Liquidation Risk

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Turmoil in financial markets is often accompanied by a significant decrease in market liquidity. Here, we investigate how such key risk measures as likelihood of insolvency, value at risk, and expected tail loss respond to bid–ask spreads that are likely to widen just when positions must be liquidated to maintain capital ratios. Our results show that this sort of illiquidity causes significant increases in risk measures, especially with fat-tailed returns. A potential strategy that a financial institution may adopt to address this problem is to sell illiquid assets first while keeping a "cushion" of cash and liquid assets for a "rainy day." Our analysis demonstrates that, although such a strategy increases expected transaction costs, it may significantly decrease tail losses and the probability of insolvency. In light of our results, we recommend that financial institutions carefully examine their strategies for liquidation during periods of severe stress.

Turmoil in financial markets is often accompanied by significant decreases in market liquidity. Financial institutions that need to liquidate positions under such stress to meet capital requirements may, therefore, face unexpectedly high bid–ask spreads, triggering additional losses in the form of transaction costs. The result may be a vicious circle of sales, which cause illiquidity losses, which necessitate further sales, and so on. Although the negative correlation between bid–ask spreads and asset prices clearly has adverse effects on financial institutions, especially those with significant leverage, the magnitude and practical relevance of this phenomenon for risk management has not previously been assessed.

We investigated the impact on key risk measures—such as the likelihood of insolvency, value at risk (VAR), and expected tail loss—of spreads that are likely to widen just when positions must be liquidated to maintain capital ratios. We consider a simple model of a leveraged financial institution that holds cash, liquid assets, and illiquid assets and that is subject to minimum capital requirements. Using a Monte Carlo analysis of 10-day trading periods, we study the link between negative return–spread correlation and these risk measures.

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The Model

For simplicity, we consider an institution with three assets—cash, a relatively liquid asset, and an illiquid asset.

Asset Price and Spread Dynamics. Let $S_{0,t}$ denote the value at time t of a position of $S_{0,0}$ that was invested in cash at Time 0. We assume that cash earns a fixed rate of return, r, with no bid–ask spread. Then,

$$S_{0,t} = S_{0,0} \exp(rt). \tag{1}$$

We assume that the mid-prices of a liquid and an illiquid asset are geometric Brownian motions. The mid-price of the liquid asset at time t is

$$S_{1,t} = S_{1,0} \exp(\mu_1 t + \sigma_1 B_{1,t}), \tag{2}$$

and the mid-price of the illiquid asset at time t is

$$S_{2,t} = S_{2,0} \exp\left[\mu_2 t + \sigma_2\left(\rho B_{1,t} + \sqrt{1-\rho^2} B_{2,t}\right)\right], \tag{3}$$

where B_1, B_2, B_3, . . . are independent standard Brownian motions, μ_i and σ_i determine the instantaneous expected return and volatility of mid-price S_i, and ρ is the instantaneous correlation between the mid-price increments of the liquid and illiquid asset.

Let $X_{i,t}$ denote the (relative) mid-to-bid spread at time t on asset i. That is, the bid price for the liquid asset is $S_{1,t}(1 - X_{1,t})$ and the bid price for the illiquid asset is $S_{2,t}(1 - X_{2,t})$. We assume that

$$X_{1,t} = X_{1,0} \exp\left[\gamma_1\left(\rho_1 B_{1,t} + \sqrt{1-\rho_1^2} B_{3,t}\right) - \frac{1}{2}\gamma_1^2 t\right] \tag{4}$$

and

$$X_{2,t} = X_{2,0}\exp\left\{\gamma_2\left[\rho_2\left(\rho B_{1,t} + \sqrt{1-\rho^2}B_{2,t}\right) + \sqrt{1-\rho_2^2}B_{4,t}\right] - \frac{1}{2}\gamma_2^2 t\right\}, \quad (5)$$

where γ_i denotes the volatility of the relative bid–ask spread on asset i and ρ_i determines the correlation between the mid-price increment of asset i and the change in the spread on asset i. With $\rho_i < 0$, spreads are expected to widen as prices fall.[1]

This formulation implies no time trend in spreads or correlation between spreads across different assets beyond that induced by mid-price movements. Reflecting the idea that Asset 1 is more liquid than Asset 2, we set initial spread values such that $X_{2,0} > X_{1,0} > 0$.

The Institution's Liquidation Behavior. At Time 0, the institution starts with the following asset and capital structure. It holds $\alpha_{0,0}$ units of cash, $\alpha_{1,0}$ units of the liquid asset, and $\alpha_{2,0}$ units of the illiquid asset. The total portfolio value evaluated at mid-prices is

$$A_0 = \alpha_{0,0}S_{0,0} + \alpha_{1,0}S_{1,0} + \alpha_{2,0}S_{2,0}. \quad (6)$$

The initial value of the liabilities is L_0. Thus, initial capital is

$$K_0 = A_0 - L_0 = \alpha_{0,0}S_{0,0} + \alpha_{1,0}S_{1,0} + \alpha_{2,0}S_{2,0} - L_0. \quad (7)$$

We suppose that—because of a regulatory requirement, for example—on any given date t, the institution attempts to attain a ratio of capital to total asset value of at least c_r.[2] That is, we liquidate the minimum amount of assets necessary to achieve $K_t = (A_t - L_t) \geq c_r A_t$. We assume that raising capital—for example, through an infusion of new equity—is not feasible during the short time horizons that we consider. Let $\lambda_{i,t}$ denote the number of units of asset i liquidated in period t. We suppose (until later analysis) that the institution liquidates cash first, then the liquid asset, and finally the illiquid asset. Details of the liquidation algorithm are provided in Appendix A. Once this process is completed, the holdings of the three asset types at the end of the period are recorded and carried over to the next period by setting, for each asset type i,

$$\alpha_{i,t+1} = \alpha_{i,t} - \lambda_{i,t}. \quad (8)$$

We assume that liabilities earn the fixed short rate r. (Because the liabilities are apparently not default free, we could assign a higher borrowing rate, $R > r$, but over short time horizons, the impact of doing so would be negligible for typical parameters.) Taking the proceeds from asset sales in period t into account, the value of the liabilities in period $t + 1$ is

$$L_{t+1} = \exp(r)[L_t - \lambda_{0,t}S_{0,t} - \lambda_{1,t}(1 - X_{1,t})S_{1,t} - \lambda_{2,t}(1 - X_{2,t})S_{2,t}]. \quad (9)$$

This asset liquidation process is repeated for 10 successive trading days. At the end of the 10th day, the terminal capital, K_{10}, is computed on the basis of current asset holdings and liabilities. The 99 percent VAR is the 99 percent critical value of the distribution of cumulative losses in capital $K_0 - K_{10}$ over the 10-day period. Expected tail loss is the expected loss in capital conditional on the event that losses exceed the 99 percent VAR. The probability of insolvency is the probability that the institution's capital is eliminated within the 10-day period.[3]

Monte Carlo Simulation Results

We conducted Monte Carlo simulations of 25,000 independent 10-day scenarios of the effect on the three risk measures of changes in spread parameters for cash-first and cash-last liquidation strategies.

Cash-First Strategies. In this section, we present the results of Monte Carlo analyses for a base-case cash-first strategy and for strategies in which fat tails, substantial price, and/or spread volatility entered the scenarios.

■ *Base case*. The (annualized) base-case parameters are

$r = 0.05$,
$\mu_1 = 0.1$,
$\mu_2 = 0.2$,
$\sigma_1 = \sigma_2 = 0.2$,
$\gamma_1 = \gamma_2 = 1$, and
$\rho = -0.5$.

To highlight the effect of liquidity, we have equated the volatility of the assets. We took the target capital ratio, c_r, to be the typical regulatory ratio of 8 percent and assumed an initial asset structure of

$\alpha_0 = 2$,
$\alpha_1 = 8$, and
$\alpha_2 = 90$,

with an initial capital ratio of 9 percent, implying initial liabilities of $L_0 = 91$. In other words, at Time 0, the institution exceeded its regulatory capital requirements by 1 percentage point and held 90 percent of its assets in illiquid form.

We studied four cases that differed as to starting values for the mid-bid spread. The base case had no spread. The other three cases assumed initial spreads for the liquid and illiquid assets of, respectively, 0.1 percent and 0.5 percent, 0.2 percent and 1 percent, and 0.5 percent and 2.5 percent.

Schultz (2001) estimated round-trip trading costs for corporate bond trades by institutional investors with dealers of approximately 0.27 percent, indicating tighter spreads than most of our cases. Our initial conditions, however, were designed to place each portfolio, in terms of leverage and spreads, in a relatively "distressed" state, in which case, a seller might anticipate predatory or conservative quotes.

For each case, we analyzed four settings, delineated by the variability of spreads and the degree of correlation between spreads and prices:

1. constant spreads,
2. random spreads uncorrelated with asset returns,
3. random spreads moderately negatively correlated with returns, $\rho_1 = \rho_2 = -0.5$, and
4. random spreads highly negatively correlated with returns, $\rho_1 = \rho_2 = -0.8$.

The resulting 99 percent VAR, expected tail loss (ETL), and probability of insolvency for a 10-day period are reported in **Table 1**. The VAR and ETL show only moderate responses to changes in the degree of illiquidity. **Figure 1** compares 10-day insolvency probabilities for the case of large initial mid-bid spreads (50 and 250 bps for, respectively, the relatively liquid and illiquid assets). Also shown is the most adverse of these cases (large negative correlations between spreads and returns) with a reversal of the order of liquidation—that is, selling the least-liquid assets first. (We discuss this liquidation strategy later in the article.)

Fat tails. That asset returns are fat tailed, especially in the short run, has been widely documented in the literature. To investigate the effect of such nonnormality on the relevance of spreads for liquidation risk, we carried out computations similar to those for Table 1 but allowed jumps in prices. To model jumps in prices, we replaced the normal distribution of the daily increment of each Brownian motion, B_i, with a mixed normal distribution that included a daily jump probability of 0.02 and a kurtosis of 10.[4] **Table 2** presents the results. The pattern is similar to that in Table 1 but the VAR and ETL values are significantly larger, with correlation between returns and spreads leading to an increase in VAR and ETL of above 7 percent. Increasing the degree of negative correlation between returns and spreads (see the shaded section) leads to a sharp increase (more than 40 percent) in the probability of insolvency—from 0.84 percent to 1.21 percent.

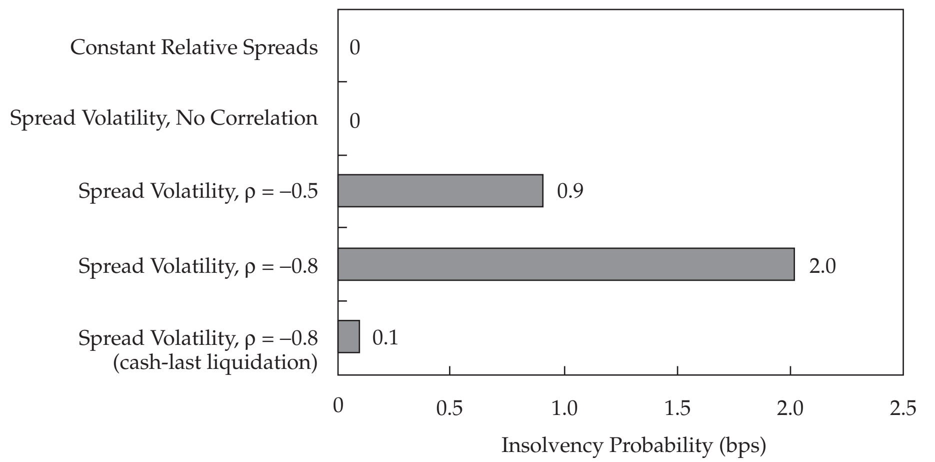
High price volatility. How does the effect of the bid–ask spread on liquidation risk depend on asset price volatility? Intuitively, increasing volatility should lead to more frequent asset sales and, therefore, to larger spread-induced losses. To

Table 1. Results of Monte Carlo Analysis of Four Cases: Cash First, Base Case

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. VAR (loss in capital as percent of initial asset value)				
Constant spreads	6.204	6.407	6.614	7.344
Variable spreads				
$\rho_1 = \rho_2 = 0$	6.204	6.398	6.595	7.380
$\rho_1 = \rho_2 = -0.5$	6.204	6.434	6.672	7.659
$\rho_1 = \rho_2 = -0.8$	6.204	6.459	6.736	7.850
B. ETL (loss in capital as percent of initial asset value)				
Constant spreads	6.635	6.825	7.030	7.740
Variable spreads				
$\rho_1 = \rho_2 = 0$	6.635	6.827	7.036	7.796
$\rho_1 = \rho_2 = -0.5$	6.635	6.868	7.125	8.087
$\rho_1 = \rho_2 = -0.8$	6.635	6.895	7.186	8.273
C. Insolvency probability (in percent)				
Constant spreads	0	0	0	0
Variable spreads				
$\rho_1 = \rho_2 = 0$	0	0	0	0
$\rho_1 = \rho_2 = -0.5$	0	0	0	0.009
$\rho_1 = \rho_2 = -0.8$	0	0	0	0.020

Note: Insolvency probability estimates based on 200,000 trials.

Figure 1. Monte Carlo Insolvency Probabilities



Note: Ten-day insolvency probabilities based on 200,000 trials; normal returns; 20 percent return volatility; 0.5 percent and 2.5 percent initial spreads; 0 or 100 percent spread volatility.

Table 2. Results of Monte Carlo Analysis of Four Cases: Cash First, Fat Tails

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. VAR (loss in capital as percent of initial asset value)				
Constant spreads	7.050	7.300	7.624	8.606
Variable spreads				
$\rho_1 = \rho_2 = 0$	7.050	7.330	7.620	8.715
$\rho_1 = \rho_2 = -0.5$	7.050	7.389	7.730	9.138
$\rho_1 = \rho_2 = -0.8$	7.050	7.423	7.826	9.326
B. ETL (loss in capital as percent of initial asset value)				
Constant spreads	8.376	8.696	9.041	10.155
Variable spreads				
$\rho_1 = \rho_2 = 0$	8.376	8.710	9.071	10.262
$\rho_1 = \rho_2 = -0.5$	8.376	8.809	9.277	10.774
$\rho_1 = \rho_2 = -0.8$	8.376	8.875	9.420	11.095
C. Insolvency probability (in percent)				
Constant spreads	0.240	0.292	0.388	0.808
Variable spreads				
$\rho_1 = \rho_2 = 0$	0.240	0.288	0.396	0.836
$\rho_1 = \rho_2 = -0.5$	0.240	0.324	0.464	1.084
$\rho_1 = \rho_2 = -0.8$	0.240	0.340	0.504	1.208

investigate this issue, we ran additional simulations using an asset price volatility of 40 percent (σ_i = 0.4). The results for normal returns, reported in **Table 3**, show that increased price volatility leads to a sizable increase in all risk measures—an especially large rise in the probability of insolvency. Although the pattern of results is similar to that in the 20 percent volatility case, the effect of spreads on liquidation risk is weaker than in the base case. For small spreads, the increases in VAR and ETL in Table 3 are only about 1.5 percent versus 3 percent in the base case. Large spreads bring increases of about 10 percent in these measures, half of the value obtained in the base case. Moreover, although negative correlation between spreads and returns still leads to an increase in VAR and ETL, this effect is weaker than it is with low volatility.

These results are driven by early asset sales. When volatility is high, the institution must liquidate assets in greater amounts, and sooner, to meet

Table 3. Results of Monte Carlo Analysis of Four Cases: Cash First, High Return Volatility

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. VAR (loss in capital as percent of initial asset value)				
Constant spreads	8.622	8.752	8.871	9.392
Variable spreads				
$\rho_1 = \rho_2 = 0$	8.622	8.755	8.872	9.414
$\rho_1 = \rho_2 = -0.5$	8.622	8.774	8.908	9.577
$\rho_1 = \rho_2 = -0.8$	8.622	8.786	8.932	9.689
B. ETL (loss in capital as percent of initial asset value)				
Constant spreads	8.864	9.014	9.183	9.935
Variable spreads				
$\rho_1 = \rho_2 = 0$	8.864	9.013	9.183	9.961
$\rho_1 = \rho_2 = -0.5$	8.864	9.041	9.247	10.173
$\rho_1 = \rho_2 = -0.8$	8.864	9.060	9.290	10.301
C. Insolvency probability (in percent)				
Constant spreads	0.136	0.272	0.512	2.668
Variable spreads				
$\rho_1 = \rho_2 = 0$	0.136	0.268	0.516	2.840
$\rho_1 = \rho_2 = -0.5$	0.136	0.308	0.620	4.052
$\rho_1 = \rho_2 = -0.8$	0.136	0.332	0.692	4.916

capital requirements. The effect is similar to that of a stop-loss strategy for sales. As more assets are sold, the institution's exposure to price fluctuations falls. As a result, VAR rises by less than the increase in asset price volatility would imply. As spreads are introduced, even more assets must be sold to meet capital requirements. The reduction in exposure thus mitigates the increase in VAR caused by larger spreads. The insolvency probability, however, is sensitive to the presence of spreads in the high-volatility case; it increases from 0.14 percent in the no-spread case to 2.67 percent for large spreads. With a strong negative correlation between spreads and returns, the insolvency probability rises farther—to almost 5 percent. As can be seen in **Table 4**, the effects of high volatility are similar in the case of fat-tailed returns.

In summary, increasing volatility actually reduces the relative impact of spreads on VAR and expected tail loss but increases the relative effect of spreads on insolvency probability.

■ *High spread volatility.* We also studied the effect of spreads on risk measures in a setting of substantial spread volatility. **Table 5** reports the results for a spread volatility of 200 percent ($\gamma_i = 2$) with a return volatility of 40 percent ($\sigma_i = 0.4$). In such a case, at the base-case correlation of –0.5 between returns and spreads, for example, spreads would widen in expectation by 2.5 percent in the face of a sudden reduction in price of 1 percent.[5]

The percentage increase in VAR caused by spreads is comparable to that in the high-volatility case, whereas the additional percentage increase in VAR caused by *correlation* between spreads and prices is comparable to that in the base case. For example, large spreads lead to an increase in VAR of about 10 percent (the value reported in the previous section), while correlation between spreads and returns leads to an additional increase of almost 6 percent in VAR (the value reported for the base-case Table 1).

Although a pattern of dependence similar to the pattern for VAR emerges for ETL, the effect of price and spread volatility *compounds* for the probability of insolvency. Both the percentage increase from spreads and the increase from correlation are substantially higher in the case of high spread volatility than in the case of high price volatility.

Cash-Last Liquidation Strategies. Thus far, we have presented results of Monte Carlo simulations based on the assumption that the institution liquidates cash first. Only when cash is exhausted does the institution sell its liquid-asset position. Coming last in the pecking order, illiquid assets are sold only in extreme cases. This cash-first liquidation strategy raises the concern, however, that in the most stressful situations, the institution may have only illiquid assets to sell. Thus, the alternative liquidation strategy of selling illiquid assets first

Table 4. Results of Monte Carlo Analysis of Four Cases: Cash First, Fat Tails and High Return Volatility

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. VAR (loss in capital as percent of initial asset value)				
Constant spreads	11.076	11.402	11.734	12.694
Variable spreads				
$\rho_1 = \rho_2 = 0$	11.076	11.402	11.711	12.669
$\rho_1 = \rho_2 = -0.5$	11.076	11.470	11.837	12.955
$\rho_1 = \rho_2 = -0.8$	11.076	11.519	11.932	13.184
B. ETL (loss in capital as percent of initial asset value)				
Constant spreads	13.903	14.198	14.488	15.350
Variable spreads				
$\rho_1 = \rho_2 = 0$	13.903	14.206	14.506	15.421
$\rho_1 = \rho_2 = -0.5$	13.903	14.287	14.666	15.811
$\rho_1 = \rho_2 = -0.8$	13.903	14.341	14.774	16.054
C. Insolvency probability (in percent)				
Constant spreads	2.136	2.356	2.616	4.232
Variable spreads				
$\rho_1 = \rho_2 = 0$	2.136	2.376	2.636	4.360
$\rho_1 = \rho_2 = -0.5$	2.136	2.424	2.724	5.144
$\rho_1 = \rho_2 = -0.8$	2.136	2.452	2.772	5.556

Table 5. Results of Monte Carlo Analysis of Four Cases: Cash First, High Spread Volatility

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. VAR (loss in capital as percent of initial asset value)				
Constant spreads	8.622	8.752	8.871	9.392
Variable spreads				
$\rho_1 = \rho_2 = 0$	8.622	8.754	8.875	9.547
$\rho_1 = \rho_2 = -0.5$	8.622	8.803	8.966	9.930
$\rho_1 = \rho_2 = -0.8$	8.622	8.831	9.021	10.111
B. ETL (loss in capital as percent of initial asset value)				
Constant spreads	8.864	9.014	9.183	9.935
Variable spreads				
$\rho_1 = \rho_2 = 0$	8.864	9.014	9.193	10.100
$\rho_1 = \rho_2 = -0.5$	8.864	9.076	9.339	10.540
$\rho_1 = \rho_2 = -0.8$	8.864	9.121	9.440	10.770
C. Insolvency probability (in percent)				
Constant spreads	0.136	0.272	0.512	2.668
Variable spreads				
$\rho_1 = \rho_2 = 0$	0.136	0.272	0.544	3.624
$\rho_1 = \rho_2 = -0.5$	0.136	0.348	0.844	6.660
$\rho_1 = \rho_2 = -0.8$	0.136	0.424	1.124	8.740

and keeping a cushion of cash and liquid assets needs to be examined.

We analyzed the effects of such a strategy on VAR, ETL, and insolvency probability. We first considered the low-volatility case ($\sigma_i = 0.2$). The results of the simulations for the four spread scenarios are summarized in **Table 6**. The picture that emerges from these calculations is similar to that for the base-

Table 6. Results of Monte Carlo Analysis of Four Cases: Cash Last, Base Case

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. VAR (loss in capital as percent of initial asset value)				
Constant spreads	5.957	6.157	6.379	7.137
Variable spreads				
$\rho_1 = \rho_2 = 0$	5.957	6.157	6.376	7.168
$\rho_1 = \rho_2 = -0.5$	5.957	6.189	6.460	7.433
$\rho_1 = \rho_2 = -0.8$	5.957	6.218	6.505	7.613
B. ETL (loss in capital as percent of initial asset value)				
Constant spreads	6.370	6.565	6.774	7.486
Variable spreads				
$\rho_1 = \rho_2 = 0$	6.370	6.568	6.782	7.544
$\rho_1 = \rho_2 = -0.5$	6.370	6.607	6.867	7.809
$\rho_1 = \rho_2 = -0.8$	6.370	6.633	6.922	7.967
C. Insolvency probability (in percent)				
Constant spreads	0	0	0	0
Variable spreads				
$\rho_1 = \rho_2 = 0$	0	0	0	0
$\rho_1 = \rho_2 = -0.5$	0	0	0	0
$\rho_1 = \rho_2 = -0.8$	0	0	0	0.001

case cash-first strategy. Comparison of Tables 1 and 6 shows that both the sizes of spreads and their correlations with asset returns have a significant impact on VAR and ETL, although VAR and ETL are significantly smaller for the cash-last strategy than for the cash-first strategy.

The improvement in VAR and ETL is accompanied, however, by higher transaction costs. **Table 7** contrasts the expected transaction costs as a percentage of initial asset value for the base-case cash-first and cash-last liquidation strategies. The cash-last strategy would cost approximately 40 percent more.

The results of similar computations for the case of fat tails are summarized in **Table 8**. A comparison with Table 2 shows that VAR, ETL, and the probability of insolvency are significantly smaller than in the case in which cash is liquidated first. The percentage decrease is strongest for the probability of

Table 7. Average Transaction Costs of Cash-First and Cash-Last Liquidation Strategies

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. Cash first				
Constant spreads	0	0.049	0.104	0.319
Variable spreads				
$\rho_1 = \rho_2 = 0$	0	0.049	0.105	0.323
$\rho_1 = \rho_2 = -0.5$	0	0.055	0.118	0.375
$\rho_1 = \rho_2 = -0.8$	0	0.059	0.127	0.410
B. Cash last				
Constant spreads	0	0.069	0.147	0.448
Variable spreads				
$\rho_1 = \rho_2 = 0$	0	0.069	0.147	0.453
$\rho_1 = \rho_2 = -0.5$	0	0.077	0.164	0.515
$\rho_1 = \rho_2 = -0.8$	0	0.081	0.174	0.554

Note: Transaction costs as a percentage of initial asset value; cost over 10-day simulation period.

Table 8. Results of Monte Carlo Analysis of Four Cases: Cash Last, Fat Tails

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. VAR (loss in capital as percent of initial asset value)				
Constant spreads	6.901	7.172	7.439	8.380
Variable spreads				
$\rho_1 = \rho_2 = 0$	6.901	7.143	7.458	8.516
$\rho_1 = \rho_2 = -0.5$	6.901	7.235	7.579	8.797
$\rho_1 = \rho_2 = -0.8$	6.901	7.261	7.668	8.976
B. ETL (loss in capital as percent of initial asset value)				
Constant spreads	8.162	8.489	8.832	9.850
Variable spreads				
$\rho_1 = \rho_2 = 0$	8.162	8.502	8.856	9.960
$\rho_1 = \rho_2 = -0.5$	8.162	8.598	9.051	10.424
$\rho_1 = \rho_2 = -0.8$	8.162	8.663	9.184	10.714
C. Insolvency probability (in percent)				
Constant spreads	0.192	0.256	0.340	0.684
Variable spreads				
$\rho_1 = \rho_2 = 0$	0.192	0.264	0.340	0.700
$\rho_1 = \rho_2 = -0.5$	0.192	0.288	0.412	0.868
$\rho_1 = \rho_2 = -0.8$	0.192	0.296	0.444	0.984

insolvency, which falls by almost 20 percent in the case of large spreads.

The case of high volatility of returns provides the following comparison of the effects of the cash-first and cash-last strategies. Consider, for example, the case of moderately large spreads (0.2 percent and 1.0 percent mid-to-bid relative prices) and moderately large correlation in spreads ($\rho_i = -0.5$). First, a comparison of **Table 9** (see shaded cell) and Table 3 shows that liquidating cash last reduces the

Table 9. Results of Monte Carlo Analysis of Four Cases: Cash Last, High Return Volatility

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. VAR (loss in capital as percent of initial asset value)				
Constant spreads	8.307	8.458	8.587	8.987
Variable spreads				
$\rho_1 = \rho_2 = 0$	8.307	8.451	8.583	8.994
$\rho_1 = \rho_2 = -0.5$	8.307	8.476	8.616	9.152
$\rho_1 = \rho_2 = -0.8$	8.307	8.495	8.637	9.270
B. ETL (loss in capital as percent of initial asset value)				
Constant spreads	8.573	8.715	8.861	9.489
Variable spreads				
$\rho_1 = \rho_2 = 0$	8.573	8.715	8.860	9.507
$\rho_1 = \rho_2 = -0.5$	8.573	8.737	8.907	9.697
$\rho_1 = \rho_2 = -0.8$	8.573	8.752	8.936	9.813
C. Insolvency probability (in percent)				
Constant spreads	0.072	0.104	0.176	0.964
Variable spreads				
$\rho_1 = \rho_2 = 0$	0.072	0.108	0.180	0.988
$\rho_1 = \rho_2 = -0.5$	0.072	0.128	0.208	1.368
$\rho_1 = \rho_2 = -0.8$	0.072	0.132	0.224	1.648

probability of insolvency by 41 bps (0.62 percent – 0.21 percent). **Table 10** shows that the cash-last strategy increases expected liquidation costs by 5 bps of assets (0.349 – 0.299 per initial 100 in assets). The implied breakeven financial insolvency distress cost is approximately 0.12 bps of assets, or roughly 1 bp of initial capital. That is, if insolvency is expected to cost more than 1 bp of the market value of the portfolio (in terms of franchise value and reorganization fees, for example), the cash-last strategy is more effective than the cash-first strategy—for this particular case. Obviously, the breakeven cost of insolvency distress depends heavily on the particular scenario of volatilities, correlations, and spreads.

Our results suggest that an important trade-off exists between the goal of minimizing expected transaction costs during stressed asset sales and the goal of reducing the probability of insolvency (with the associated costs of overall financial distress).

Conclusion

Using a simple model, we analyzed the effect of spreads and their variability on various measures of liquidation risk. If spreads are expected to increase as prices fall, then the effect of market liquidity on liquidation risk can be dramatic, especially with fat-tailed returns.[6]

If the goal is to minimize expected transaction costs, cash and liquid assets should be sold first. This liquidation strategy raises a concern, however, that in the most dramatic cases, the institution will have only illiquid assets left to sell, thus triggering large losses. An alternative strategy is to sell illiquid assets first and keep a cushion of cash and liquid assets for a rainy day. Such a strategy, while increasing expected transaction costs, significantly decreases tail losses and especially the probability of insolvency. In light of our results, financial institutions would be wise to carefully examine their strategies for liquidation during periods of severe stress.

Our analysis assumed that a given target capital ratio (8 percent in our case) would be maintained as long as possible. Relaxation of this target ratio would presumably increase the probability of insolvency while reducing expected transaction costs. Optimal liquidation strategies for given risk–reward objectives remain an interesting subject for future research.

We are grateful for conversations with Bob Litzenberger, Bob Litterman, and Thaleia Zariphopoulou and for research funding from Gifford Fong Associates.

Table 10. Average Transaction Costs of Cash-First and Cash-Last Liquidation Strategies: High Return Volatility

	Spread: Liquid and Illiquid Asset			
Spread Behavior	No Spread	0.1% and 0.5%	0.2% and 1.0%	0.5% and 2.5%
A. Cash first				
Constant spreads	0	0.131	0.274	0.802
Variable spreads				
$\rho_1 = \rho_2 = 0$	0	0.131	0.275	0.807
$\rho_1 = \rho_2 = -0.5$	0	0.142	0.299	0.886
$\rho_1 = \rho_2 = -0.8$	0	0.149	0.315	0.933
B. Cash last				
Constant spreads	0	0.154	0.323	0.938
Variable spreads				
$\rho_1 = \rho_2 = 0$	0	0.154	0.323	0.943
$\rho_1 = \rho_2 = -0.5$	0	0.166	0.349	1.022
$\rho_1 = \rho_2 = -0.8$	0	0.173	0.365	1.068

Appendix A. Liquidation Algorithm

For the strategy of liquidating the most-liquid asset first, the recipe for liquidation is as follows.

First, if

$$\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t} - (L_t - \alpha_{0,t}S_{0,t}) \geq c_r(\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t}),$$

the institution's cash holdings are sufficient to meet the capital requirement. In this case, the institution's cash is reduced by $\lambda_{0,t}$ to satisfy the capital requirement. Solving produces

$$\lambda_{0,t} = \frac{L_t - (1-c_r)(\alpha_{0,t}S_{0,t} + \alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t})}{S_{0,t}c_r}.$$

By assumption, none of the liquid or illiquid asset holdings is to be sold in this case. That is, $\lambda_{1,t} = \lambda_{2,t} = 0$.

Second, whenever

$$\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t} - (L_t - \alpha_{0,t}S_{0,t}) < c_r(\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t}),$$

the $\alpha_{0,t}$ units of cash available are not sufficient to meet the capital requirement. Some of the liquid asset is, therefore, liquidated. If

$$\alpha_{2,t}S_{2,t} - [L_t - \alpha_{0,t}S_{0,t} - \alpha_{1,t}(1 - X_{1,t})S_{1,t}] \geq c_r\alpha_{2,t}S_{2,t},$$

the current holdings of the liquid asset and cash together are sufficient to meet the capital requirement. In this case, cash is reduced first. That is, $\lambda_{0,t} = \alpha_{0,t}$. The number of units of the liquid asset to be sold is based on the bid price, $S_{1,t}(1 - X_{1,t})$. Thus,

$$\frac{(\alpha_{1,t} - \lambda_{1,t})S_{1,t} + \alpha_{2,t}S_{2,t} - [L_t - \alpha_{0,t}S_{0,t} - \lambda_{1,t}S_{1,t}(1 - X_{1,t})]}{(\alpha_{1,t} - \lambda_{1,t})S_{1,t} + \alpha_{2,t}S_{2,t}} = c_r,$$

yielding

$$\lambda_{1,t} = \frac{(L_t - \alpha_{0,t}S_{0,t}) - (1-c_r)(\alpha_{1,t}S_{1,t} + \alpha_{2,t}S_{2,t})}{S_{1,t}(c_r - X_{1,t})}.$$

Because none of the illiquid assets must be sold in this case, we have $\lambda_{2,t}$ of zero.

Third, if

$$\alpha_{2,t}S_{2,t} - [L_t - \alpha_{0,t}S_{0,t} - \alpha_{1,t}S_{1,t}(1 - X_{1,t})] < c_r\alpha_{2,t}S_{2,t},$$

current holdings of cash and liquid assets are not sufficient to meet the regulatory capital requirement and some of the illiquid asset holdings must also be sold. In this case, all cash and liquid asset positions are liquidated ($\lambda_{0,t} = \alpha_{0,t}$ and $\lambda_{1,t} = \alpha_{1,t}$) and

$$\lambda_{2,t} = \min\left\{\frac{[L_t - \alpha_{0,t}S_{0,t} - \alpha_{1,t}S_{1,t}(1 - X_{1,t})] - (1-c_r)\alpha_{2,t}S_{2,t}}{S_{2,t}(c_r - X_{2,t})}, \alpha_2\right\}.$$

If $\lambda_2 = \alpha_2$, the institution is effectively insolvent.

Notes

1. This parameterization admits the possibility of negative bid prices, but at typical parameters, the likelihood of this possibility over short horizons is negligible.
2. Capital ratio c_r need not be a regulatory minimum. For example, a policy that allows excess capital would have c_r larger than the regulatory minimum capital ratio.
3. VAR is not a coherent risk measure in the sense of Artzner, Delbaen, Eber, and Heath (1999), but the expected tail loss is coherent and, although not as commonly reported, is preferred conceptually as a risk measure. The use of a 99 percent confidence level rather than some other quantile is arbitrary but conventional.
4. To simulate a random variable of zero mean and unit variance with fat tails (excess kurtosis), we proceeded as follows. Let Y be the outcome of a Bernoulli trial that takes the value 1 with probability p and the value 0 with probability $1 - p$. Let Z denote a standard normal random variable. Then, the random variable $X = \left[\alpha Y + \sqrt{(1 - p\alpha^2)/(1-p)} \times (1 - Y)\right] Z$ has zero mean, unit variance, and a kurtosis of $k = [3/(1-p)](p\alpha^4 - 2p\alpha^2 + 1)$. Using this result, one can find values for p and α that achieve the desired degree of kurtosis $\bar{k}$ by setting $\bar{k}$ equal to k. Solving, we find $\alpha = \sqrt{1 + \sqrt{[(\bar{k}/3) - 1][(1/p) - 1]}}$.
5. The expected response of the spread to an unexpected return of Z percent is to scale it up by approximately $\exp(Z\rho_i\gamma_i/\sigma_i)$.
6. We have not treated the case of market impact, under which the act of selling itself lowers bid prices, which could be critical if the asset holdings are large relative to the market.

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