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Geosuite / Settlement calculation

PROGRAM SETTLE

THEORETICAL PRINCIPLES

1. Preface

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2. The settlement calculation

In program SETTLE the excess pore pressure development in primary consolidation phase is calculated by finite element method based on Terzaghi 1-dimensional consolidation theory. There has been made an additional feature for the basic theory, which enables time depending loads (load history). The differential equation for primary consolidation is following:

$$c_v \frac{\partial^2 u(z,t)}{\partial z^2} = \frac{\partial u(z,t)}{\partial t} - \frac{\partial q(z,t)}{\partial t} \tag{1}$$

$$c_v = \frac{kM}{\gamma_w} \tag{2}$$

u the excess pore pressure at depth z and time t

z the depth coordinate

t the time from the beginning of the consolidation process

q the vertical distributed additional load at depth z and time t

c_v the vertical coefficient of consolidation

k the coefficient of permeability

M the modulus of compressibility (oedometer)

γ_w the volume weight of water ($\cong$ 10 kN/m^3)

Differential equation (1) is converted to FEM equation by method of Galerkin and integration of time is done by implicit difference method, which is mostly used in this kind of tasks where the result is convergent. When using the implicit method the time increment must be large enough, then the error of the result oscillation is cut out (chapter 7). As a result the excess pore pressure u is calculated in the stress points of the elements as a function of time.

The initial condition for excess pore pressure is needed for solution, and it is following:

$$u(z,t=0)=\Delta\sigma(z) \tag{3}$$

$\Delta\sigma$ the change in total vertical stress at depth z

Effective stress in ground is calculated as follows:

$$\Delta\sigma'(z,t)=-\Delta u(z,t) \tag{4}$$

$\Delta\sigma'$ the change in effective vertical stress at depth z and time t

Δu the change in excess pore pressure

Now the effective vertical stress increases as much as excess pore pressure decreases.

Effective vertical stresses σ' in ground:

$$\sigma'=\sigma'_0+\Delta\sigma'=\sigma'_0+\Delta\sigma-u \tag{5}$$

σ'_0 the effective vertical initial stress

Vertical deformation ε_{zp} in primary consolidation phase is calculated with chosen material model (connection between stresses and deformations) in stress points of the elements at time t (chapter 2). Primary consolidation settlement S_p at time t is calculated by adding vertical displacements of the elements:

$$S_p(t)=\sum_1^E \varepsilon_{zp} L^e \tag{6}$$

ε_{zp} the vertical deformation
E the number of elements
L^e the length of the element

The initial settlement in undrained condition is not included in program SETTLE. If there is importance with initial settlement, the influence must be estimated separately. In program SETTLE there is included primary consolidation settlement calculation and secondary consolidation will be added too. Secondary consolidation calculation is going to be based possibly on Buisman's classical theory or developed version of it.

$$\varepsilon_{zs} = C_{\alpha\varepsilon} \log\left(\frac{t}{t_p}\right) \tag{7}$$

ε_{zs} the vertical deformation caused by secondary consolidation

$C_{\alpha\varepsilon}$ the coefficient of time (secondary consolidation)

t_p the comparison time (the time at the end of primary consolidation)

t time from the beginning of the primary consolidation process

The settlement caused by secondary consolidation is calculated as follows:

$$S_s(t) = \sum_{1}^{E} \varepsilon_{zs} L^e \tag{8}$$

Finally both primary and secondary consolidation settlement is added together as total settlement S at time t.

$$S(t) = S_p(t) + S_s(t) \tag{9}$$

3. The material models of primary consolidation and parameters

3.1. Tangent modulus method

The modulus of compressibility M is defined by tangent modulus method (also called Ohde-Janbu method) as follows (figure 1):

$$M = m_1 \sigma_v \left(\frac{\sigma'}{\sigma_v}\right)^{1-\beta_1} \qquad ,\sigma' > \sigma'_p \tag{10a}$$

$$M = m_2 \sigma_v \left(\frac{\sigma'}{\sigma_v}\right)^{1-\beta_2} \qquad ,\sigma'_0 < \sigma' < \sigma'_p \tag{10b}$$

m_1 the modulus number (material parameter), normally consolidated part

m_2 the modulus number (material parameter), over-consolidated part

β_1 the stress exponent (material parameter), normally consolidated part

β_2 the stress exponent (material parameter), over-consolidated part

σ' the effective vertical stress (formula (5))

σ_v the equivalent pressure (100 kPa)

σ'_p the effective consolidation pressure

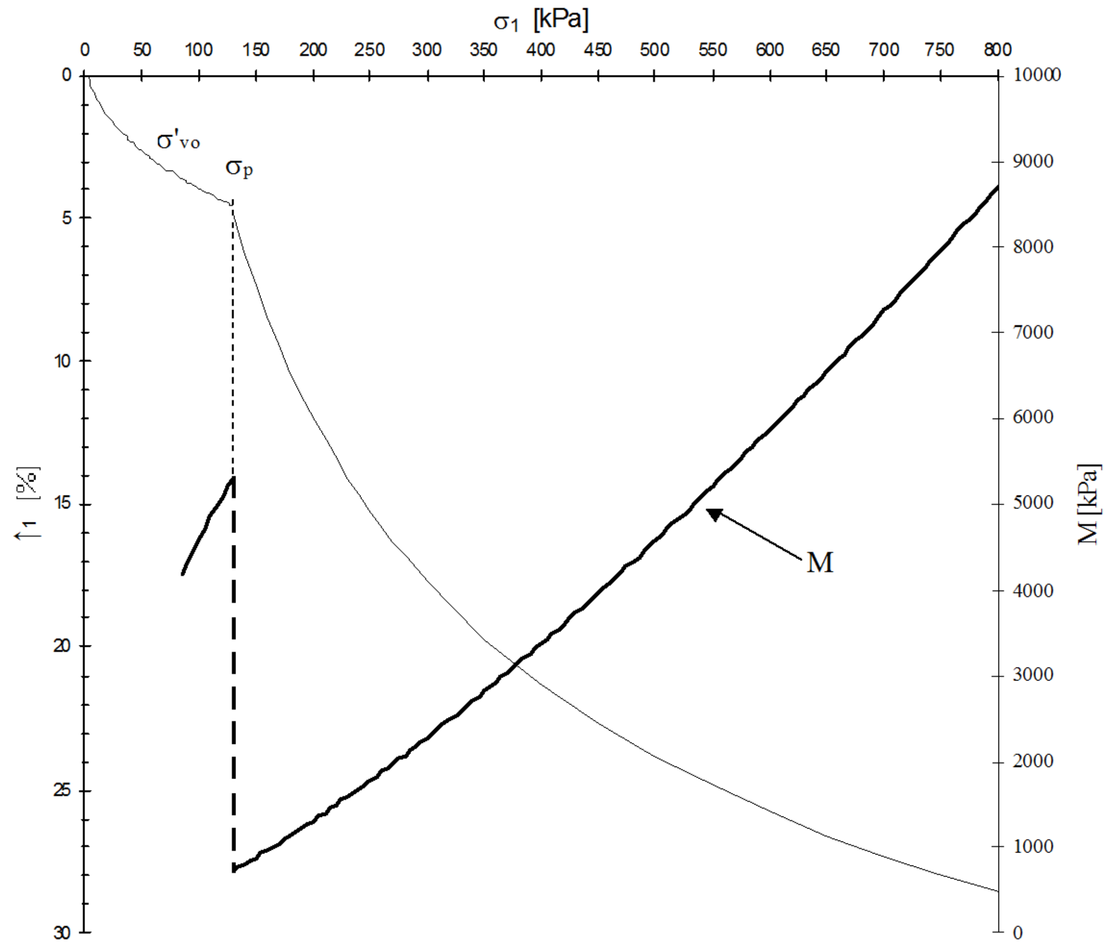

Figure 1. The modulus of compressibility M as a function of stress, example.

Vertical deformations are calculated by tangent modulus method as follows (Helenelund 1974):

Normally consolidated part, $\sigma' > \sigma'_p$, parameters m_1 and β_1:

$$\varepsilon_{zp} = \frac{1}{m_1 \beta_1}\left[\left(\frac{\sigma'}{\sigma_v}\right)^{\beta_1} - \left(\frac{\sigma'_p}{\sigma_v}\right)^{\beta_1}\right] \tag{11a}$$

$$\varepsilon_{zp} = \frac{1}{m_1}\ln\left(\frac{\sigma'}{\sigma'_p}\right) \quad ,\beta_1 = 0 \qquad (11b)$$

Over-consolidated part, $\sigma'_0 < \sigma' < \sigma'_p$, parameters m_2 and β_2:

$$\varepsilon_{zp} = \frac{1}{m_2\beta_2}\left[\left(\frac{\sigma'}{\sigma_v}\right)^{\beta_2} - \left(\frac{\sigma'_0}{\sigma_v}\right)^{\beta_2}\right] \qquad (12a)$$

$$\varepsilon_{zp} = \frac{1}{m_2}\ln\left(\frac{\sigma'}{\sigma'_0}\right) \quad \beta_2 = 0 \qquad (12b)$$

The modulus of compressibility M is calculated separately for normally consolidated and over-consolidated part with formulas (10a) and (10b). In that case there is discontinuation point with magnitude of module at effective consolidation pressure area. Modulus of compressibility is needed for Eq. (2) when calculating the coefficient of consolidation and the speed of consolidation process in both normally and over-consolidated phases. Vertical deformations are calculated with equations (11) and (12) also separately in normally and over-consolidated phases.

Near ground surface, when effective initial stress σ'_0 is rather low, the value of modulus of compressibility M for the over-consolidated part is going to be quite small with conventional stress exponent β_2 values, and also the vertical deformation is unrealistic high. This special case can be handled as follows:

- Stress exponent $\beta_2 = 1$, the modulus of compressibility is then $M=m_2\sigma_v$. The modulus of compressibility is now constant. Deformations can be calculated with Eq. (12a).

3.2. Compression index method

The deformation parameter for compression index method in normally consolidated part is compression index C_c and in over-consolidated part the parameter is unloading and reloading phase compression index C_r. Between the modulus number parameters m_1 and m_2 (tangent module method) and parameters C_c and C_r (compression index method) is possible to conduct the following relation:

$$m_1 = \frac{2.3(1+e_0)}{C_c} \qquad (13a)$$

$$m_2 = \frac{2.3(1+e_0)}{C_r} \tag{13b}$$

In which case the values of stress exponents β_1 and β_2 are zero. This must be taking into consideration when defining compression indexes from oedometer tests.

e_0 the void ratio in current effective stress σ'_0

The modulus of compressibility M and vertical deformation ε_{zp} can be calculated with tangent modulus method equations (10),(11) and (12).

3.3. The methods based on water content

When using methods based on water content, the empiric connections between water content and compression index is tried to find out. There is no way to develop common connections between those parameters because they are linked to clay layer geological development and local circumstances. In figure (2) is introduced some observations between water content w and normally consolidated part compression index C_c, dispersion is noticeable large.

There are two methods based on water content in program SETTLE: method by Helenelund and method by Janbu.

The method by Helenelund (Helenelund 1951):

$$C_c = 0.85\left(\frac{w}{100}\right)^{1.5} \tag{14}$$

The method by Janbu:

$$m_1 = \frac{700}{w} \qquad , \beta_1 = 0 \tag{15}$$

w the water content, %

The void ratio e_0 is calculated with fully saturated soil water content w and solid density. The value for solid density is chosen as 2.7.

$$e_0 = 2.7\,\frac{w}{100} \tag{16}$$

The compression index for normally consolidated part is calculated with equation (14). The needed void ratio in formula (13a) is calculated with Eq. (16). After that the modulus of compressibility M and the vertical deformation ε_{zp} for normally consolidated part can be calculated with tangent modulus method formulas (10) and (11b).

This method is suitable for only normally consolidated ground settlement calculation. If the ground is over-consolidated, the consolidation pressure σ'_p must be estimated separately for example by vane shear test. Likewise the modulus number for over-consolidated part m_2 must be separately estimated and also coefficient of permeability k or coefficient of consolidation C_v for normally and over-consolidated parts has to be estimated.

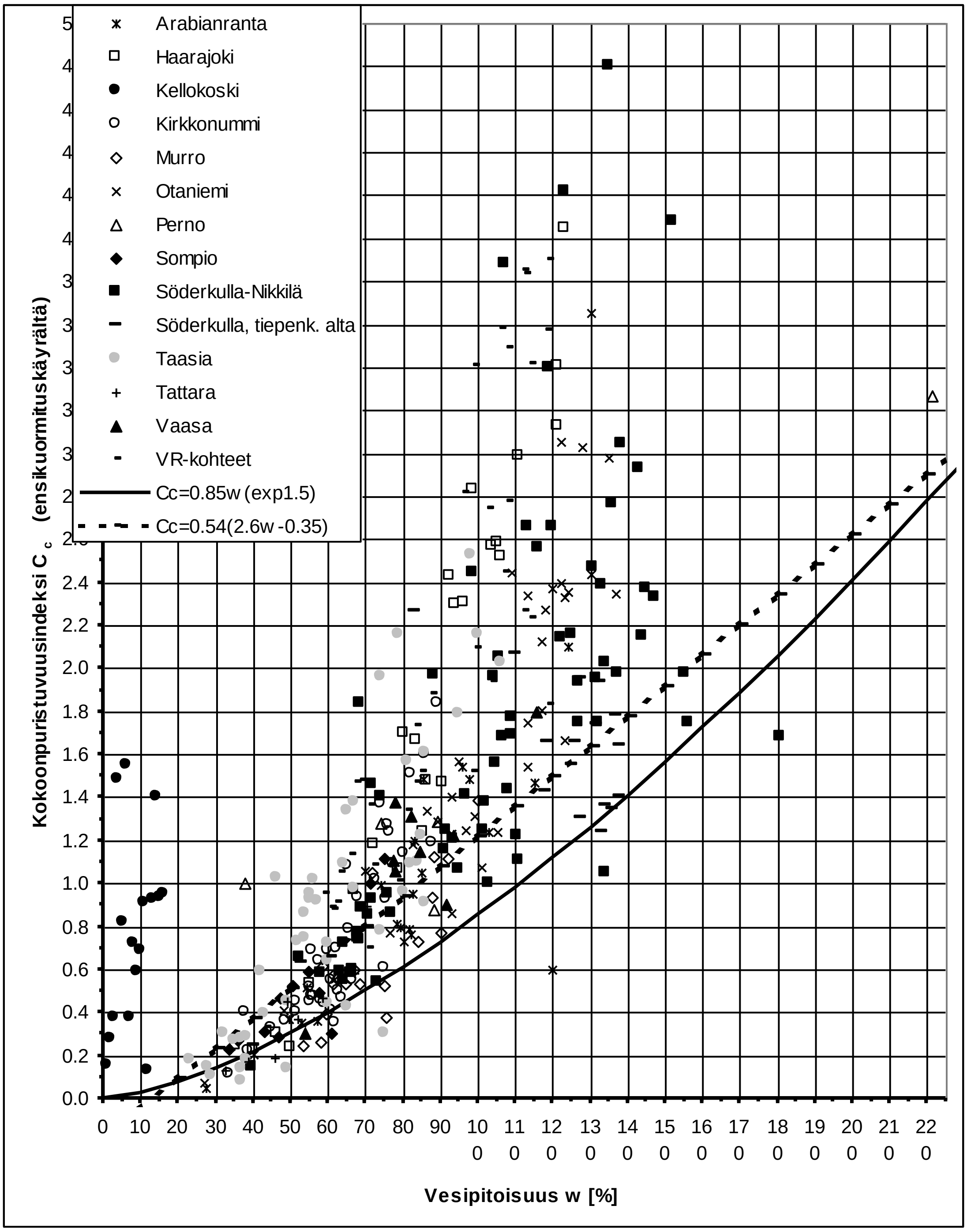

Figure 2. The observed connection between water content and compression index.

3.4. The Swedish settlement calculation method

The interdependence of the modulus of compressibility M and effective stress is illustrated in figure (3).

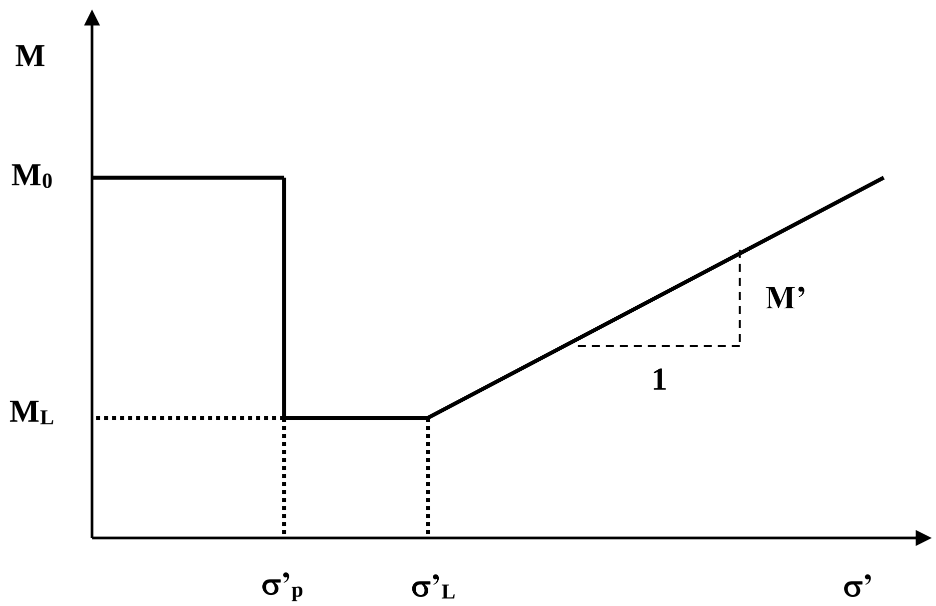

Figure 3. The Swedish settlement calculation method. The connection between modulus of compressibility and effective vertical stress.

Parameters for the method is recommend to define by CRS oedometer tests.

M_0 the modulus of compressibility (over-consolidated), $\sigma'_0 < \sigma' < \sigma'_p$

σ'_0 the effective vertical initial stress

σ'_p the effective consolidation pressure

M_L the constant modulus of compressibility between consolidation pressure σ'_p and stress σ'_L, $\sigma'_p < \sigma' < \sigma'_L$

σ'_L the effective stress where the modulus of compressibility begins to increase

M' the modulus number, when effective stress exceeds stress σ'_L, $\sigma' > \sigma'_L$

$$M' = \frac{\partial M}{\partial \sigma'} \tag{17}$$

Modulus number M' is equivalent to tangent modulus method's modulus number m_1, when stress exponent β_1 is zero.

Modulus of compressibility M which exceeds stress σ'_L is calculated with equation (18)

$$M = M_L + M'\left(\sigma' - \sigma'_L\right) \tag{18}$$

Primary consolidation ε_{zp} is calculated as follows:

$$\varepsilon_{zp} = \frac{\sigma' - \sigma'_0}{M_0} \quad , \sigma'_0 < \sigma' < \sigma'_p \tag{19a}$$

$$\varepsilon_{zp} = \frac{\sigma'_p}{M_0} + \frac{\sigma' - \sigma'_p}{M_L} \quad , \sigma'_p < \sigma' < \sigma'_L \tag{19b}$$

$$\varepsilon_{zp} = \frac{\sigma'_p}{M_0} + \frac{\sigma'_L - \sigma'_p}{M_L} + \frac{1}{M'}\ln\left[\frac{M'\left(\sigma' - \sigma_L\right)}{M_L} + 1\right] \quad , \sigma' > \sigma'_L \tag{19c}$$

3.5. The time – settlement parameters

The time-settlement parameters can be given in three different ways in program SETTLE:

- The coefficient of consolidation c_v is given for over-consolidated and normally consolidated part (two parameters). The over-consolidated part c_v is typically approximately 10 times larger than normally consolidated part c_v in Finnish soft clays.

- The coefficient of permeability k is given. The parameter k is same for both over-consolidated and normally consolidated part. The coefficient of consolidation c_v is calculated with equation (2). The coefficient of consolidation changes the same way like modulus of compressibility M depends on the stress level.

- The parameters k_0 and α for the coefficient of permeability k, which is changing by deformation, is given. The model is developed at HUT (Ravaska & Vepsäläinen 2001).

$$k = k_0 (1-\varepsilon)^{\alpha} \tag{20a}$$

$$c_v = \frac{kM}{\gamma_w} \tag{2bis}$$

The coefficient of consolidation can be given for the tangent modulus method as follows:

$$c_v = \frac{k_0 (1-\varepsilon)^{\alpha}\, m\sigma_v}{\gamma_w} \left(\frac{\sigma'}{\sigma_v} \right)^{1-\beta} \tag{20b}$$

k_0 The initial value for coefficient of permeability

α the deformation exponent

ε the vertical deformation in stress point

m the modulus number, normally and over-consolidated part

β the stress exponent, normally and over-consolidated part

The coefficient of consolidation depends on effective stresses but also vertical deformations (equations (11a),(11b),(12a),(12b)). When using The Swedish settlement calculation method, the modulus of compressibility M and vertical deformation is defined from the figure 3 for the eq. (20a) and (2bis) and with formulas (18), (19a), (19b) and (19c).

4. The consolidation pressure

The consolidation pressure can be given in four different ways (figure 4):

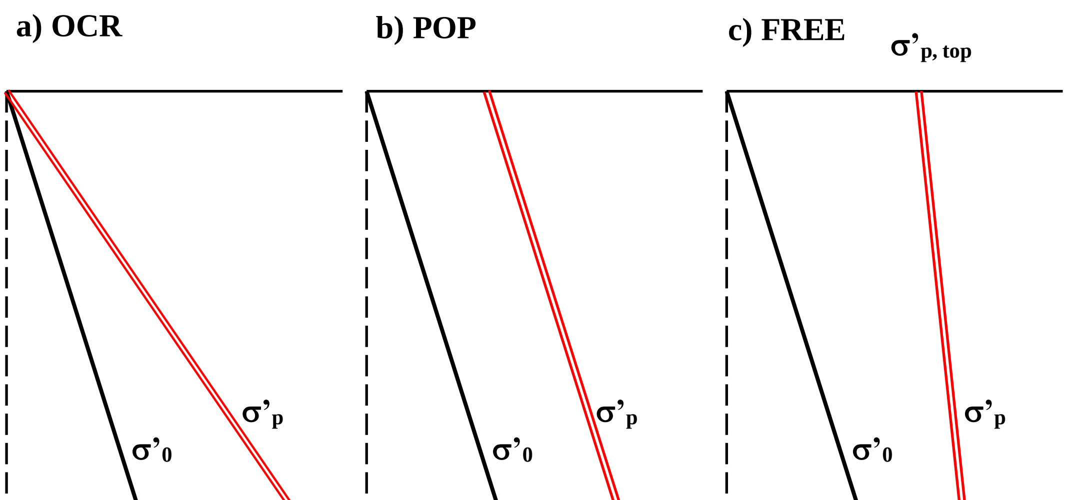

Figure 4. The given consolidation pressures. a) OCR b) POP c) values for the top and bottom of the layer.

- If the value for consolidation pressure is given as $\sigma'_p = 0$, then it is normally consolidated case: $\sigma'_p = \sigma'_0$ where σ'_0 is effective vertical initial stress. A constant value for consolidation pressure can be given also.

- OCR (over-consolidation ratio):

$$OCR = \frac{\sigma'_p}{\sigma'_0} \tag{21}$$

- POP (pre-overburden pressure):

$$POP = \sigma'_p - \sigma'_0 \tag{22}$$

- FREE: consolidation pressure σ'_p is given for the top and bottom of the layer (figure 4), and intermediate values will be interpolated.

5. The load types and additional stresses caused by loads

The following load types can be in the same calculation:

1. Equally distributed extensive load
2. Equally distributed rectangular load, which sides are parallel to coordinate axes
3. Equally distributed rectangular load, which sides are in arbitrary direction.
4. Linearly changing rectangular load, which sides are parallel to coordinate axes
5. Linearly changing rectangular load, which sides are in arbitrary direction
6. Linearly changing triangular load, which sides are in arbitrary direction
7. Equally distributed strip load, y-axis direction
8. Arbitrary strip load, y-axis direction

Vertical stress $\Delta\sigma_z$ caused by load is calculated by Boussinesq theory for different load types as follows:

1. Equally distributed extensive load p:

$$\Delta\sigma_z = p \tag{23}$$

2. – 5. Rectangular loads:

The calculation points for rectangular loads, which sides are in arbitrary direction, will be placed on local coordinate system which axes are parallel to the sides of the rectangular load and the origin is situated in the center of the rectangle.

Linearly changing rectangular loads are divided to part-loads which are shaped as rectangular and equally distributed. The accuracy of the calculation is better the more there are part-loads.

Vertical stress distribution is calculated right under the corner point of the rectangular load (figure 5):

$$\Delta\sigma_z = \frac{p}{2\pi}\left[\arctan\frac{lb}{zR_3} + \frac{lbz}{R_3}\left(\frac{1}{R_1^2} + \frac{1}{R_2^2}\right)\right] \tag{24a}$$

$$R_1 = \sqrt{l^2 + z^2} \tag{24b}$$

$$R_2 = \sqrt{b^2 + z^2} \tag{24c}$$

$$R_3 = \sqrt{l^2 + b^2 + z^2} \tag{24d}$$

The angles are in radians.

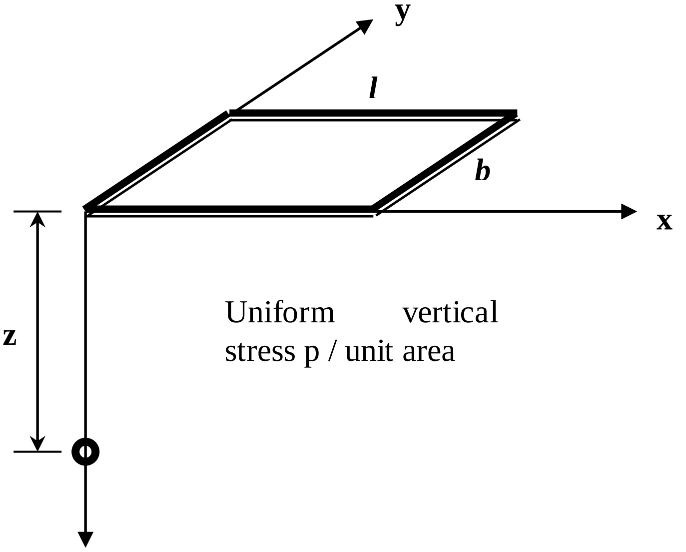

Figure 5. The vertical stress distribution under the corner point of the rectangular load (markings).

σ_z the vertical stress caused by the load

p the equally distributed load

l the length of the slab

b the width of the slab

z the depth

The vertical stress distribution in arbitrary point inside or outside of the load section is calculated by principle of superposition.

The rigid rectangular slab settlement can be calculated as well with load types 2.-5. In the case of rigid slab the additional stresses and settlements are calculated in four defined points, which are situated from the center of the slab in a distance as follows:

$$(0.37\,l,\ 0.37\,b) \qquad (25)$$

When calculating the rigid slab, the program defines automatically the calculating points. In other cases the calculating points have to be defined manually.

6. Linearly changing triangular load, which sides are in arbitrary direction

The triangle loads are divided to small part-triangles. At the center of gravity point of the part-triangle is situated a point force. The intensity of the point force is defined by the area and the average load intensity of the part-triangle. The accuracy of the calculation is better the more there are part-triangles.

The vertical stress caused by the point force is calculated with equation (26):

$$\Delta\sigma_z = \frac{3P}{2\pi} z^3 \left(r^2 + z^2\right)^{-5/2} \qquad (26)$$

z the depth under the influence point of the load

r the horizontal distance from the influence point of the load

The polygon loads are created by combining triangular loads and if necessary by adding rectangular loads also.

7. - 8. Strip loads, y-axis direction

The vertical stress distribution caused by strip load is illustrated in figure 6 with all the symbols.

$$\Delta\sigma_z = \frac{p}{\pi}\left[\alpha + \sin\alpha\,\cos(\alpha + 2\delta)\right] \tag{27}$$

Radian system is applied.

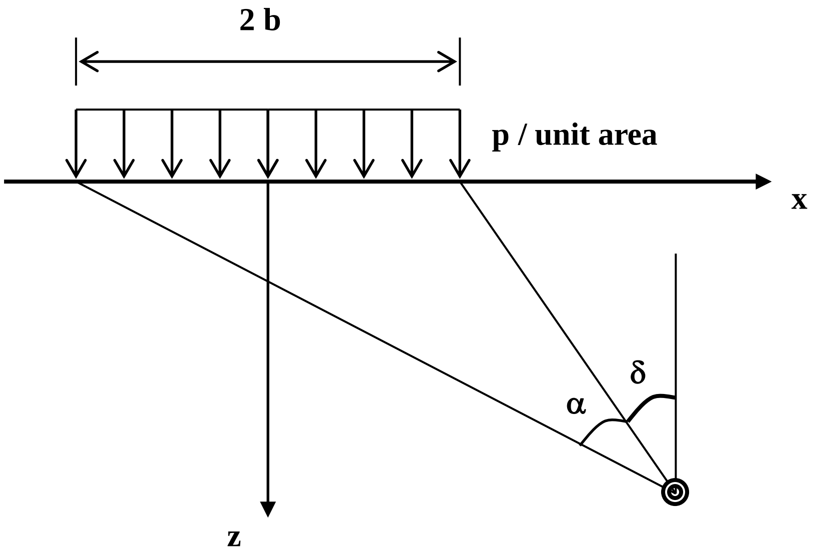

Figure 6. The vertical stress caused by strip load (markings).

The arbitrary strip load (figure 7) is distributed equally to sheeted strip loads. The stress distribution caused by the load in arbitrary point under or beside of the load is calculated from sheeted strip load effects by superposition method.

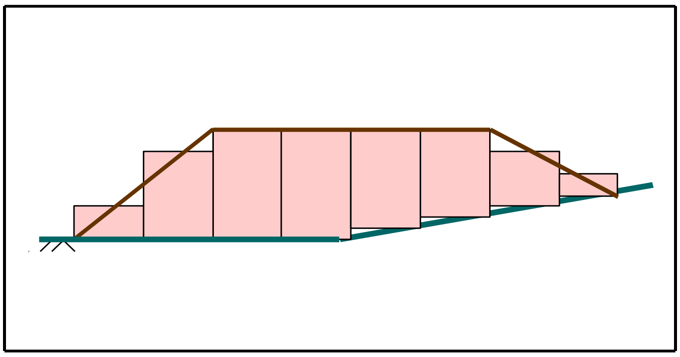

Figure 7. The principle of the arbitrary strip load distribution to sheeted strip loads.

6. The load history

There can be given an unique load history for each load not depending on the load type. Load can be constant all the time or it can increase or decrease linearly as a function of time (figure 8). If the load is changing as a function of time then the vertical additional stress is time dependent as well and it is calculated in the last term of the equation (1).

The time, when there is a change in load history is automatically also a calculation time.

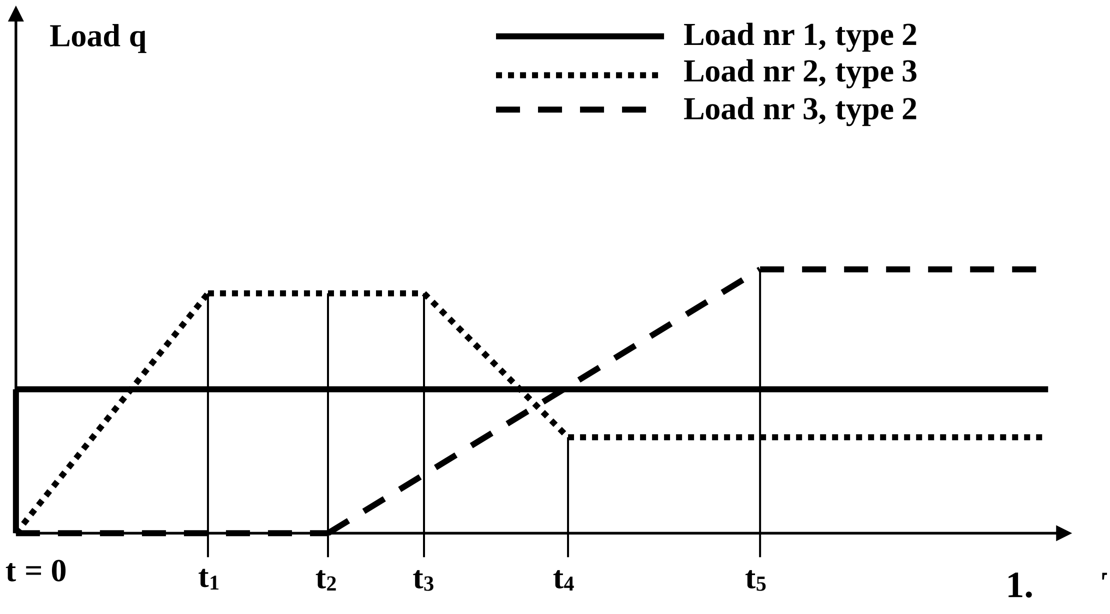

Figure 8. An example of load history and obligatory calculation times.

7. The consolidation boundaries

There are given consolidation boundaries for each calculation point both upper edge (foundation level or surface) and bottom edge (the bottom edge of the settlement layers). These values are constant during the whole consolidation process time. Options are for upper and bottom edge as follows:

- The edge is defined as fully permeable, then the excess pore pressure at the edge is zero.

-The edge is defined as impermeable.

The edge can contain only one of those options.

The program doesn't take into account if the upper edge is situated at groundwater level or above of it. If the upper edge is situated above groundwater level, for example in solum, then the coefficient of consolidation or coefficient of permeability has to be given as high value for this layer.

In settlement layers it is also possible to define for each calculation point the depth position of thin water permeable layers. The thin water permeable layer is defined as fully permeable and the excess pore pressure is zero in the layer. If the layer as above has high value for coefficient of consolidation or coefficient of permeability and the layer is not defined as permeable then the layer only balances differences in excess pore pressures.

8. The calculation times

As written in chapter (5) the obligatory calculation times are the situations when there is a change in load history. Other calculation times can be given manually or automatically, if the load history is simple enough. In manual case the program checks that the smallest time increment calculated by the calculation time will not be under critical time increment Δt_{crit} (Vermeer et al 1981):

2-knot segment of line element:

$$\Delta t_{crit} = \frac{L^2}{6c_v} \tag{28a}$$

3-knot segment of line element (not at the moment in program SETTLE):

$$\Delta t_{crit} = \frac{L^2}{10\, c_v} \tag{28b}$$

L the length of the largest individual element in element mesh

If the time increment used in the calculations is less than the critical time increment, then there will be numerical inaccuracy with small time values caused by oscillation. The situation is mainly in the load history where the load influence is immediate. The program prints out the value of the critical time increment, and after that the end user can either change the density of the element mesh or change calculation times.

9. The principles of the numerical solution

The numerical solution is based on solving the consolidation equation (1) by element method. The time integration is done by implicit difference method. The excess pore pressure u in stress points of the elements is get as a result as a function of time and depth.

The matrices for the element method are following:

$$[M]\{U(t+\tau)\} = \{Q\} \tag{29}$$

[M]	the global stiffness matrix which is defined by the time integration
$\{U(t+\tau)\}$	the excess pore pressure matrix at time t + τ
$\{Q\}$	the global load matrix which is defined by the time integration
t	the previous calculation time moment
τ	the time increment

$$[M] = [K] + \frac{1}{\tau}[P] \tag{30a}$$

$$\{Q\} = \{R*\} + \frac{1}{\tau}[P]\{U(t)\} \tag{30b}$$

[K]	the global stiffness matrix
{R*}	the global load matrix
[P]	the global mass matrix
{U(t)}	the excess pore pressure matrix at time t (initial condition when t = 0)

Stiffness, mass and load matrices by elements:

$$\left[K^e\right] = c_v \int_0^L \left[\frac{\partial N}{\partial z}\right]^T \left[\frac{\partial N}{\partial z}\right] dz \tag{31a}$$

$$\left[P^e\right] = \int_0^L [N]^T [N] dz \tag{31b}$$

$$\left\{R*^e\right\} = \left\{R^e\right\} + \{F(L)\} + \{F(0)\} \tag{31c}$$

$$\left\{R^e\right\} = \int_0^L [N]^T [N] dz \frac{\partial [Q]}{\partial t} \tag{31d}$$

$$\{F(L)\} = \frac{M}{\gamma_w} q_2 [N(L)]^T \tag{31e}$$

$$\{F(0)\} = -\frac{M}{\gamma_w} q_1 [N(0)]^T \tag{31f}$$

[N] the matrix of shape function

[Q] knot-specific additional stress matrix at time t (the load history is included)

q_2 edge condition: the flow rate through the bottom edge, $q_2 = 0$ in program SETTLE

q_1 edge condition: the flow rate through the top edge, $q_1 = 0$ in program SETTLE

The element mesh in program SETTLE is created by using 2-knot linear line elements. Element-specific matrices are combined to global matrices by standard compilation method. The matrix of excess pore pressure is solved after edge conditions placements with the elimination method by Gauss.

10. Literature

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