CONVEX AND ABSTRACT POLYTOPES [1]
Thursday, May 19, 2005 to Saturday, May, 21, 2005

MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Friday & Saturday.
Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Friday & Saturday.
Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Thursday & Friday.
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall.
For other lighter meal options at the Banff Centre, there are two other options: Gooseberry's Deli, located in the Sally Borden Building, and The Kiln Cafe, located beside Donald Cameron Hall. There are also plenty of restaurants and cafes in the town of Banff, a 10-15 minute walk from Corbett Hall.

MEETING ROOMS

All lectures are held in Max Bell 159. Hours: 6 am - 12 midnight. LCD projector, overhead projectors and blackboards are available for presentations. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

SCHEDULE

Thursday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
19:30	Informal gathering in 2nd floor lounge, Corbett Hall. Beverages and small assortment of snacks available in the lounge on a cash honour-system basis.

Friday

9:15-10:05	*C.Lee — Some Construction Techniques for Convex Polytopes.*
10:15-10:35	*J.Lawrence — Intersections of Convex Transversals.*
10:45-11:15	Coffee Break, 2nd floor lounge, Corbett Hall.
11:15-11:35	*R.Dawson — New Triangulations of the Sphere.*
11:45-12:05	*V.Soltan — The Characteristic Intersecting Property of Generalized Simplices.*
12:15-2:00	Lunch Break.
2:00-2:50	*B.Monson — Regular and Chiral Polytopes, Orthogonal Groups and Symmetric Graphs.*
3:00-3:20	*M.Hartley — An Atlas of Small Regular Abstract Polytopes.*
3:30-4:00	Coffee Break, 2nd floor lounge, Corbett Hall.
4:00-4:20	*I.Hubard — Petrie-Coxeter Polyhedra Revisited.*
4:30-4:50	*G.Williams — Petrie Polygons and Petrie Schemes.*
5:00-5:20	*A.Herman — Rationality Questions for Coxeter Groups.*
5:30	Dinner.

[1] Organizers: Ted Bisztriczky, Egon Schulte, Asia Weiss

Saturday

9:00-9:50	*K.Bezdek — Ball-Polytopes.*
10:00-10:20	*M.Gavrilova — Thiessen Polytopes for Analysis of Crystalline Structures.*
10:30-11:00	Coffee Break, 2nd floor lounge, Corbett Hall.
11:00-11:20	*K.Rybnikov — TBA.*
11:30-11:50	*R.Erdahl — Structure Theorem for Voronoi Polytopes for Lattices.*
12:00-12:45	Lunch.
1:00-5:00	Bus Excursion (Banff - Lake Lousie - Village Park Inn).

Checkout by 12 noon.
** 2-day workshops are welcome to use the BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 16:00 on Saturday, although participants are still required to checkout of the guest rooms by 12 noon. There is no coffee break on Saturday afternoon, but self-serve coffee and tea are always available in the 2nd floor lounge, Corbett Hall. **

POLYTOPES DAY IN CALGARY [1]
Sunday, May 22, 2005

SCHEDULE

Sunday

All events on 4'th Floor, Math Sciences Building, University of Calgary.

8:30-9:30	Breakfast, Math Lounge.
9:30-10:20	*R.Ehrenborg — Linear Inequalities for Flag f-Vectors of Polytopes.*
10:30-10:50	*H.Burgiel — Signed Associahedra.*
11:00-11:30	Coffee Break.
11:30-11:50	*M.Bayer — Shelling and the h-Vector of the (Extra)ordinary Polytope.*
12:00-12:30	*Convex Polytopes — state of the field discussion.*
12:30-2:00	Lunch Break, Math Lounge.
2:00-2:50	*H.Martini — Geometric Properties of Simplices.*
3:00-3:20	*A.Edmonds — The Geometry of Equifacetal Simplices.*
3:30-4:00	Coffee Break.
4:00-4:20	*N.Johnson — Uniform Polychora.*
4:30-4:50	*E.Schulte — Chiral and Regular Polytopes in Low Dimensions.*
5:00-5:30	*Abstract Polytopes — state of the field discussion.*

CONVEX AND ABSTRACT POLYTOPES
May 19–21, 2005

and

POLYTOPES DAY IN CALGARY
May 22, 2005

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Margaret Bayer** (University of Kansas)
Title: *Shelling and the h-Vector of the (Extra)ordinary Polytope*
Abstract: Of great importance in the study of simplicial polytopes is the combinatorial interpretation via shellings for the h-vector of the polytope. Until now cubical polytopes formed the only class of nonsimplicial polytopes for which the h-vector is known to have a combinatorial interpretation (shown by Clara Chan). Ordinary polytopes were introduced by Bisztriczky as a (nonsimplicial) generalization of cyclic polytopes. In this talk I will discuss the colex shelling of the ordinary polytope, and show how to use it to compute the h-vector. I will also give a shallow triangulation of the (boundary of the) ordinary polytope related to the shelling. As a consequence, the entries of the h-vector of any ordinary polytope are simple sums of binomial coefficients.

Speaker: **Karoly Bezdek** (University of Calgary)
Title: *Ball-Polytopes*
Abstract: Ball-polytopes are intersections of finitely many balls of the same radius. The talk will survey results on the following topics:

1. Isoperimetric-type inequalities for circle-polygons and Alexander's conjecture;
2. Rigidity of ball-polyhedra in 3-space;
3. Ball-polyhedra with symmetric sections;
4. (Quantitative) illumination of ball-polyhedra;
5. Caratheodory's theorem and the Euler-Poincare formula for ball-polytopes;
6. On Maehara's conjecture.

Speaker: **Heidi Burgiel** (Bridgewater State College)
Title: *Signed Associahedra*
Abstract: An associahedron has vertices corresponding to the triangulations of a convex polygon and edges corresponding to relationships between triangulations. By assigning signs (+ or -) to the vertices of the polygon and choosing appropriate definitions of adjacency we define two different types of "signed associahedron". Examples of all three of these objects will be presented.

Speaker: **Robert Dawson** (Saint Mary's University)
Title: *New Triangulations of the Sphere*
Abstract: In the 1920's, Sommerville classified the tilings of the sphere with congruent spherical triangles under certain assumptions. In the 1960's, Davies did the same, under only the restriction that the triangles should meet edge-to-edge. Other tilings and tiles arise if we drop that assumption; this paper is a progress report on the classification of such tilings.

Speaker: **Allan Edmonds** (Indiana University)
Title: *The Geometry of Equifacetal Simplices*
Abstract: A discussion of recent results and open questions on the existence, classification, and characterization of simplices all of whose facets are congruent to one another.

Speaker: **Richard Ehrenborg** (University of Kentucky)
Title: *Linear Inequalities for Flag f-Vectors of Polytopes*
Abstract: The f-vector enumerates the number of faces of a convex polytope according to dimension. The flag f-vector is a refinement of the f-vector since it enumerates face incidences of the polytope. To classify the set of flag f-vectors of polytopes is an open problem in discrete geometry. This was settled for 3-dimensional polytopes by Steinitz a century ago. However, already in dimension 4 the problem is open.

The known linear inequalities for the flag f-vector of polytopes are the non-negativity of the toric g-vector, that the simplex minimizes the cd-index and inequalities obtained from the Kalai convolution and the lifting technique. We also present open questions in the area such as: examples of inequalities to prove and examples of inequalities to prove that they are sharp.

Speaker: **Robert Erdahl** (Queen's University)
Title: *Structure Theorem for Voronoi Polytopes for Lattices*
Abstract: It is natural to ask whether the Voronoi polytope for a lattice can be written as the Minkowski sum of simpler Voronoi polytopes. It turns out that this is possible if and only if the corresponding Delaunay tilings for the two simpler Voronoi polytopes are commensurate. The Minkowski sum of two polytopes is a familiar notion in convexity theory, but the notion of commensurate Delaunay tiling is new and will be explained in the course of the talk. This Structure Theorem is the main result that will be reported, and generalizes an earlier result of S. S. Ryshkov.

This Structure Theorem allows an arbitrary Voronoi polytope to be written as a Minkowski sum of simpler irreducible Voronoi polytopes, which are the building blocks and correspond through duality to "edge forms" in Voronoi's theory of lattice types. I will report what is known about the numbers of types of building blocks, or edge forms, and how these numbers of types grow with dimension. The problem of describing these elementary building blocks, and characterizing when they can combine through Minkowski sums to form more complicated Voronoi polytopes is an interesting new problem in geometry of numbers.

Speaker: **Marina Gavrilova** (University of Calgary)
Title: *Thiessen Polytopes for Analysis of Crystalline Structures*
Abstract: The partitioning of a plane with n points into n convex polygons such that each polygon contains exactly one point and every point in a given polygon is closer to its central point than to any other is called a Voronoi diagram. The cells resulting from such subdivision are called Dirichlet regions, Thiessen polytopes, or Voronoi polygons. The talk is focused on classification of Voronoi polygons according to types of studied crystalline structures in computer models of dense packings of atoms.A shape of the Delaunay simplex and the shape of its neighbors are used to determine whether the Delaunay simplex belongs to a given crystalline structure. Implications for analysis of other models are also discussed.

Speaker: **Michael Hartley** (University of Nottingham)
Title: *An Atlas of Small Regular Abstract Polytopes*
Abstract: This talk will announce the creation of an atlas of small regular abstract polytopes. The atlas

contains information on 32550 polytopes with at most 2000 flags. This is all except for those with $2^8.k$ flags, where $k > 1$. The atlas, while a work in progress, should be useful to researchers in abstract polytopes, both as a reference, and as a tool for conjecture formation. While introducing the atlas, the talk will explain the methods used to generate it.

Speaker: **Allan Herman** (University of Regina)
Title: *Rationality Questions for Coxeter Groups*
Abstract: An irreducible representation of a group G as a group of square complex matrices is said to be realizable over the reals if it is similar to a group of matrices whose entries are all real. It has been known for some time that all irreducible representations of all finite Coxeter groups are realizable over the field of real numbers. In this talk, we will consider such rationality questions for the automorphism groups of abstract regular polytopes, which are certain finite dimensional images of infinite Coxeter groups. I will present a recent result of myself and Barry Monson that shows that whenever a minimal realization cone of an abstract regular polytope has dimension greater than one, it must arise from an irreducible representation of its automorphism group that is realizable over the field of real numbers.

Speaker: **Isabel Hubard** (York University)
Title: *Petrie-Coxeter Polyhedra Revisited*
Abstract: A chiral polytope is a non-reflexible polytope with maximal symmetry by rotation. Self-dual chiral polytopes can be properly or improperly. Properly self-dual chiral 4-polytopes can be twisted to obtain regular maps. A similar operation can be done for improperly self-dual chiral 4-polytopes which will give us chiral quotients of the Petrie-Coxeter polyhedra.

Speaker: **Norman W. Johnson** (Wheaton College)
Title: *Uniform Polychora*
Abstract: In addition to the five Platonic solids, the regular polyhedra include the four starry figures discovered by Kepler and Poinsot. Other uniform star polyhedra, analogous to the thirteen Archimedean solids, were discovered in the nineteenth century by Hess, Badoureau, and Pitsch and in the twentieth by Coxeter, Longuet-Higgins, and Miller. There are also infinite families of uniform prisms and antiprisms. Schlfli and Hess found the six convex and ten starry regular 4-polytopes, or *polychora*. Forty other uniform convex polychora were found by Thorold Gosset and Alicia Boole Stott and one more by John H. Conway and Michael Guy. Until recently little was done to extend these results to uniform star polychora. But two nonprofessional mathematicians, Jonathan Bowers and George Olshevsky, have found hundreds of new figures, so that, exclusive of infinite families, there are now 1845 known uniform polychora.

Speaker: **James Lawrence** (George Mason University)
Title: *Intersections of Convex Transversals*
Abstract: Let $\{U_1, ..., U_n\}$ be a family of n nonempty sets in R^d. A "convex transversal" of $\{U_i\}$ is a convex set T such that $T \cap U_i$ is nonempty, for each index i. Let X denote the intersection of the convex transversals of $\{U_i\}$. It is easily seen that X is a compact convex set. In fact, X is a (possibly empty) convex polytope (and hence one of the subjects of this conference). We describe this result, some related results, and some related questions.

Speaker: **Carl Lee** (University of Kentucky)
Title: *Some Construction Techniques for Convex Polytopes*
Abstract: Joint work with Matt Menzel (Marietta College) and Laura Schmidt (University of Wisconsin-Stout).

We will discuss some construction techniques for convex polyhedra, with an eye toward realizing some classes of f-vectors and flag f-vectors.

1. Billera and Lee describe a set of necessary conditions for f-vectors of antistars in simplicial polytopes, and hence for regular triangulations and (by duality) for unbounded, simple polyhedra. It is not yet

known whether these conditions are sufficient. In joint work with Laura Schmidt we construct certain classes of regular triangulations to demonstrate the sufficiency of these conditions in low dimensions. The construction exploits some of the combinatorial structure of the simplicial polytopes used in the proof of the g-Theorem.

2. The set of flag f-vectors of four-dimensional polytopes has not yet been characterized. Extending the sewing technique of Shemer (dual to Barnette's facet-splitting), in joint work with Matt Menzel we discuss a generalized sewing method to construct nonsimplicial polytopes. One encouraging feature of this method is that it encompasses the construction of ordinary polytopes by Bisztriczky and Dinh. We present some results on the set of flag f-vectors of four-dimensional polytopes achievable by this process.

Speaker: **Horst Martini** (Technische Universität Chemnitz)
Title: *Geometric Properties of Simplices*
Abstract: In this lecture we will survey geometric properties of simplices which play a role in convex geometry, disvrete and combinatorial geometry as well as classical geometry. In particular, we will discuss properties that describe or characterize general simplices as special convex bodies or polytopes in convex geometry and that refer to notions like affine diameters, sharp shadow-boundaries, cross-section measures or reducedness (also in Minkowski spaces). On the other hand, we will study classes of special simplices, with the regular ones as the most symmetric case, that are obtained by the coincidence of analogues of classical triangle centers (such as in- and circumcenters, or centroids) in higher dimensional spaces. These classes can be described by their facial structures, e.g. yielding equifacetal or equiareal simplices. But also (special) orthocentric simplices will be taken into consideration.

Speaker: **Barry Monson** (University of New Brunswick)
Title: *Regular and Chiral Polytopes, Orthogonal Groups and Symmetric Graphs*
Abstract: Rather than attempting to cover the union of the above topics (thank goodness!), I will instead poke around a very pretty corner of their intersection.

I'll first visit some key ideas concerning abstract regular or chiral polytopes, and their realizations, with mention of some open problems. As time permits, I will then look at some recent work of my own on 4-polytopes: first (with Egon Schulte) a classification of polytopes constructed through the modular reduction of crystallographic Coxeter groups; second (with Asia Weiss) t-transitivity in the 'medial graphs' of 4-polytopes.

Speaker: **Konstantin Rybnikov** (University of Massachusetts)
Title: *TBA*
Abstract: Our algorithm checks convexity of a piecewise-linear (PL) immersion into R^n of an $(n-1)$-dimensional manifold, of unknown topology, using $O(f_{n-3,n-2})$ arithmetic $(+,-,*)$ operations and $O(f_{n-3,n-2})$ evaluations of polynomial predicates of degree at most n. Here, as usual, $f_{n-3,n-2}$ denotes the number of incidences between $(n-3)$- and $(n-2)$-dimensional faces. This implies that for a surface in 3-space convexity can be verified in O(#vertices) operations (we assume that we are given the numbers of edges, vertices, and faces a priori). In this basic case one just has to check if the surface is convex at each vertex. The ratio of the number of vertices at which the surface is convex to the total number of vertices gives a combinatorial convexity measure for a PL-surface.

How "convex" an intrinsically non-convex surface, e.g. a torus, can be? A theorem of Gritzmann and Betke says that any orientable closed surface of positive genus can be PL-embedded into R^3 with only 5, but not fewer, non-convex vertices. I will talk about the following questions: Where does 5 come from? What is the minimal possible number of non-convex vertices in a PL-immersion of a sphere with m Möbius films?

Speaker: **Egon Schulte** (Northeastern University)
Title: *Chiral and Regular Polytopes in Low Dimensions*
Abstract: There are two main thrusts in the theory of chiral and regular polytopes: the abstract, purely

combinatorial aspect, and the geometric one of realizations. We summarize recent progress on the realization theory in euclidean spaces of low dimensions, focusing on the complete enumeration of the discrete chiral polyhedra in ordinary 3-space. The latter are the chiral analogues of the Grünbaum-Dress polyhedra, these being the regular polyhedra in 3-space.

Speaker: **Valeriu Soltan** (George Mason University)
Title: *The Characteristic Intersecting Property of Generalized Simplices*
Abstract: According to Rockafellar (1970), a generalized d-simplex in E^d is a direct sum of an k-simplex and a $(d-k)$-simplicial cone, $0 \leq k \leq d$. Fourneau (1977) proved that a line-free, closed convex set $M \subset E^d$ with nonempty interior is a generalized d-simplex if and only if for any two homothetic copies of M, their intersection, if d-dimensional, is again a homothetic copy of M.

We show that for a pair of line-free convex bodies M_1 and M_2 in E^d, the following two conditions are equivalent:

(a) the d-dimensional intersections $M_1 \cap (x + M_2)$ belong to a unique homothety class of convex bodies in E^d,

(b) both M_1 and M_2 are generalized d-simplices, and there is a generalized d-simplex M such that any d-dimensional intersection $M_1 \cap (x + M_2)$ is homothetic to M.

A complete description of M_1, M_2, and M in terms of simplices and simplicial cones is given.

Speaker: **Gordon Williams** (Moravian College)
Title: *Petrie Polygons and Petrie Schemes*
Abstract: Petrie polygons, especially as they arise in the study of regular polytopes and Coxeter groups, have been studied by geometers and group theorists since the early part of the twentieth century. They are named for H. S. M. Coxeter's friend J. F. Petrie, and were popularized by Coxeter, most notably in his classic text *Regular Polytopes*. An open question is the determination of which polyhedra and polytopes possess Petrie polygons that are simple closed curves. In this talk we will discuss the status of this problem; we will also introduce a related combinatorial structure called a Petrie scheme that provides added insight into the problem and also poses new challenges.