The Taming of Roelofs Polyhedra

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Abstract

Roelofs polyhedra form a vast collection of polyhedra containing many interesting solids and including very irregular ones. The purpose of this paper is to consider two special subsets: polyhedra with the symmetry of the prism and polyhedra with just two different types of vertices. Beside the figures in the paper PowerPoint pictures, all made by Rinus Roelofs, will be presented.

1. Introduction

Some time ago Rinus Roelofs defined the following class of polyhedra:
a) The faces are convex regular polygons;
b) Some of the faces are intersecting.

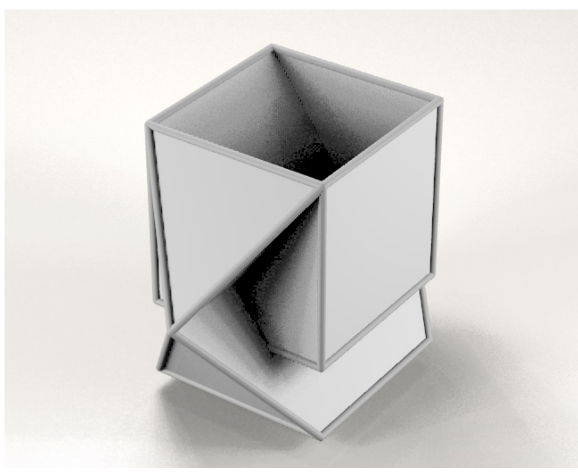

Figure 1: *Example of a Roelofs polyhedron.*

An example is shown in Figure.. These solids are now generally known as Roelofs polyhedra. From the definition it will be clear that there exists overlap with other categories. For example, all non-convex uniform polyhedra without star polygons belong to the class of Roelofs polyhedra. However, the latter contains a great deal more, including many irregular, not to say wild polyhedra. It is true that many of these irregular solids have interesting properties. Nevertheless, the subject of this paper to present a restricted subset of the Roelofs polyhedra with stress on simplicity, beauty, and symmetry.

In the next section we consider prism-like polyhedra, and in Section 3 we examine polyhedra with two types of vertices.

2. Prism-like Roelofs polyhedra

Archimedean prisms and antiprisms form two infinite series among the convex Archimedean solids. They are quite simple in their symmetry and are generally considered as the ugly ducklings. With the Roelofs polyhedra, several other series with the same symmetry arise, which makes them more interesting. Consider the following example.

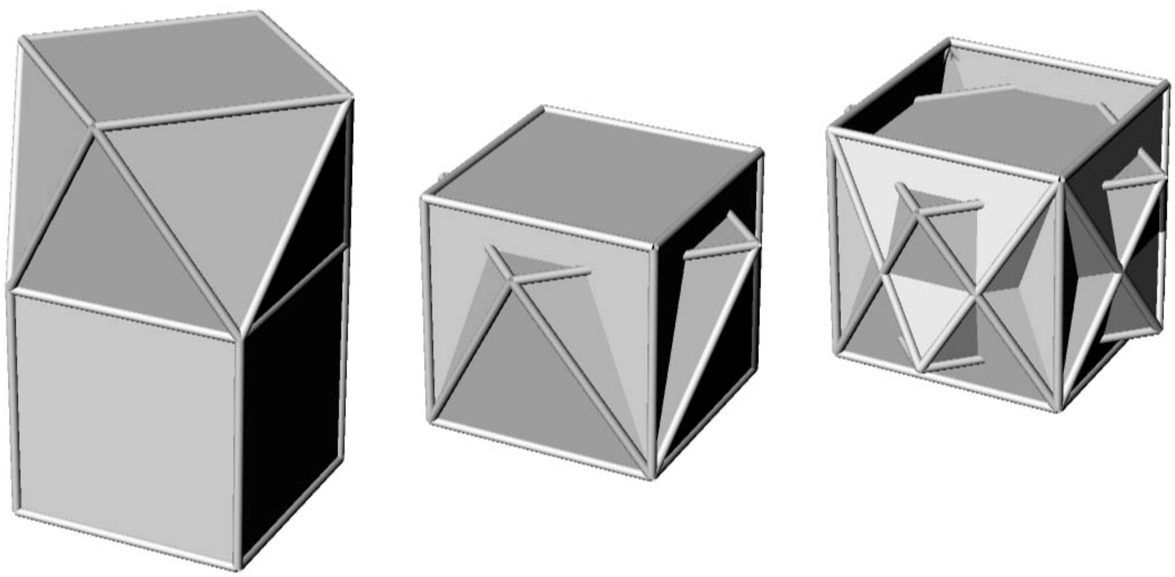

Figure 2a, 2b, 2c: *Building a prism-like Roelofs polyhedron.*

Start with a four-sided antiprism on top of a cube; see Fig. 2a. The square half-way up is not one of the faces. Now push the cube upward; the faces of the two solids will intersect. The result is the 14-hedron shown in Fig. 2b. Repeating the process with an antiprism on the opposite face gives the solid in Fig. 2c; this one has the full prism symmetry D4. Please note that the solids in Fig. 2a, b, and c are not compounds.

The trick also works for n-sided prisms and antiprisms. And it can be repeated: form a pile of prisms and antiprisms, now and then applying intersections as above. Star prisms and star antiprisms (grammic or grammic-crossed) can be used provided the two extremal stars are covered by star-shaped pyramids. In the next section, where we consider a restricted subset, conditions on the stars will be given.

An alternative variation of the prisms is obtained when the lateral faces are used as bases for other polyhedra.

3. Two types of vertices

The uniform polyhedra have, by definition, only one type of vertex; they are vertex-transitive. The polyhedra we consider in this section are *next-to-uniform* in that they have two types of vertices.

As a first category we consider a subset of the prism-like solids mentioned in the previous section. The polyhedron in Fig. 2b has three types of vertices. After adding the second antiprism, as in Fig. 2c, the number of types drops to two. When allowing star prisms and antiprisms, the star faces must be covered by a star pyramid. Of course, the side length of the star has to be sufficiently large to make this possible. In detail: for an (n,k)-star (see Fig. 3) we must have gcd(n,k)=1, k>1 and moreover 2k<n<6k for a prism and 2k<n<3k for an antiprism.

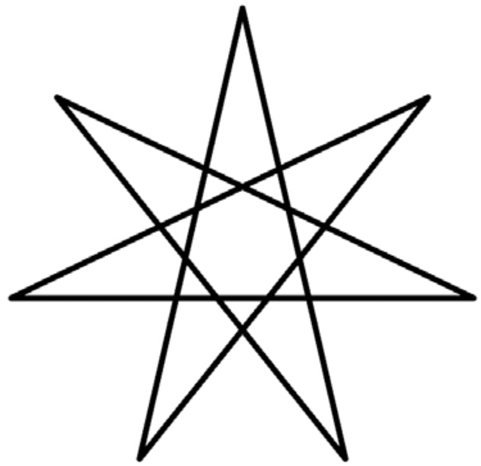

Figure 3: *the 7/3 star.*

When working this out, nine infinite series of next-to-uniform solids with prism symmetry are obtained.

Other types of symmetry appear when starting with the Platonic solids. The simplest example is the tetrahedron with on each face a prism pushed inside. Applying the same trick on the cube, it turns out that the number of new solids need not be 6; also with 2 or 4 of them, appropriately placed, a next-to-uniform solid is obtained. The other Platonic solids have similar properties. An interesting example is the following. On the icosahedron choose four faces such that twelve vertices are all different. (This can be done, in fact in only one way up to isomorphism.) On each of these triangles push a tetrahedron inside. The result is a Roelofs polyhedron with 28 triangular faces with "microscopic" intersections.

Finally we consider Roelofs polyhedra with a small number of faces. Roelofs investigated the possible cases with 6 to 12 faces. For 12 faces he found about 60 solids, with no claim for completeness! From his list, eleven next-to-uniform cases can be extracted. These will be shown in the power-point presentation. The unique one with 11 faces is shown in Fig. 4.

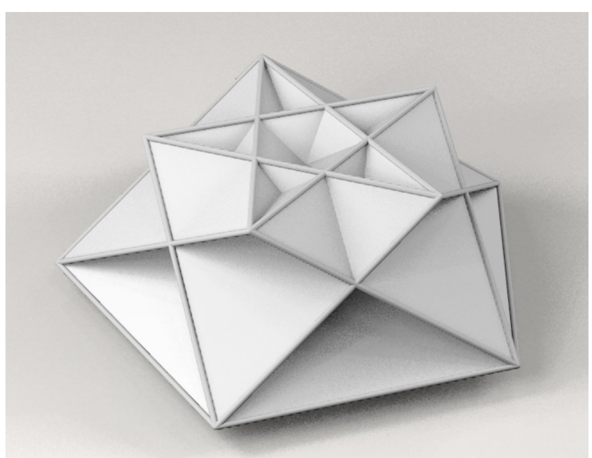

Figure 4: *the next-to-uniform 11-hedron.*

Roelofs also found a 30-hedron bounded by six dodecagons and 24 squares; see Figure 5. Johan van de Konijnenberg noted that this solid still has only two types of vertices when the dodecagons are replaced by 16-gons; the number of squares then increases to 36. These two polyhedra are the start of a remarkable infinite series with cube-symmetry. However, the number of types of vertices increases.

Figure 5: *the 12(6)4(24)-hedron.*

4. Conclusion

You have seen attempts to define subsets of Roelofs polyhedra which contain fewer irregular specimen. The restriction to next-to-uniform solids seems promising. Further research is necessary to find out whether certain characteristics of the irregular solids, such as invisible faces, will occur in the restricted set.

Acknowledgement

I am grateful to Rinus Roelofs who made all figures in the text as well as all power point pictures..