23. $y'_x = f(x)y^2 + g(x)y + h(x)$.

General Riccati equation.

1°. Given a particular solution $y_0 = y_0(x)$ of the Riccati equation, the general solution can be written as:

$$y = y_0(x) + \Phi(x)\left[C - \int f(x)\Phi(x)\,dx\right]^{-1},$$

where

$$\Phi(x) = \exp\left\{\int \left[2f(x)y_0(x) + g(x)\right]\,dx\right\},$$

C is an arbitrary constant. To the particular solution $y_0(x)$ there corresponds $C = \infty$.

2°. The substitution

$$u(x) = \exp\left[-\int f(x)y(x)\,dx\right]$$

reduces the general Riccati equation to a second-order linear equation:

$$f(x)u''_{xx} - \left[f'_x(x) + f(x)g(x)\right]u'_x + f^2(x)h(x)u = 0,$$

which often may be easier to solve than the original Riccati equation.

3°. See also special cases of the Riccati equation:

- special Riccati equation ,
- Riccati equation, special case 1 ,
- Riccati equation, special case 2 ,
- Riccati equation, special case 3 ,
- Riccati equation, special case 4 ,
- Riccati equation, special case 5 ,
- Riccati equation, special case 6 ,
- Riccati equation, special case 7 ,
- Riccati equation, special case 8 ,
- Riccati equation, special case 9 ,
- Riccati equation, special case 10 ,
- Riccati equation, special case 11 ,
- Riccati equation, special case 12 ,
- Riccati equation, special case 13 ,
- Riccati equation, special case 14 ,
- Riccati equation, special case 15 ,
- Riccati equation, special case 16 .

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