# THE CHORD FUNCTION

One of the earliest trigonometric functions was the chord function **crd**; given an angle $\theta$ between 0 and $\pi$ radians, the value of $\mathbf{crd}\,\theta$ is given by constructing an isosceles triangle $\Delta ABC$, where $\mathbf{d}(A,B) = \mathbf{d}(A,C) = 1$ and $|\angle BAC| = \theta$, and taking the length of the third side $[AC]$.

**1.** Prove that $\mathbf{crd}\,\theta \;=\; 2\sin\frac{1}{2}\theta$ and use this to find a Maclaurin series $C(\theta)$ for the chord function.

**2.** Let $C_n(\theta)$ be the $n^{\text{th}}$ degree polynomial approximation to the power series $C(\theta)$ in the preceding exercise. Find a value of $n$ such that $|C_n(\theta) - C(\theta)| < 0.0005$ for all $\theta$ in the interval $[0, \frac{1}{2}\pi]$. [*Hint:* Use the remainder term in Taylor's Theorem together with the estimate for the error term in the Leibniz test for alternating series.]