**2.** $ax'_t = (b-c)yzf(x,y,z,t)$, $by'_t = (c-a)zxf(x,y,z,t)$, $cz'_t = (a-b)xyf(x,y,z,t)$.

First integrals:

$$ax^2 + by^2 + cz^2 = C_1,$$
$$a^2x^2 + b^2y^2 + c^2z^2 = C_2,$$

where $C_1$ and $C_2$ are arbitrary constants. On solving the integrals for $y$ and $z$ and on substituting the resulting expressions into the first equation of the system, one arrives at a first-order equation (if the function $F$ is independent of $t$, this equation is separable).

 