36)	-.51 (.53)	-.71 (.62)	-.21 (.54)	.56 (.20)	-.055 (.055)	-.04 (1.08)
RS_{t-2}	-.52 (.40)	-.70 (.59)	-1.64 (.69)	.09 (.60)	-.10 (.22)	.093 (.061)	4.01 (1.20)
RS_{t-3}	-.13 (.50)	-.08 (.73)	-.55 (.86)	-.30 (.75)	.31 (.27)	-.024 (.076)	-4.12 (1.50)
RS_{t-4}	-.38 (.53)	-.15 (.77)	-.34 (.90)	.26 (.78)	-.25 (.29)	.074 (.079)	2.30 (1.57)
RS_{t-5}	.06 (.43)	-.84 (.62)	-.72 (.73)	-1.05 (.64)	-.21 (.23)	-.159 (.065)	-.61 (1.28)
$R\ell_{t-1}$	-.11 (1.14)	-1.54 (1.66)	2.72 (1.94)	2.91 (1.69)	.51 (.62)	1.303 (.172)	-.78 (3.41)
$R\ell_{t-2}$	4.56 (1.97)	.96 (2.87)	.88 (3.35)	.78 (2.92)	-.54 (1.07)	-1.111 (.297)	-12.31 (5.89)
$R\ell_{t-3}$	-.07 (2.05)	-1.72 (2.98)	3.67 (3.49)	3.36 (3.04)	-.06 (1.12)	.678 (.309)	8.26 (6.13)
$R\ell_{t-4}$	4.00 (2.13)	1.80 (3.11)	-2.49 (3.64)	1.52 (3.17)	1.50 (1.16)	-.046 (.323)	-9.49 (6.39)
$R\ell_{t-5}$	-1.98 (1.41)	-.36 (2.05)	.06 (2.40)	-2.51 (2.09)	-1.01 (.77)	-.046 (.213)	8.35 (4.22)
e_{t-1}	-.01 (.05)	-.10 (.08)	.01 (.09)	.06 (.08)	.01 (.03)	-.003 (.008)	.87 (.16)
e_{t-2}	.02 (.07)	.03 (.10)	-.02 (.12)	-.08 (.10)	.01 (.04)	.002 (.010)	-.46 (.21)
e_{t-3}	-.05 (.07)	.12 (.10)	.02 (.12)	.15 (.11)	.02 (.04)	.015 (.011)	.21 (.21)
e_{t-4}	-.11 (.06)	-.07 (.08)	-.09 (.09)	-.18 (.08)	.00 (.03)	.004 (.008)	.17 (.16)
e_{t-5}	.01 (.05)	.04 (.07)	-.09 (.08)	.06 (.07)	-.01 (.02)	-.007 (.007)	-.06 (.14)
D_2	-2.29 (1.91)	-2.48 (2.78)	-1.22 (3.25)	-7.13 (2.83)	-.61 (1.04)	-.049 (.288)	4.82 (5.71)
Constant	-1.19 (.27)	.74 (.39)	.08 (.45)	-1.16 (.39)	.11 (.14)	.029 (.040)	1.86 (.79)
T	93	93	93	93	93	93	93
R^2	.99991	.99981	.99848	.99944	.90007	.98709	.98273
$100\hat{\sigma}$	1.535	2.237	2.617	2.280	.8375	.2321	4.598
$\eta_1(10,32)$	.43	.75	.39	.30	.30	4.33	1.77
$\xi_2(7)$	11.4	8.0	6.1	9.7	8.8	24.0	7.7
$\xi_4(1)$	.0	4.0	5.8	2.1	.3	1.5	.9
$\eta_3(5,42)$	18.69	1.10	1.05	1.00	.98	1.20	1.78
$\eta_3(30,42)$	2.02	1.62	1.86	4.23	1.42	1.71	2.85

Table G.IV. Estimates of equation (11) with p*: observations for WWI and WWII excluded (i.e., 1914-1919, 1939-1946)

dependent variable / regressor	m_t	p_t^*	i_t	p_t	RS_t	$R\ell_t$	e_t
m_{t-1}	1.57 (.17)	.47 (.38)	.26 (.25)	.48 (.26)	.00 (.08)	.005 (.022)	.95 (.46)
m_{t-2}	-.64 (.29)	-.18 (.66)	-.54 (.44)	-.13 (.46)	-.11 (.13)	-.049 (.038)	-.83 (.80)
m_{t-3}	.30 (.30)	.19 (.69)	.79 (.46)	-.11 (.49)	.20 (.14)	.089 (.040)	-.12 (.84)
m_{t-4}	.03 (.27)	-.17 (.62)	-1.03 (.41)	.26 (.43)	-.13 (.13)	-.065 (.036)	-.11 (.75)
m_{t-5}	-.03 (.13)	.23 (.30)	.47 (.20)	-.03 (.21)	.04 (.06)	.013 (.017)	.20 (.37)
p_{t-1}^*	-.07 (.10)	1.13 (.22)	.28 (.14)	.02 (.15)	.02 (.04)	.019 (.012)	.72 (.26)
p_{t-2}^*	.20 (.12)	.26 (.27)	-.34 (.18)	.24 (.19)	.04 (.05)	-.001 (.015)	-.47 (.33)
p_{t-3}^*	-.06 (.13)	-.22 (.31)	-.27 (.20)	-.05 (.21)	.03 (.06)	-.002 (.018)	-.60 (.37)
p_{t-4}^*	-.05 (.11)	-.35 (.26)	.22 (.17)	-.24 (.18)	-.08 (.05)	-.010 (.015)	.25 (.31)
p_{t-5}^*	-.10 (.09)	-.03 (.21)	.16 (.14)	-.02 (.15)	-.01 (.04)	-.004 (.012)	.10 (.26)
i_{t-1}	-.02 (.10)	-.18 (.23)	.66 (.15)	-.08 (.16)	.08 (.05)	.018 (.013)	-.25 (.27)
i_{t-2}	-.11 (.13)	-.35 (.30)	.04 (.20)	-.06 (.21)	-.05 (.06)	-.003 (.017)	-.39 (.36)
i_{t-3}	-.03 (.12)	-.08 (.27)	-.04 (.18)	-.09 (.19)	-.12 (.06)	-.021 (.016)	.56 (.33)
i_{t-4}	.03 (.12)	.03 (.26)	.20 (.17)	-.10 (.18)	.14 (.05)	.008 (.015)	.30 (.32)
i_{t-5}	-.06 (.10)	.12 (.23)	.19 (.15)	-.06 (.16)	-.06 (.05)	.005 (.013)	-.36 (.28)
p_{t-1}	-.06 (.15)	-.37 (.33)	-.57 (.22)	.98 (.23)	.06 (.07)	.003 (.019)	-1.06 (.40)
p_{t-2}	-.09 (.19)	-.34 (.43)	.71 (.29)	-.64 (.30)	-.07 (.09)	-.004 (.025)	.41 (.53)
p_{t-3}	-.19 (.20)	.17 (.46)	-.27 (.30)	.18 (.32)	-.08 (.09)	-.001 (.026)	.71 (.56)
p_{t-4}	-.03 (.19)	.12 (.42)	.03 (.28)	-.03 (.30)	.12 (.09)	-.002 (.024)	-.07 (.51)
p_{t-5}	.06 (.11)	-.10 (.25)	.06 (.16)	-.05 (.17)	-.05 (.05)	.011 (.014)	-.22 (.30)
RS_{t-1}	-.22 (.40)	-.29 (.91)	-.16 (.60)	-.74 (.64)	.54 (.19)	-.057 (.052)	1.21 (1.10)
RS_{t-2}	-.26 (.43)	1.21 (.99)	-1.82 (.65)	.46 (.69)	-.15 (.20)	.066 (.057)	2.90 (1.20)
RS_{t-3}	-.06 (.51)	-1.53 (1.17)	-.48 (.77)	-.77 (.82)	.41 (.24)	.002 (.068)	-3.74 (1.42)
RS_{t-4}	-.84 (.58)	-1.45 (1.33)	-.65 (.88)	-.21 (.93)	-.38 (.27)	.060 (.077)	.63 (1.61)
RS_{t-5}	.01 (.50)	-.42 (1.15)	-.09 (.76)	-.60 (.80)	-.12 (.23)	-.124 (.066)	2.30 (1.39)
$R\ell_{t-1}$	-.77 (1.28)	.71 (2.90)	2.87 (1.91)	2.30 (2.04)	.05 (.59)	1.184 (.167)	-4.01 (3.52)
$R\ell_{t-2}$	4.74 (2.14)	-.88 (4.86)	2.49 (3.21)	.29 (3.41)	.08 (.99)	-.936 (.280)	-3.58 (5.90)
$R\ell_{t-3}$	-.69 (2.17)	9.58 (4.94)	2.52 (3.26)	3.72 (3.47)	-.14(1.01)	.674 (.285)	5.85 (6.00)
$R\ell_{t-4}$	4.29 (2.25)	-1.44 (5.11)	-2.16 (3.37)	2.64 (3.59)	1.90(1.04)	-.001 (.295)	-4.64 (6.20)
$R\ell_{t-5}$	-1.36 (1.63)	1.98 (3.70)	-3.18 (2.44)	-1.03 (2.60)	-.48 (.76)	.100 (.214)	4.45 (4.50)
e_{t-1}	-.05 (.06)	-.06 (.14)	-.01 (.09)	-.03 (.10)	-.03 (.03)	-.013 (.008)	.69 (.17)
e_{t-2}	.07 (.07)	-.07 (.17)	.12 (.11)	-.04 (.12)	.01 (.03)	.002 (.010)	-.17 (.20)
e_{t-3}	-.10 (.07)	.06 (.17)	.03 (.11)	.00 (.12)	-.00 (.03)	.011 (.010)	-.00 (.20)
e_{t-4}	-.04 (.06)	.00 (.14)	-.13 (.09)	.01 (.10)	.02 (.03)	.007 (.008)	.20 (.17)
e_{t-5}	.05 (.05)	.05 (.11)	-.13 (.07)	.04 (.08)	-.01 (.02)	-.008 (.007)	-.21 (.14)
D_2	-6.74 (1.93)	-16.81 (4.38)	.67 (2.89)	-14.37 (3.07)	-1.02 (.90)	-.087 (.253)	4.32 (5.32)
Constant	-.39 (.16)	-.66 (.36)	.18 (.24)	-.52 (.25)	.00 (.07)	.000 (.021)	.91 (.44)
T	93	93	93	93	93	93	93
R^2	.99989	.99746	.99860	.99923	.91331	.98838	.98247
$100\hat{\sigma}$	1.678	3.817	2.518	2.678	.7800	.2201	4.632
$\eta_1(10,32)$	.43	.47	.35	.73	.43	5.37	1.59
$\xi_2(7)$	12.7	13.5	9.5	10.7	4.3	22.9	12.4
$\xi_4(1)$	.9	.5	7.2	1.2	.6	5.4	.0
$\eta_3(5,42)$	53.55	1.64	1.47	2.55	.58	1.16	1.20
$\eta_3(30,42)$	1.47	1.69	2.12	2.68	1.85	2.06	2.79

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