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Tesla Coil – Double Tuned Resonant Transformer - Analysis and Design

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ABSTRACT: Tesla coil also known as a double tuned resonanate transformer was proposed by a Serbian - American inventor Nicholas Tesla in around 1891. It is basically an electrical transformer that can produce high AC voltages at high frequecny and low currents.This prototype proposed by Nicholas Tesla has been accepted by the world for its other features most importantly his renmrks toward wireless transfer of electricity. This paper describes the basics in the construction of a simple tesla coil model, working principle and its circuit analysis to detemine the optimum constructional parameters and output voltage equations.

KEYWORDS : Resonant Transformer , Toroid , Streamer arc , spark gap, Tuning ratio , Rotary spark.

I. INTRODUCTION

Tesla coil is an electrical transformer capable of producing very high AC voltages of high frequency and low current. Tesla coil was proposed by Nikola Tesla, an American scientist who worked in the field of alternating currents which lead to war of currents between Edison and Tesla in the 19th century. Tesla coil originally termed as a device for transmitting electrical energy without wires was patented to Tesla in 1914 .Tesla coil works on the basic principle of electrical resonance and hence it is also termed as a simple resonant transformer. Tesla conducted many experiments in the field of alternating current and majority of his experiments were focused on method to transmit wireless electricity to consumers. His vision was to wirelessly distribute power over large distances using the earth's ionosphere. Due to lack of financial assistance many of his ideas still remain in notebooks which are currently examining by young engineers for unexploited clues into the field of wireless electricity. This paper presents a detailed analysis of tesla coil with electrical analysis and design part.

II. LITERATURE REVIEW

- James C. Maxwell in 1864 predicted the existence of radio waves by means of mathematical model.
- John H. Poynting in 1884 realized that the Poynting Vector would play an important role in quantifying the electromagnetic energy.
- Nikola Tesla started his work on wireless transmission in 1891 at his "experimental station" at Colorado. A small incandescent resonant circuit, grounded on one end was successfully lighted. Wardenclyffe tower designed by Tesla for trans- Atlantic wireless telephony and also for demonstrating wireless electrical power transmission.
- William C. Brown contributed much to the modern development of microwave power transmission which dominates research and development of wireless transmission today.

III. CONSTRUCTION

Tesla coil or a Tesla Transformer is capable of producing high voltages ranging from several hundred kilo volts to several mega volts in high frequency ranges. Tesla coil is a double tuned resonant circuit consisting of following key components:

- Primary coil - Inductive part of the primary circuit and form a resonant circuit. The primary coil is powered by the control circuitry and generates the magnetic field that the secondary use to create the high voltage. It is the few

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turns of thick wire at the base of the secondary coil. Usually constructed from copper tubing or heavy gauge wire. It is easier to vary the inductance than the primary capacitance.

- Primary capacitance - The capacitance value used to determine the resonant frequency Spark gap - High powers switch and it is a brain to Tesla Coil. Initiating the discharge of the tank capacitor into the primary winding. It turns-on when sufficient voltage exists across the spark gap. The air in the gap ionizes and begins to conduct electricity like a closed switch.
- Secondary coil - It is a single air cored inductor wound with thousand of turns of insulated wire. The secondary coil is a long cylinder. It is PVC pipe covered by an enameled wire. One side is connected to ground high voltage comes through the other side. The high voltage terminal is connected to a toroidal electrode
- The Top load Toroid: The top load is the metallic object at the top of the secondary coil. It provides a capacitance to the Tesla coil. Known as top loads form the capacitor for the secondary circuit. Any lump of metal will form a capacitance. It forms a resonant circuit at the same frequency as the primary circuit.
- Power source.

IV. WORKING PRINCIPLE

Tesla coil or Tesla Transformer works on the principle of electrical resonance. The primary is supplied through a capacitor C1. A spark gap G is connected across the primary winding which can be triggered at a desired voltage. The primary and secondary coils form two resonance circuits. The primary winding inductance with capacitor C1 and the primary winding resistance element forms a resonant circuit in the primary, the secondary winding inductance $_2$, its resistance and capacitance C2 of the toroid electrode forms a resonant circuit in the secondary. The primary and secondary windings are tuned to resonance such that the primary resonant frequency is equal to the secondary resonant frequency. As the primary is supplied from an external source and the when voltage across the spark gap exceeds the desired level that is the breakdown voltage of the spark gap a spark stars which reduces the resistance of the gap and the spark gap starts conducting. When the gap starts conducting, a closed circuit is formed in the primary and the current from the capacitor C1 start flowing from one plate of the capacitor to another through the inductor L1. Hence electrostatic energy stored in the capacitor is transferred to the inductor and stored as electromagnetic energy. Once the capacitor is empty the current flow stops and the current starts to flow in the opposite direction into the capacitor through the inductor L1. The current continues to flow back and forth from one plate of the capacitor to another until the spark gap ceases due to insufficient energy due to energy loss occur during the current flow as heat. As the current oscillates at resonance frequency through the primary coil the oscillating magnetic field created in the primary coil induces an oscillating current in the secondary coil by Faraday's law of induction since the two coils circuits are tuned to the same resonance frequency and the energy in the primary circuit is transferred to the secondary circuit. The oscillating voltage in the secondary circuit increases in amplitude this process is known as a ring up or secondary ring up and oscillations in the set primary die out as the energy become insufficient to maintain the conducting spark in the spark gap. This decrease in primary oscillations is known as primary Ring down. The secondary current flowing through the secondary circuit will also create an oscillating magnetic field that induces a voltage to the primary winding and hence the energy is transferred back to the primary coil and energy shift rapidly back and forth between the primary and secondary circuits until the spark lab ceases due to energy dissipation as heat in the spark gap and resistance of the coil. The secondary circuit capacitance C2 is due to the capacitance of the toroid electrode and the parasitic capacitance between the terms of the coil and the capacitance C2 is of very small value and as result of the oscillating current in the secondary circuit a high voltage appears across a secondary capacitance C2 as compared to the primary voltage. After the spark ceases, again the input supply voltage charges the capacitor C1 and again the cycle repeats. The oscillations dies out in a very few milliseconds and for each spark, High Voltage damped sinusoidal wave is obtained at the output terminal of the coil. each pulse die out before the occurrence of the next pulse and hence the coil generate a pattern of damped and not a continuous sinusoidal wave the Spark forms and dies out in very short time and as a result the Spark appears to be continuous and a continuous High Voltage streamer is obtained at the top of the coil that is at the torrid electrode. across As the output voltage increases, it reaches the point where the air next to the high voltage terminal ionizes and corona discharges, brush discharges and streamer arcs break out from the secondary the toroid structure of the system.

How it differs from a conventional transformer:

Tesla coil differs from an ordinary Transformer in many ways:

- Tesla coil like ordinary Transformers does not have a solid iron core to increase the magnetic coupling between the coils instead Tesla coil consist of air core cylindrical core of insulated material as a result and core losses are not occurring in Tesla coil due to absence of Eddy current and magnetic hysteresis.
- Ordinary transformers are tightly coupled in order to increase the efficiency of energy transfer between the primary and secondary in contrast the Tesla transformer is loosely coupled. The coefficient of coupling ranges from 0.05 to 0.20 approximately.

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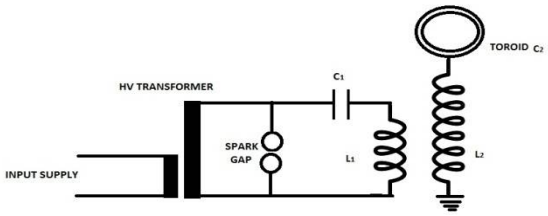
- It consists of single layer of winding unlike ordinary Transformers which are large number of layers of windings, and hence it reduces proximity effect losses as in case of ordinary Transformers.
- Tesla coil works with open circuit secondary but an ordinary Transformer works with a secondary load.

The output from a Tesla coil is not proportional to the transformation ratio k as in case of ordinary transformers which works on the principle of electromagnetic induction, Tesla coil works on the principle of electrical resonance and its output voltage is proportional to the square root of the ratio of capacitance C1 and C2 that we will discuss later.

V.ANALYSIS OF TESLA COIL



Operation of Tesla coil can be analyzed by considering the tesla coil equivalent circuits to inductively coupled resonant circuits. The primary and secondary circuits are shown in the figure. The primary circuit is closed by the conduction of the spark gap and it consists of the primary capacitor C1 the primary winding inductance L1 and its equivalent resistance R1. Also the secondary circuit is formed by the secondary winding inductance L1, the resistance R2 of the winding coil and the toroid capacitance C2. Also the primary and secondary coils are inductively coupled with a mutual inductance M.
According to Kirchhoff's Voltage law, the algebraic sum of EMF and voltage drop around a closed circuit is zero. Then for the primary and secondary resonant circuits;

$$R_1 i_1 + \frac{1}{C_1}\int i_1 .dt + L_1 .\frac{di_1}{dt} + M.\frac{di_2}{dt} = 0 \ldots..(1)$$

$$R_2 i_2 + \frac{1}{C_2}\int i_2 .dt + L_2 .\frac{di_2}{dt} + M.\frac{di_1}{dt} = 0 \ldots\ldots.(2)$$

Let q_1 and q_2 be the instantaneous charge on capacitors C_1 and C_2 , then

$$\frac{dq_1}{dt} = i_1, \quad \frac{dq_2}{dt} = i_2$$

Then equations (1) and (2) becomes;

$$R_1 \frac{dq_1}{dt} + \frac{1}{C_1}q_1 + L_1 .\frac{d^2 i_1}{dt^2} + M.\frac{d^2 q_2}{dt^2} = 0$$
$$R_2 \frac{dq_2}{dt} + \frac{1}{C_2}q_2 + L_2 .\frac{d^2 i_2}{dt^2} + M.\frac{d^2 q_1}{dt^2} = 0$$

Let, D be a differential operator with respect to time t, then the above equations can be represented as;

$$[D^2 + \frac{R_1}{L_1}D + \frac{1}{C_1 L_1}]q_1 + \frac{M}{L_1}Dq_2 = 0$$
$$[D^2 + \frac{R_1}{L_1}D + \frac{1}{C_2 L_2}]q_1 + \frac{M}{L_1}Dq_2 = 0$$

The above equation gives characteristic equation of the form,

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$$1-k^2)D^4+\frac{R_1}{L_1}+\frac{R_2}{L_2}D^3+(\omega_1{}^2+\omega_2{}^2+\frac{R_1}{L_1}.\frac{R_2}{L_2}D^2+\frac{R_1}{L_1}\omega_2{}^2+\frac{R_2}{L_2}\omega_1{}^2D+\omega_1{}^2\omega_2{}^2=0 \quad ..(3)$$

where,

$k=\frac{M}{\sqrt{L_1L_2}}$ coefficient of coupling between the coils

$\omega_1=\frac{1}{\sqrt{L_1C_1}}$ And $\omega_2=\frac{1}{\sqrt{L_2C_2}}$, ω_1 *and* ω_2 are the angular resonance frequencies of the uncoupled primary and secondary circuits.

Equation (3) is a linear homogenous differential equation which has 4 complex roots (D_i)

If the roots are distinct then the four functions

$$y_i=e^{D_i t} \qquad For,\ i=1....4$$

The general solution for this system of equations is given,

$$q_1=\sum_{i=1}^{4}A_i.e^{D_i t}\ and\ q_2=\sum_{i=1}^{4}B_i.e^{D_i t}$$

A_i *and* B_i are constants that can be determined from initial conditions at t = 0,

$$q_1=0\ and\ q_2=q_0$$

i.e,

$$Dq_2=Dq_2=0$$

where q_0 is the charge initially on the primary capacitor C_1.

The primary and secondary capacitor voltages are given by

$$v_1=\frac{1}{C_1}.q_1=\frac{1}{C_1}\sum_{i=1}^{4}A_i.e^{D_i t}$$

$$v_2=\frac{1}{C_2}.q_2=\frac{1}{C_2}\sum_{i=1}^{4}B_i.e^{D_i t}$$

For ideal case, there is no damping, ($R_1=R_2=0$) , the roots of equation (3) have only imaginary parts and secondary voltage is given by

$$v_2t=\frac{2kV_0}{\sqrt{(1-T)^2+4k^2T}}\sqrt{\frac{L_1}{L_2}}sin(\frac{(u_1+u_2)}{2}t)\,sin(\frac{(u_2-u_1)}{2}t) \quad .....(4)$$

where,

$T=\frac{L_2C_2}{L_1C_1}=\frac{\omega_1{}^2}{\omega_2{}^2}$ is defined as the tuning ratio of the circuit

V_0 = the initial voltage across primary capacitor C_1

Also u_1 *and* u_2 are the resonant frequencies of the primary and secondary circuits when coupled and are given by equations.

By proper designing of k *and* T , u_1 *and* u_2 and are always real.

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$$u_1 = \omega_2 \sqrt{\frac{1+T-\sqrt{(1-T)^2+4k^2T}}{2(1-k^2)}} \quad \text{.....(5)}$$

$$u_2 = \omega_2 \sqrt{\frac{1+T+\sqrt{(1-T)^2+4k^2T}}{2(1-k^2)}} \quad \text{.....(6)}$$

From equation (4), it is clear that, the secondary voltage v_2 is a high frequency oscillation and the output voltage v_2 is higher than V_0 at time $t = 0$. This is the basic principle used by tesla coil to generate high secondary voltage.

For maximum output voltage, the maximum voltage gain G, can be obtained from equation (4) when both the sine terms in equation are equal to +1 and -1 simultaneously.

$$G = [\frac{V_2}{V_0}] = \frac{2k}{\sqrt{(1-T)^2+4k^2T}} \sqrt{\frac{L_2}{L_1}}$$

i.e. $\frac{(u_1+u_2)}{2}t = \frac{\pi}{2} + m\pi$ and $\frac{(u_1-u_2)}{2}t = \frac{\pi}{2} + n\pi$, where m and n are integers.

For n=0, we have $\frac{u_2}{u_1} = \frac{1+m}{m}$ ..... (7)

Substituting equation (5) in equation (7) we get

$$k = \sqrt{\frac{\gamma^2(1+T)^2-1-T^2}{4T}} \quad \text{.....(8)}$$

where,

$$\gamma = \frac{2m+1}{2m^2+2m+1} \quad \text{.....} \quad (9)$$

Then maximum gain G is then given by

$$G = [\frac{V_2}{V_0}] = \sqrt{\frac{\gamma^2(1+T)^2-(1-T)^2}{\gamma^2T(1+T)^2}} \cdot \sqrt{\frac{L_2}{L_1}}$$

$$V_2 = V_0.G = V_0.G.\sqrt{\frac{L_2}{L_1}} \quad \text{.....(10)}$$

Hence by choosing a value for m, calculating γ from equation 9, taking suitable value for T, and calculating k using (8), the sine terms product will be maximum and so will be the output voltage.

We have, $T = \frac{L_1C_2}{L_2C_1} = g^2 . \frac{C_2}{C_1}$

i.e, $g = \sqrt{\frac{C_1}{C_2}T} \propto \sqrt{T}$ .....(11)

From (10), and (11) $V_2 \propto G.\sqrt{T}$

Therefore V_2 is maximum when $T = 1$

In order to achieve the optimal performance, two conditions are important:

- Tune the primary coil to achieve $T = 1$.
- Increase k as much as possible.

Tuning ratio T can be adjusted by adjusting the primary winding and primary capacitor

When T=1,the output voltage is given by the equation,

VI. DESIGN SPECIFICATIONS OF TESLA COIL.

A Tesla coil may be a basic two coil arrangement of pricol inductive coupling arrangement. In a basic two coil Tesla coil energy from the primary circuits transferred to the secondary circuit by inductive coupling. In a coil arrangement the primary coil is connected in series and inductively coupled to the secondary primary and secondary therefore represent an autotransformer connection that fits the extra series coil.

Spark gap used in Tesla coil maybe static or Rotary Spark gap in a static Spark static couple of electrodes separated by a suitable dielectric material like nitrogen is used Spark starts when voltage level exceeds the threshold limit and Spark extinguish when

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current through the gap decreases below a certain specified value. static Spark apps are more expensive to purchase and they are not suitable for high power application in such cases Rotary Spark games are employed .They basically consists of a set of stationery electrodes and a set of a rotating electrodes when the rotating electrode comes in proximity to the stationary electrode the clearance between them becomes minimum and Spark starts. Spark gaps can be easily manufactured by using brushless DC Motors.

By using a Rotary Spark gap the triggering voltage tolerance can be eliminated by adjusting the clearance of the electrode to a minimum value and hence the dielectric breakdown can be made to order in every condition also precise timing of spark occurrence can be achieved by employing speed control of the spark gap motor.

Basically the output of a Tesla coil is a combination of three
1. An Electromagnetic field and an electrostatic field generated by the secondary coil and its top toroid terminal
2. Arc formation into free air
3. Discharges to solid grounded poems

1. Secondary Design:

According to wheeler equation the secondary inductance of a Tesla coil can be estimated by using the equation

$$L_S = \frac{R^2 N^2}{2540(9R+10H)}$$

where,

R = *Secondary radius(cm)*, L_s = *Secondary Inducstance (mH)*, H = *secondary height (cm)* and N = *number of turns*

The secondary self capacitance is estimated using the medhurst formula

$$C_S = KD$$

where,

C_S = secondary self capacitance (Pf), D =secondary diameter(cm), K =constant depending on the H/D ratio

Top terminal design:

The selected top terminal is a toroid with an outer diameter $d1$ equal to the secondary height and a cross section diameter $d2$ big enough to provide a large radius of curvature. The toroid capacitance is calculated from the following empirical equation

$$C_T = 2.8(1.278 - \frac{d_2}{d_1}\sqrt{0.1217(d_2 d_1 - d^2_2)}$$

where,

C_T - toroid capacitance (pF), $d1$ - toroid outer (cm). $d2$ - toroid cross section diameter (cm)

The self-resonance frequency of the secondary with the top load mounted drops down to

$$f_r = \frac{1}{2\pi\sqrt{L_S(C_S+C_T)}}$$

2. Primary Design:

Primary capacitor choice:

Capacitance values for the primary capacitor usually vary between 0.05 µF and 0.2 µF.
Optimum value from practical experience is 0.1 µF.

Primary Winding design:

Optimal performance can be achieved if secondary and primary circuits have got the same Resonance frequency

$$L_p = \frac{1}{C_T}\frac{1}{(2\pi f_r)^2}$$

Width of windings on one side is given by

$$W_P = N_p - 1(S_T + D_T) + D_T$$

where, Wp= width of windings on one side (cm), Np= number of turns, St= turn spacing (cm), Dt =pipe outer diameter (cm)

Also,

$$R_{AVG} = \frac{D_i + W_p}{2}$$

where, Wp =width of windings on one side (cm),Di =inner diameter (cm),Rav= average radius (cm)

$$L_P = \frac{R_{AVG}{}^2 N_P{}^2}{20.32 R_{AVG} + 27.94 W_p}$$

where, Lp primary inductance (cm),Wp width of windings on one side (cm),Rav average radius (cm)

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VII. CONCLUSION

A detail analysis of the working operation and design analysis of practical tesla coil is presented in this paper. The working principle of a basic tesla coil circuit is as explained to produce high voltage streamer arcs across the toroid capacitor. It is an efficient method for developing high voltage low current high frequency output voltage. As explained the design of the tesla coil circuit with the specified components is very useful in experimental study and small tewla coil model design works.

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