



Exact Solutions > Ordinary Differential Equations > First-Order Ordinary Differential Equations > Special Riccati Equation

6. $y'_x = ay^2 + bx^n$.

Special Riccati equation, n is an arbitrary number.

1°. Solution for $n \neq -2$:

$$y = -\frac{1}{a} \frac{w'_x}{w}, \quad w(x) = \sqrt{x} \left[C_1 J_{\frac{1}{2k}} \left(\frac{1}{k} \sqrt{ab} x^k \right) + C_2 Y_{\frac{1}{2k}} \left(\frac{1}{k} \sqrt{ab} x^k \right) \right],$$

where $k = \frac{1}{2}(n+2)$; $J_m(z)$ and $Y_m(z)$ are the Bessel functions; C_1 and C_2 are arbitrary constants.

2°. Solution for $n = -2$:

$$y = \frac{\lambda}{x} - x^{2a\lambda} \left(\frac{ax}{2a\lambda + 1} x^{2a\lambda} + C \right)^{-1},$$

where λ is a root of the quadratic equation $a\lambda^2 + \lambda + b = 0$.

References

Kamke, E., *Differentialgleichungen: Lösungsmethoden und Lösungen, I, Gewöhnliche Differentialgleichungen*, B. G. Teubner, Leipzig, 1977.

Polyanin, A. D. and Zaitsev, V. F., *Handbook of Exact Solutions for Ordinary Differential Equations, 2nd Edition*, Chapman & Hall/CRC, Boca Raton, 2003.

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