Total Harmonic Distortion Calculation by Filtering for Power Quality Monitoring

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Abstract--**Measuring and monitoring quality parameters of AC power systems requires several calculations, such the Total Harmonic Distortion (THD) of voltage and current. This calculation is performed with samples of the monitored waveforms, at sample frequency equal to a power of two multiple of the frequency of waves. The samples are converted to digital values by Analog-to-Digital Converters, with a finite number of bits. Numeric algorithms applied to these digital values insert some errors in the final results, due to the number of bits used in calculations. The most used algorithm to obtain the THD is FFT. Conventional 16-bit DSP FFT algorithms use only 16-bit calculations, despite the DSP 40-bit hardware accumulator. The results are errors in amplitude of each harmonic order. Using these amplitudes to calculate the THD, these errors are summed, and the final error is very greater. The purpose of this article is to present an algorithm to calculate the THD using the DSP 40-bit hardware accumulator and resulting more accurate results.**

Index Terms -- **Algorithms, Discrete time systems, Discrete Fourier transforms, Distortion, Harmonic analysis, Harmonic distortion, Measurement errors, Power quality, Sampled data systems, Signal analysis.**

I. INTRODUCTION

IN AC power energy systems, measurements of quality parameters must be performed at several network points.

Some measurements are expressed by waveform parameters of voltage and current, such the Total Harmonic Distortion (THD), expressing a waveform deviation from the ideal sinusoidal pattern [1].

The THD is obtained by calculations with samples of the monitored voltage or current waveforms, at sample frequency equal to an integer multiple, generally a power of two, of the nominal frequency of waves. Samples are converted to numeric values by Analog-to Digital Converters, which digitalize the values to a finite bit number. Numeric algorithms are applied to the digitalized samples, introducing more errors, due to the limited bit number used in calculations. In this case, we used 64 samples per cycle and 14-bit signed analog to digital conversion.

The most used numeric algorithm is FFT, because the computational time is reduced. However, each final value is a result of another value with added error, due to the nature of

FFT, which applies several calculations in steps, each step depending on the results from the preceding one, with error propagation. In our implementation, using a commercial FFT library, we obtained a 0,5% error in each individual result of the FFT, resulting a 0,7% error each harmonic order. This error is reasonable for the individual results, but the THD error is very great, if calculated by these values. For the THD, the error is multiplied by a square root of the harmonic order number, approximately 5,6 for up to the harmonic order 32, resulting a 4% error. This method is not suited to calculate the THD.

The second tried method is the real DFT, with a 64 by 64 integer Q15 matrix. The samples are putted in a column 64 elements vector, and multiplied by the real DFT matrix, that is, each real DFT matrix line, using a scalar product support DSP instruction "repeat MAC". This feature uses a 40-bit accumulator, instead a 16-bit variables used in FFT library, and the errors are very reduced. This method uses a very great amount of memory to accommodate the vectors and matrices to perform the calculations, and is not suitable for small systems.

The third method is filtering the input waves. The first filter is to obtain the fundamental component, and the second, to obtain the wave without the fundamental component. These two resulting waves have their RMS value calculated, thus resulting the FFT.

II. THD CALCULATION

The THD may be calculated by one the following expressions.

$$THD_1 = \frac{\sqrt{\sum_{h=2}^{N/2} v_h^{\,2}}}{v_1} \qquad (1)$$

$$THD_2 = \frac{\sqrt{\sum_{h=2}^{N/2} v_h^{\,2}}}{\sqrt{\sum_{h=1}^{N/2} v_h^{\,2}}} \qquad (2)$$

Here, h is the harmonic order, N is the amount of samples per period, and v_h is the harmonic order h amplitude. $N/2$ is the maximum harmonic order.

Equation (1) calculates the THD related to the fundamental

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(first order) power, while (2) calculates it related to the total signal power.

In both cases, it is possible to define the distortion amplitude, by (3).

$$v_d = \sqrt{\sum_{h=2}^{N/2} v_h^2} \qquad (3)$$

Due to (3), (1) and (2) becomes (4) and (5).

$$THD_1 = \frac{v_d}{v_1} \qquad (4)$$

$$THD_2 = \frac{v_d}{\sqrt{v_1^2 + v_d^2}} \qquad (5)$$

The final goal is the most precisely distortion amplitude calculation.

III. FILTER DEVELOPMENT

Each line of the real DFT matrix defines a filter, one pair to each harmonic order, a real filter and an imaginary filter. The real filter is defined by a cosine sequence, and the set of real filters corresponds to a cosine transform of the signal.

Each filter matches the value one in the center frequency, and zero value at the another harmonic frequencies, and the amplitude response shape is a *sinc(f/f_s-h)* function, where f_s is the signal frequency. For integer f/f_s and h values, this function corresponds to a unitary discrete impulse at $f/f_s = h$.

Thus, each real DFT matrix line that corresponds to a real filter is the desired filter at each harmonic order.

A. Fundamental Filter

The fundamental, first order, filter $F_1(k)$ is the one period cosine line, given by (6).

$$F_1(k) = \frac{2}{N} \cos\left(\frac{2k\pi}{N}\right) \qquad 0 \le k < N \qquad (6)$$

The value *2/N* is used to normalize the result to the input signal amplitude.

Using a *N* element *x(k)* vector, the fundamental wave is given by (7).

$$x_1(n) = \sum_{k=0}^{N-1} \left(F_1(k)x(n-k)\right) \qquad (7)$$

B. Distortion Filter

All the cosine lines of the real DFT matrix are orthogonal in one signal period. It allows the summation of remainder cosine lines to obtain the band filter from harmonic order *2* to *N/2*. The Nyquist term, order *N/2* must have special treatment, because it represents a summation of both actual and aliasing terms, and must be divided by two. The corresponding filter $F_2(k)$ is given by (8).

$$F_2(k) = \frac{2}{N} \sum_{h=2}^{N/2-1} \cos\left(\frac{2hk\pi}{N}\right) + \frac{1}{N}\cos(k\pi) \quad 0 \le k < N \quad (8)$$

The last term, $h = N/2$, is the Nyquist term, divided by two.

The distortion wave is given by the same way as the fundamental wave, by (9).

$$x_2(n) = \sum_{k=0}^{N-1} \left(F_2(k)x(n-k)\right) \qquad (9)$$

IV. ALGORITHM

Upon the last *N* discrete numeric results from the two filters, the RMS value are calculated, and applied (4) and (5) to obtain the THD. The denominator of (5) is the total AC RMS amplitude, and may result from the AC RMS calculation from the original input signal or by the quadratic summation of the RMS values form the two filtered waves.

The performance result is not very well, because the filter calculations (7) and (9) are performed at each sample, resulting an amount of calculation the same as the DFT application. These calculations uses the DSP instruction "repeat MAC" to improve calculation performance.

V. RESULTS

To verify the accuracy of this method, it was applied to several real and simulated signals.

First, it was used to measure the THD from three real power line AC signals. Fig. 1 shows the original signal, in solid line, the filtered fundamental result, in dashed line, and the filtered summation of harmonics 2 to 32, for the first real power line AC signal. Note the elimination of DC value, the basic difference between the original and fundamental waves. Fig. 2 shows the same for the second real power line AC signal, which have a greater DC offset. Fig. 3 shows the same for the third real power line AC signal.

It is very difficult to obtain a real signal with significant distortion to show here. Thus, the next three cases are for simulated signals, with well-known characteristics to compare the results with theory. Fig. 4 shows the same set of curves for a 33% amplitude third harmonic; the THD calculated from (4) will be theoretically 33%. Fig. 5 shows the same for a 10% amplitude fifth harmonic, with theoretical THD of 10%. Fig. 6 shows the same for a square wave, with theoretical THD of 48,34% if calculated by (4).

Note the very good discrimination of the different frequencies from the three last signals. This pair of filters is a perfect reconstruction filter pair, that is, the summation of the output signals from the two filters is almost exactly the input original signal, except the DC value and numeric errors.

Table 1 shows the THD values calculated for the six signals, and the ideal characteristics, if exist, to compare results.
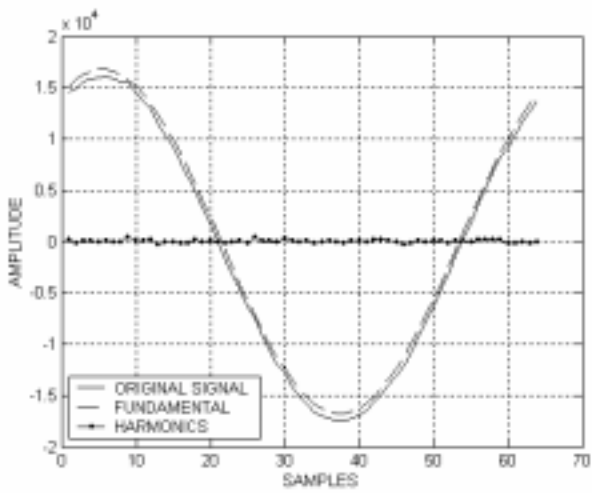
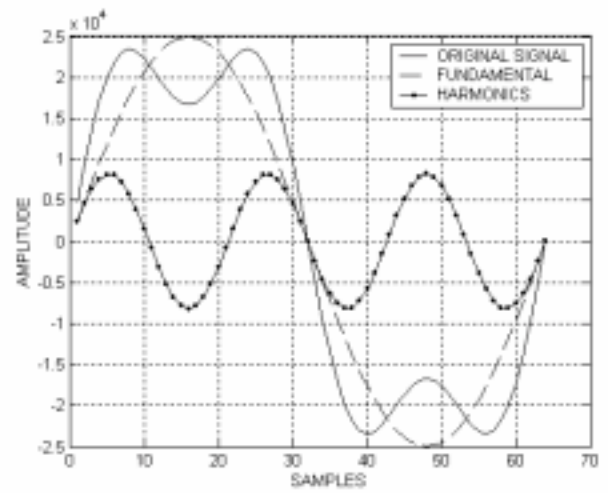Fig. 1. Filter results from the first real power line AC signal

Fig. 4. Filter results from the simulated signal with 33% third harmonic
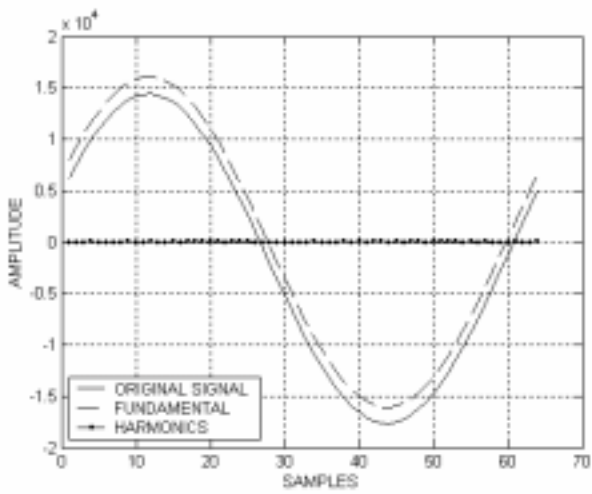

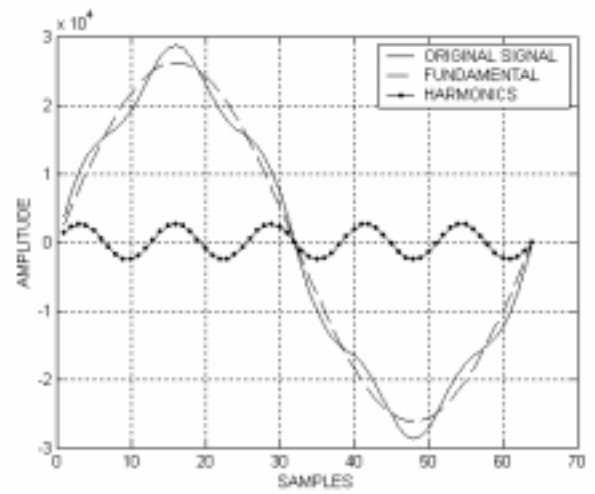Fig. 2. Filter results from the second real power line AC signal

Fig. 5. Filter results from the simulated signal with 10% fifth harmonic
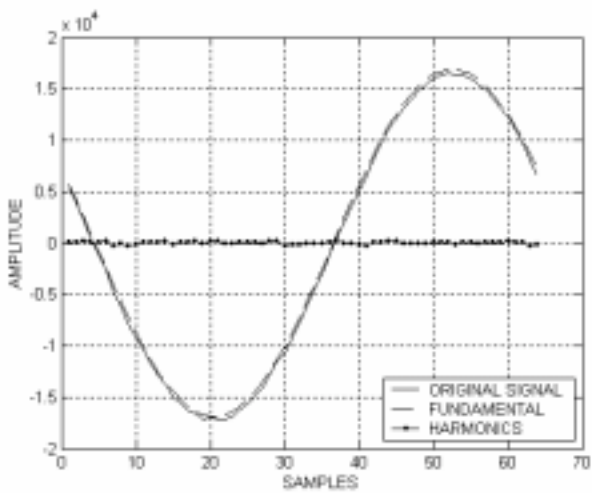

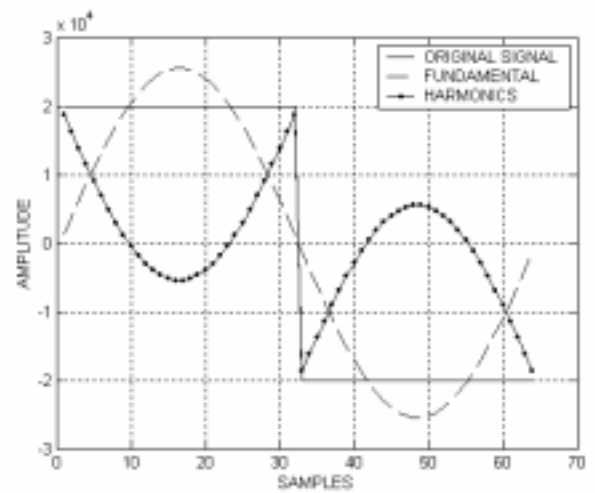Fig. 3. Filter results from the third real power line AC signal

Fig. 6. Filter results from the square wave simulated signal

Table 1: Calculated THD for several signals
and their ideal characteristics

Signal	Description	THD (%)	
		Ideal	Calculated
1	Real power line AC	-	1,18
2	Real power line AC	-	0,27
3	Real power line AC	-	1,18
4	33% third harmonic	33	32,95
5	10% fifth harmonic	10	9,98
6	square wave	48,34	48,15

VI. CONCLUSIONS AND FURTHER DEVELOPMENTS

As shown in results, this method is powerful to calculate the THD for several signals. The final accuracy of the results for the shown examples may be considered as a good accuracy indicator of the method.

The filters here described are easy to develop and are suitable for sampled signals without frequency deviation from the nominal one. In this case, it is possible to develop a set of filters with perfect signal reconstruction, by choosing the cosine lines of DFT matrix.

The orthogonal DFT matrix lines allow the band-pass filter development by summation of the desired harmonic orders corresponding cosine lines.

Due to the continuous filtering, each sample, the amount of calculations performed in this method is no better than the DFT method.

By using the imaginary (sine) lines of the real DFT matrix, it might be possible to develop algorithms to calculate the THD without a continuous filtering, and by less calculations as the DFT method, with the same accuracy.

VII. REFERENCES

[1] *IEEE Standards Coordinating Committee 22 on Power Quality. IEEE recommended practice for monitoring electric power quality* - IEEE Std. 1159, June 14, 1995.