6. $y'_x = ay^2 + bx^n$.

Special Riccati equation, n is an arbitrary number.

1°. Solution for $n \neq -2$:

$$y = -\frac{1}{a}\frac{w'_x}{w}, \qquad w(x) = \sqrt{x}\left[C_1 J_{\frac{1}{2k}}\left(\frac{1}{k}\sqrt{ab}\,x^k\right) + C_2 Y_{\frac{1}{2k}}\left(\frac{1}{k}\sqrt{ab}\,x^k\right)\right],$$

where $k = \frac{1}{2}(n+2)$; $J_m(z)$ and $Y_m(z)$ are the Bessel functions: C_1 and C_2 are arbitrary constants.

2°. Solution for $n = -2$:

$$y = \frac{\lambda}{x} - x^{2a\lambda}\left(\frac{ax}{2a\lambda+1}x^{2a\lambda} + C\right)^{-1},$$

where λ is a root of the quadratic equation $a\lambda^2 + \lambda + b = 0$.

References

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