THE APRIL MEETING IN NEW YORK

The four hundred thirty-fourth meeting of the American Mathematical Society was held at Columbia University on Friday and Saturday, April 16–17. The attendence was over three hundred, including the following three hundred six members of the Society:

C. R. Adams, E. B. Allen, C. B. Allendoerfer, R. L. Anderson, R. G. Archibald, L. A. Aroian, Emil Artin, Natascha Artin, R. N. Ascher, Max Astrachan, M. C. Ayer, W. L. Ayres, Joshua Barlaz, P. T. Bateman, G. E. Bates, M. F. Becker, Richard Bellman, Stefan Bergman, P. G. Bergmann, Lipman Bers, A. A. Blank, W. E. Bleick, M. L. Boas, R. P. Boas, Salomon Bochner, H. W. Bode, H. A. Bohr, G. L. Bolton, Samuel Borofsky, Paul Boschan, C. B. Boyer, A. D. Bradley, H. R. Brahana, Paul Brock, A. B. Brown, E. F. Buck, R. C. Buck, Hobart Bushey, J. H. Bushey, S. S. Cairns, W. R. Callahan, J. D. Campbell, P. G. Carlson, Alan Cathcart, K. Chandrasekharan, K. K. Chen, K. T. Chen, Sarvadaman Chowla, Alonzo Church, Randolph Church, Edmund Churchill, J. A. Clarkson, L. W. Cohen, R. M. Cohn, T. F. Cope, Richard Courant, M. D. Darkow, D. A. Darling, F. H. Davidson, M. D. Davis, J. B. Díaz, M. P. Dolciani, J. L. Doob, C. H. Dowker, Y. N. Dowker, Arnold Dresden, Roy Dubisch, James Dugundji, William H. Durfee, Jacques Dutka, Aryeh Dvoretzky, J. E. Eaton, A. W. Ebin, Samuel Eilenberg, Churchill Eisenhart, Paul Erdös, G. C. Evans, R. M. Exner, J. M. Feld, William Feller, Emanuel Fischer, Irwin Fischer, D. A. Flanders, Erling Folner, R. M. Foster, M. F. Freeman, Bernard Friedman, K. O. Friedrichs, Orrin Frink, P. R. Garabedian, L. L. Gavurin, H. M. Gehman, Abe Gelbart, P. W. Gilbert, B. P. Gill, Leonard Gillman, Wallace Givens, O. E. Glenn, Sidney Glusman, V. D. Gokhale, H. H. Goode, R. E. Goodman, M. J. Gottlieb, H. J. Greenberg, Harriet Griffin, M. M. Gutterman, P. R. Halmos, Carl Hammer, R. W. Hamming, G. H. Handelman, O. G. Harrold, K. E. Hazard, A. E. Heins, M. H. Heins, Alex Heller, Eric Hemmingsen, L. A. Henkin, L. H. Herbach, A. D. Hestenes, Einar Hille, W. M. Hirsch, I. I. Hirschman, Banesh Hoffmann, R. B. Hofstra, E. M. Hull, Witold Hurewicz, L. C. Hutchinson, M. A. Hyman, R. E. Ingram, R. L. Jeffery, Fritz John, R. E. Johnson, A. W. Jones, Mark Kac, Aida Kalish, Samuel Kaplan, M. E. Kellar, J. B. Kelly, P. W. Ketchum, H. S. Kieval, R. S. Kingsbury, F. L. Kiokemeister, S. A. Kiss, J. W. Kitchens, J. R. Kline, Morris Kline, E. R. Kolchin, B. O. Koopman, H. L. Krall, M. S. Kramer, H. G. Landau, R. E. Langer, Mario Laserna, P. D. Lax, Solomon Lefschetz, Howard Levene, Howard Levi, Samuel Linial, M. A. Lipschutz, Marie Litzinger, Charles Loewner, E. R. Lorch, Lee Lorch, A. N. Lowan, Eugene Lukacs, Brockway McMillan L. A. MacColl, G. W. Mackey, G. R. Magee, Dis Maly, Murray Mannos, A. J. Maria, M. H. Maria, M. H. Martin, W. T. Martin, Imanuel Marx, J. L. Massera, D. G. Mead, A. E. Meder, Paul Meier, A. N. Milgram, K. S. Miller, W. H. Mills, S. H. Min, Don Mittleman, Morris Monsky, J. T. Moore, T. W. Moore, K. A. Morgan, C. R. Morris, Marston Morse, I. R. Moses, H. H. Mostafa, G. D. Mostow, T. S. Motzkin, G. W. Mullins, F. J. Murray, C. A. Nelson, O. E. Neugebauer, John von Neumann, P. B. Norman, Morris Ostrofsky, E. R. Ott, O. G. Owens, J. C. Oxtoby, Edward Paulson, A. M. Peiser, Anna Pell-Wheeler, Everett Pitcher, Harry Pollard, E. L. Post, M. H. Protter, Hans Rademacher, H. W. Reddick, Irving Reiner, Eric Reissner, Daniel Resch, R. G. D. Richardson, John Riordan, J. F. Ritt, I. F. Ritter, H. E. Robbins, G. de B. Robinson, S. L. Robinson, I. H. Rose, M. E. Rose, J. B.

Rosenbach, Edward Rosenthall, L. R. Rubashkin, Herman Rubin, Herbert Ruderfer, C. W. Saalfrank, Charles Saltzer, H. E. Salzer, Arthur Sard, A. T. Schafer, R. D. Schafer, Robert Schatten, Albert Schild, I. J. Schoenberg, Lowell Schoenfeld, Abraham Schwartz, G. E. Schweigert, C. H. W. Sedgewick, H. N. Shapiro, Max Shiffman, S. S. Shü, D. N. Silver, James Singer, L. L. Smail, P. A. Smith, J. J. Sopka, E. H. Spanier, W. H. Spragens, George Springer, J. J. Stachel, N. E. Steenrod, S. K. Stein, Fritz Steinhardt, Wolfgang Sternberg, J. J. Stoker, R. W. Stokes, M. H Stone, Walter Strodt, M. M. Sullivan, Fred Supnick, J. H. Taylor, J. M. Thomas, D. L. Thomsen, C. J. Titus, M. M. Torrey, C. C. Torrance, H. I. Treiber, A. W. Tucker, J. W. Tukey, J. R. Van Andel, H. E. Vansant, S. I. Vrooman, B. L. van der Waerden, J. L. Walsh, R. M. Walter, J. B. Walton, W. G. Warnock, Alan Wayne, M. A. Weber, J. V. Wehausen, Alexander Weinstein, Louis Weisner, F. P. Welch, M. E. White, A. L. Whiteman, A. M. Whitney, Hassler Whitney, John Williamson, Jacob Wolfowitz, H. A. Wood, M. A. Wurster, Bertram Yood, J. W. Young, Arthur Zeichner, J. J. Zeig, Daniel Zelinsky, J. A. Zilber, H. J. Zimmerberg, Leo Zippin, O. J. Zobel.

On Friday evening Professor O. E. Neugebauer of Brown University gave an address on *Mathematical methods in ancient astronomy.* Professor P. A. Smith presided.

On Saturday afternoon Professor Charles Loewner of Syracuse University gave an address on *Some classes of functions defined by difference or differential inequalities.* Professor G. C. Evans presided.

On Friday afternoon, at the kind invitation of the International Business Machines Corporation, members of the Society inspected the I.B.M. Selective Sequence Electronic Calculator.

On Saturday morning there were two sections, one for papers in Analysis in which Professor Salomon Bochner, President Hille, and Professor G. C. Evans presided, and one for papers in Algebra in which Professor Hans Rademacher presided. On Saturday afternoon there were two sections, one for papers in Geometry, Topology, and Applied Mathematics, in which Professor Samuel Eilenberg presided, and one for papers in Statistics and Probability, in which Professor J. L. Doob presided.

The Council met at 4:00 P.M. on Friday in the Faculty Club and at 12:00 M. on Saturday in the School of Business.

The Secretary announced the election of the following forty-six persons to ordinary membership in the Society:

Professor Andrew Wilson Ashburn, Texas State College for Women, Denton, Tex.;
Mr. Howard William Baeumler, University of Buffalo;
Mr. Albert Walter Bailey, Casper Junior College, Casper, Wyo.;
Miss Aurora Belsky, Polytechnic Institute of Brooklyn;
Mr. William Sheffield Bryan, University of Washington;
Mr. José Angel Colon, University of Puerto Rico;
Mr. Ralph Pat Crouch, Texas Technological College;
Mr. Roy Edward Dawson, Jacksonville Junior College;

Mr. Burton Victor Dean, Columbia University;
Mr. J. T. Duprat, University of Ottawa;
Dr. Fred G. Elston, Seton Hall College, South Orange, N. J.;
Mr. Marvin Phelps Epstein, University of California;
Mr. Avner Herman Ferester, Seton Hall College, South Orange, N. J.;
Mr. Robert Fitzpatrick, University of Pittsburgh;
Mr. Herbert Parrish Galliher, Yale University;
Mr. Sidney Glusman, Seton Hall College, South Orange, N. J.;
Professor Victor Goedicke, Ohio University, Athens, Ohio;
Professor David Cann Haley, Acadia University, Wolfville, N. S.;
Dr. Theodore Edward Harris, Douglas Aircraft Company, Santa Monica, Calif.;
Mr. Henry John Harrje, Harrje Associates, Architects and Engineers, Jacksonville Beach, Fla.;
Mr. William Walden Kester, Yakima Valley Junior College, Yakima, Wash.;
Professor Vivian Wright Kline, Northeast Missouri State Teachers College, Kirksville, Mo.;
Professor Carl Robert Kossack, State Teachers College, New Haven, Conn.;
Mr. Saul Kravetz, Brooklyn, N. Y.;
Miss Leonore Morey Laderman, New York, N. Y.;
Mr. Gordon Eric Latta, University of British Columbia;
Mr. John Burr Lennes, University of Oklahoma;
Mr. Arthur Davenport Lyles, Presbyterian Junior College, Maxton, N. C.;
Mr. Ralph A. Mauller, Central College, Fayette, Mo.;
Mr. Edward Francis O'Shea, University of Scranton;
Mr. Windsor Lewis Sherman, Providence, R. I.;
Mr. Jordan H. Siedband, Illinois Institute of Technology;
Mr. Thomas Francis Singleton, Regis College, Denver, Colo.;
Mr. Thomas Harrison Slook, Temple University;
Mr. Otis Sheldon Spears, University of Oklahoma;
Miss Vivian Spurgeon, Southwest Baptist College, Bolivar, Mo.;
Mr. William Duane Stahlman, Massachusetts Institute of Technology;
Mr. Walter Boyd Stovall, Jr., University of Florida;
Mr. C. Robert Swenson, Georgia School of Technology;
Mr. Andrew J. Terzuoli, Polytechnic Institute of Brooklyn;
Mr. Francis Bernard Taylor, Manhattan College;
Mr. William Bell Thompson, University of Toronto;
Miss Bess B. Vogel, New York University;
Miss Mamie Smith Ware, Clark College, Atlanta, Ga.;
Ina Warren Welmers (Mrs. E. T.), University of Buffalo;
Mr. Arthur Zeichner, Harvard University.

It was reported that the following had been elected to membership on nomination of institutional members as indicated:

Haverford College: Mr. Murray Fox Freeman.
Institute for Advanced Study: Drs. Sarvadaman Chowla, Robert Gustave Debever, Paul Turán.

The Secretary reported that a reciprocity agreement had been established between the Society and the Wiskundig Genootschap te Amsterdam.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: London Mathematical Society: Dr. Wolfgang Heinrich Fuchs, University of Liverpool; Professor Philip Gerard Gormley, University College, Dublin; Matematisk Forening i København: Dr. Mogens Pihl, Institute for Theoretical Physics, Copenhagen; Wiskundig Genootschap te Amsterdam: Dr. Christoffel Jacob Bouwkamp, N. V. Philips' Laboratory, Eindhoven, Netherlands; Professor Adriaan Cornelis Zaanen, University of Indonesia, Bandoeng, Java.

The following appointments by President Einar Hille were reported: Professor H. J. Ettlinger as representative of the Society at the inauguration of William Richardson White as President of Baylor University on April 13, 1948; Professor E. J. Purcell as representative of the Society at the inauguration of James Byron McCormick as President of the University of Arizona on May 5, 1948; Professor Eric Reissner as representative of the Society on the Cooperating Committee for the International Congress of Theoretical and Applied Mechanics, to be held in London in September, 1948; Professor H. A. Meyer as representative of the Society at the inauguration of Richard Vernon Moore as President of Bethune-Cookman College on March 19, 1948; Professor John von Neumann as Chairman of the Financial Committee for the International Congress of Mathematicians, to replace Professor M. H. Stone (Professor Stone to continue as member of committee until May 1, 1948); Professor Jerzy Neyman as representative of the Society at the Sierpinski Jubilee of the Polish Mathematical Society on September 20–23, 1948; Dr. R. P. Boas (Chairman), Professors Samuel Eilenberg, D. H. Lehmer, William Prager, and G. Y. Rainich as a committee on the project of translation of Russian mathematical articles; Professors Samuel Eilenberg (Chairman), Lipman Bers, P. R. Halmos, G. Y. Rainich, and A. E. Ross as a committee to prepare a pamphlet for aid to mathematicians in reading Russian mathematical articles.

It was reported that Professor B. P. Gill had been added as an ex officio member of the Financial Committee for the International Congress.

The Secretary reported that the Mathematics Branch of the Office of Naval Research had approved a request from the Society for funds to inaugurate a project whereby translations of articles in Russian and other unfamiliar languages will be made available and a pamphlet to aid mathematicians in reading Russian mathematical articles prepared. The request has been referred to the Contracts Division of the Office of Naval Research for negotiation.

The Council elected Professor J. W. T. Youngs as Associate Secretary for the midwest, to fill the unexpired term of Professor R. H. Bruck. This term is to begin after the 1948 Summer Meeting and to extend to December 31, 1949. (This election was subsequently approved by the Board of Trustees.)

The Council voted to invite Professor Casimir Kuratowski of the University of Warsaw to act as Visiting Lecturer of the Society during the academic year 1948–1949.

A report from the Committee on Reorganization was presented by Dean W. L. Ayres, Chairman. This committee was authorized by the Council at the 1947 Summer Meeting to study the present organization of the Society. President Hille subsequently appointed the following committee: Dean W. L. Ayres (Chairman), Professors S. S. Cairns, B. P. Gill, J. R. Kline, W. T. Martin, P. A. Smith, and M. H. Stone. Changes in the organization of the Society, as recommended by this committee and adopted by the Council, appear in the following excerpts from the report of the Committee on Reorganization:

I. *President Elect.* A new office of President Elect shall be created and this office shall be filled during the even numbered years only. The person designated as President Elect shall automatically become President of the Society at the end of his one year term as President Elect. He shall be a member of the Council and of its Executive Committee during this one year term and thus should be well acquainted with the affairs of the Society at the time he assumes office as its President.

II. *Executive Committee of the Council.* An Executive Committee of the Council shall be formed and this Committee shall be empowered to act for the Council on (1) matters which have been delegated to the Executive Committee by the Council (such as election of members, approval of dates of meetings, etc.) and (2) such other matters of policy or administration of Society business that require immediate decision, it being understood that if three members of the Executive Committee request postponement of any matter until the next meeting of the Council the matter shall be so postponed. It may consider the agenda for the meetings of the Council and make recommendations on important matters to the Council. It shall be responsible to the Council and shall report its actions to the Council. The Executive Committee shall consist of seven members, the President, Secretary, President Elect (even numbered years) and Ex-President (odd numbered years), and four members of the Council elected in January for two year terms with staggered periods of office. These elected members of the Executive Committee shall be elected directly by the Council, and any member of the Council shall be eligible for election to this Committee. In case an elected member of the Executive Committee shall be elected to serve for a term beyond his term of membership on the Council he shall automatically continue as a member of the Council during his period on the Executive Committee.

The Executive Committee shall meet at the call of the President or of any two of its members. The members of the Executive Committee shall have expenses paid by the Society in attending these meetings and, in general, it would be expected that members of the Executive Committee would attend all meetings unless prevented by illness or duties of extreme importance.

Comment: The proposed Executive Committee should form a small responsible group which can meet several times a year to guide the policies of the Society. There are many times in the affairs of the Society when a decision must be made before the next possible meeting of the entire Council. Such a small sub-committee could be called together quickly or could be consulted by mail or telegram. The payment of travel expenses is believed necessary to insure full attendance at these meetings, particularly of those members of the committee at some distance from the office of the Society.

III. *The Council.* The membership of the Council shall be changed as follows:

1. The number of Associate Secretaries shall be changed from four to three, eliminating the Associate Secretary who has been in charge of financial matters.
2. The President of the Society shall serve as a member of the Council and its Executive Committee for one year following his period of office.
3. Each Secretary of the Society shall serve as a member of the Council for two years following his term of office.
4. At the time of this change the Ex-Presidents and Ex-Secretaries who are members of the Council shall continue to the end of their usual term.

All members of the Council shall be voting members of the Council under the following provisions: in case of a roll call vote, the group of Associate Secretaries, the editorial committees of the Society publications which now have representation on the Council, and the representatives on the Board of Editors of the American Journal of Mathematics shall be entitled to one vote each. This vote shall be divided among the members of the committee who are present at the Council meeting. No member of the Council shall have more than one vote, and if a member of an editorial committee is also entitled to a vote on the Council due to another office, the remaining members of that editorial committee present shall divide the vote equally between them.

Comment: Your Committee has heard considerable criticism of the existence of membership of the Council without vote and the above proposal is intended to improve this situation. The objection raised is that non-voting members of the Council resent this status and do not take as active a part in the deliberations of the Council. Many of these individuals are persons of considerable experience in the affairs of the Society and the Council needs the benefit of their wisdom.

Several years ago the Society established the principle that the members of the Council elected by the membership of the Society should have a voting majority in the Council. The Vice-Presidents of the Society and the members-at-large of the Council are so elected. The reduction of the number of Ex-Presidents on the Council and the specification of the term of the Ex-Secretaries, together with the proposed method of voting of the editorial members of the Council are designed to maintain this principle. The above proposal would give the elected members eighteen votes as compared with fifteen ex-officio votes.

IV. *The Secretariat.* The present office of Secretary shall be divided into two offices, an Executive Director and a Secretary.

The Executive Director shall be a full-time paid employee of the Society. He shall be in charge of the central office of the Society and shall be charged with the general administration of the affairs of the Society once policies have been set by the Council and its Executive Committee. He will work under the immediate direction of the Council and its Executive Committee. He will attend meetings of the Council and its Executive Committee, but not be a member of these bodies. The person holding this office shall be nominated by the Council and approved by the Trustees. Due to the nature of his work it seems desirable that he be a mathematician.

The Executive Director shall take over the functions of the present Associate Secretary who deals with financial affairs, and this office shall be abolished. Three individuals have held this Associate Secretaryship and all three believe that the office should be abolished and the duties performed by an officer located in the main office of the Society.

All abstracts shall be mailed directly to the office of the Society and the programs will be arranged by the Executive Director subject to review by the Associate Secretary in charge of the meeting.

The Secretary of the Society will continue to be the principal officer of the Society concerned with policy making. He will work with the President, the Council, and its committees in setting the policies of the Society. While the Secretary and Executive Director must cooperate closely in their work, the division of their functions can be described briefly in the two phrases "policy making" and "administration." Once the Council has set the policy of the Society it will be the duty of the Executive Director to carry out the administration.

Comment: The growth in membership and activities of the Society has produced a pressure on the Secretary, which must be relieved. The present Secretary estimates that he devotes well over fifty per cent of his time to the affairs of the Society, and there is every indication that this pressure will continue to grow in the immediate years ahead. Your committee feels that it is imperative that a new full-time office be planned at this time to relieve the Secretary of many of the present duties of his office, permitting him to concentrate on policy matters. There would remain the type of position that a professor as Secretary could reasonably be expected to handle along with his university work.

On recommendation of the Committee on Reorganization, the Council voted to refer the question of the possible establishment of a federation of various mathematical organizations to the Policy Committee for Mathematics.

The President was authorized to appoint: (1) a committee to formulate the changes in By-Laws made necessary by the adoption of the above changes in organization; (2) a committee to nominate to the Council an Executive Director of the Society.

The Council voted to schedule annual meetings in the southeastern section of the country, the meetings to be held at a time most likely to encourage southeasterners to attend both southeastern and national (Summer, Annual) meetings.

The Birkhoff Editorial Committee reported it had completed the compilation and arrangement of the material to be published as the *Collected Mathematical Papers* of G. D. Birkhoff.

On recommendation of the Committee on Reorganization of the Society's Programs, the Council voted to advance by one week the deadlines for the submission of contributed papers, to consult the Committee on Applied Mathematics regarding the titles of sections on abstract blanks, to recommend to the Committees to Select Hour

Speakers that they take advantage of opportunities to invite distinguished foreign visitors who happen to be in the United States, to allow publishers to display books at or above the graduate level at meetings of the Society.

It was reported that Professor R. D. James would deliver an hour address at the June, 1948, meeting at the University of British Columbia.

The Secretary reported plans for the publication of the Canadian Journal of Mathematics by the Canadian Mathematical Congress, the first number of which will appear in January, 1949. Members of the Society will receive a discount of fifty per cent on subscriptions to this Journal.

The Council adopted the following resolution regarding the Committee on Un-American Activities of the House of Representatives which was subsequently released to the newspapers and sent to appropriate members of Congress:

The Council of the American Mathematical Society, a national scientific organization comprising 3700 members, at the 434th meeting of the Society in New York adopted the following resolution:

Whereas, the Committee on Un-American Activities of the House of Representatives has recently made public a statement derogatory to a distinguished American scientist, Dr. Edward U. Condon, the present Director of the Bureau of Standards in the Department of Commerce; and

Whereas, a careful examination of these statements in the light of public information discloses that the Committee has made insinuations unsubstantiated by the evidence submitted, and resorted (in the furtherance of its own purposes) to misrepresentation of even the most innocent acts, and

Whereas, it is inevitable that the procedure adopted by the said Committee in attacking Dr. Condon will, if tolerated, prove an effective obstacle to the entry of scientists into public service; and

Whereas, furthermore these procedures will, if continued, enable congressional committees to usurp and abuse functions properly reserved under our Constitution to the executive and judicial branches of our government; now therefore be it resolved

That, this Council publicly express its grave concern over the implications of the actions followed by the Committee on Un-American Activities of the House of Representatives in attacking Dr. Edward U. Condon; and

That the Council petition the House of Representatives and the Senate of the United States forthwith to adopt rules of procedure for their committees which will safeguard the constitutional separation of powers, secure our citizens in their individual right to just treatment by their government, and reflect Congressional understanding of the necessity of self-restraint in the exercise of the powers of legislative inquiry.

The Council approved the recommendation of the Organizing Committee for the International Congress that Russian be added to the list of official languages as stated in the Eisenhart report (the fundamental document on which the organization of the Congress is based, adopted by the Council in 1937). It was also agreed that the Organizing Committee need not adhere to the time limits set in the Eisenhart report, due to the difference in the interval between the inauguration of the plans and the opening date of the Congress.

Abstracts of the papers read follow below. Papers whose abstract numbers are followed by the letter "*t*" were read by title. Paper number 241 was read by Dr. Chandrasekharan, paper number 249 by Professor Frink, and paper number 252 by Professor Schatten.

Algebra and Theory of Numbers

226*t*. Eckford Cohen: *Sums of an odd number of squares in* $GF[p^n, x]$.

Let F be a polynomial of $GF[p^n, x]$, $p>2$, of degree $f<2k$, and let $\alpha_1, \cdots, \alpha_{2s+1}$ be nonzero elements of $GF(p^n)$. The object of the author is to find the number of solutions N^k_{2s+1} of $F=\sum_{i=1}^{2s+1} \alpha_i X_i^2$ in polynomials X_i $(i=1, 2, \cdots, 2s+1)$ of degree less than k. The solution of the present problem is obtained in terms of Artin's σ-function, $\sigma_i(F)=\sum(F/H)$, where (F/H) indicates the quadratic character of F with respect to H and the summation ranges over primary H of degree i. Although the formula obtained is rather complicated, it simplifies greatly for certain special cases. For example, when $k>f$ and F is quadratfrei, $N^k_{2s+1}=\sum_{z=0}^{k} p^{-nzs}(\sigma_z(\theta F)-p^n\sigma_{z-2}(\theta F))+(p^n-1)\sum_{z=0}^{k-1} p^{-nzs}\sigma_z(\theta F)$ where $\theta=(-1)^s\alpha_1\alpha_2\cdots\alpha_{2s+1}$. (Received March 11, 1948.)

227. R. M. Cohn: *Manifolds of difference polynomials.*

The author studies the manifold of a single difference polynomial and the theory of elimination among systems of difference polynomials. Let A be an algebraically irreducible difference polynomial in an unknown y, with coefficients in an abstract difference field. Let A be of order n and degree r in y_n and let y_0 appear effectively in A. It is shown that A has at least one, and at most r, essential irreducible manifolds not held by polynomials of order less than n. A may also possess essential irreducible manifolds held by polynomials of order less than n, and termed, by analogy with differential polynomials, essential singular manifolds. Thus, if $A=yy_2-y_1$, then $yA_1+A=y_1(yy_3-1)$, so that $y=0$ constitutes an essential irreducible manifold. In the theory of elimination and the structure of difference fields theorems are developed bearing a close analogy to those known in algebra, and the theory of differential equations, and culminating in the proof that, given a prime reflexive difference ideal Σ, one may find a resolvent unknown w, in terms of which the unknowns of Σ may be expressed uniquely. It is a peculiarity of difference equations that the system of equations which furnishes this unique expression need not be linear. (Received February 16, 1948.)

228. R. E. Johnson: *Equivalence rings.*

If in a ring R two elements are related by $a=pbq$ for some p, q in R, then $a\leqq b$. R is called an equivalence ring if for every pair of elements a, b of R either $a\leqq b$ or $b\leqq a$. In case $a\leqq b$ and $b\leqq a$ for every pair of nonzero elements of R, R is called a 1-equivalence ring. The relation $<$ gives rise to a linearly-ordered lattice, L, of subsets of R.

The ideals of R are elements of L if L is well-ordered. A commutative equivalence ring has a unit element and a nonzero radical if it is not a field. A simple nonradical ring is either a regular equivalence ring or a subring of a regular 1-equivalence ring. (Received March 12, 1948.)

229t. E. R. Kolchin: *Existence theorems connected with the Picard-Vessiot theory of homogeneous linear ordinary differential equations.*

It is shown how a theorem on algebraic differential equations due to J. F. Ritt (Trans. Amer. Math. Soc. vol. 48 (1940) pp. 543–544) yields two results concerning the differential equation $L(y)=y^{(n)}+p_1y^{(n-1)}+\cdots+p_ny=0$ ($p_1, \cdots, p_n$ in an ordinary differential field $\mathcal{J}$ of characteristic 0 with algebraically closed field of constants $\mathcal{C}$). 1. *$L(y)=0$ always has a fundamental system of solutions $\eta_1, \cdots, \eta_n$ such that the field of constants of $\mathcal{J}\langle\eta_1, \cdots, \eta_n\rangle$ is $\mathcal{C}$.* This solves a problem proposed by R. Baer (pp. 332–334 of F. Klein's *Vorlesungen über hypergeometrische Funktionen*, Berlin, 1933). 2. *If $L(y)$ is linearly irreducible* [that is, not properly of the form $L_1(L_2(y))$] *over $\mathcal{J}$ and if $L(y)=0$ has one solution contained in an extension of $\mathcal{J}$ by integrals, exponentials of integrals, and algebraic functions, then $L(y)=0$ has a fundamental system of solutions $\eta_1, \cdots, \eta_n$ such that $\mathcal{J}\langle\eta_1, \cdots, \eta_n\rangle$ is a liouvillian extension of $\mathcal{J}$* (that is, an extension by integrals, exponentials of integrals, and algebraic functions, of which the field of constants is $\mathcal{C}$). (Received March 11, 1948.)

230t. E. R. Kolchin: *On certain concepts in the theory of algebraic matric groups.*

For an algebraic matric group $\mathfrak{G}$ over an algebraically closed field of characteristic p the concepts "component of the identity," "solvable," "anticompact," and "quasicompact" (previously discussed by the author, Ann. of Math. (2) vol. 49(1948) pp. 1–42) are further studied. Among the new results are the following: 1. $\mathfrak{G}$ is anticompact if and only if $\mathfrak{G}$ is reducible to special triangular form. 2. A connected $\mathfrak{G}$ is quasicompact if and only if $\mathfrak{G}$ is reducible to diagonal form. 3. $\mathfrak{G}$ is solvable as an algebraic group if and only if $\mathfrak{G}$ is solvable as an abstract group. 4. The component of the identity $\mathfrak{G}^0$ of $\mathfrak{G}$ is the smallest subgroup of $\mathfrak{G}$ of finite index (provided $p=0$; for $p\neq 0$ purely group-theoretic characterization of $\mathfrak{G}^0$ is impossible). 5. "Quasicompact and connected" is characterized in terms of dim $\mathfrak{G}$ and purely group-theoretic concepts (no purely group-theoretic characterization exists). (Received March 11, 1948.)

231t. Joseph Lehner: *Divisibility properties of the Fourier coefficients of the modular invariant $j(\tau)$.*

The following theorem is proved: let c_n ($n=0, 1, 2, \cdots$) be the Fourier coefficients of the absolute modular invariant $j(\tau)=x^{-1}+744+196,884x+\cdots=x^{-1}+c_0+\sum_{n=1}^{\infty} c_nx^n$, $x=\exp 2\pi i\tau$ ($j((a\tau+b)/(c\tau+d)))=j(\tau)$ for every integral a, b, c, d, with $ad-bc=1$). Then for $\alpha=1, 2, 3, \cdots$, and $n=1, 2, 3, \cdots$, $5^\alpha | n$ implies $5^{\alpha+1} | c_n$; $7^\alpha | n$ implies $7^\alpha | c_n$. Also for $\beta=1, 2$, and $n=1, 2, 3, \cdots$, $11^\beta | n$ implies $11^\beta | c_n$. The methods used are those of the theory of modular functions and are described in the author's earlier paper (Amer. J. Math. (1943)). The difficulty in extending the congruences to powers of 11 beyond the second comes from the fact that the subgroup $\Gamma_0(n)$ of the full modular group (defined by $c\equiv 0 \pmod{n}$), which occurs in an essential way in the proof, is of genus one when $n=11$, whereas it is of genus zero when $n=5, 7$. (Received February 9, 1948.)

232*t*. N. H. McCoy: *Prime ideals in general rings.*

Some of the known results about prime ideals in commutative rings are extended to arbitrary rings. In place of the multiplicative systems used in the commutative case an important role is played by *m-systems*. An *m*-system M of elements of the ring R is a set with the property that $a\in M$, $b\in M$ imply the existence of an x in R such that $axb\in M$. It is shown that a two-sided ideal $\mathfrak{p}$ in R is a prime ideal if and only if its complement in R is an *m*-system. The radical of an ideal $\mathfrak{a}$ is defined, and shown to be the intersection of all prime ideals which contain $\mathfrak{a}$. The methods are based on those of Krull (Math. Ann. vol. 101 (1929) pp. 729–744). The radical N of the zero ideal has all the familiar "radical-like" properties, and is actually a generalization of the classical radical of a ring. The Jacobson radical (Amer. J. Math. vol. 67 (1945) pp. 300–320) is the intersection of a certain class of prime ideals, as is also the radical defined by Brown and McCoy (Amer. J. Math. vol. 69 (1947) pp. 46–58). (Received March 11, 1948.)

233. I. F. Ritter: *Solution of algebraic equations.*

This paper describes an iterative process yielding approximate values for the roots, complex or real, of an algebraic equation of any degree. Practically useful approximations with "slide rule accuracy" are obtained by it, using as tool nothing more elaborate than the slide rule. Such tools as logarithms or an adding machine make the process yield as many correct significant digits of the roots as wanted. The effort required is less than that needed in Graeffe's or other methods. It is not at all increased by the presence of complex roots and the iteration involved in the process makes most steps in it self-checking. These two facts are the main reasons for the practical advantage of this new method. (November 6, 1947.)

234. G. de B. Robinson: *On the representations of the symmetric group.*

If $[\alpha]$ be any irreducible representation of the symmetric group $\mathfrak{S}_n$, then the Kronecker product $[\alpha]\times[\alpha]\times\cdots$ (*m* factors) on *m* different sets of variables yields a representation of the sub-group $H=\mathfrak{S}_n\times\mathfrak{S}_n\times\cdots$ (*m* factors) which induces a representation of $\mathfrak{S}_{mn}$; the problem under consideration is the reduction of this representation arising from the interchangeability of the *m* factors. The case is parallel to Schur's derivation of the irreducible representations of the full linear group of order *m*. The components, here reducible, have been designated $[\alpha]\otimes[\beta]$, where $[\beta]$ is an irreducible representation of S_m. For $[\alpha]=[n]$ (cf. D. E. Littlewood, Proc. London Math. Soc. vol. 49 (1947) pp. 282–306), the irreducible representation $[\lambda]$ of S_{mn} appears in $[n]\otimes[\beta]$ with a multiplicity $(1/m!)\sum_S\chi_\beta(S^*)(n!)^{-m}\phi_\lambda^{HS}$ where ϕ_λ^{HS} denotes the sum of the characters of the elements of the coset HS in $[\lambda]$, and S is a substitution of $\mathfrak{S}_{mn}$ which permutes the *m* sets of variables (or symbols used to define H); $\{S\}$ is a subgroup of $\mathfrak{S}_{mn}$ simply isomorphic to $\mathfrak{S}_m^*$. This formula generalizes to the case of an arbitrary $[\alpha]$. (Received March 12, 1948.)

235. R. D. Schafer: *The Wedderburn principal theorem for alternative algebras.*

The following generalization of the so-called Wedderburn principal theorem for associative algebras is proved. Let $\mathfrak{A}$ be an alternative algebra over an arbitrary field $\mathfrak{F}$ which is neither semisimple nor nilpotent, and let $\mathfrak{N}$ be the radical of $\mathfrak{A}$. Then, if

$\mathfrak{A}-\mathfrak{N}$ is separable (that is, semisimple for every scalar extension), $\mathfrak{A}=\mathfrak{S}+\mathfrak{N}$ where $\mathfrak{S}$ is equivalent to $\mathfrak{A}-\mathfrak{N}$. (Received January 26, 1948.)

236. Jean B. Walton: *Theta series in the Gaussian field.*

The functions $\vartheta_r(\tau, \rho, Q) = \sum_{\mu \equiv \rho (2Q)} \mu^r e^{2\pi i \tau \mu \mu'/4Q}$ for μ and ρ integers in the Gaussian field, Q a positive rational integer, τ a complex number with positive imaginary part, are defined and transformation formulae derived showing their behavior when $Q=1$ under arbitrary modular substitutions. Positive values of r are considered. The cases with $r=0$, 1 have been discussed by Hecke (Math. Ann. vol. 97 (1927) pp. 210–242). The functions are all invariant in the principal congruence subgroup modulo 4. If the concept of invariance is generalized to include as multiplicative factor a root of unity, the functions $A(\vartheta_r(\tau, 0, 1)+\vartheta_r(\tau, 1+i, 1))$ if $r\equiv 0 \bmod 4$ and $\vartheta_r(\tau, 1, 1)$ if $r\equiv 2 \bmod 4$ are found to be the only linear combinations of the theta series being invariant in a larger subgroup than before. By means of a general method suggested by Hecke (*Lectures on Dirichlet series, modular functions and quadratic forms*, Edwards Brothers, 1938) formulae are derived expressing these functions in terms of Jacobi's theta functions. (Received March 12, 1948.)

237*t*. Daniel Zelinsky: *On ordered loops.*

In a previous paper (Bull. Amer. Math. Soc. vol. 54 (1948) pp. 175–183) was stated the following necessary and sufficient condition for an ordered loop L to be the value loop of some (not necessarily associative) algebra with a unity quantity and of finite order over a field: L contains in its center a subloop G of finite index. In the present paper the author determines all such loops L as special loop extensions of finite centrally nilpotent loops (L/G) by order abelian groups (G). The central theorem proves the central nilpotence of the quotient loop L/G and of L itself by exhibiting a uniquely defined central series for L. The proof leans heavily on the fact that L is a topological loop in the interval topology. (Received March 2, 1948.)

238. Daniel Zelinsky: *Topological characterization of fields with valuations.*

Necessary and sufficient conditions for a topological field F to have a nonarchimedean valuation (values in any ordered group) preserving the topology of the field are (i) some neighborhood of zero in F generates a bounded additive group; and (ii) if $A\subset F$ and A is bounded away from zero, then A^{-1} is bounded. The fields satisfying (i) alone are the quotient fields of integral domains, topologized by designating the nonzero ideals of the integral domain as neighborhoods of zero. (Received March 2, 1948.)

Analysis

239. Lipman Bers: *Isolated singularities of minimal surfaces.*

Isolated singularities of functions satisfying the minimal surface equation (1) $(1+\phi_y^2)\phi_{xx}-2\phi_x\phi_y\phi_{xy}+(1+\phi_x^2)\phi_{yy}=0$ are classified. Only functions possessing single-valued gradients are considered. It turns out that, unlike harmonic functions, solutions of (1) possess only relatively simple singularities. Single-valued solutions possess at finite points only removable isolated singularities. A multiple-valued solution behaves in the neighborhood of a finite isolated singularity like the velocity potential of a flow due to a vortex. A solution ϕ regular in the neighborhood of infinity behaves there either like a harmonic function regular at infinity or like the velocity potential of a flow due to a source-vortex-doublet; in either case ϕ_x and ϕ_y possess finite limits at infinity. Parametric representations for these singularities are derived and a new proof is given for S. Bernstein's theorem which asserts that a solution of (1) regular at all finite points is a linear function. The author uses Chaplygin's gas-dynamical interpretation of the minimal surface equation, and some properties of one-to-one mappings by means of a pair of nonconjugate harmonic functions. It seems probable that similar results hold for a wide class of nonlinear partial differential equations. (Received March 11, 1948.)

240t. R. P. Boas, R. C. Buck, and Paul Erdös: *The set on which an entire function is small.*

Let $f(z)$ be an entire function, and let E_λ be the set of points for which $\log |f(z)| \leqq (1-\lambda) \log M(r)$, where $\lambda > 0$. Denote by $D^*(S)$ the upper density of the measurable set S, and by $D_*(S)$ the lower density. Then, there is a number K, dependent only upon λ, such that $D^*(E_\lambda) \leqq K$; moreover, $c^2/(1+c)^2 \leqq K \leqq \lambda^{-1}$, where c is the positive root of $c(2+c)^{\lambda-1} = 1$. It is also shown that $D_*(E_\lambda) = o(\lambda^{-1})$ as $\lambda \to \infty$. (Received March 3, 1948.)

241. Salomon Bochner and K. Chandrasekharan: *Gauss summability of trigonometric integrals.*

If $f(x_1, \cdots, x_k)$ is of period 2π in each variable and belongs to the Lebesgue class L_1, and if $f_p(t)$ and $S^\delta(R)$ denote respectively the pth spherical mean of f and the δth Riesz mean of its Fourier series at a given point $(x_1, \cdots, x_k)$, where $p \geqq 0$, $\delta \geqq 0$, then it is known that for $\delta > p+(k-1)/2$, $S^\delta(R) = R\int_0^\infty f_p(t)H(tR)dt$ where $H(x) = x^{\nu-1-2\delta}J_\nu(x)$, $\nu = p+\delta+k/2$ and J_ν is the Bessel function of order ν; it is also known that for large p and δ, $S^\delta(R)$ and $f_p(t)$ could be interchanged in the formula (Proc. Indian Acad. Sci. vol. 24 (1946) pp. 229–232). It is now proved that $S^\delta(R) = \lim_{\epsilon\to 0} R\int_0^\infty \exp(-\epsilon^2 t^2) f_p(t)H(tR)dt$ for *every* $\delta \geqq 0$, and that the inverse formula could also be similarly summed for every $p \geqq 0$. (Received March 4, 1948.)

242t. Orrin Frink: *A ratio test.*

The series $\sum a_n$ is absolutely convergent if lim sup $|a_n/a_{n-k}|^n < e^{-k}$. If $|a_n/a_{n-k}|^n \geqq e^{-k}$ for n sufficiently large, $k \neq 0$, then it is *not* absolutely convergent. If $\lim |a_n/a_{n-1}|^n = r$, then the abscissa of absolute convergence of the Dirichlet series $\sum a_n n^{-s}$ is $1+\log r$. (Received March 11, 1948.)

243t. P. R. Garabedian: *Schwarz's lemma and the Szegö kernel function.* Preliminary report.

Let D be a finite domain of connectivity n bounded by analytic curves C. The regular function $F_0(z)$ maximizing $|F'(z_1)|$ under the conditions $F(z_1)=0$, $|F(z)| \leqq 1$ in D is found directly by a method of contour integration analogous to that introduced by Grunsky. The zeros $z_2, \cdots, z_n$ of $F_0(z)$ are located by means of a Jacobi inversion problem. The function $f_0(z) = (z-z_1)^{-2} + a_{-1}(z-z_1)^{-1} + a_0 + a_1(z-z_1) + \cdots$ minimizing $l(f) = \int_C |f(z)| ds$ for functions with a pole of the type shown is found by the same method, using the fundamental relation $iF_0 f_0 z' < 0$ on C, where z' is the derivative of the parametric representation $z(s)$ of C. The Szegö kernel function of D (Math. Zeit. vol. 9 (1921) pp. 218–270) is found to be given by $4\pi^2 K(z, z_1)^2 = F_0(z)^2 f_0(z)$, and this leads to a formula for $K(z, z_1)$ in terms of Green's function and harmonic measures. The identities $2\pi |F_0'(z_1)| = l(f_0) = 4\pi^2 K(z_1, z_1)$ are proved. Thus there is a close relationship between the Szegö kernel function and Schwarz's lemma in multiply-connected domains. An application of these results is made to the problem of Painlevé concerning the existence of bounded analytic functions in the exterior of a compact set. (Received March 5, 1948.)

244t. P. R. Garabedian and M. M. Schiffer: *Identities in the theory of conformal mapping.*

The Green's function, Neumann's function, harmonic measures and canonical mapping functions associated with a multiply-connected domain of the complex plane and their derivatives satisfy various simple boundary conditions. These conditions are used in a method of contour integration similar to that exploited by Grunsky in his thesis and analogous to that employed in the classical theory of Abelian integrals on a Riemann surface to prove a large number of identities relating the domain functions named. A few of these identities have already appeared in the papers of Bergman and Schiffer concerning the theory of kernel functions. Here a systematic treatment of the identities is given, while at the same time geometric and extremal properties of the functions involved are discussed. Applications of the results of the paper are made to the theory of the variation of domain functions with their domain of definition. In particular, it is shown how variations of a multiply-connected domain can be made which preserve conformal type or which preserve boundary configurations. (Received March 4, 1948.)

245t. A. P. Hillman: *On the reality of zeros of Bessel functions.*

A necessary and sufficient condition for the existence of real zeros on any branch of a Bessel function of real order is obtained. This condition is applied for special cases such as real cylinder functions and Bessel functions of the first kind, of the second kind, of integral order, and of rational order. (Received March 8, 1948.)

246. Witold Hurewicz: *On stability of nonlinear operators.*

Let T be a continuous linear transformation of Hilbert space E_∞, defined over a neighborhood of the origin 0, and let $T(0)=0$. Suppose T is "differentiable" at the point 0, that is, there exists a continuous linear transformation L such that $\|L(x)-T(x)\|/\|x\|$ tends uniformly to zero when $x\rightarrow 0$. Let S be the spectrum of L (S is a point set in the complex plane). Assume that $\lambda\in S$ implies $\lambda\neq 0$ and $|\lambda|\neq 1$. Under these conditions the following theorem holds: If $|\lambda|<1$ for every $\lambda\in S$, the transformation T is *stable* at the origin, that is, $T^n(x)\rightarrow 0$ with $n\rightarrow\infty$, for all points x sufficiently near to the origin. If $|\lambda|>1$ for at least one point $\lambda\in S$, T is *unstable*, that is, given any pair of positive numbers (ϵ, δ) there exists a point $x\in E_\infty$ and a positive integer n, such that $0<\|x\|<\delta$, $\|T^n(x)\|>\epsilon$ (or $T^n(x)$ undefined). This result can be regarded as an extension of classical results of Poincaré-Liapounoff to the case of infinite number of dimensions. The proof is based on recent results of E. R. Lorch concerning linear operators. (Received March 22, 1948.)

247t. Meyer Karlin: *Note on the expansion of confluent hypergeometric functions in terms of Bessel functions of integral order.* Preliminary report.

An expansion of the confluent hypergeometric series $F(\alpha, \gamma; x)$ was obtained in terms of modified Bessel functions of the first kind $I_k(x)$ of integral orders by means of a recursion formula, that is well adaptable for computational purposes. This expansion has decided advantages over the power series, especially for large values of α/γ. An illustration is cited where eight terms in this expansion yield five significant figures; whereas hundreds of terms in the power series are required for a similar accuracy. (Received March 19, 1948.)

248*t*. V. K. Klee: *On the support property of a convex set in a linear normed space.*

Suppose that C is a closed convex set in the linear normed space S. The following theorems are proved: (1) if C is a cone $\neq S$, then C has at least one plane of support. (2) If C has an interior point, then C is supported at each of its boundary points. (3) If any one of the following conditions is satisfied, C is supported at a set of points dense in its boundary: (i) C is weakly compact; (ii) E is weakly compact, $S=E^*$, and C is weakly compact as a set of functionals; (iii) $S=E^*$ and C is transfinitely closed in S. (1) is used to prove (2) and (3). (2) is already known, but (1) and (3) have previously been proved only under considerably more restrictive assumptions. (Received March 12, 1948.)

249. H. L. Krall and Orrin Frink: *A new class of orthogonal polynomials: the Bessel polynomials.*

The *Bessel polynomials* are solutions of the differential equation $x^2y''+(2x+2)y' = n(n+1)y$, and are closely related to the Bessel functions of half-integral order. They are orthogonal with weight function $e^{-2/x}$ and path of integration the unit circle in the complex plane. This paper contains a detailed treatment of their theory, including the derivation of their recurrence relations, generating function, Rodrigues' formula, and normalizing factors. Applications to the theory of traveling waves are also given. In the second part of the paper a similar theory is developed for the *generalized Bessel polynomials*, which satisfy the differential equation $x^2y''+(ax+b)y' = n(n+a-1)y$. (Received March 9, 1948.)

250. J. L. Massera: *On the existence of periodic solutions of differential equations.*

Systems of the form $\dot{x}=X(x, t)$, where x is a vector of m-dimensions and X depends periodically on t with period 1, are considered. It is proved that: (a) If $m=1$, the existence of a solution which is bounded "in the future" (that is, for $0\leqq t<\infty$) implies the existence of a periodic solution of period 1; and any almost periodic solution is actually periodic of period 1; (b) If $m=2$, the existence of a bounded solution is not sufficient for the existence of a periodic solution of period 1, but it is so if, moreover, all the solutions exist in the future; (c) If $m>2$, the existence of a bounded solution and the existence of all the solutions in the future is not sufficient for the existence of a periodic solution. Furthermore, it is shown that the only necessary conditions on the number of the different harmonics and subharmonics are the compatibility with Theorems (a) and (b) and the obvious restriction that the number of solutions of period p should be a multiple of p. (Received March 5, 1948.)

251*t*. C. N. Moore: *On the relationship between the integral formulas of Cauchy and Poisson.*

A method is given of deriving Poisson's formula directly from that of Cauchy. By taking real parts on both sides of Cauchy's formula a preliminary expression for $u(x, y)$ is obtained in terms of an integral involving both u and v. The v is then eliminated by means of known relationships between conjugate functions. (Received March 12, 1948.)

252. Robert Schatten and John von Neumann. *The cross-space of linear transformations.* III.

The present notation is that of Bull. Amer. Math. Soc. Abstracts 51-9-162 and 52-1-22. Let $\mathfrak{B}_1$ and $\mathfrak{B}_2$ denote two Banach spaces and $\mathfrak{B}$ a Banach space of some linear transformations A from $\mathfrak{B}_1$ into $\mathfrak{B}_2^*$. An element A in $\mathfrak{B}$ possesses a norm $\|A\|$ which in general is different from its bound $|||A|||$. $\mathfrak{B}$ is termed an "ideal" if: (i) $A\in\mathfrak{B}$ implies $YAX\in\mathfrak{B}$ for any pair of linear transformations X and Y on $\mathfrak{B}_1$ and $\mathfrak{B}_2^*$ respectively, and (ii) $\|YAX\|\leqq|||Y|||\ \|A\|\ |||X|||$. The trace-class, the Schmidt-class, the space of all linear transformations on a Hilbert space $\mathfrak{H}$ furnish such ideals. The class of crossnorms α for which $(\mathfrak{B}\otimes_\alpha\mathfrak{B}_2)^*$ is an ideal is characterized. Furthermore, $(\mathfrak{H}\otimes_\alpha\mathfrak{H})^*$ is an ideal if and only if α is unitarily invariant. If in addition $\alpha\leqq\sigma$ (where σ represents the Schmidt-crossnorm on $\mathfrak{H}\odot\mathfrak{H}$) we have $(\mathfrak{H}\otimes_\alpha\mathfrak{H})^*=\mathfrak{H}\otimes_{\alpha'}\mathfrak{H}$ whenever for a complete orthonormal set (ϕ_i) and every sequence of real numbers $\{a_i\}$ with $\lim a_i=0$, the condition $\lim\alpha'(\sum_{i=1}^n a_i\phi_i\otimes\phi_i)<+\infty$ as $n\to\infty$ implies $\lim\alpha'(\sum_{i=m}^n a_i\phi_i\otimes\phi_i)=0$ as m and $n\to\infty$. In particular, we have $(\mathfrak{H}\otimes_\lambda\mathfrak{H})^*=\mathfrak{H}\otimes_\gamma\mathfrak{H}$. (Received February 13, 1948.)

253*t*. I. M. Sheffer. *On the theory of sum-equations.*

A system of sum-equations is one for the form $\sum_{j=0}^\infty a_{n,j}x_{n+j}=c_n$ $(n=0,1,\cdots)$. The k-periodic case, that is, where there is a positive integer k such that $a_{p+sk,j}=a_{p,j}$ for $p=0,1,\cdots,k-1$; j, $s=0,1,\cdots$ was treated earlier, as also the k-periodic case with perturbations. It is now shown: (a) that solutions of the k-periodic case can be expressed in closed form by means of contour integrals; and (b) that under certain conditions the nonperiodic case has a solution obtained from (a) by a limiting process. (Received March 12, 1948.)

254. W. H. Spragens. *On the absolute Cesàro summability of double Fourier series.*

Let $\sum a_{mn}$ be a double series and denote by $s_{mn}^{\alpha\beta}$ the m, nth Cesàro partial sum of order α, β. The series is said to be summable $|C,\alpha,\beta|$ if $\sum|s_{mn}^{\alpha\beta}-s_{m-1,n}^{\alpha\beta}-s_{m,n-1}^{\alpha\beta}+s_{m-1,n-1}^{\alpha\beta}|$ converges. Sufficient conditions and a necessary condition on $f(x,y)$ for the absolute summability of its double Fourier series are obtained. These results are analogues of those for single series proved by Bosanquet (Proc. London Math. Soc. (2) vol. 41 (1936) pp. 519–527) and Hyslop (Proc. London Math. Soc. (2) vol. 43 (1937) pp. 475–483). (Received March 11, 1948.)

255*t*. J. L. Walsh. *On the critical points of real rational functions.*

Let the circular region C: $|z-bi|\leqq r$ and $\overline{C}$: $|z+bi|\leqq r$ contain all zeros of a real rational function $R(z)$ whose poles are all real. The critical points of $R(z)$ lie on the set consisting of the axis of reals plus (i) the closed interiors of C and $\overline{C}$ if we have $b\geqq 2r$; (ii) the closed interiors of the circles of the coaxal family determined by C and $\overline{C}$ passing through the points $\pm i[(b^2-2r^2)/2]^{1/2}$ if we have $2r>b>2^{1/2}r$. (Received April 8, 1948.)

256. Alexander Weinstein: *Separation theorems for the eigenvalues of partial differential equations.*

It is shown by a variational method introduced in previous papers (A. Weinstein, C. R. Acad. Sci. Paris (1935) and Mémorial des Sciences Mathématiques, No. 88, 1937, Bull. Amer. Math. Soc. vol. 49 (1943) p. 368) that some results connected with the oscillation theorems of the theory of ordinary differential equations can be extended to the field of partial differential equations. Let S be a domain in m dimensions and let f be a given function on its boundary C with a vanishing mean value $M(f)$. It is shown that the eigenvalues $\lambda=\lambda_k$ of the equation $\Delta u+\lambda u=0$ with the boundary conditions: (1) $M(fu)=0$; (2) $\partial u/\partial n$ congruent zero modulo f, separate the eigenvalues $\lambda=\omega_k$ of the same equation with the boundary condition $\partial u/\partial n=0$, namely that $\omega_k\leqq\lambda_k\leqq\omega_{k+1}\leqq\lambda_{k+1}$. For $m=1$ this theorem reduces to the corresponding result about the eigenvalues of the periodic eigenfunctions of the equation $u''+\lambda\mu=0$. (Received March 9, 1948.)

Applied Mathematics

257*t*. Garrett Birkhoff: *Dimensional analysis and its generalizations.*

Buckingham's pi theorem is proved without any restrictions on the functions involved. The assumption of differentiability in Bridgman's characterization of homogeneous variables by the property of "absolute invariance of relative magnitude" is replaced by a weak continuity assumption. Dimensional analysis can be applied to problems whose formulation is invariant under a group of transformations of the form $q_i\rightarrow\lambda_i q_i$ $(i=1,\cdots,n)$. It ordinarily permits a reduction by n in the number of variables or parameters which need be considered. It is shown that this is true of any group, and that the same principle can be applied to systems of partial differential equations. This general principle, whose explicit formulation seems to be new, is then applied to obtain as special cases well known solutions of the heat equation and nonlinear partial differential equations of fluid dynamics. (Received February 4, 1948.)

258*t*. Garrett Birkhoff and Milton Plesset: *Wall corrections for cavity flow.*

Using formulas previously derived by Rethy, Cisotti, and Mises, wall corrections are obtained for two-dimensional flow with cavity ("wake") past a vertical flat plate in the middle of a water tunnel with fixed walls, in a free jet, and in a jet issuing from a channel. It is shown that the method of Mises can be extended to any flow in which the hodograph is a circular sector, and the diagram in the complex potential plane is a plane, half-plane, or infinite strip, with or without cuts. The indefinite integrals involved are the same, and can be evaluated explicitly in terms of complex logarithms if the central angle of the circular sector is commensurable with 2π. The problems of numerical computation are discussed, and a "principle of stability of the pressure coefficient" is obtained in the fixed wall case. (Received March 19, 1948.)

259. A. E. Heins: *The effect of a submerged barrier on water waves in a channel of finite depth.*

The author is concerned with the solution of (*) $\nabla^2\phi=0$ in the strip $-\infty<x<\infty$, $0\leqq y\leqq a$ subject to the conditions (i) $\partial\phi/\partial y=0$ for all x, and $y=0$, (ii) $\partial\phi/\partial y=0$, $y=b$ $(b<a)$, $x<0$, (iii) $\partial\phi/\partial y=\beta\phi$ $(\beta>0)$ $y=a$, for all x. A travelling wave solution in the regions $b\leqq y\leqq a$, $x<0$, and $0\leqq y\leqq a$, $x>0$, is found. The plane $y=b$, $x<0$ is the submerged barrier. The problem is formulated as an integral equation of the Wiener-Hopf type in terms of a Green's function kernel which may be exhibited explicitly. An explicit solution as an infinite series or a contour integral is derived and the asymptotic properties of the solution are discussed. (Received March 1, 1948.)

260. Brockway McMillan: *Realizability of passive networks*. Preliminary report.

A $2n$-pole is a network of lumped elements having n terminal pairs T_r, T_r' $(1 \leqq r \leqq n)$ across which external generators may be connected. If the currents $j_r e^{pt}$ in these external meshes are unrestricted, then the $2n$-pole has an impedance operator $Z(p)$ such that $f = Z(p)j$, where $f_r e^{pt}$ is the voltage between T_r and T_r', and the f_r and j_r are components of vectors f and j in a unitary space. R. M. Foster (1924) gave conditions necessary and sufficient that a scalar $Z(p)$ be the impedance of a 2-pole of reactances only. O. Brune (1932) extended these to a general passive 2-pole. Using ideal transformers: (i) W. Cauer (1932) proved that the operator $Z(p)$ is the impedance of a reactance $2n$-pole if and only if $\phi_j(p) = [Z(p)j, j]$ satisfies Foster's scalar criterion for every vector j; (ii) in the present work the author proves that $Z(p)$ is the impedance of a general passive $2n$-pole if and only if $\phi_j(p)$ satisfies Brune's criterion for every j. Some simple side conditions common to (i) and (ii) are unstated. (Received March 12, 1948.)

261*t*. H. E. Salzer: *Coefficients for facilitating trigonometric interpolation*.

The Gauss formula (1) $f(x) = \sum_{i=0}^{2n} [\prod_{j=0}^{\prime 2n} \sin 2^{-1}(x - x_j) / \prod_{j=0}^{\prime 2n} \sin 2^{-1}(x_i - x_j)] f_i$ for trigonometric interpolation, the symbol $\prod'$ denoting the absence of a factor having $j = i$, involves a large amount of computational labor, even when the points x_i are equally spaced. By generalization of a method originally given by W. J. Taylor for Lagrangian interpolation only, the work in employing (1) is reduced considerably with the aid of the auxiliary coefficients $A_i^{(2n+1)} \equiv 1/\prod_{j=0}^{\prime 2n} \sin 2^{-1}(x_i - x_j)$. Unlike the corresponding quantities for Lagrangian interpolation, separate sets of $A_i^{(2n+1)}$ are required for each interval in x. The coefficients $A_i^{(2n+1)} \equiv A_{2n-i}^{2n+1}$ are tabulated to 8*S* for the 3-, 5-, 7-, 9-, and 11-point cases, all at intervals in x equal to 1.0, 0.5, 0.2, 0.1, 0.05, 0.02 and 0.01; also $A_i^{(2n+1)}$ are given to 8*S* for functions tabulated at $2n+1$ equally spaced points over a range of π and $\pi/2$, for $2n+1 = 3(2)11$. The extension of this method to interpolate for other special types of functions besides polynomials and trigonometric sums is briefly indicated. (Received February 2, 1948.)

262*t*. H. E. Salzer: *Formulas for complex Cartesian interpolation of higher degree*.

In interpolating by Lagrange's formula for an analytic function tabulated over a square grid, $f(z) = f(z_0 + Ph)$ where h is the length of a square, if in $P = p + iq$ either p or q is finer than 0.1, or if the interpolation requires eight or nine points, no Lagrangian coefficients (polynomials in P) are available, and their calculation involves a prohibitive amount of labor. But generalization of a scheme originated by W. J. Taylor, to hold for complex interpolation based upon any set of fixed points, leads to a shorter method of interpolation which employs auxiliary coefficients A_k. These quantities A_k are tabulated for various configurations of the fixed points z_k, ranging from the three- to nine-point cases. In the five- to nine-point cases, the quantities A_k are given for at least two configurations which are applicable everywhere in a table, and for one to three additional configurations (for each degree) which will locate the argument z more centrally with respect to the z_k's. The quantities A_k are also useful for (a) checking purposes, since they are the coefficients of f_k in the complex divided differences, and for (b) performing complex extrapolation. (Received February 13, 1948.)

263t. Alexander Weinstein: *On surface waves.*

The mixed boundary value problems considered by H. Lewy (Bull. Amer. Math. Soc. vol. 52 (1946) p. 737), J. J. Stoker (Quarterly of Applied Mathematics vol. 5 (1947) p. 1) and others in connection with the theory of surface waves are reduced by a conformal transformation to the following problem: to find all harmonic functions vanishing on the boundaries of a parallel strip (see A. Weinstein, *Zum Phragmen-Lindeloefschen Ideenkreis*, Abh. Math. Sem. Hamburgischen Univ., 1928). This procedure allows one to find a more general type of solutions than those considered previously. (Received March 18, 1948.)

Geometry

264t. Edward Kasner and John DeCicco: *Generalization of Appell's transformation.*

The authors extend Appell's theorem concerning the transformation theory of the dynamical trajectories of a field of force from positional fields to generalized fields where the force vector depends not only on the position of the point but also on the direction through the point. The required transformations for generalized fields are those of the mixed projective group of eight-parameters consisting of collineations and correlations together with a time factor. This time factor depends in the case of a collineation only on the particular collineation up to an arbitrary function of two variables; and in the case of a correlation, it depends not only on the particular correlation but also on the particular generalized field under consideration up to an arbitrary function of two variables. This is a new phenomenon. If both fields are positional, the authors' theorems give Appell's theorem as a special case. Then the only appropriate transformations are the collineations and the time factor depends only on the collineation used up to an arbitrary constant factor. (Received December 30, 1947.)

265t. Edward Kasner and John DeCicco: *Physical curves in generalized fields of force.*

The authors study the geometry of systems S_k of ∞^3 curves in a generalized field of force where the force vector depends not only on the position of the point but also on the direction through the point. A system S_k consists of curves along which a constrained motion is possible so that the pressure P is proportional to the normal component N of the force vector. The important systems of physical interest are (a) the system S_0 of dynamical trajectories, (b) the system S_1 of generalized catenaries, (c) the system S_{-2} of generalized brachistrochrones, (d) the systems S_∞ of generalized velocity systems. Every system S_k is given by a differential equation of the type (G) and thus possesses the Geometric Property I. Velocity systems are curvature trajectories. The ratio of the curvature of the curves of S_k at the points where the speed of the particle is zero to the curvature of the tangent line of force is studied. Finally the transformation theory of systems S_k is developed. (Received February 4, 1948.)

266t. Edward Kasner and John DeCicco: *Supplementary theorems in dynamics.*

The authors discuss geometric properties I, II, III of triply infinite families of trajectories in an arbitrary positional field of force. New proofs in parametric form of the known properties are given and additional properties are discussed. The distinction between actual and virtual trajectories is carefully considered. (Received February 4, 1948.)

267t. Edward Kasner and John DeCicco: *The osculating conics of physical families of curves.*

Consider the ∞^3 curves of a system S_k connected with any arbitrary positional field of force in the plane. A curve of a system S_k is a locus along which a constrained motion is possible so that the pressure P is proportional to the normal component N of the force vector. Thus $P=kN$. The four important physical subcases are (1) the systems S_0 of trajectories, (2) the systems S_1 of catenaries, (3) the systems S_{-2} of brachistochrones, (4) the systems S_∞ of velocity curves. Construct the osculating conics of fourth order contact to the ∞^3 curves of a system S_k. Through any general lineal-element E, there pass ∞^1 conics. Their centers describe a conic and also the envelope is a conic. By varying k, it is found that the central conics and also the enveloping conics form quadratic families. The system S_0 of trajectories are of special interest. The osculating conics of S_0 passing through a given lineal-element E always touch two fixed lines which intersect on the line of the force vector. The centers describe a straight line. (Received March 17, 1948.)

268. Don Mittleman: *The unions of trajectorial series of lineal elements generated by the plane motion of a rigid body.*

The motion of a plane, rigid body moving freely in a plane under the influence of any force which depends solely on the position of the body may be described by the motion of a lineal element composed of the center of gravity and a principal axis of inertia. If the maximum diameter of the body approaches zero, limiting forms of the equations of motion of the rigid body, called the equations of motion of an *element-particle*, are obtained. The concept of *microscopic body* is introduced also, and the equations of motion for such a body are determined. A trajectory in each of the three cases is a series of lineal elements. For each case, the totality of trajectories in a given field is ∞^5 series; among these series there may be trajectorial unions. The unions associated with the motion of an element-particle are studied in detail: for no field of force can one find ∞^3 unions among the ∞^5 series; in some fields of force one can find ∞^2 unions. Necessary and sufficient conditions that ∞^2 curves be trajectorial unions are given geometrically. (Received March 10, 1948.)

Logic and Foundations

269. E. L. Post: *Degrees of recursive unsolvability.* Preliminary report.

The author's canonical sets (Amer. J. Math. vol. 65) are generalized to S-canonical sets by hypothetically adding primitive assertions representing the membership or non-membership of $1, 2, 3, \cdots$ in set of positive integers S. Set S_1 of positive integers is proved (Turing) reducible to S (Post, Bull. Amer. Math. Soc. vol. 50) when and only when S_1 and its complement are S-canonical sets. A "complete" S-canonical set S' is set up, and it is proved that each S-canonical set is reducible to S', while S' is not reducible to S. With S, say, the null set K_0, a scale of increasing degrees of unsolvability is thus furnished by $K_1, K_2, K_3, \cdots, K_{n+1}=K_n'$. The main completed result is a strengthened Kleene Theorem II, Trans. Amer. Math. Soc. vol. 53: each class of sets in Kleene's $(n+1)$st column includes a set of higher degree of unsolvability than any set common to both classes. Work is in progress on further equivalence proofs, further applications of the K_n-scale, incomparable degrees of unsolvability related to the K_n-scale, extension of the main theorem to Mostowski, Fund. Math. vol. 34, and extension of the K_n-scale into the constructive transfinite. (Received March 17, 1948.)

270*t*. A. R. Schweitzer: *On the principles of correspondence and continuation in heuristic mathematics.*

In his monograph, *Theorie und Anwendung der Laplace-Transformation* (Berlin, 1937; New York, 1943) Gustav Doetsch mentions two principles which guide the development of his discussion, namely, (1) "Abbildungsprinzip" (p. 146; see also pp. 174, 222, 223), (2) "Fortsetzungsprinzip" (pp. 285–286). In this paper principle (1) is found to be similar to the principle of correspondence stated by the author in his article, *Les idées directrices de la logique génétique des mathématiques* (Rev. de Mét. et de Mor., Paris, 1914, pp. 174–196) and which is attributed by the author to W. K. Clifford (loc. cit. p. 189). Principle (2) is interpreted as a special case of the author's principle of continuation (loc. cit. p. 188) to which the preceding principle of correspondence is subordinated. The effect of these relationships is to extend the application of the author's principle of continuation to mathematical physics as outlined by Doetsch in his treatise. Reference is made to the author's interpretation of the principle of Dirichlet (loc. cit. p. 192; Doetsch, loc. cit. p. 378) in the theory of the potential. Also the author's principle of continuation is again seen to act as a bond connecting analysis with other branches of mathematics. (Received March 9, 1948.)

Statistics and Probability

271*t*. T. W. Anderson: *The theory of testing serial correlation.*

Tests of significance of $\lambda=0$ are considered when the observations have a normal distribution with exponent $-2^{-1}\alpha\{(X-M)'\Psi(X-M)+\lambda(X-M)'\Theta(X-M)\}$, where X relates to the vector of observations, M is its expected value (a regression on vectors of "fixed variates"), and Ψ and Θ are specified matrices such that for values of λ under consideration $\Psi+\lambda\Theta$ is positive definite. The class of all similar regions is deduced. The uniformly most powerful tests for one-sided alternatives and the B_1 test for two-sided alternatives are derived under the condition that the fixed variate vectors are characteristic vectors of Θ normalized with respect to Ψ. This theory is applied to cases where Ψ and Θ are defined so that the density is one of a serially correlated population. If the fixed variate vectors consist of certain trigonometric terms, the best tests for a circular population are given by inequalities on the circular serial correlation coefficient of residuals. In each of two other cases a serial correlation coefficient treated in the literature is obtained for defining the best tests. Significance levels are given for two cases of special interest. (Received March 10, 1948.)

272*t*. Garrett Birkhoff: *Note on Poincaré's recurrence theorem.*

Kac has recently extended Poincaré's recurrence theorem to stochastic processes involving discrete states (Bull. Amer. Math. Soc. vol. 53 (1947) pp. 1002–1010). The recurrence theorem is now extended to arbitrary stochastic processes, in the following form. Let T be any transition operator, with $eT=T$ and $0\leqq f\leqq e$. Then

$\lim_{m\to\infty} f \cap m \limsup_{n\to\infty} \{fT^n\} = f$. This is obviously equivalent to the classical recurrence theorem in the deterministic case. The simpler statement $f \cap \limsup_{n\to\infty} \{fT^n\} = f$ is unfortunately not generally true, except in the deterministic case. (Received March 19, 1948.)

273*t*. K. L. Chung: *Asymptotic distribution of the maximum cumulative sum of independent random variables.*

Certain distributions considered by Erdös-Kac and Wald are derived from a classical formula due to DeMoivre. An estimate of the remainder is obtained in each case. For Bernoullian variables this is of the order of magnitude $n^{-1/2}$. For more general cases a recent theorem due to H. Bergström is used and the order of magnitude is a certain negative power of n. This seems amply sufficient for any "strong" theorem we may wish to prove. (Received March 1, 1948.)

274. Paul Erdös: *On a theorem of Robbins and Shu.*

Let $f_1(x)$, $f_2(x)$, $\cdots$ be an infinite sequence of independent functions in (0, 1) all having the same distribution function $G(x)$. Assume that $\int_{-\infty}^{\infty} x dG(x) = 0$, $\int_{-\infty}^{\infty} x^2 dG(x) < \infty$. A theorem of Robbins and Shu state that if M_n denotes the measure of the set in x for which $\left| \sum_{k=1}^{n} f_k(x) \right| > n$ then $\sum_{n=1}^{\infty} M_n$ converges. They also conjecture the converse, namely that if $\sum_{n=1}^{\infty} M_n$ converges then $\int_{-\infty}^{\infty} x dG(x) = 0$ and $\int_{-\infty}^{\infty} x^2 dG(x) < \infty$. The author succeeded in proving this conjecture and also obtained a simpler proof of the theorem of Robbins and Shu. (Received March 9, 1948.)

275. Bernard Friedman: *A simple type of Markoff chain.* Preliminary report.

Let W_n, X_n $(n = 1, 2, \cdots)$ be random variables such that $W_{n+1} = W_n + X_n$ and such that the probability distribution of X_n depends linearly on W_n. If $f_n(t)$ is the generating function of W_n, we find that $f_{n+1}(t) = m_0(t) f_n(t) + m_1(t) f_n'(t)$ where $m_0(t)$ and $m_1(t)$ are known functions. A method for solving this difference-differential equation is given. The results can be specialized to obtain the probabilities in an urn problem considered by Pólya and Eggenberger (Zeitschrift für Angewandte Mathematik und Mechanik vol. 3 (1923) p. 279) and also the probabilities in a random walk with elastic restoring force (M. Kac. Amer. Math. Monthly vol. 54 (1947) p. 369). (Received March 12, 1948.

276*t*. Wassily Hoeffding and Herbert Robbins: *A central limit theorem for dependent random variables.*

A sequence of random variables or vectors is called m-dependent if any two sets of terms are independent whenever they are separated by more than m intervening terms, $m \geqq 0$. Under certain mild conditions (*not* involving conditional expectations) the central limit theorem for sums holds for m-dependent sequences. (Received March 19, 1948.)

277*t*. W. G. Madow: *On the limiting distributions of statistics based on samples of finite populations.*

Let us suppose that a random sample is selected without replacement from a finite population. Then under very general conditions concerning the "growth" of the population, it is shown that the limiting distribution of the sample mean is normal. This result is then applied to obtain the limiting distribution of more general statistics. The method of obtaining the basic distribution is through the calculation of the moments. Once the basic distribution is obtained conclusions are drawn concerning the convergence of the characteristic functions of other statistics. (Received March 11, 1948.)

278*t*. F. J. Murray: *On locally dependent chance variables.* Preliminary report.

Consider a set R of chance variables ρ_T such that each ρ_T is independent of all but a certain subset $U(\rho_T)$ of R. The number of elements in $U(\rho_T)$ is supposed to be bounded by a number k which is small relative to N, the number of elements in R. Let Σ denote the sum of the ρ_T. The objective of the present paper is to extract the Nth root of the characteristic function of Σ and for this purpose it is shown that the semi-invariants of Σ are essentially proportional to N under these circumstances. Convergence questions are also considered. This result is applicable to the order-disorder problem in a crystal lattice and this connection is also given. (Received April 12, 1948.)

279. Edward Paulson: *Some limiting distributions associated with the two box problem.*

Let balls be thrown into either box $B^{(1)}$ or $B^{(2)}$ with probability p of falling in $B^{(1)}$. Let n denote the random number of throws required to reach a prescribed pattern of balls. McCarthy (Ann. Math. Statist. vol. 18 (1947) pp. 349–384) obtained the moments of n for several prescribed patterns. The limiting distribution of n required to get at least k_1 in $B^{(1)}$ and at least k_2 in $B^{(2)}$ is now shown to be normal. In the same problem, let n_1 equal number of throws needed until one box contains required number, and n_2 the additional number needed to complete experiment; the limiting distribution of n_1 and n_2 is shown to be bivariate normal. Consider probability $C_{n-1,k-1}p^k(1-p)^{n-k}$ that n throws are required to get k balls in $B^{(1)}$. Put $p=1-1/s$, let k be a random variable depending on parameter s in such a manner that $E(k)=As$, $\sigma^2(k)=Bs$ (A, B constants), and the limiting distribution of k as $s\to\infty$ is normal. It is shown that $\lim_{s\to\infty}$ [Probability $\{n-k=i\}]=e^{-A}A^i/i!$. In same situation, keep p fixed, then the limiting distribution of n is normal. This last result has been generalized. (Received February 28, 1948.)

280. Herman Rubin: *Some extensions of the central limit theorem to dependent variables.*

The method of Lindeberg and Lévy is applied to the distribution of real-valued functions of the values of a discrete stochastic process with values in an arbitrary space. The special case in which the space is the unit interval with Borel field the ordinary Borel sets is first considered, and the results are obtained in that case by a modification of the proofs given by Lévy and Loève. The results in the general case follow immediately from those of the special case. (Received March 10, 1948.)

281. J. W. Tukey: *Asymptotic moments and expectations.*

Let $Z_n\|I$ denote the chance quantity Z_n *restricted* to I, that is to say, the chance quantity whose values all lie in I with probabilities proportional to those of Z_n. If $\Pr\{Z_n \text{ not in } I\}$ vanishes more rapidly than the variance of $Z_n\|I$, the variance of the sequence $\{Z_n \| I\}$ is the *asymptotic variance* of $\{Z_n\}$. The choice of another such *adequate* interval does not change the order of the asymptotic variance. If every open interval containing z_0 is adequate for $\{Z_n\}$, then, under mild conditions: (I) the asymptotic variance of $\{g(Z_n)\}$ is the product of the asymptotic variance of $\{Z_n\}$ and $(g'(z_0))^2$; (II) the *asymptotic mean* (in a similar sense) of $\{g(Z_n)\}$ depends on $g(z_0)$, $g'(z_0)$, and $g''(z_0)$ in a well defined way; (III) the higher *asymptotic moments* vanish (to the order to which they are unique); (IV) if the *asymptotically standardized* distributions corresponding to $\{Z_n\}$ converge to a distribution with unit variance, then the asymptotically standardized distributions corresponding to $g\{Z_n\}$ converge to the same limiting distribution. These results are extended to the case of a sequence of functions $g_n(Z)$, and applied to the special case of estimation of scale by equating the average pth power of its expected value. (Received March 15, 1948.)

Topology

282t. Jean M. Boyer and D. W. Hall: *On a certain Peano space.*

The authors show that any Peano space M can be regarded as the continuous image of a rather simple Peano space L containing the unit interval I in such a way that I maps under the transformation onto a preassigned Peano space P contained in M, while the image of each of the at most denumerable collection of components of $L-I$ has at most one point in common with P. Applications to structure theory of Peano spaces are drawn including a very simple proof of the cyclic connectivity theorem along lines suggested by W. L. Ayres. (Received March 25, 1948.)

283. C. H. Dowker: *Retracts of metric spaces.*

A metric space is called an absolute neighborhood retract (ANR) if it is a neighborhood retract of every metric space containing it as a closed subset. Let R be a metric (but not necessarily separable) space. The following conditions hold if and only if R is an ANR: (1) Every mapping into R of a closed subset A of a metric space B can be extended over a neighborhood of A in B. (2) There is a homeomorphism f of R into a generalized Hilbert space H such that $f(R)\times 0$ is a neighborhood retract of the subspace $f(R)\times 0+H\times(0,1]$ of $H\times[0,1]$. (3) For every $\epsilon>0$ there is a natural polytope P and mappings $f: R\rightarrow P$, $g: P\rightarrow R$ such that $gf: R\rightarrow R$ is ϵ-homotopic to the identity. (4) Every open covering σ of R has a refinement τ such that a partial realization of a singular complex in τ (that is, with each partial cell contained in a set of τ) can be extended to a full realization in σ. (Received March 17, 1948.)

284t. Beno Eckmann: *Coverings and Betti numbers.*

Let the finite polyhedron P be a regular covering of the polyhedron $\overline{P}$, and let G be the corresponding group of covering transformations of P. The nth homology group H_n of P with real coefficients is a real vector group of finite rank, and G acts on H_n as a group of linear transformations. It is proved that the homology group $\overline{H}_n$ of $\overline{P}$ with real coefficients is determined by H_n and by the operation of G on H_n. Namely, when $s_n(x)$, $x\in G$, denotes the character of the linear representation of G in H_n, and g the order of G, the nth Betti number $\overline{p}_n$ of $\overline{P}$ is equal to $1/g\sum_{x\in G}s_n(x)$. As an application, relations are established between the Betti numbers of a closed nonorientable manifold $\overline{M}$ and those of a two-sheeted orientable covering of $\overline{M}$. (Received February 24, 1948.)

285*t*. C. J. Everett and S. M. Ulam: *On the problem of determination of mathematical structures by their endomorphisms.*

As examples of the general problem the following are noted: (1) Given the transformations $n \rightarrow kn$ for all integers n, the endomorphisms of the additive group of integers, what are all abelian groups definable on the set of integers, admitting the given class of transformations as endomorphisms? A general class of such groups is obtained. The same problem can be studied for the case of the rational, real, and complex fields. (2) Given the class of all homeomorphisms of a topological space, what other topologies exist on the same set which have these transformations as the class of all their homeomorphisms? It is easy to find examples (including the line) where the topology is uniquely determined by the class of its homeomorphisms. The problem in an algebraic formulation originated in conversations with E. Teller and J. v. Neumann. (Received March 12, 1948.)

286*t*. M. K. Fort: *A unified theory of semi-continuity.* Preliminary report.

Let M be a metric space with metric ρ, and let P be the set of all positive real numbers. A *continuity structure* for M is an ordered triple $(\bigcirc, F, \square)$, such that: $\bigcirc$ is one of the relations $>$ or $<$, F is a function whose domain is $P \times M$, $\square$ is a relation which partially orders the range of F, and $F(h, x) \square F(k, y)$ for all $h, k \in P$ and all $x, y \in M$ such that $h \bigcirc k$ and $\rho(x, y) < |h-k|$. A function f on a topological space into M is $(\bigcirc, F, \square)$-continuous at a point p if corresponding to each pair $h, k \in P$ such that $h \bigcirc k$ there is a neighborhood V of p such that $F(h, f(p)) \square F(k, f(q))$ for all $q \in V$. Some theorems about $(\bigcirc, F, \square)$-continuous functions are proved. If the proper continuity structures are chosen, this theory specializes to yield some of the classical theorems about semi-continuous real-valued functions (including a theorem concerning the category of the set of continuity points of a semi-continuous function). It is possible to choose other continuity structures and, by making use of hyperspaces, obtain theorems about semi-continuous set-valued functions (including semi-continuous decompositions). (Received February 27, 1948.)

287. A. N. Milgram: *Isomorphisms of the semigroup of continuous functions on a bicompact Hausdorff space.* II.

Let X be a bicompact Hausdorff space and $C(X)$ the semigroup of continuous functions with multiplication defined pointwise; $f \cdot g(x) = f(x) \cdot g(x)$. If $p(x)$ is a positive function in $C(X)$ denote by E_p the automorphism E_p: $f \rightarrow f$ where $f(x) = \operatorname{sgn} f(x) \cdot |f(x)|^{p(x)}$. Call such an automorphism an exponential automorphism of the semigroup $C(X)$. Let $\tau_1, \cdots, \tau_n$ be n automorphisms of the semigroup of the real numbers, and let $x_1, \cdots, x_n$ be n isolated points of X. Define the automorphism T of $C(X)$ by $T: f \rightarrow \bar{f}$ where $\bar{f}(x_i) = \tau_i(f(x_i))$ and $\bar{f}(x) = f(x)$ for all $x \neq x_1, \cdots, x_n$. Call T a finite automorphism. If H is a homeomorphism of X on X', define the isomorphism H^* of $C(X)$ on $C(X')$ by $H^*: f \rightarrow f'$, $f(x) = f'(H(x))$. The theorem can now be stated: If X and X' are spaces satisfying the first denumerability axiom of Hausdorff and σ is an isomorphism of $C(X)$ on $C(X')$ there exists a homeomorphism H of X on X', a finite automorphism T, and an exponential automorphism E_p such that $\sigma = T \cdot E_p \cdot H^*$. (Received March 11, 1948.)

288. C. W. Saalfrank: *Retraction properties for normal Hausdorff spaces.*

The author develops a retraction theory for normal Hausdorff spaces, NHS. AR and ANR are defined for NHS and are characterized as homeomorphs of retracts and closed neighborhood retracts, respectively, of Tychnoff cubes. It is shown that the topological product of any set of AR or any finite set of ANR is an AR or an ANR, respectively. It is proved that AR have the fixed point property and that for ANR any null-homotopic map has a fixed point. Borsuk's theorem is extended as follows. Let C be a closed subset of a compact Hausdorff space X and let B be a retract of an AR set A. Then for any map f such that f: $(X\times B)\cup(C\times A)\rightarrow N$, where N is an ANR, there exists an extension F of f over $X\times A$ such that F: $X\times A\rightarrow N$. A similar result is proved in which N is any topological space, but C is a closed neighborhood retract of an ANR set X and $X\times A$ is perfectly normal. The author exhibits a finite-dimensional, compact, locally contractile, Hausdorff space which is not an ANR and thus shows that a rather natural conjecture for the characterization of finite-dimensional ANR is impossible. (Received February 24, 1948.)

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