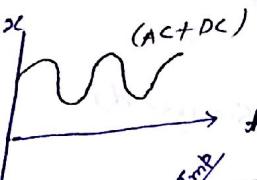
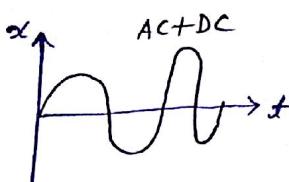
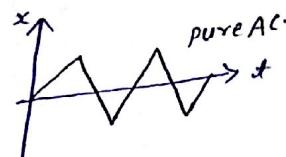
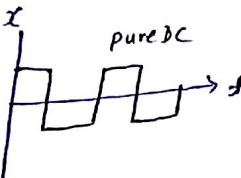
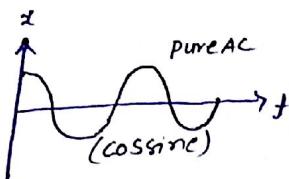
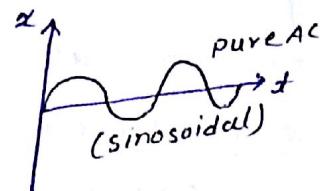
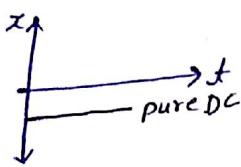


# ALTERNATING CURRENT

Electrical signal

→ Direct (unidirectional,  $\phi$  const)

→ Alternating (Bidirectional, periodic, positive peak equal to negative peak position.)



\* AC measured by Hot wire instrument  
↳ Heating Effect of current prove AC.

\* DC measured by Moving coil galvanometer.  
↳ proved by magnetic Effect of current.

Fundamental Alternating signal.

$$x = x_0 \sin(\omega t \pm \phi)$$

$x$  → Instantaneous value

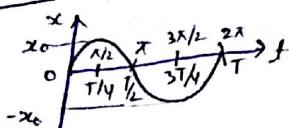
$x_0$  → Peak value

$2x_0$  → Peak to peak value

Phase Angle →  $\omega t + \phi$

$\omega$  → Angular frequency

$$\text{case-I } x = x_0 \cos \omega t$$



$$2\pi = T$$

$$\text{eg} \rightarrow \frac{\pi}{3} = \frac{T}{6}$$

$$\frac{\pi}{4} = \frac{T}{8}$$

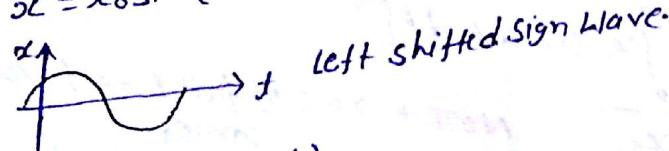
$$\frac{\pi}{6} = \frac{T}{12}$$

In one cycle direction change twice

0 -  $T/2$  +ve Half cycle.

$T/2 - T$  -ve half cycle.

$$\text{case-II } x = x_0 \sin(\omega t + \phi)$$

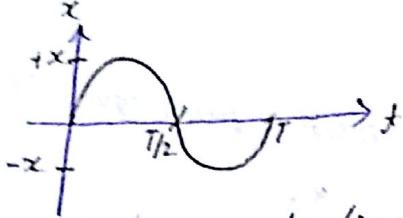


$$\text{case-III } x = x_0 \sin(\omega t - \phi)$$



## Fundamental Alternating signal

$$x = x_0 \sin \omega t$$



|I| → Average/mean value/DC voltage

case-I → For complete cycle.

$$\langle x \rangle = \frac{1}{T} \int_0^T x dt$$

$$= \frac{1}{T} \int_0^T x_0 \sin \omega t dt$$

$$\boxed{\langle x \rangle = 0}$$

NOTE → For any AC signal  
Average for one cycle is always zero.

case-II → For Half cycle

$$\langle x \rangle_{+1/2} = \frac{1}{T/2} \int_0^{T/2} x dt$$

$$= \frac{2}{T} \int_0^{T/2} x_0 \sin \omega t dt$$

$$\langle x \rangle_{+1/2} = \frac{2x_0}{\pi}$$

$$\boxed{\begin{aligned}\langle x \rangle_{+1/2} &= +\frac{2x_0}{\pi} \\ \langle x \rangle_{-1/2} &= -\frac{2x_0}{\pi}\end{aligned}}$$

NOTE → For half cycle average value for sinusoidal voltage is may be +ve, -ve, zero.

\* Average voltage/current  
 $\langle \bar{v} \rangle_{cycle} = 0$   
 $\langle i \rangle_{cycle} = 0$

|II| → Root mean square velocity ( $v_{rms}$ , Apparent, Effective, virtual)

$$x_{rms} = \left\{ \frac{1}{T} \int_0^T x^2 dt \right\}^{1/2}$$

$$x_{rms} = \left\{ \frac{1}{2} \cdot \int_0^T x_0^2 \sin^2 \omega t dt \right\}^{1/2}$$

$$\boxed{x_{rms} = \frac{x_0}{\sqrt{2}}}$$

NOTE →  $v_{rms}$  of full & half cycle is same.

## POWER LOSS

$$P_{\text{inst}} = VI = V_0 \sin(\omega t) \cdot I_0 \sin(\omega t + \phi)$$

$$= \frac{V_0 I_0}{2} [\cos \phi - \cos(2\omega t + \phi)]$$

$$P = I^2 R$$

$$H = I^2 R t$$

$$\langle P_{\text{avg}} \rangle_{\text{cycle}} = \frac{V_0 I_0}{2} \cos \phi = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi$$

$$= V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos \phi$$

NOTE → \* If nothing is mentioned then given AC voltage consider rms only.  
 \* For Heat & power calculation only RMS value is used.  
 \* 220 volt AC is more dangerous than 220 volt DC (bc peak value of AC is  $311$ . (After  $\sqrt{2}$  &  $311$  is  $220$  on  $\pi$ ))

## Form Factor (FF)

$$FF = \frac{x_{\text{rms}}}{x_{\text{average}}} = \frac{\frac{x_0}{\sqrt{2}}}{\frac{2x_0}{\pi}} = \boxed{\frac{\pi}{2\sqrt{2}}}$$

NOTE → \*  $\langle \sin \omega t \rangle_T = 0$       \*  $\langle \sin^2 \omega t \rangle_T = 1/2$   
 \*  $\langle \cos \omega t \rangle_T = 0$       \*  $\langle \cos^2 \omega t \rangle_T = 1/2$   
 \*  $\langle \sin^2 \omega t \rangle_T = 0$   
 \*  $\langle \cos^2 \omega t \rangle_T = 0$

Ex →  $I = I_1 + I_2 \sin \omega t$

Short trick      Average for one cycle.

$$\textcircled{1} \quad \langle I \rangle_T = \langle I_1 + I_2 \sin \omega t \rangle_T$$

$$I_1 + I_2 \times 0$$

$$= I_1$$

iii) rms for one cycle

$$I_{\text{rms}} = \sqrt{\langle I^2 \rangle_T}$$

$$= \sqrt{I_1^2 + I_2^2 \sin^2 \omega t + 2I_1 I_2 \sin \omega t}$$

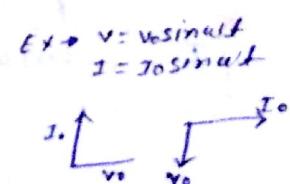
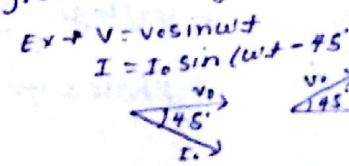
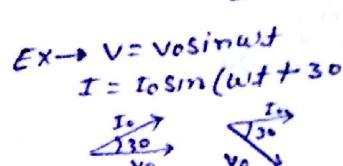
$$I_{\text{rms}} = \sqrt{I_1^2 + \frac{I_2^2}{2} + 0}^{1/2}$$

$$= \sqrt{I_1^2 + \frac{I_2^2}{2}}$$

## Phase

When a physical parameter changes sinusoidally with time then it can be represented by a straight line & this representation is called phase. Here length of line represent peak value.

Phaser diagram It is diagram having phase current & voltage both.

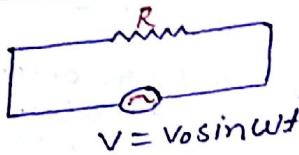


Phase difference ( $\phi$ )  $\rightarrow$  It is difference b/w phase of  $V$  &  $I$ .

$$\text{Power factor} = \cos \phi$$

## AC CKTS

|1|  $\rightarrow$  Pure Resistive CKT (R)



$$V = V_0 \sin \omega t$$

By KVL at any instant.

$$* I = \frac{V}{R} = \frac{V_0}{R} \sin \omega t$$

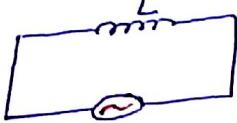
$$I = I_0 \sin \omega t$$

|i|  $\rightarrow$  Phase diagram

|ii|  $\rightarrow$  Phase difference  $= 0$

|iii|  $\rightarrow$  Power factor  $\cos \phi = 1$  (max)  
('R' consumes the total power of CKT).

|2|  $\rightarrow$  Pure Inductive CKT (L)



$$V = I_0 \sin \omega t$$

By KVL

$$* I = I_0 \sin(\omega t - \pi/2)$$

\* Voltage leading the current by  $\pi/2$

\* Phase difference  $= \pi/2$

\* Phasor diagram

\* Power factor  $= \cos \phi = 0$

\* Power  $= V_{\text{rms}} I_{\text{rms}} \cos \phi = 0$

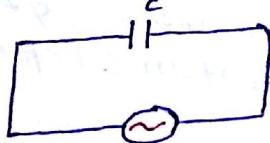
Inductive Reactance ( $X_L$ )

$$* X_L = \omega L = 2\pi f L$$

$$* X_L \propto f$$

\* Unit  $\rightarrow$  ohm ( $R, X_L, X_C$ )

|3|  $\rightarrow$  Pure Capacitive CKT (C)



$$V = V_0 \sin \omega t$$

$$V = \frac{Q}{C} = 0$$

$$Q = CV$$

$$* I = I_0 \sin \omega t + \pi/2$$

Capacitive Reactance ( $X_C$ )

$$* X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

\* Unit  $\rightarrow$  ohm

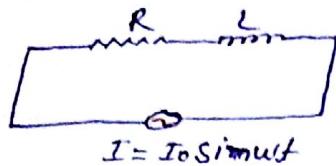
$$* X_C \propto \frac{1}{f}$$



\* Voltage lagging by current by  $\pi/2$  angle  
so  $\phi = \pi/2$

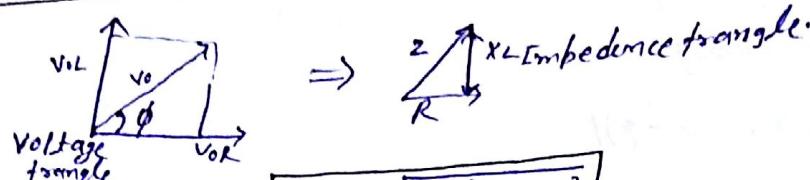
\* Phasor diagram

14) Series RL →



Let,  $I = I_0 \sin \omega t$   
 $\checkmark V_R = V_0 R \sin \omega t$   
 $\checkmark V_L = V_0 L \sin (\omega t + \pi/2)$   
When  $V_0 R = I_0 R$   
 $V_0 L = X_0 L$

Phase Diagram

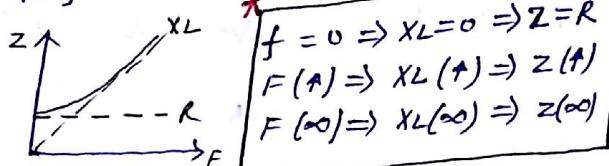


\* i)  $\rightarrow V_0 = \sqrt{V_0 R^2 + V_0 L^2}$

\* ii)  $\rightarrow$  Impedance ( $Z$ )

\*\* iii)  $\rightarrow$  Unit = [ohm]

iv)  $\rightarrow Z$  v/s f



$f = 0 \Rightarrow X_L = 0 \Rightarrow Z = R$   
 $F(\infty) \Rightarrow X_L(\infty) \Rightarrow Z(\infty)$   
 $F(\infty) \Rightarrow X_L(\infty) \Rightarrow Z(\infty)$

Imp  $V$  Leading

\* phase difference ( $\phi$ ) = Acute Angle

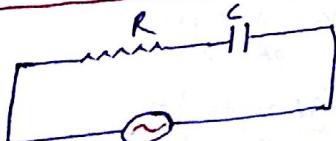
$\phi = \tan^{-1} \left( \frac{V_0 L}{V_0 R} \right) = \tan^{-1} \left( \frac{X_L}{R} \right)$

\* power factor =  $\cos \phi = \frac{V_0 R}{V_0} = \frac{R}{Z}$

Imp Equation of supply voltage

$V_{\text{Supply}} = V_0 \sin(\omega t + \phi)$

15) Series RC



Let, ckt current

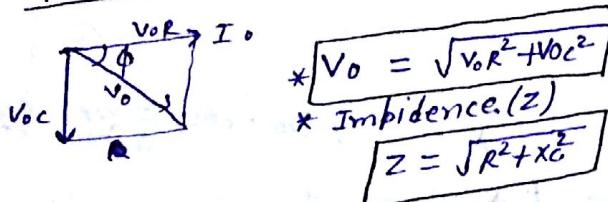
$I = I_0 \sin \omega t$

$V_R = V_0 R \sin \omega t$

$V_C = V_0 C \sin (\omega t - \pi/2)$

When  $V_0 R = I_0 R$   
 $V_0 C = X_0 C$

Phase diagram



\*  $V_0 = \sqrt{V_0 R^2 + V_0 C^2}$

\* Impedance ( $Z$ )

$Z = \sqrt{R^2 + X_C^2}$

\*  $Z$  v/s f  
 $* f = 0 \quad X_C = \infty \Rightarrow Z = \sqrt{R^2 + X_C^2} = \infty$   
 $* f(\infty) \quad X_C = 0 \Rightarrow Z \downarrow$   
 $* f = \infty \quad X_C = \infty \Rightarrow Z = R$



\* Voltage Jaging.

\* Phase difference ( $\phi$ )  $\Rightarrow$  Acute Angle.

$$\phi = \tan^{-1} \left( \frac{V_{oC}}{V_{oR}} \right) = \tan^{-1} \left( \frac{X_C}{R} \right)$$

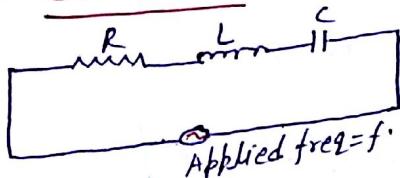
\* Power factor

$$PF = \cos \phi = \frac{V_{oR}}{V_o} = \frac{R}{Z}$$

\* Equation of power supply

$$V_{\text{Supply}} = V_o \sin(\omega t - \phi)$$

161  $\rightarrow$  Series RLC

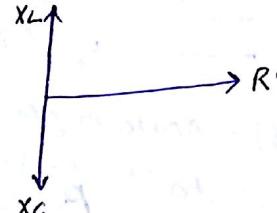
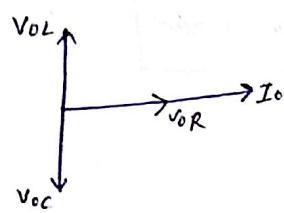


Let  $I = I_0 \sin \omega t$

$$V_R = V_{oR} \sin \omega t$$

$$V_L = V_{oL} \sin(\omega t + \pi/2)$$

$$V_C = V_{oC} \sin(\omega t - \pi/2)$$



case-I If  $V_{oL} > V_{oC}$

\* Series R-L behaviour.

$$* V_o = \sqrt{V_{oR}^2 + (V_{oL} - V_{oC})^2}$$

$$* Z = \sqrt{R^2 + (X_L - X_C)^2}$$

\* Voltage leading.

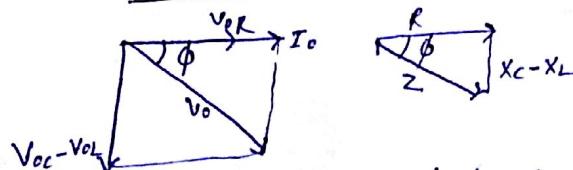
$$* \phi = \tan^{-1} \left( \frac{V_{oL} - V_{oC}}{V_{oR}} \right)$$

$$= \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$$

\* power factor

$$P.F = \cos \phi = \frac{V_{oR}}{V_o} = \frac{R}{Z}$$

case-II If  $V_{oC} > V_{oL}$



\* Series R-C behaviour

$$* V_o = \sqrt{V_{oR}^2 + (V_{oC} - V_{oL})^2}$$

$$* Z = \sqrt{R^2 + (X_C - X_L)^2}$$

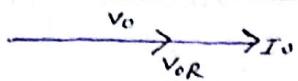
$$* \text{Power factor } \cos \phi = \frac{V_{oR}}{V_o} = R/Z$$

\* 'V' Lagging.

$$* \phi = \tan^{-1} \left( \frac{V_{oC} - V_{oL}}{V_{oR}} \right)$$

$$= \tan^{-1} \left( \frac{X_C - X_L}{R} \right)$$

### Case - III → Voltage Resonance ( $V_{oL} = V_{oC}$ )



- \* pure 'R'
- \*  $V_o = V_{oR}$
- \*  $\phi = 0$
- \* power factor =  $\cos \phi = 1$  (Max)
- \*  $Z = R$  (min)
- \*  $I = \frac{V}{Z} = \frac{V}{R}$  (Max)

At Resonance

$$(X_L)_r = (X_C)_r$$

$$(V_{oL})_r = (V_{oC})_r$$

$$(I_o X_L)_r = (I_o X_C)_r$$

$$(W_L)_r = \left(\frac{1}{\omega C}\right)_r$$

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ (Rad/sec)}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} (XL)$$

⇒  $Z$  v/s  $f$

$$|i| \rightarrow f = 0 \\ R = R \\ X_L = 0 \\ X_C = \infty \\ Z = \infty = X_C$$

$$|iii| \rightarrow 0 < f < f_r \\ R = R \\ X_L \uparrow \\ X_C \downarrow \\ X_C > X_L$$

$$|iii| \rightarrow f = f_r \\ R = R \\ X_L = X_C$$

] pure 'R'

$$|iv| \rightarrow f_r < f < \infty$$

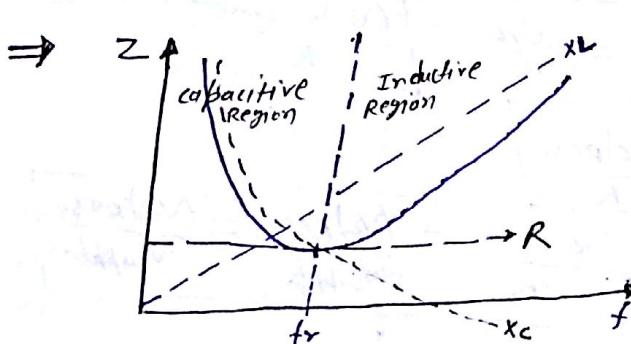
$$R = R \\ X_L > X_C$$

] Series R-L

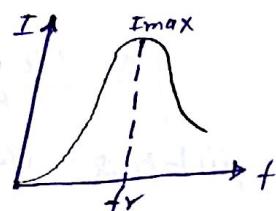
$$|v| \rightarrow f = \infty$$

$$R = R \\ X_L = \infty \\ X_C = 0$$

]  $Z = \infty = X_L$   
pure 'L'

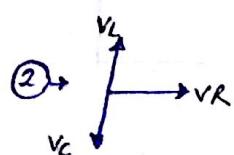


$$\text{Graph} \\ I = V/Z$$



General Knowledge

$$① \rightarrow \text{Resistance} = R \\ \text{Reactance} = X \rightarrow X_L = \omega L \\ X_C = \frac{1}{\omega C} \quad Z \Rightarrow \begin{array}{c} X_L \\ \diagdown \\ R \\ \diagup \\ X_C \end{array}$$

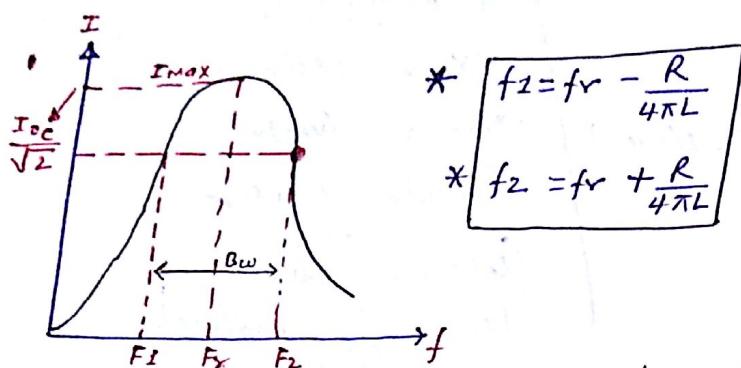


$$③ \rightarrow \phi = \tan^{-1} \left( \frac{V}{R} \right)$$

$$|4| \rightarrow \cos \phi = \frac{R}{Z} = \text{Power factor.}$$

## # Half power frequency

At this freq. current become  $\frac{1}{\sqrt{2}}$  times of  $I_{max}$  so power become max.



\* Bandwidth (BW)

$$\Delta f = f_r - f_1$$

$$\Delta f = \frac{R}{2\pi L} \text{ (Hz)}$$

$$\Delta \omega = \frac{R}{L} \text{ (Rad/sec)}$$

\* Fractional Bandwidth

$$\frac{\Delta f}{f_r} = \frac{\Delta \omega}{\omega_r}$$

$$\frac{R/L}{2/\sqrt{2}LC}$$

$$= \boxed{R\sqrt{\frac{C}{L}}}$$

\* Quality factor (Q)

→ Measure of sharpness of 'I' v/s 'f' curve.

→ If 'Q' ↑ ⇒ Sharpness ↑  
Q ↓ ⇒ Sharpness ↓

→ It is inverse of fractional Bandwidth.

$$i) \rightarrow Q = \frac{1}{f_r \cdot BW} = \boxed{\frac{1}{R} \sqrt{\frac{L}{C}}}$$

$$ii) \rightarrow Q = \frac{\omega_r}{\Delta \omega} = \frac{\omega_r}{R/L} = \boxed{\frac{\omega_r L}{R}} = \boxed{\frac{(XL)_{resonance}}{R}}$$

$$Q = \frac{(XL)_{resonance}}{R} = \frac{(XC)_{resonance}}{R}$$

$$iii) \rightarrow Q = \frac{(VL)_{resonance}}{VR} = \frac{(VC)_{resonance}}{VR} = \frac{(VL)_{resonance}}{V_{supply}} = \boxed{\frac{(VC)_{resonance}}{V_{supply}}}$$

## POWER in AC Ckt

$$i) \rightarrow P_{inst} = V_{inst} \cdot I_{inst}$$

$$ii) \rightarrow P_{peak} = V_{peak} \cdot I_{peak}$$

$$iii) \rightarrow P_{apparent} = V_{rms} \cdot I_{rms}$$

$$iv) \rightarrow \text{Real/Average power of ckt.}$$

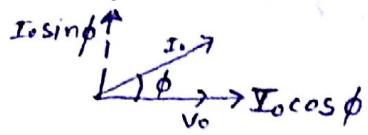
$$P = V_{rms} I_{rms} \cos \phi$$

$$P = I_{rms}^2 R = \left(\frac{V_R}{R}\right)^2 R$$

\* Ckt voltage & current  
 $P = V_{rms} I_{rms} \cos \phi$   
\*  $P = I_{rms}^2 R = \frac{V_R^2}{R}$   
 (Resistance voltage.)

## # Wattless current / Workless current / Powerless current

If it is part of ckt current which is not responsible for any power consumption.



$$\begin{aligned} *I_{\text{Wattless}} &= I_{\text{rms}} \sin \phi \\ *I_{\text{Wattfull}} &= I_{\text{rms}} \cos \phi \\ *I_{\text{ckt}} &= \text{Pythagorean of both.} \end{aligned}$$

# Skin Effect → \* High freq. Ac does flow near the surface of wire.  
\* So, Ac cable is made by multiple wire of smaller cross-sectional area rather than thick cable.



$$R = \frac{sL}{A} \quad : A (\downarrow \downarrow \downarrow) \Rightarrow R \text{ is } (\uparrow \uparrow \uparrow)$$

$\therefore \text{Power Loss} = I^2 R \text{ Loss } (\uparrow \uparrow)$



$$A_{\text{eff}} (\uparrow) \Rightarrow R_{\text{eff}} (\downarrow)$$

$\therefore P_{\text{Loss}} \Rightarrow \downarrow$

## # Hot Wire Ammeter & voltmeter

i) → Moving coil or galvanometer does not work on AC Ckt. since its reversed direction continuously.

So, measure AC, A device is made based on heating effect of current called Hot-wire Ammeter & voltmeter.

Heating effect of current is direction independent.

\* Its direction is direction independent.

Deflection  $\propto H$

$$\begin{aligned} \theta &= i_{\text{rms}}^2 \\ \theta &\propto V_{\text{rms}}^2 \end{aligned}$$

$$\begin{aligned} H &\propto i_{\text{rms}}^2 \\ H &\propto V_{\text{rms}}^2 \end{aligned}$$

iii) → It measures r.m.s value.

iv) → It has non-linear scale.

v) → It can work in AC & DC both.

NOTE → i) In case of Resonance in R-L-C Ckt

Amplitude of current is max.  
For Max Amplitude,  $X_C = X_L$

Resonance Angular Frequency.

$$\omega = \frac{J}{\sqrt{LC}}$$

Linear Frequency.

$$f = \frac{1}{2\pi\sqrt{LC}}$$

iii) → Avg. Power

$\cos \phi$  → power factor.

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2 \cdot R}{12I^2}$$

\* pure 'R'  $\Rightarrow \phi = 0$   
 $\cos \phi = \max = 1$

\* pure 'L' or 'C'  $\Rightarrow \phi = \frac{\pi}{2} \text{ or } -\frac{\pi}{2}$   
 $\cos \phi = 0$   
(Wattless current found in ckt).

* In India	* In USA
220V	110V
50Hz	60Hz

- \*  $1 \text{ unit} = 1 \text{ kWh} = 1000 \times 60 \times 60 = 3.6 \times 10^6 \text{ Jule.}$
- \* Any scalar quantity come with phase diff. in physics then vector addition occur. not simple addition.

- \*  $R \rightarrow \text{Power } P = I^2 R$
- \*  $C \rightarrow \text{Power } P = \frac{I^2}{R}$
- \*  $L-C \rightarrow \text{Power } P = I^2 R$

### # Phase & Amplitude Relation for Alternating current & voltage

<u>SYMBOL</u>	<u>Impedance</u>	<u>Phase of current</u> In phase w/ $V_R$	<u>Phase Angle (<math>\phi</math>)</u>	<u>Amplitude Relation</u>
R	R		0°	$V_R = IR$
C	$X_C$	Leads $V_C$ by 90°	-90°	$V_C = I X_C$
L	$X_L$	Lags $V_L$ by 90°	+90°	$V_L = I X_L$

Mnemonic  $\Rightarrow$  designed to assist the memory.  
 ↳ (ELI the ICE man)

- \* L  $\Rightarrow$  Inductance
- \* C  $\Rightarrow$  capacitance
- \* E  $\Rightarrow$  voltage
- \* I  $\Rightarrow$  current.

- \* In Inductive ckt (ELI)  $\rightarrow$  the current (I) lags the voltage (E)
- \* In Capacitive ckt (ICE)  $\rightarrow$  the current leads the voltage.