



# 1 ISO/IEC JTC1/SC2/WG2 N2816

2004-06-18

Universal Multiple Octet Coded Character Set  
International Organization for Standardization  
Organisation internationale de normalisation  
Международная организация по стандартизации

**ISO/IEC JTC 1/SC 2/WG 2  
PROPOSAL SUMMARY FORM TO ACCOMPANY SUBMISSIONS  
FOR ADDITIONS TO THE REPERTOIRE OF ISO/IEC 10646<sup>1</sup>**

**Please fill all the sections A, B and C below.**

Please read Principles and Procedures Document (P & P) from <http://www.dkuug.dk/JTC1/SC2/WG2/docs/principles.html> for guidelines and details before filling this form.

Please ensure you are using the latest Form from <http://www.dkuug.dk/JTC1/SC2/WG2/docs/summaryform.html>.

See also <http://www.dkuug.dk/JTC1/SC2/WG2/docs/roadmaps.html> for latest Roadmaps.

### A. Administrative

- 1. **Title:** Proposal to add counting rod numerals to Unicode and ISO/IEC 10646
- 2. Requester's name: Christopher Cullen and John H. Jenkins
- 3. Requester type (Member body/Liaison/Individual contribution): Individual contribution
- 4. Submission date: 18 June 2004
- 5. Requester's reference (if applicable): N/A
- 6. Choose one of the following: This is a complete proposal

### B. Technical - General

- 1. Choose one of the following:
    - a. This proposal is for a new script (set of characters):
      - Proposed name of script: Counting Rod Numerals
  - 2. Number of characters in proposal: 19
  - 3. Proposed category (select one from below - see section 2.2 of P&P document):
    - A-Contemporary  B.1-Specialized (small collection)  B.2-Specialized (large collection)
    - C-Major extinct  D-Attested extinct  E-Minor extinct
    - F-Archaic Hieroglyphic or Ideographic  G-Obscure or questionable usage symbols
  - 4. Proposed Level of Implementation (1, 2 or 3) (see Annex K in P&P document): 3
    - Is a rationale provided for the choice? Yes
      - Negative numbers are indicated by overlaying a diagonal slash
  - 5. Is a repertoire including character names provided? Yes
    - a. If YES, are the names in accordance with the "character naming guidelines" in Annex L of P&P document? Yes
    - b. Are the character shapes attached in a legible form suitable for review? Yes
  - 6. Who will provide the appropriate computerized font (ordered preference: True Type, or PostScript format) for publishing the standard? John H. Jenkins
    - If available now, identify source(s) for the font (include address, e-mail, ftp-site, etc.) and indicate the tools used: FontLab 4, available from jenkins@apple.com
  - 7. References:
    - a. Are references (to other character sets, dictionaries, descriptive texts etc.) provided? Yes
    - b. Are published examples of use (such as samples from newspapers, magazines, or other sources) of proposed characters attached? Yes
  - 8. Special encoding issues:
    - Does the proposal address other aspects of character data processing (if applicable) such as input, presentation, sorting, searching, indexing, transliteration etc. (if yes please enclose information)? Yes
  - 9. Additional Information: See below
- Submitters are invited to provide any additional information about Properties of the proposed Character(s) or Script that will assist in correct understanding of and correct linguistic processing of the proposed character(s) or script.

<sup>1</sup> Form number: N2652-F (Original 1994-10-14; Revised 1995-01, 1995-04, 1996-04, 1996-08, 1999-03, 2001-05, 2001-09, 2003-11)

Examples of such properties are: Casing information, Numeric information, Currency information, Display behaviour information such as line breaks, widths etc., Combining behaviour, Spacing behaviour, Directional behaviour, Default Collation behaviour, relevance in Mark Up contexts, Compatibility equivalence and other Unicode normalization related information. See the Unicode standard at <http://www.unicode.org> for such information on other scripts. Also see <http://www.unicode.org/Public/UNIDATA/UCD.html> and associated Unicode Technical Reports for information needed for consideration by the Unicode Technical Committee for inclusion in the Unicode Standard.

**C. Technical - Justification**

- |  |   |
|--|---|
| 1. Has this proposal for addition of character(s) been submitted before?   | No  |
| 2. Has contact been made to members of the user community (for example: National Body, user groups of the script or characters, other experts, etc.)?          | Yes   |
| If YES, with whom?   | Scholars studying the history of East Asian mathematics |
| If YES, available relevant documents:  | N/A   |
| 3. Information on the user community for the proposed characters (for example: size, demographics, information technology use, or publishing use) is included? |   |

The user community is limited to scholars studying pre-modern East Asian mathematical texts

- |   |      |
|---|------|
| 4. The context of use for the proposed characters (type of use; common or rare)   | Rare |
| 5. Are the proposed characters in current use by the user community?  | No   |
| 6. After giving due considerations to the principles in the P&P document must the proposed characters be entirely in the BMP? | No   |
| 7. Should the proposed characters be kept together in a contiguous range (rather than being scattered)?                       | Yes  |
| 8. Can any of the proposed characters be considered a presentation form of an existing character or character sequence?       |      |

Yes; some are similar to some of the “Hangzhou” numerals or Han numeric ideographs; but since this is a small set which is with overall distinct shapes and combining behavior, it would be best to encode them as a block

- |  |    |
|--|----|
| 9. Can any of the proposed characters be encoded using a composed character sequence of either existing characters or other proposed characters? | No |
| 10. Can any of the proposed character(s) be considered to be similar (in appearance or function) to an existing character?                       |    |

Yes; some are similar to some of the “Hangzhou” numerals or Han numeric ideographs; but since this is a small set which is with overall distinct shapes and combining behavior, it would be best to encode them as a block

- |  |     |
|--|-----|
| 11. Does the proposal include use of combining characters and/or use of composite sequences? | Yes |
| If YES, is a rationale for such use provided?  |     |

Negative numbers were indicated by overlaying a diagonal slash

- |   |    |
|---|----|
| Is a list of composite sequences and their corresponding glyph images (graphic symbols) provided?                   | No |
| 12. Does the proposal contain characters with any special properties such as control function or similar semantics? | No |
| 13. Does the proposal contain any Ideographic compatibility character(s)?   | No |

Counting rod numerals were used in pre-modern East Asian mathematical texts in conjunction with counting rods used to represent and manipulate numbers. The counting rods were a set of small sticks, several centimeters long which were arranged in patterns on a counting board to represent numbers. Counting rods and the counting board provided a flexible system for mathematicians to manipulate numbers, allowing for considerable sophistication in mathematics.

The specifics of the patterns used to represent various numbers using counting rods varied, but there are two main constants :

Two sets of numbers were used, for alternate columns (e.g., ones/hundreds/ten-thousands vs. tens/thousands). The latest forms for the numerals, used when Chinese mathematics had reached its peak, are as given here; during Han dynasty times, the glyphs for the unit digit forms had swapped with those for the tens digit forms. This should be considered a font difference.

Zero was indicated by a blank square on the counting board and avoided in written texts or represented with U+3007 IDEOGRAPHIC NUMBER ZERO. (Historically, U+3007 IDEOGRAPHIC NUMBER ZERO originated as a dot; as time passed, it increased in size until it became the same size as an ideograph. The actual size of U+3007 IDEOGRAPHIC NUMBER ZERO in mathematical texts varies, but this variation should be considered a font difference.) Written texts could also take advantage of the alternating shapes for the numerals to avoid having to explicitly represent zero. Thus, 6708 can be distinguished from 678, because the former would be 一 十 〇, whereas the latter would be 一 七 〇.

Negative numbers were originally indicated on the counting board by using rods of a different color. In written texts, a diagonal slash from lower-right to upper-left is overlaid upon the right-most digit. On occasion, the slash might not be actually overlaid. (See the ninth column from the right in Figure 2 below.)

The predominant use of counting-rod numerals in texts was as part of diagrams of counting boards. They are occasionally, however, used in other contexts, and are occasionally, even in modern texts, occasionally placed within the body of the text itself. As such, they could be part of the equivalent of modern equations, used with ideographs and in complex layouts which are not necessary to represent in plain text. (Again, see Figure 2 below.) Needham, *Science and Civilisation in China*, vol. 3, p. 45 gives as an example:

$$\begin{array}{r}
 \parallel \\
 - \text{||||} \\
 | \perp \top \quad \text{元} \\
 \equiv \text{||||} \perp \text{〇} \quad \text{太}
 \end{array}$$

Here, the 元 marks the x term in the equation and the 太 marks the constant, giving the equation  $2x^3 + 15x^2 + 166x - 4460 = 0$ .

The current use of counting rod numerals is limited to discussions of the history of East Asian mathematics and representations or digitizations of the early texts.

We propose encoding counting rod numerals using nineteen characters: nine even-column digits, nine odd-column digits, and one combining negation marker. The proposed code points are U+1D360 through U+1D372.

There is no need to account for the precise variations in shape of the counting rod numerals from time to time; this can be handled as a font difference.

Note from the figures that the precise layout of the numerals could be moderately complicated in actual texts. We consider that the default behavior would be to lay them out in a fashion similar to ideographs, and any complications for specific texts can be handled by higher-level protocols.

The nineteen characters requested are:

U+1D360		COUNTING ROD UNIT DIGIT ONE
U+1D361		COUNTING ROD UNIT DIGIT TWO
U+1D362		COUNTING ROD UNIT DIGIT THREE
U+1D363		COUNTING ROD UNIT DIGIT FOUR
U+1D364		COUNTING ROD UNIT DIGIT FIVE
U+1D365	┌	COUNTING ROD UNIT DIGIT SIX
U+1D366	┐	COUNTING ROD UNIT DIGIT SEVEN
U+1D367	└	COUNTING ROD UNIT DIGIT EIGHT
U+1D368	┘	COUNTING ROD UNIT DIGIT NINE
U+1D369	—	COUNTING ROD TENS DIGIT ONE
U+1D36A	==	COUNTING ROD TENS DIGIT TWO
U+1D36B	≡	COUNTING ROD TENS DIGIT THREE
U+1D36C	≡	COUNTING ROD TENS DIGIT FOUR
U+1D36D	≡	COUNTING ROD TENS DIGIT FIVE
U+1D36E	⊥	COUNTING ROD TENS DIGIT SIX
U+1D36F	⊥	COUNTING ROD TENS DIGIT SEVEN
U+1D370	≡	COUNTING ROD TENS DIGIT EIGHT
U+1D371	≡	COUNTING ROD TENS DIGIT NINE
U+1D372	∖	COUNTING ROD NEGATIVE NUMBER SIGN

It might be possible to use U+20E5 COMBINING REVERSE SOLIDUS OVERLAY instead of a new character for the negative sign. The main objection to this is that the metrics for U+20E5 may not be appropriate for an East Asian display style, where every character occupies a square.


Examples:

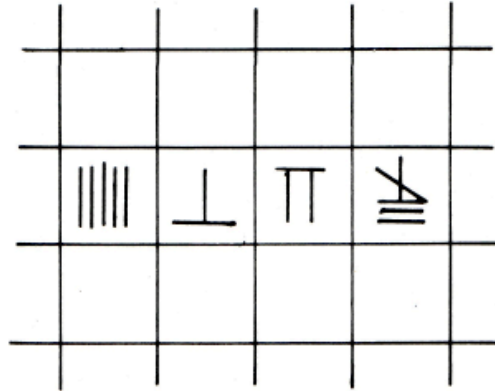


Fig. 17 Using counting-rods to set up negative numbers.

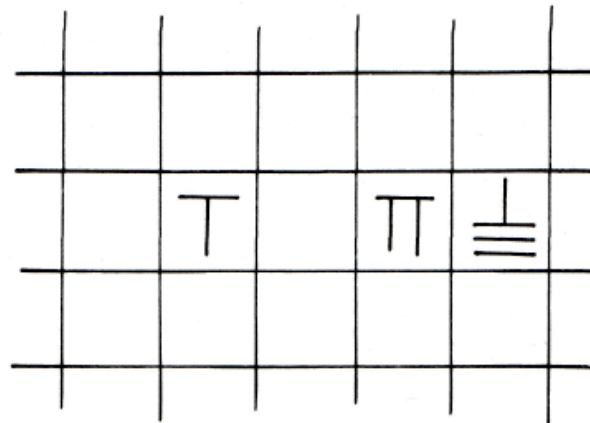


Fig. 18 Using counting-rods to indicate the symbol of zero.

confusion, although in many cases – such as this one – one can easily make inference from the way the digits are written, as 678 is written as  $\perp \parallel \equiv$  and not  $\top \parallel \equiv$ . The first zero sign in a Chinese mathematical text appears in the work of Qin Jiushao 秦九韶 in the mid-thirteenth century AD. For example, he wrote the number 1,470,000 as  $| \equiv \parallel \bigcirc \bigcirc \bigcirc \bigcirc$ .

From page 58 of *Li, Qi, and Shu* by Ho Peng Yoke (Hong Kong University Press: 1985). The discussion in the paragraph is of how 6078 and 678 could be distinguished without a sign for 0.

行羈於上又以斜步乘二之甲南行加於上  
為從方四之甲南行為益廉四步常法開立  
方得半徑

草曰別得斜步為小弦也以斜步減圓徑餘  
為小和也乃立天元為半徑以二之減於甲  
南行得恆<sup>上</sup>為大差也以自之得<sup>三</sup>為  
大差羈也置甲南行羈<sup>內</sup>加大差羈而半  
之得<sup>二</sup>為大弦也<sup>內帶大差</sup>又置甲南  
行羈內減大差羈而半之得<sup>一</sup>為大勾也

測圓海鏡卷五

至知不足齋叢書

<sup>帶大差</sup>分母 又以大差乘股六百步得<sup>二</sup>併入

大勾得<sup>一</sup>為大和也<sup>帶大差</sup>分母 乃先以小

弦乘大和得下式<sup>元</sup>寄左 又以小和阮

乘大弦得<sup>三</sup>為同數與左相消得

下<sup>三</sup>開立方得一百二十步即半徑

也合問

依前問假令乙出東門南行丙出南門東行各

不知步數而立<sup>只云丙行步</sup>甲從乾隅南行

六百步望乙丙與城參相直乙復斜行就丙

From the Ciyuan Haijing.



方面二分之一問四事各幾何

答曰立圓徑一十六尺

立方面二十四尺

平圓徑一十四尺

平方面四十八尺

術曰立天元一為立圓徑。一減二尺餘為平圓徑。一自之就以二十二乘之為二十八段積。就分四之為一百一十二段圓容積。又列立圓徑加八尺為立方面。一再自乘又以一百一十

算學啓蒙卷下

置

三乘之為一百一十。又列立圓徑

十二段立方積也。再自乘九因

為十六段積。又七之為一百一

十二段立圓積。又列立方面二

之為平方面。一自乘又以一百一十二

乘之亦為一百一十二。四位共併為一

十二段平方積也。百一十二段積

寄左列共積一萬八千五百八

十六尺以一百一十二乘之得

二百八萬一千六百三十二與寄左相消

From the Suanxue Qimeng.



# 沈钦裴的四元术补草

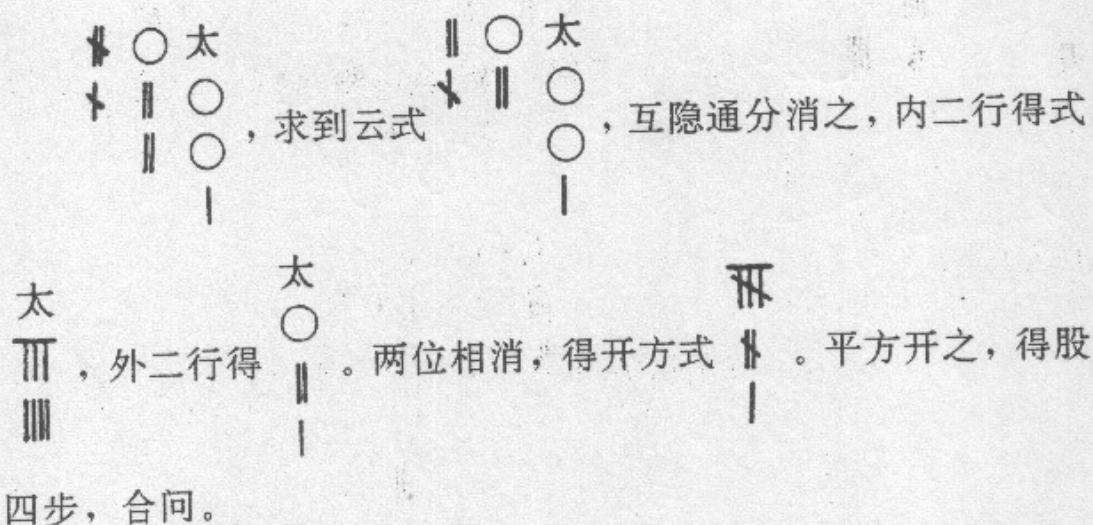
朱世杰的“假令四草”为四元术之范例，然约而不详。清代罗士琳、沈钦裴、陈棠等，都曾为其补草，他们的工作有利于读者理解朱世杰的思想。其中罗士琳草影响较大，但相比之下，笔者以为沈钦裴草更符合朱世杰原意。现根据北京图书馆藏抄本，将沈钦裴《四元玉鉴细草》中“两仪化元”、“三才运元”、“四象会元”三题的细草照录如下。为便于读者比较，将朱世杰的草一并给出。

## 一、两仪化元

今有股幂减弦较较与股乘勾等，只云勾幂加弦较和与勾乘弦同，问股几何？

答曰：四步。

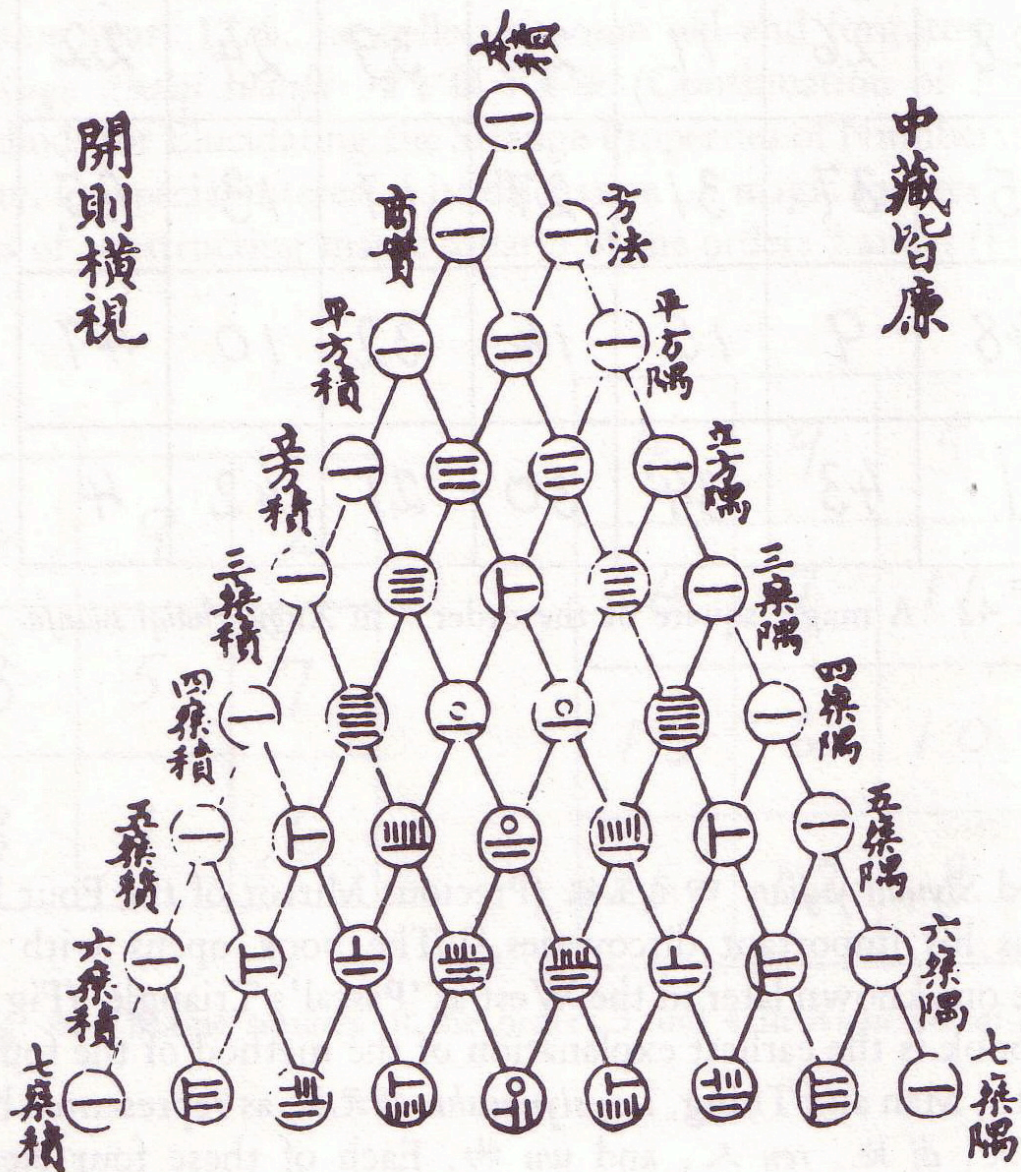
草曰：立天元一为股，地元一为勾弦和，天地配合求之，得今式



From the *Siyuanshu Bucuo*.

On the next page is Pascal's triangle using counting rod numerals.

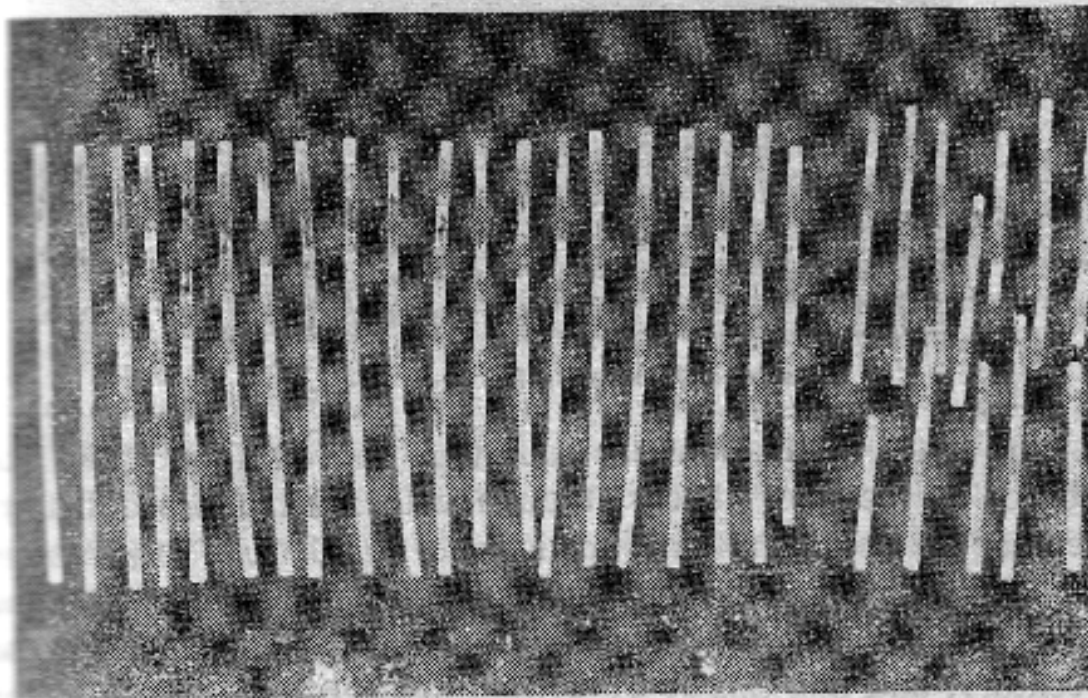
# 古法七乘方圖



一	二	三	四	五	六	七
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積方法 一廉 二廉 三廉 四廉 五廉 六廉 七廉

Qianyang County of Shaanxi Province. Dating back to the reign of the Emperor Xuan Di (73-49 B.C.) of the Western Han Dynasty, they are true to the descriptions in *History of the Han Dynasty* except that they are made of bone. The bundle of rods unearthed



Counting-rods of the Western Han Dynasty unearthed in Qianyang County of Shaanxi Province. They are made of animal bones.

in early 1975 in the No. 168 Han tomb at Fenghuangshan in Jiangling County of Hubei Province are made of bamboo, but a bit longer than the Shaanxi rods. They date from the reign of the Emperor Wen Di (179-157 B.C.).

In rod arithmetic the rods could be placed either upright with  $\text{I} \text{ II} \text{ III} \text{ IIII} \text{ IIII} \text{ T} \text{ II} \text{ III} \text{ IIII}$  equivalent to modern 1, 2, 3, 4, 5, 6, 7, 8, 9 respectively, or horizontally with  $\text{—} \text{=} \text{≡} \text{≡} \text{≡} \text{⊥} \text{⊥} \text{≡} \text{≡}$  playing the same roles. In fact, the rod digits were always placed alternately upright or horizontally to avoid confusion in reading; the upright for units and hundreds, the horizontal for tens and thousands, etc. For instance, 6,708 would be  $\text{⊥} \text{ II} \text{ III}$ , the blank space standing for "zero". All arithmetical operations and even algebraic extractions of square and cubic roots could be thus carried out and traced step by step with rods.

Figure 7. *Ancient China's Technology and Science*, Beijing, Foreign Languages Press: 1983.