Structural Assessment and Historical Review of the Dome at Soltaniyeh

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Table of Contents

V. Analysis Methods and Precedents

VI. Results and Discussion

I. Overview

Part of a larger mausoleum complex in the Iranian province of Zanjan, the dome at Soltaniyeh was constructed between 1302 and 1312. The project was commissioned by the local ruler, Oljeitu, and it is considered by historians as a prime example of the architectural style pioneered in the Il-Khanid dynasty. The dome has a diameter of approximately 25 meters and reaches a height of 49 meters. The double shelled dome rests on a vertical drum which is supported on an octagonal base with a series of pointed arches and detailed *muqarnas*. 8 short minarets are radially arrayed around the dome. The dome is constructed entirely out of herringbone-patterned brick with no wooden or iron tension rings. A layer of thin decorative tile adorns the outside of the dome. For the past half-century, the dome has received intensive renovation work after centuries of disrepair.

Figure 1: The Soltaniyeh dome as seen in 2011 after modern reconstruction efforts on the dome. Repairs are ongoing on restoring other features of the dome, such as the minarets which were heavily damaged and eroded.

The main structural and aesthetic features of the Soltaniyeh dome can be visualized in renderings and drawings of the structure. The 3D model of the building also highlights and reveals all the features and details which are missing or eroded in the present structure. These models were constructed using extensive historical research, site visits, and restoration work extending from 1974 to the present day. All models and drawings of the Soltaniyeh dome were generously provided by Marco Brambilla, who has devoted many years to the detailed study of the building. His colleagues at *Sultaniyya.org* have also been extremely helpful in procuring resources and research material on the structure.

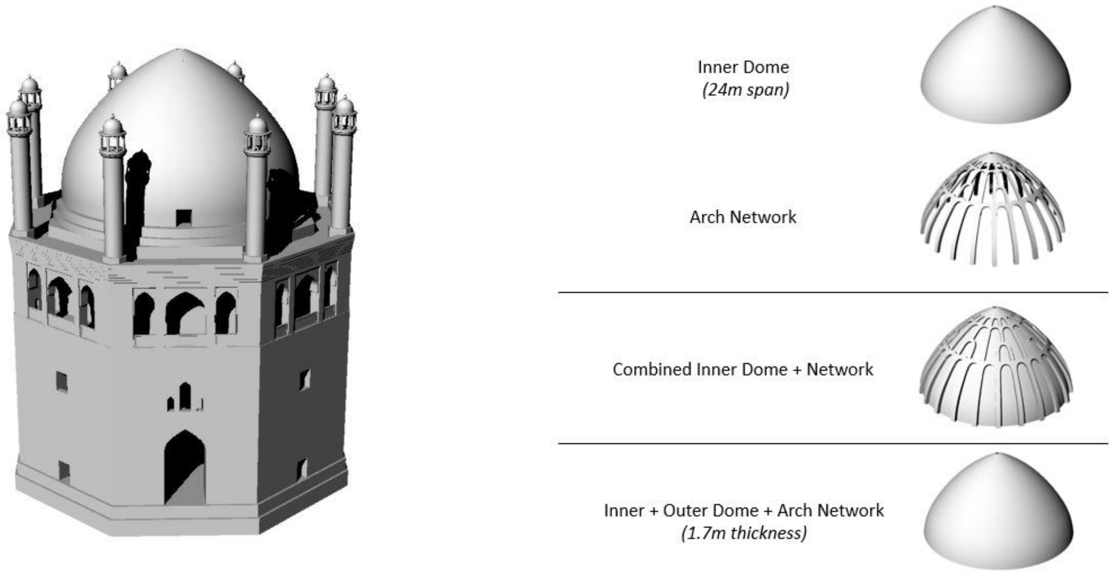


Figure 2: The rendering of the 3D model of the Soltaniyeh dome reveals key details which are not visible in the present structure such as the fully intact minarets. The model also shows important details such as the arch network between the two shells of the dome (Source: Models provided by Marco Brambilla)

A key structural feature of the Soltaniyeh dome is the use of a detailed arch network between the two brick shells (as can be seen in figure 2) which acts as ribbing. The total thickness of the inner and outer dome is about 1.7 meters. Because of its use of a ribbing system between the domes (as well as the use of herringbone pattern brick, *spina-di-pesce*), the Soltaniyeh dome has been compared with the dome of the Santa Maria del Fiore in Florence, Italy (constructed in the 15th century and designed by Filipo Brunelleschi). The Soltaniyeh dome however, remains unique in its use of arches for ribbing and stability. There are several academic inquiries into whether the Florence cathedral was inspired by the Soltaniyeh dome however, there is currently no definite proof for this hypothesis. Rather, it appears that Brunelleschi and the builders of the Soltaniyeh independently produced the same structural solutions given similar constraints and precedents.

The structure is considered one of the earliest double-shells in the Islamic world and it directly inspired several domes in Mughal India. Furthermore, the dome remains the third largest spanning unreinforced brick dome in the world (after the Hagia Sophia in Istanbul and the Santa Maria del Fiore). However, even given this importance, there is currently very limited research and analysis of the Soltaniyeh dome. This paper seeks to provide both a historical review of the building as a product of a unique architectural context as a well as a detailed structural assessment of the dome and its key features.

II. Historical and Architectural Review

The Soltaniyeh dome is particularly emblematic of the Il-Khanid period architecture in Iran. Medieval Persian architecture is best described as an amalgam of several precedents and styles which were adopted and combined given the strategic economic and geographic location of Iran.

One of the oldest elements influencing Medieval Persian architecture (and what would come to be defined as traditional Persian architecture by the time of the Safavid dynasty in the 15th century) is Sassanian architecture. Developed during the Sassanid Empire (224-651 CE), this architectural style was defined by large imperial porches, *iwans*, as well as large vaulted ceilings which can be seen prominently in the ruins of Ctesiphon, in modern day Iraq. The use of brick as both structural and decorative elements was pioneered in the Sassanid Empire. These elements were quickly adopted into early-Islamic architecture. Brought from the Arabian peninsula in the mid-7th century, early-Islamic style prototyped hypostyle mosque architecture (large flat-roofed prayer halls supported with a multitude of columns). This open program was quite flexible as there were few religious restrictions on mosque design (e.g. strategic orientation toward Mecca). By the 11th century, elements of Sassanian architecture prominently appeared in mosque design in Iran and elsewhere in the Islamic world. The Jameh Mosque of Isfahan (constructed and rebuilt in several stages) reveals the addition of high vaulted porches and patios to the traditional hypostyle design. The influence of Seljuk architecture, however, can also be seen in the Jameh Mosque. The Seljuk Empire had extensive contact with the Byzantine Empire and their architecture quickly grew to incorporate a prominent feature of Byzantine design, the dome. As such, the Seljuk influence on Persian architecture can be seen in the two prominent domes in the Jameh Mosque. Indigenous domes had existed previously in the ancient near east, however the spread of the Seljuk Empire saw the proliferation of domed mosques through the Islamic world. The Il-Khanid Empire, which lacked its own distinct architecture, quickly spread and adopted the vernacular architecture of Iran. Il-Khanid rulers invested much in aesthetics and innovative structural features such as transitional zones (going from an octagonal or square base to a circular dome through a series of pendatives and squinches). Indeed, the traditional Persian dome (as seen in the Safavid dynasty through the 17th century) is the Il-Khanid model with few changes. Due to the spread of the Il-Khanid Empire (as part of a larger Mongol advance into the near east), dome architecture spread to India where it was quickly picked up by the Mughal Empire, explaining the similarity in aesthetics and design between Persian and Medieval Indian domes[1].

The dome at Soltaniyeh represents all of the elements of Il-Khanid architecture (which itself adopted an amalgam of several previous styles). As the largest dome constructed under that empire, the structure is both culturally and historically significant. The dome featured prominently in Iranian architecture for a long period of time, until it ultimately fell into disrepair as a result of political turmoil and shifting geopolitics (as centers of powers moved towards the capitals of Isfahan and later Tehran). As figure 3 shows, the dome and mausoleum were the centerpieces of a larger complex of buildings and mosques in the Il-Khanid city of Soltaniyeh.

Figure 3: A 16th century book illustration of the city of Soltaniyeh with the dome and mausoleum clearly visible in the center (building with 8 minarets).

III. Literature Review

There currently exist limited resources on the Soltaniyeh dome. This is mostly due to limited access to the site (a product of geopolitics) as well as limited general knowledge of the building and its historical significance.

General Information, Historical and Structural Review

Andre Godard (1881-1965), a preeminent architect and historian of Iranian architecture, provides a general background on the structure. He cites the Soltaniyeh dome in several of his publications (e.g. *A Survey of Persian Art, Vol. III*). Godard also includes some preliminary sketches of the building and its structural system. In his discussion, Godard notes that the outer dome is purely decorative and not needed structurally. Godard does not provide any calculations or include a more detailed structural analysis of the building or dome.

Marco Brambilla, an important resource in the development of this paper who provided all the necessary models and drawings of the building, has published several papers on the Soltaniyeh dome. His paper *"Large scale building techniques in Ilkhanid Iran"* (2012)[2] looks at the dome within the larger context of Il-Khanid architecture. The paper provides important site information as well as a qualitative assessment of the major structural features of the building. Brambilla's paper, *"Construction techniques of the dome of the mausoleum of Uljaytu in Sultaniyya"* (2012)[3] looks closer at the specific structural details of the dome as well as noting important construction methods. Brambilla also points out the significance of the arch networks in the structural stability of the building as well as providing detailed drawings of their configuration.

Restoration Work

The documentation and work of Piero Sanpaolesi are considered in the work of Brambilla. Sanpaolesi, who was tasked with restoration strategies for the building in the 1970's, published, "*La cupola di Santa Maria del Fiore ed il mausoleo di Soltanieh. Rapporti di forma e struttura fra la cupola del Duomo di Firenze ed il mausoleo del Ilkhan Ulgiaitu a Soltanieh*" (1972). Sanpaolesi includes some of the first detailed sections and plans of the dome. In this paper Sanpaolesi also provides some very important photographs of the dome before modern restoration efforts. These images not only reveal an immense state of disrepair but highlgiht some of the important structural features of the building. The arch network is clearly visible in these images. In addition, images of the interior of the building highlgiht expected vertical cracking patterns (outward movement of walls separate the dome slices or lunes causing distinctive cracking). The images are provided in the following page in figures 4 and 5. Sanpaolesi's paper also devotes much time to the comparison of the Soltaniyeh dome with other domes in the Il-Khanid period and near east as well as the dome of the Santa Maria del Fiore in Florence. Here, significantly, Sanpaolesi points at precedents that inspired the major features of the dome and draws similarities between the Soltaniyeh dome and the dome in Florence[4].

Seismic Analysis

The only widely available detailed structural calculation on the Soltaniyeh dome is provided by Akbar Vasseghi in "*Preliminary seismic evaluation of the historic Sultaniyeh dome*" (2007). Vasseghi and his team of researchers in Iran performed a finite element seismic analysis of the entire building. Vasseghi considered the whole building and dome as one unit. This method may be insufficienct since a finite element model considers tension in masonry structures and the different components of the building were not modelled and tested separately. The paper provides conclusive results:

- An earthquake with a return period of 75 years will not cause significant damage to the structure. This is equivalent to a horizontal ground acceleration of 0.23g.
- An earthquake with a return period of 475 years will produce heaving cracking in the building. This is equivalent to a horizontal ground acceleration of 0.44g.
- An earthquake with a return period of 2500 years will cause complete failure and collapse. This is equivalent to a horizontal ground acceleration of 0.76g.

This is a logical result given that the building, which is a seismically active zone, remains standing[5].

Figure 4: ca 1970's image of the interior of the Soltaniyeh dome revealing distinctive vertical cracking patterns associated with outward movements of support walls. This image comes directly out of Piero Sanpaolesi's paper, "La cupola di Santa Maria del Fiore ed il mausoleo di Soltanieh. Rapporti di forma e struttura fra la cupola del Duomo di Firenze ed il mausoleo del Ilkhan Ulgiaitu a Soltanieh" (1972)

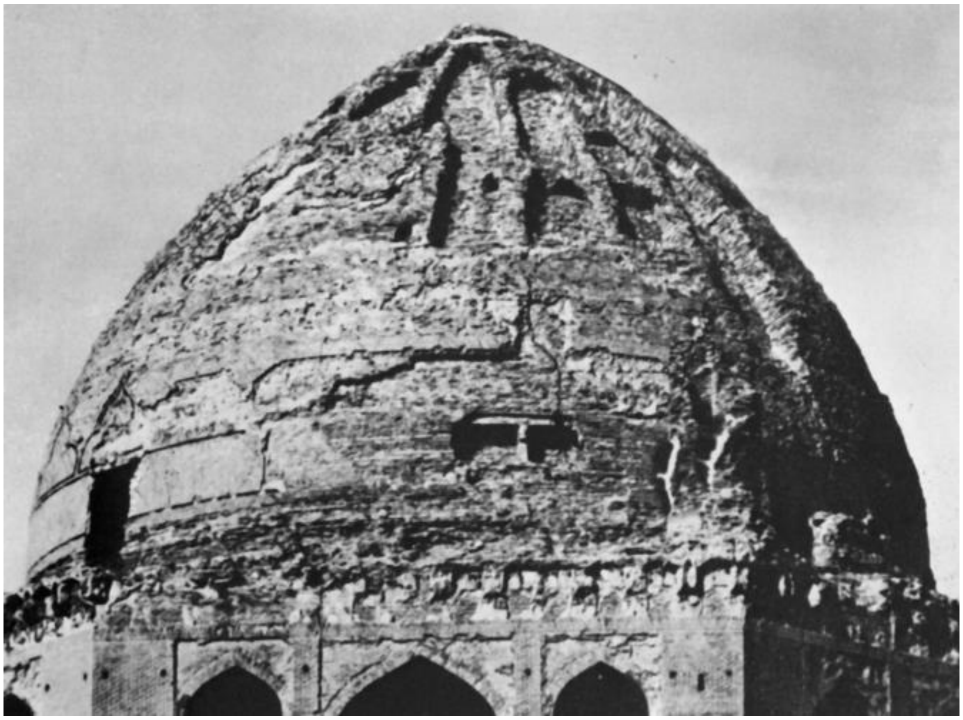


Figure 5: In this image from Sanpaolesi's paper, the weathered exterior of the dome reveals the arch network between the inner and outer shell of the dome. A large part of the outer dome remains intact even though its tile has quite obviously eroded.

IV. Research Questions

Given the significance of the Soltaniyeh dome and the lack of sources with detailed structural analysis of the building, the following research questions motivated the project and are documented in the remainder of the paper. The research questions and subsequent project are divided into two broad categories, static stability and dynamic/seismic stability, and focus exclusively on the dome.

Static Stability
- What thrust line solutions exist and what are the role of the double dome and arches in the structural stability of the dome?

Dynamic/Seismic Stability
-Given that the structure is located in a seismically active zone, what range of earthquake magnitudes will cause damage to the dome and how will it collapse?

V. Analysis Methods and Precedents

a. Static Analysis

In order to establish the static stability of the Soltaniyeh dome and to understand the role of the double shells and arch network, a graphic statics approach was used.

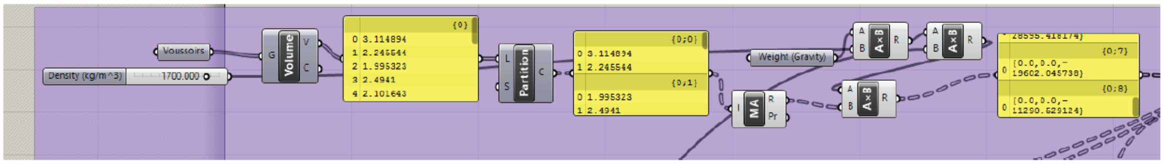
The precedent of looking at double domes with graphic statics exists since at least 1748 when Giovanni Poleni conducted a detailed study of the dome of St. Peter's Basilica in response to concerns caused by serious cracking. Poleni's method accounted for both shells by allowing for the line of thrust to pass through the void between the inner and outer dome. William S. Wolfe, in 1921, looked in depth at the use of graphic statics in construction in his book, *Graphic Statics, A Textbook*. Wolfe considered a slice of the dome, known as a lune, and included assumptions for materials, with and without tensile capacity. The methods described by Wolfe were expanded and applied to Mamluk domes by Wanda Lau in 2006[6].

To evaluate the Soltaniyeh dome using graphic statics, several assumptions and considerations were made. First, a lune of 10° was selected and it was subsequently divided into 9 voussoirs or sections, radially divided (Lau's thesis stipulated that the lune must be between 5° and 15° for a relevant result to be produced). No hoop forces were considered in the dome; this was based on some of the analysis by Wolfe on masonry as well as the consideration of vertical cracking patterns which would not transmit hoop forces as seen in Sanpaolesi's photograph. Finally, only the weight of the voussoirs, acting downwards at their centroids, were considered; live loading and asymmetrical loading conditions were disregarded.

As a means to quickly and effectively evaluate numerous horizontal thrust values and thrust line positions and scaling, the parametric scripting language Grasshopper was used with a lune constructed from the 3D Rhinoceros model provided by Marco Brambilla.

The process is summarized in the figures below:

STEP 1: A 10^{o} lune was selected from the complete model of the Soltaniyeh dome (the 3D modelling program Rhinoceros was used). The dome was subdivided into 9 radially distributed voussoirs.



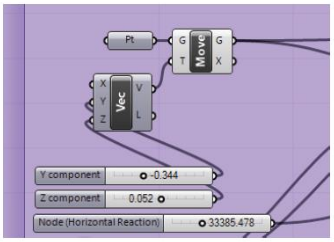
Density (kg/m^3) 1700.000

```
{0}
0 3.114894
1 2.245544
2 1.995323
3 2.4941
4 2.101643
```

```
{0;0}
0 3.114894
1 2.245544
{0;1}
0 1.995323
1 2.4941
```

```
 28555.418174}
{0;7}
0 {0.0,0.0,-
  19602.045738}
{0;8}
0 {0.0,0.0,-
  11290.529124}
```

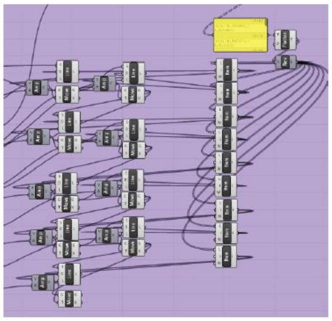
STEP 2: The volume and weight of each voussoir was calculated (this includes the segments both in the inner and outer dome). The weights were considered to be point loads acting in the centroid of the voussoir and were arranged to form a load line.

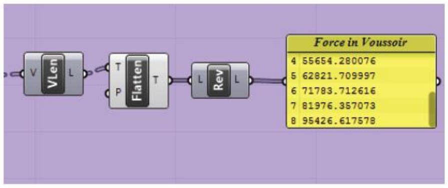


Y component -0.344

Z component 0.052

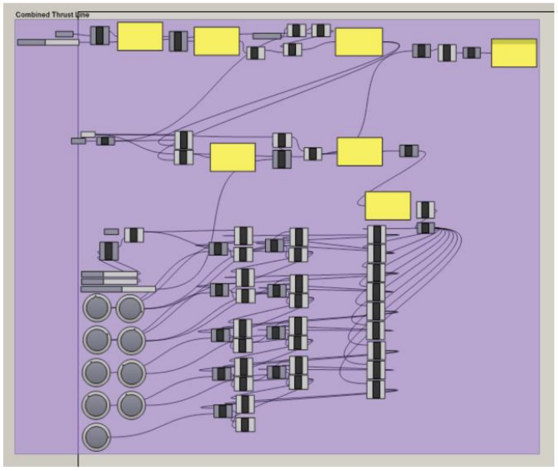
Node (Horizontal Reaction) 33385.478





```
Force in Voussoir
4 55654.280076
5 62821.709997
6 71783.712616
7 81976.357073
8 95426.617578
```

STEP 3: A force polygon was drawn with parametric controls for the horizontal thrust value. Subsequently the force in each member was calculated.



STEP 4: The line of thrust was superimposed onto the model of the lune. Horizontal thrust values were modified as well as scaling of the thrust line and its point of origin to find a thrust line solution which fit within the geometry.

The process was repeated for both the inner and outer dome considered separately. In all cases the arch network was not included since its discontinuous geometry would overcomplicate the graphic statics. The force polygons and values for horizontal thrust and force in members are provided in the appendix of this paper (see appendix).

b. Dynamic/Seismic Analysis

The dynamic analysis of the Soltaniyeh dome will primarily focus on physical experiments performed on a 3D printed, scaled version of the dome.

Jennifer Zessin in her 2012 thesis, "*Collapse Analysis of Unreinforced Masonry Domes and Curving Walls,"* considered several experiments to look at the dynamic performance of a 3D printed dome such as a spreading support test[7]. Matthew DeJong, in "*Seismic Assessment Strategies for Masonry Structures"(*2009) looked at the use of the tilt-table to establish the horizontal ground acceleration necessary to topple masonry structures[8]. The combination of methods will be used to analyze the dynamic performance of the Soltaniyeh dome. Ultimately, relevant values will be provided to match with literature established seismic performance of the dome.

The process to 3D print the dome begins with the 3D model of the building provided by Marco Brambilla. Only the inner dome with arch network will be considered (not including vertical drum beneath the dome) in order to simplify and reduce the cost of printing. The tests performed will be important in determining the significance and role of the arch network. A scale of 1:87 is arbitrarily chosen, given that this will generate a 3D printed model that is about 1 foot in diameter (desirable for cost and convenience). The model is then discretized into 64 pieces (8 straight horizontal cuts and 8 staggered vertical radial cuts). The number of pieces is reasonable since it preserves the ease of construction, but still delivers accurate model performance without having blocks which are too fragile. The discretized pieces are then printed using the ZCorp 3D printer and coated in clear cyanoacrylate. This material performs closely to masonry (as established by Zessin) and the clear glue further protects the pieces.

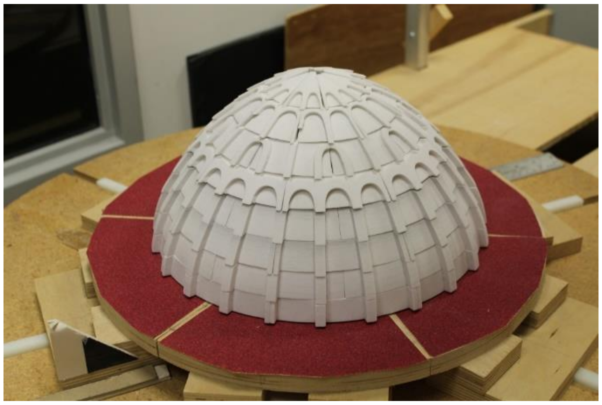


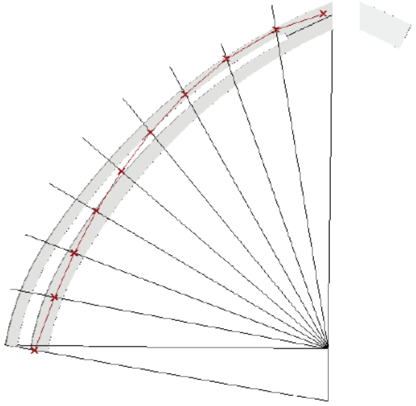
Figure 6: The completed 3D printed dome on the spreading support table.

Figure 7: The 3D printed dome on the tilt table. The smallest pieces on the top row have been removed for ease of construction.

VI. Results and Discussion

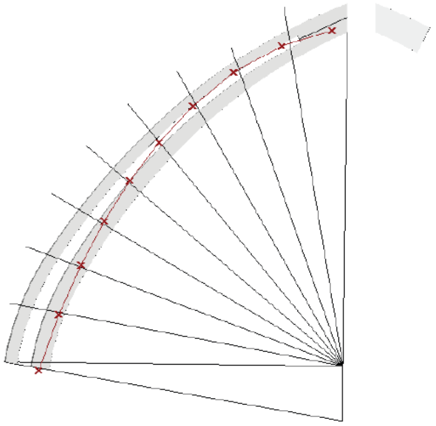
a. Parametric Graphic Statics

The results of the graphic statics calculations can be summarized in the figures below:



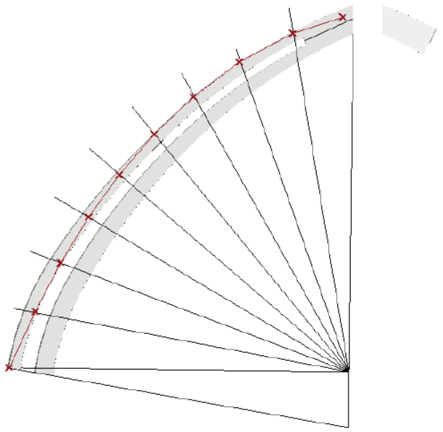
Inner and Outer Dome Combined

When considering both the inner and outer dome together, the line of thrust passes through the structure, ensuring that it is statically stable. Parametric adjustment of the model indicates that there are multiple possible thrust line solutions.



Inner Dome

When only considering the inner dome by itself, there was no thrust solution that fit through the structure indicating that the inner dome is not statically stable by itself.



Outer Dome Combined

No thrust line solution was found for the outer dome, considered by itself. This indicates that the outer dome is not statically stable.

When considering the vertical drum, in addition to the combined inner and outer arch system, a satisfactory thrust line was also found. This is summarized in figure 8 below:

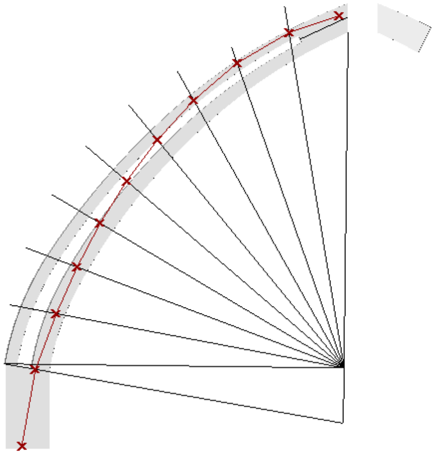


Figure 8: The vertical drum was modelled as a rectangular block beneath the selected lune. The weight of the block was calculated and added to the load line. Using the same Grasshopper parametric model a new thrust line was generated. This thrust line was found to pass through the entire structure. As a result, it can be said that the structure is statically stable even without considering hoop forces.

The results indicate that the structure is stable only when the inner and outer dome are considered together. Neither the inner or outer dome is stable by itself, seemingly contradicting the assessment made by Godard that the outer dome is only for decorative purposes. However, the historical images provided by Sanpaolesi indicate that before modern reconstruction the dome remained standing even when parts of the outer dome was missing. This would indicate that the dome is indeed stable with only the inner dome. This contrast can be explained through the assumptions considered in the Grasshopper parametric model. First, the model did not consider the arch network in place in the actual due to its complex geometry. However, even though the weight of arch network contributes to heavier sections (voussoir), the arches increase the depth of the section considerably. A revised thrust line will indeed pass through this thickened section (as can clearly be seen in the diagram showing the thrust line for inner dome by itself). Furthermore, the model did not consider hoop forces which are active in the actual structure. These forces contribute to stability of a thinner section. Finally, the arch network provides alternate load paths which cannot be considered through a 2D method of graphic statics.

b. 3D Printed Model, Spreading Supports

The spreading support test, meant to simulate the outward movement of supports holding the dome, involves placing the 3D printing dome six outward moving wedges. The wedges move outward at an equal rate.

The 3D printed model of the Soltaniyeh dome performed unusually in the spreading supports test and the dome did not collapse as expected. The lowermost pieces of the dome rotated and the top pieces of the dome slid on top of each other. As a result, the dome was able to withstand much more outward support movement than other experimental domes.

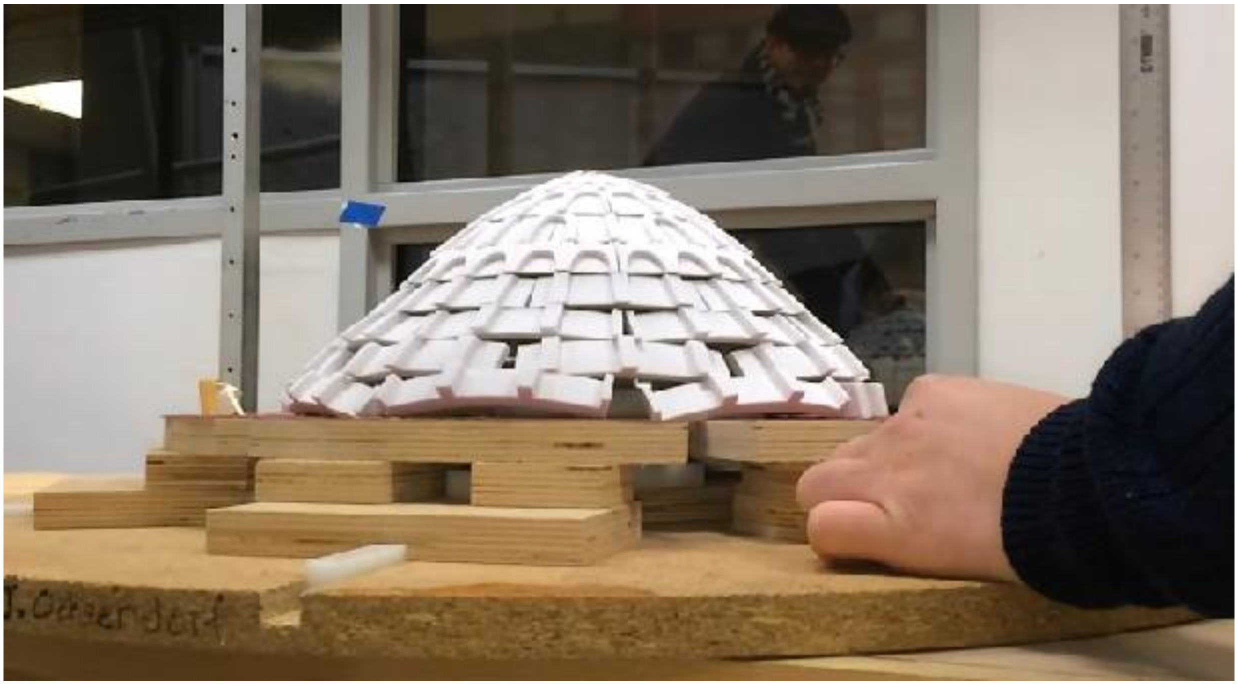


Figure 9: The screen capture from a video of the test reveals the collapse mechanism of the dome. It can clearly be seen that the lower pieces have rotated and some of the upper pieces have slid on top of each other.

The behavior of the domes can be explained through several factors. First, the lower pieces were too long and not heavy enough and rotated as a result. Furthermore, the sliding behavior can be explained through the horizontal discretization strategy used to model the dome. The horizontal cuts promoted sliding. Sliding and the rotation at the base were further increased by the lack of friction between the blocks which can be attributed to the glue coating applied on them. Of course, part of the collapse mechanism may also be explained through the unique dome geometry and the arch network which may have affected the dome specific behavior.

<u>c. 3D Printed Model, Tilt Table</u>

The tilt table test involved placing the dome on a table that was gradually and slowly inclined. This test was indicative of the horizontal ground acceleration that would be experienced by the dome in an earthquake.

The dome failed at an angle of 29-30^o. This is equal to a horizontal ground acceleration of 0.55g, based on the method established by DeJong. Failure occurred through a combination of sliding blocks at hinging which occurred at around the 5th row of blocks (a modelling and printing discrepancy added small dents in this row). This value falls within the range established by Vasseghi in his seismic analysis of the Soltaniyeh dome. Vasseghi indicates cracking at 0.44g and ultimate failure at 0.76g. The difference in values can be explained through several points. First, Vasseghi utilized one FEA model for a complete masonry structure (allowing for tension and not accounting completely for individual section and brick movements). The 3D model of the dome also utilized horizontal cuts which introduced sliding (which began at the light top pieces at around 27°). Sliding failures produced premature failure in the dome. Finally, 3D printed blocks, especially those coated with glue, have a different performance than bricks especially with regards to friction between blocks. That being said, the value of 0.55g for ultimate dome failure is a conservative estimate of the dome's performance while Vasseghi's FEA model may be an overconfident model that does not account for individual variations within the masonry structure.

Figure 10: Three frames from the collapse of the dome. First, sliding failure occurred at the top blocks since they were the smallest and lightest. Ultimate failure occurred at 29-30° which corresponds to a horizontal ground acceleration of 0.55g. Hinging occurs at around the 5th row of the dome (due to specific qualities of the model) and the dome collapses in on itself. The collapsed blocks can be seen in the third frame.

VII. Conclusion and Future Work

The research and experimentation on the Soltaniyeh dome revealed very important and interesting information on this extremely important historic structure. The results add on to and enrich the existing research conducted on the dome.

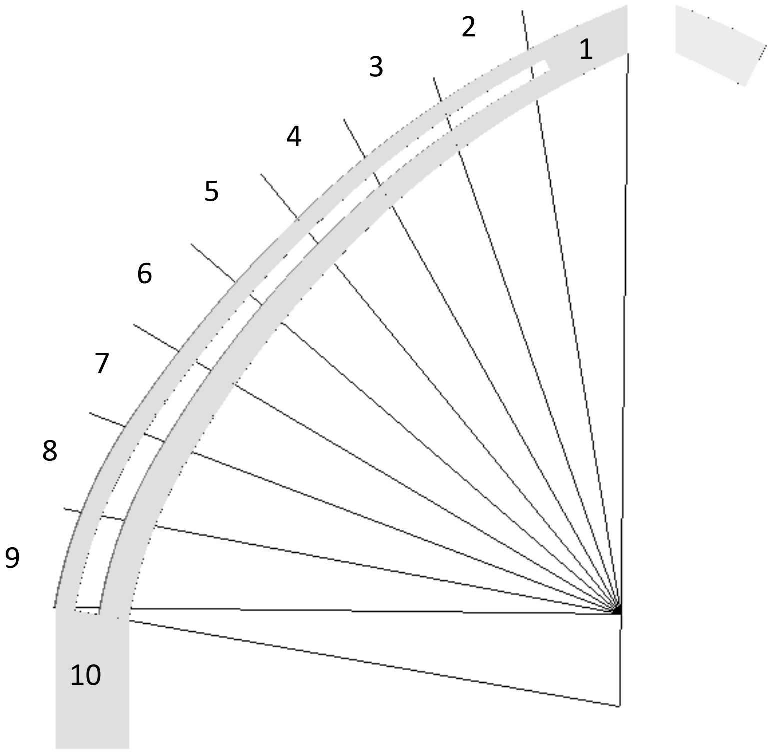
1. The dome is only stable when both the inner and outer dome are considered together and the arch network is ignored. Under no condition is the outer dome stable by itself. The inner dome is only stable by itself when the arch network is included in calculation. Hoop forces and alternate load paths (as part of a more rigorous 3D analysis of the structure) must also be included for a complete static analysis of the building.
2. The model of the Soltaniyeh dome collapses with a horizontal ground acceleration of 0.55g. This is within the range established by FEA models of the building however, limitations for both the FE analysis and the experimental model must be considered.

These results prompt future work and modifications to the methods of analysis. In regards to graphic statics calculation, a more rigorous analysis will include consideration of several lunes with different arch geometries included. Hoop forces must also be considered based on the methods established by Wolfe. For the experimental model, a major modification will include printing the dome with radial instead of horizontal cuts. The comparison of the results will indicate which discretization method is more effective in conveying dome behavior. Furthermore, the vertical wall may also be printed and included in both the tilt test and spreading support test to better indicate the collapse of the actual dome. In addition, another simulation method such as discrete element analysis may be used instead of FEA to model the collapse of the dome given dynamic conditions.

VIII. Appendix

Numbered Voussoir and Lune Section

From a total dome with a volume of 1042.76 m^3, a 10^o degree lune was considered with a brick density of 1700 kg/m^3. The figure below indicates the lune and the numbered voussoirs which correspond to numbering in the load line and force polygon.



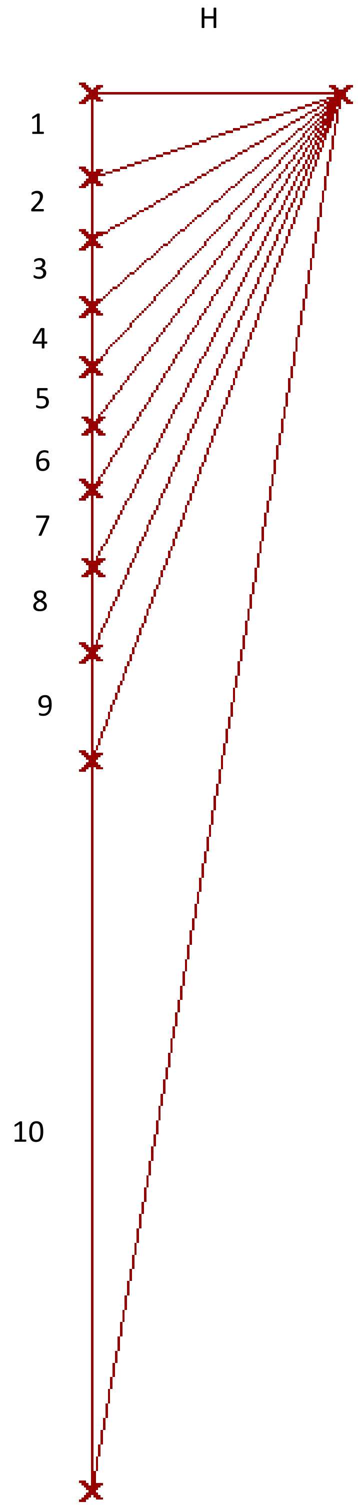
Force in each Voussoir (Section)

Force in Each Section (kN)	1	2	3	4	5	6	7	8	9	10
	35.2	38.7	44.0	49.6	55.7	62.8	71.8	82.0	95.4	190.4

The above table summarizes the force in the voussoirs considering both the inner and outer domes.

Force Polygon

The following is a representation of the force polygon generated through graphic statics in Grasshopper. For reference, the horizontal reaction force (labelled as H in the diagram) is 33.4 kN.



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