Quantitative Description of Robot-Environment Interaction Using Chaos Theory[1]

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Abstract. Any theory (such as, for example a theory of robot-environment interaction) is dependent upon quantitative descriptions of behaviour. In this paper we present experiments on the application of chaos theory to describe a robot's behaviour quantitatively.
Computing the Lyapunov exponent of robot trajectories observed in a number of experiments, we find that a change in task influences robot behaviour far more noticeably than a change in environment.

1 Introduction

1.1 Motivation

Research in mobile robotics to date has, with very few exceptions, been based on trial-and-error experimentation and the presentation of existence proofs. Task- achieving robot control programs are obtained through a process of iterative refinement, typically involving the use of computer models of the robot, the robot itself, and program refinements based on observations made using the model and the robot. This process is iterated until the robot's behaviour resembles the desired behaviour to a sufficient degree of accuracy. Typically, the results of these iterative refinement processes are valid within a very narrow band of application scenarios, they constitute "existence proofs". As such, they demonstrate that a particular behaviour can be achieved, but not how that particular behaviour can in general be achieved for any experimental scenario.

1.2 A Theory of Robot-Environment Interaction

The operation of an autonomous mobile robot — robot-environment interaction — is governed by three major components: i) the robot itself, its sensors, actuators and hardware morphology in general, ii) the environment, its perceptual properties, environmental conditions *etc.*, and iii) the task, usually the control program being executed by the robot. The behaviour of the robot emerges through the interaction of these three aspects (see figure 1).

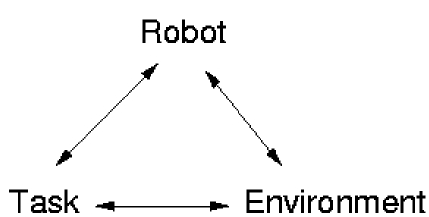

Fig. 1. The fundamental triangle of robot-environment interaction.

Any theory of robot-environment interaction, i.e. any coherent body of hypothetical, conceptual and pragmatic generalisations and principles that forms the general frame of reference within which mobile robotics research is conducted, will be fundamentally dependent upon quantitative descriptions of the robot's

[1] Proc. ECMR 03, Warsaw 2003

behaviour. While we investigate a general theory of robot-environment interaction in ongoing work at Essex and Point Loma, the purpose of this paper is to introduce the idea that chaos theory can be applied to describe a robot's behaviour quantitatively.

1.3 Quantitative Analysis of Robot-Environment Interaction

An autonomous mobile robot, interacting with its environment through some control program, can be viewed as an analog computer. Similar to an optical lens that can be used to compute the Fourier spectrum of an input image, a mobile robot "computes" behaviour (the output) from environmental conditions, the control program and the robot's morphology (the inputs). This is shown diagrammatically in figure 2.

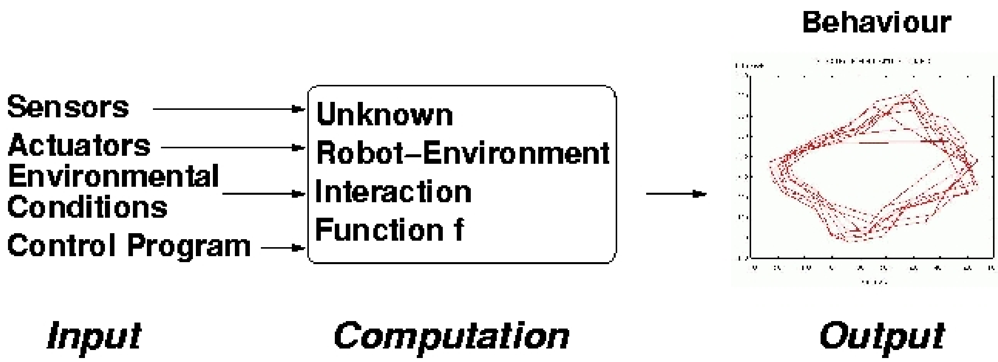

Fig. 2. A mobile robot interacting with its environment can be described as an analog computer, taking environmental, morphological and task-related data as input, and "computing" behaviour as output (see also figure 1).

As a first approximation, we argue that the robot's *trajectory* encapsulates the important aspects of the robot's behaviour. In this paper, we therefore focus on the application of dynamical systems theory (chaos theory) to the analysis of robot trajectories.

The Lyapunov Exponent One of the most distinctive characteristics of a chaotic system is its sensitivity to a variation in the system's variables: Two trajectories in phase space that started close each other will diverge from one another as time progresses, the more chaotic the system, the greater the divergence.

Consider some state of a deterministic dynamical system and its corresponding location S_o in phase space. As time progresses the state of the system follows a deterministic trajectory in phase space. Let another state S_1 of the system lie arbitrarily close to S_o, and follow a different trajectory, again fully deterministic. If d_o is the initial separation of these two states in phase space at time $t = 0$, then their separation d_t after t time steps can be expressed as $d = d_o e^{\lambda t}$ [1, p. 27].

Or, stated differently, consider the average logarithmic growth of an initial error E_0 (the distance $|x_0 - (x_0 + \epsilon)|$, where ϵ is some arbitrarily small value) between two points in phase space [7, p. 709]. If E_k is the error at time step k, and E_{k-1} the error at the previous time step, then the average logarithmic error growth can be expressed by equation 1.

$$\lambda = \lim_{n\to\infty} \lim_{E_0\to 0} \frac{1}{n} \sum_{k=1}^{n} log|\frac{E_k}{E_{k-1}}|. \tag{1}$$

λ (which is measured in Hz or s^{-1}))is known as the Lyapunov exponent. For an m-dimensional phase space there will be m Lyapunov exponents, one for each dimension. If any one or more of those exponents are positive, then the trajectories of nearby states diverge exponentially from each other in phase space and the system is deemed chaotic. Since any system's variables of state are subject to uncertainty, a knowledge of what state the system is in can quickly become unknown if chaos is present. The larger the positive Lyapunov exponent, the quicker knowledge about the system is lost. One only knows that the state of the system lies somewhere on one of the trajectories traced out in phase space, i.e., somewhere on the strange attractor.

The Lyapunov exponent is one of the most useful quantitative measures of chaos, since it will reflect directly whether the system is indeed chaotic, and will quantify the degree of that chaos. Also, knowledge of the Lyapunov exponents becomes imperative for any analysis on prediction of future states. For example, in a system exhibiting highly chaotic deterministic behaviour (large Lyapunov exponent), predictions within a defined error margin of future system states are only possible for a few time steps ahead. The movement of a billiard ball in a game of pools is an example of this. On the other hand, for systems exhibiting no or very little deterministic chaos (small Lyapunov exponent, for example a pendulum), predicitions within the same defined error margin are possible for a larger number of time steps.

Determination of the Lyapunov exponents from a time series such as the robot trajectories used in this research or, more importantly, the existence and value of any positive exponents, has been discussed extensively in the literature [9, 7, 3, 1, 5, 2]. Several academic and commercial software packages have been made available to compute Lyapunov exponents from time series, for example [9, 8, 4]. In our analyses we have used the method proposed by Wolf *et al.* [9], as well as Abarbanel's commercially available software package [8] to determine the largest Lyapunov exponent. Figure 3 shows a typical computation of the Lyapunov exponent, using the method described in [9].

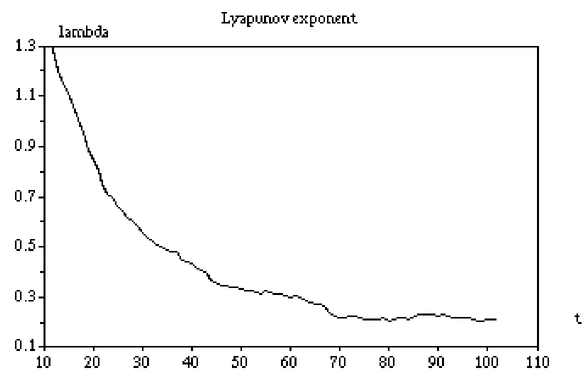

Fig. 3. Computation of the Lyapunov exponent of a "robotic billiard ball" in an environment without central obstructions (see figure 7), using the method outlined by Wolf *et al.* [9]. After 100 iterations the algorithm converges to $\lambda \approx 0.27s^{-1}$.

2 Experiments

2.1 Experimental Procedure and Results

Based on the hypothesis stated earlier that a robot's behaviour emerges through the interaction of robot, task and environment (figure 1), and provided a quantitative description (such as the Lyapunov exponent) exists, any experiment in which *one* of the three components of figure 1 is changed in a systematic manner would reveal the influence of that component upon the robot's overall behaviour.

We therefore conducted two sets of experiments:

1. Using the same robot, we let the robot perform *different* tasks in *the same* environment, and
2. again using the same robot, we let the robot perform *the same* task in *different* environments.

The robot we used throughout all experiments was a Pioneer II mobile robot (see figure 4), the robot's trajectory was logged using an overhead camera. The data obtained in this manner were two timeseries per experiment, the x and y position of the robot over time. The robot's position was logged every 250 ms, figure 5 shows part of a typical timeseries obtained.

A view of the arena, taken with the overhead camera used for data logging, is shown in figure 6.

Fig. 4. The Pioneer II mobile robot used.

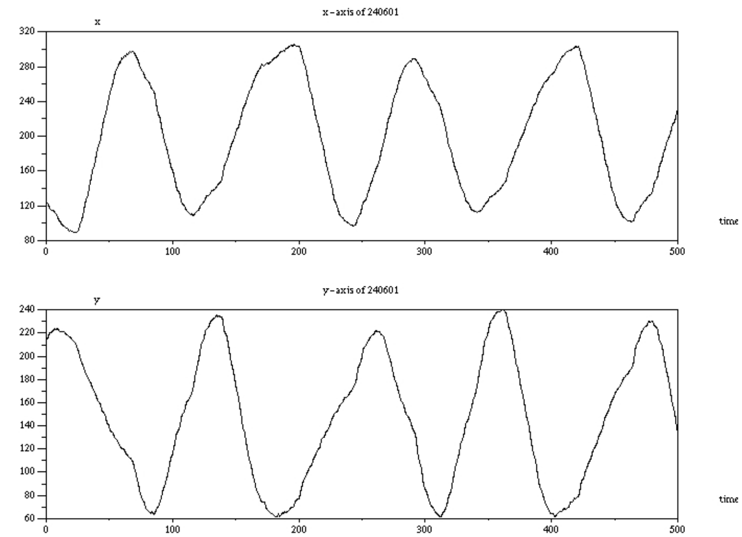

Fig. 5. Part of the logged x and y coordinates of the "robotic billiard ball" behaviour (see also figure 7). Abscissa units are arbitrary pixel coordinates, time is given in units of 250 ms.

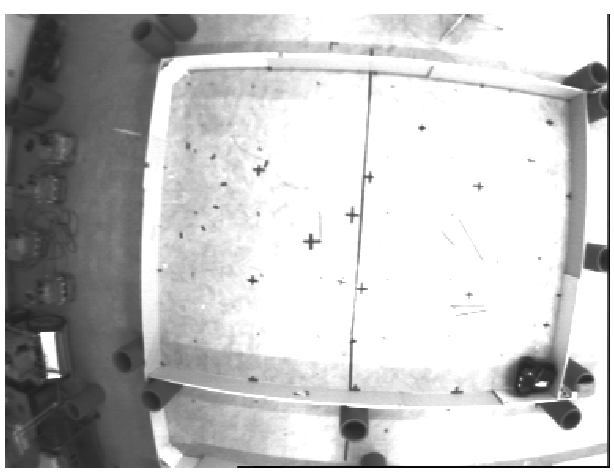

Fig. 6. View of the experimental arena, taken by the overhead camera used for data logging. The robot is visible at the bottom right hand corner. The cylinders used as obstructions in some experiments are visible adjacent to the arena.

2.2 Experiment 1: Same Environment, Different Tasks

In the first set of experiments, the robot operated in a square laboratory environment, containing smooth, light coloured walls. One task of the robot was to act like an "active billiard ball", moving randomly through the environment and avoiding obstacles. The second task was that of wall following. 54 minutes of wall-following behaviour were logged (13041 data points), and 109 minutes of billiard ball behaviour (26196 data points). Figure 7 shows both trajectories.

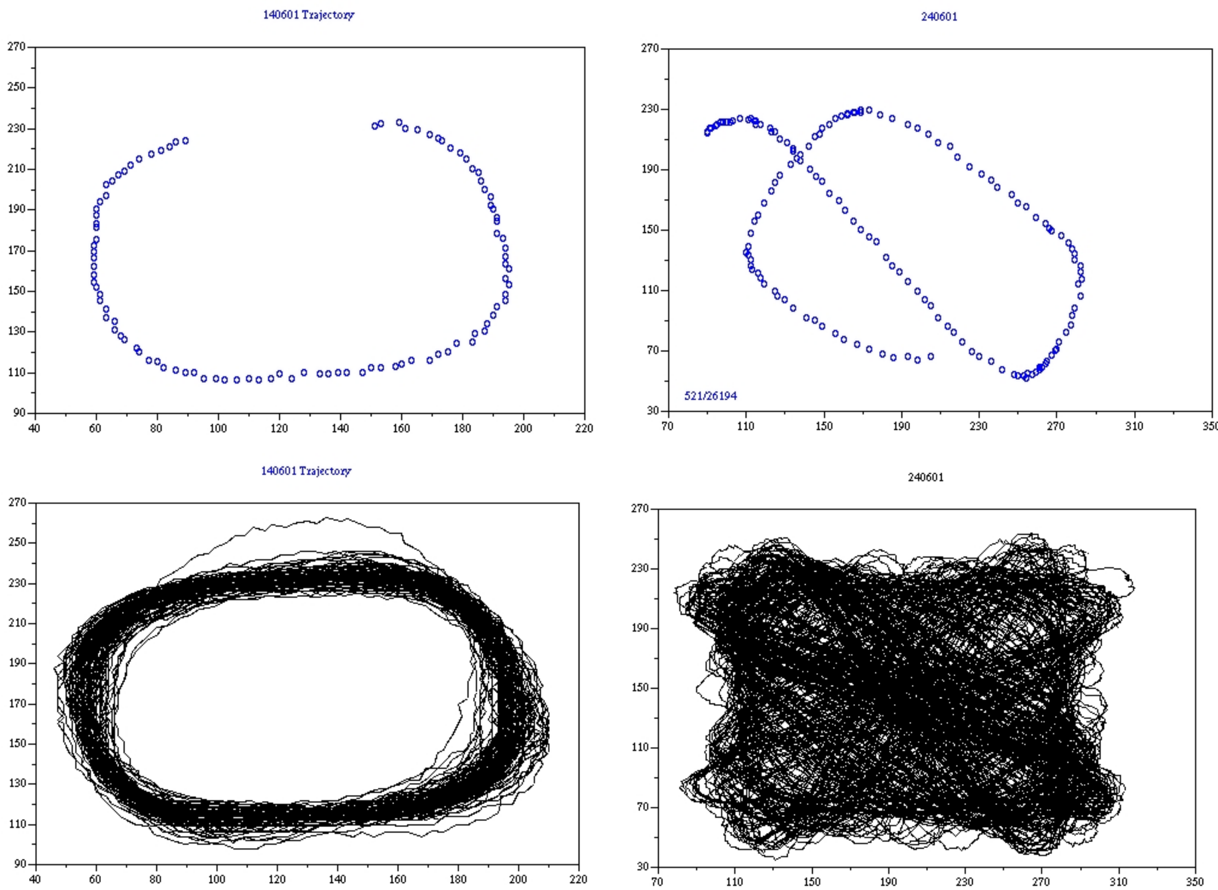

Fig. 7. Wall following (left column) and billiard ball behaviour (right column). Short segments of the trajectories are shown in the top row, whole trajectories are shown in the bottom row.

The Lyapunov exponent computed for the wall-following behaviour was $\lambda \approx 0.1 s^{-1}$, i.e. a very small number, indicating good predictability. For the billiard ball behaviour the computation yielded $\lambda \approx 0.3 s^{-1}$, i.e. a considerably higher degree of sensitivity to initial conditions, indicating a much shorter prediction horizon. This result is entirely within our expectations, as the wall following behaviour is considerably more orderly, and should result in a longer prediction horizon. Under ideal circumstances (for instance in simulation), wall following behaviour should yield a Lyapunov exponent very close to zero, because the wall acts as a "physical attractor", resulting in predictable and repeatable trajectories.

2.3 Experiment 2: Different Environments, Same Task

In a second set of experiments, we let the Pioneer II execute the "billiard ball" program in three different environments: the square environment introduced in section 2.2, the same environment with an additional central obstruction, and the same environment again with an off-centre obstruction.

The two trajectories obtained with obstructions present are shown in figure 8, the trajectory obtained when no obstruction is present is shown in figure 7 (right column).

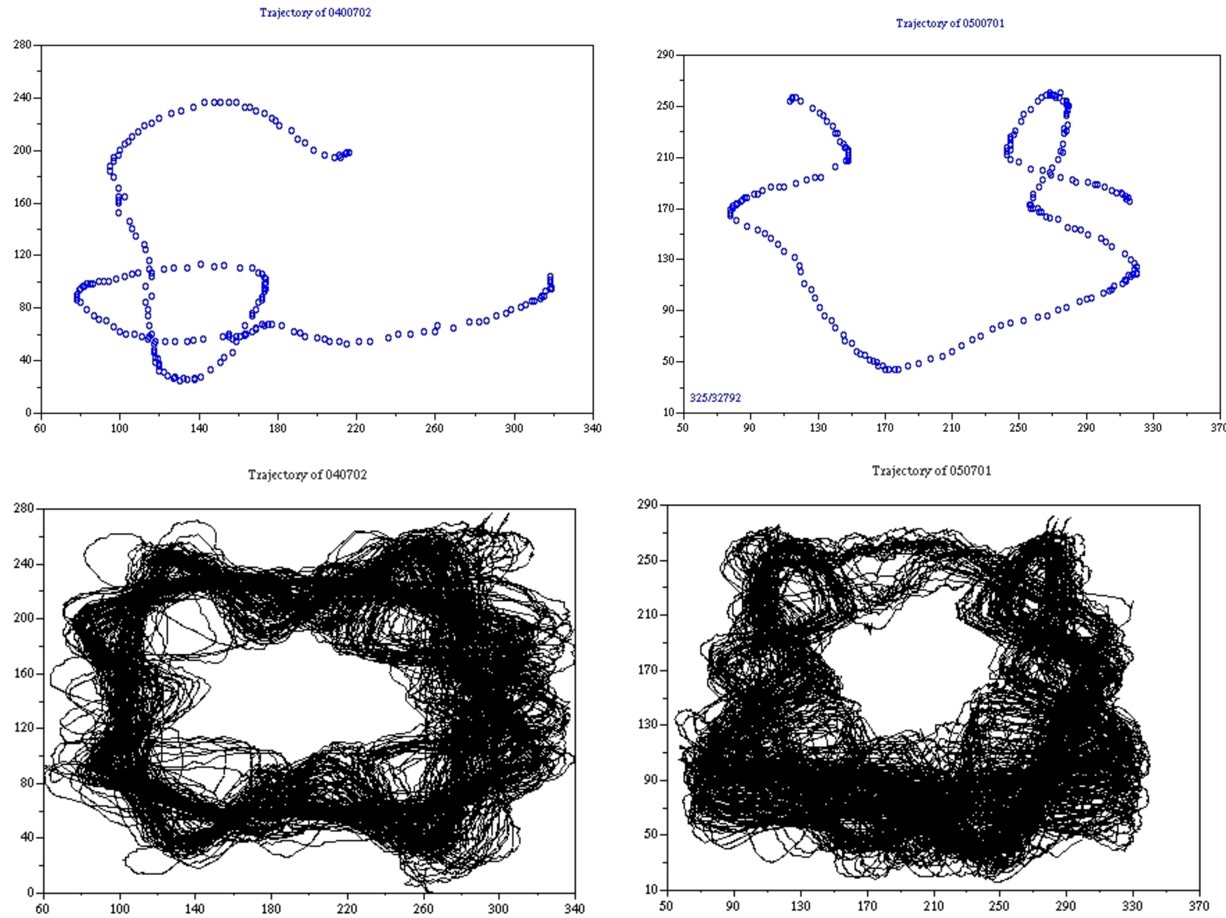

Fig. 8. Billiard ball behaviour in an environment with a central obstruction (left column) and in an environment with an off-centre obstruction (right column). Segments of 200 data points are shown in the top row, whole trajectories in the bottom row.

For the analysis, we logged 26196 data points (109 minutes of operation) in the environment without obstruction, 29996 data points (125 minutes) in the environment with central obstruction, and 32794 data points (136 minutes) in the environment with off-centre obstruction.

The computed Lyapunov exponents were very similar in all cases, and computed in all three cases as $\lambda \approx 0.3s^{-1}$. This is a highly interesting observation, as it demonstrates quantitatively that the robot's behaviour is governed much more by the task the robot is executing than by the environment the robot is operating in.

3 Summary and Conclusion

3.1 Summary

While certain scientific disciplines, such as physics, have a theoretical framework to formulate and evaluate hypotheses — a theory — at their disposal, mobile robotics to-date is still dependent upon existent proofs and trial-and-error experimentation. In order to accelerate the design process of robot controllers, as well as for principled experimental investigation, it would therefore be highly desirable to develop a theory of robot-environment interaction.

The foundation of any theory is a *quantitative* description of experimental results. In this paper, we present experiments with an autonomous mobile robot, in which we analyse the robot's behaviour quantitatively, using concepts from chaos theory.

3.2 Conclusion

Assuming that there are three main aspects that influence the overall behaviour of the robot — robot, task and environment — we conducted two experiments in which two of the three components were kept unchanged, whilst the third component was modified.

In a first set of experiments we let the robot conduct two different tasks in the same environment. Our finding was that the Lyapunov exponent differed markedly between the two tasks, signifying that the overall behaviour of the robot differed between the two experiments, and that this must have been due to the changed control program (because this was the only changed parameter in our experimentation).

In a second set of experiments we kept robot and task constant, and varied the environment. The Lyapunov exponent was the same in any of the three environments we used, indicating that the overall behaviour of the robot was not noticeably influenced by the environment the robot was operating in.

While these results require further consolidation, they are nevertheless interesting and encouraging. They seem to indicate that robot-environment interaction is far more influenced by the control program — a parameter that the user has influence over — than the environment — a parameter the user often has no influence over. This raises hopes that it might eventually be possible to develop a theory that will allow to design robot control programs without trial-and-error experimentation, and to make predictions regarding the robot behaviour *before* an experiment is conducted.

3.3 Future Work

While there are a number of experiments that have either already been conducted at Essex and Point Loma or are subject to ongoing investigation [6], there is one particular experiment that should be conducted, given the results we have obtained.

If the wall following behaviour was executed in an environment such as the one shown in figure 9, one would expect to obtain a Lyapunov exponent of $\lambda \approx 0.1 s^{-1}$, whereas if the billiard ball behaviour was executed in that environment, the expected Lyapunov exponent would be $\lambda \approx 0.3 s^{-1}$. It remains to be seen whether this prediction will be proven correct or not.

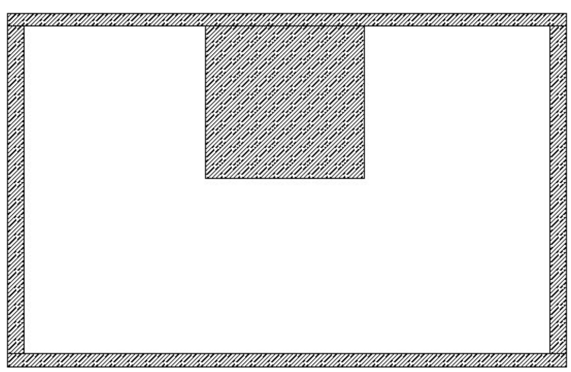

Fig. 9. Alternative environment for investigating wall-following behaviour.

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