SCHOOL OF ECONOMICS

AUCKLAND PARK KINGSWAY CAMPUS

SUPPLEMENTARY EXAM 2022

Module Name: **Stochastic Processes in Financial Engineering** **Date: 2022**

Module Code: SPFE9X00

Master of Financial Economics **Duration: 3 H**

Mark: 70

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Instructions:

1) The exam contributes 50 percent of the course mark.
2) Answer all questions.
3) This paper consists of 3 pages.
4) Please round off to 2 decimal places.

Initials & Surname: ______________________________

Student number: ______________________________

Telephone number: ______________________________

SECTION	TOTAL	MARK	EXTERNAL
Q1	10		
Q2	15		
Q3	15		
Q4	05		
Q5	15		
Q6	10		
TOTAL	70		

Q1) **[10 marks]**

Three months ago, a 10-years bond was issue with a par value of R1,000. Its annual coupon rate is 10%, and its coupons are paid semi-annually.

A 6-months European call option on the bond has a strike of R1,100, and its premium is R55.05.

The continuously compounded interest rate is 8%.

Find the value of a 6-months European put option on the bond that has a strike of R1,100.

Q2) **[15 marks]**

Assume the Black-Scholes framework. You are given:
(i) For $t \geq 0$, $S(t)$ denotes the time-t price of a stock.
(ii) $S(0) = 1$
(iii) $\frac{dS(t)}{S(t)} = 0.08dt + 0.40d\tilde{Z}(t)$,
where $\{\tilde{Z}(t)\}$ is a standard Brownian motion under the risk-neutral probability measure.
(iv) For $t \geq 0$, the stock pays dividends of amount $0.04S(t)dt$ between time t and time $t + dt$.
(v) For a real number c and a standard normal random variable Z,
$E\left[Z^2 e^{cZ}\right] = \left(1 + c^2\right) e^{c^2/2}$.
Consider a derivative security that pays
$1 + S(1)\{\ln[S(1)]\}^2$ at time $t = 1$, and nothing at any other time.

Calculate the time-0 price of this derivative security.

Q3) **[15 marks]**

In a simple random walk model of stock prices, the price of a stock is assumed to change by \$0.01 during each trading day. The probability that the price moves up on a given day is 51% and the probability that it moves down is 49%. Movements on different days are assumed to be statistically independent.

Stock VAR has a current price of \$5.00. A trader is concerned about potential losses on the stock over the next year, which consists of 250 trading days. He wishes to determine the smallest value of L such that:

$$P[S - 5.00 < -L] \leq 0.01,$$

where S denotes the stock price at the end of the year.

Use a normal distribution to approximate the value of L.

Q4) **[05 marks]**

Assume the Black-Scholes framework. Eight months ago, an investor borrowed money at the risk-free interest rate to purchase a one-year 75-strike European call option on a nondividend-paying stock. At that time, the price of the call option was 8. Today, the stock price is 85. The investor decides to close out all positions. You are given:

(i) The continuously compounded risk-free rate interest rate is 5%.

(ii) (ii) The stock's volatility is 26%.

Calculate the eight-month holding profit.

Q5) **[15 marks]**

You use a binomial interest rate model to evaluate a 7.5% interest rate cap on a $100

three-year loan.You are given:

(i) The interest rates for the binomial tree are as follows:

*r*0 = 6.000%

ru = 7.704%

rd = 4.673%

ruu = 9.892%

rud = *rdu* = 6.000%

rdd = 3.639%

(ii) All interest rates are annual effective rates.

(iii) The risk-neutral probability that the annual effective interest rate moves up or down is ${}^{1}\!/_{2}$.

(iv) The loan interest payments are made annually.

Using the binomial interest rate model, calculate the value of this interest rate cap.

Q6) **[10 marks]**

$r(t)$, the continuously compounded interest rate at time is modelled by the diffusion process:

$$dr(t) = [0.06 - r(t)]dt + 0.01dZ(t),$$

where is Brownian motion.

The theoretical price, $V(t)$, of a derivative whose value is linked to is:

$$V(t) = \exp[-t * r(t)]$$

Determine the diffusion model satisfied by $V(t)$.