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of the American Mathematical Society

April 2012

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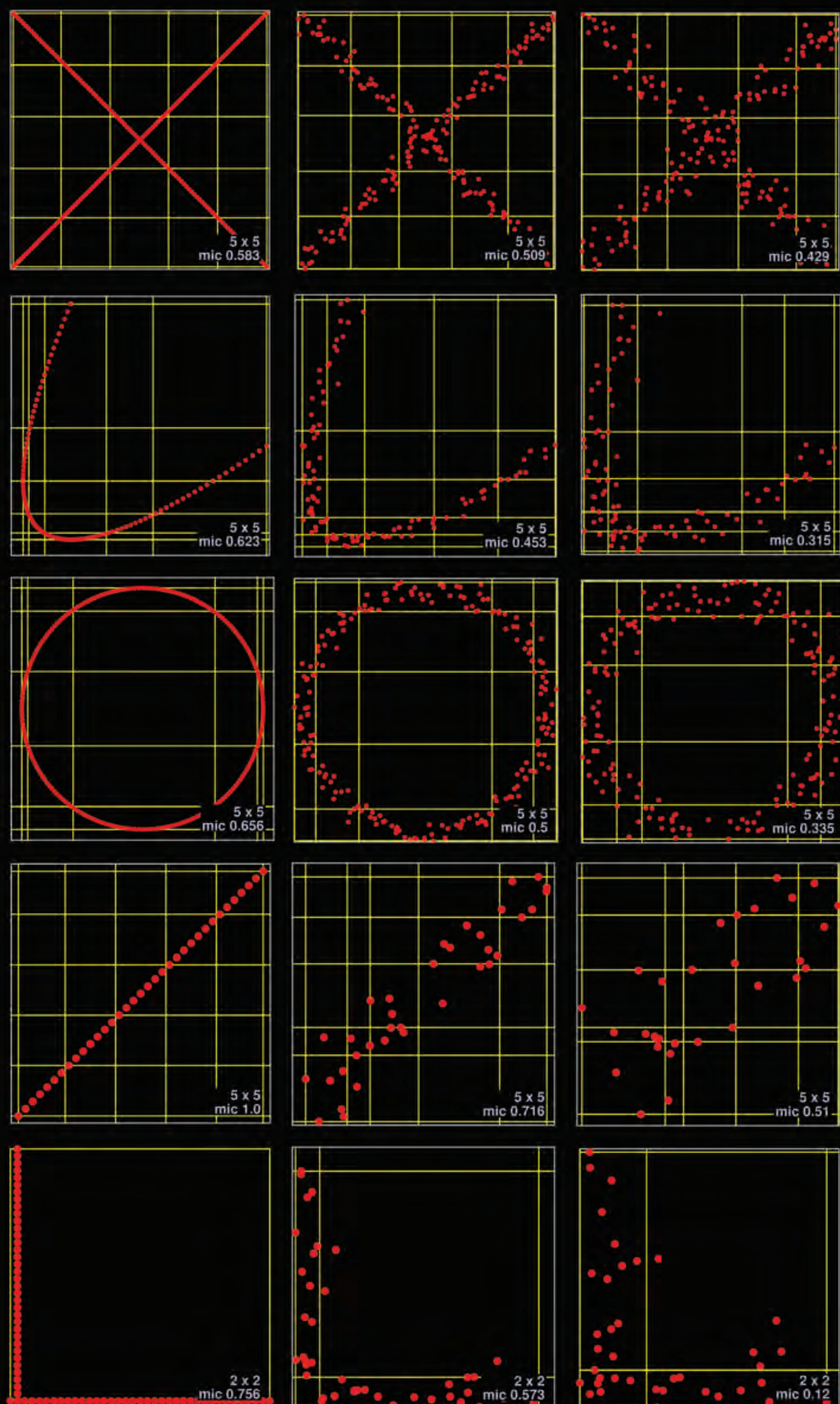

About the Cover:
Data MINEing
See page 598

Notices
of the American Mathematical Society

April 2012

Communications

Commentary

In this issue we treat topics both historical and contemporary. There is an article about the first American woman Ph.D. in mathematics. An article about the remarkable contemporary mathematician Vladimir Arnold. A piece about home schooling and another about privacy and electronic data. We also explore data mining—a hot current topic. And there is an article about AIDS in India. April is Mathematics Awareness Month, and the topic this year is data and statistics. We endeavor to present subjects that bear on this theme.

—*Steven G. Krantz, Editor*

Features

Notices
of the American Mathematical Society

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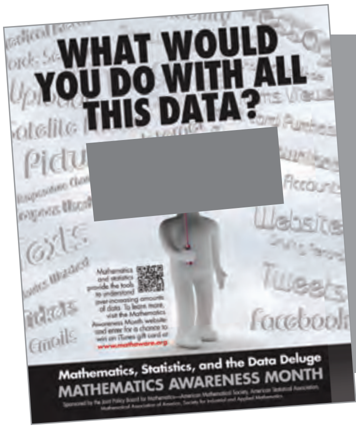
From the AMS Secretary

American Mathematical Society

April is Mathematics Awareness Month



Find out more about mathematics, statistics, and the data deluge at **mathaware.org**

Tensors: Geometry and Applications

Applied Mathematics

J. M. Landsberg, *Texas A&M University, College Station, TX*

Graduate Studies in Mathematics, Volume 128; 2012; 439 pages; Hardcover; ISBN: 978-0-8218-6907-9; List US$74; SALE US$44.40; Order code GSM/128

Data Mining and Mathematical Programming

Applied Mathematics

Panos M. Pardalos, *University of Florida, Gainesville, FL*, and **Pierre Hansen**, *HEC Montréal, QC, Canada*, Editors

Titles in this series are co-published with the Centre de Recherches Mathématiques.

CRM Proceedings & Lecture Notes, Volume 45; 2008; 234 pages; Softcover; ISBN: 978-0-8218-4352-9; List US$88; SALE US$52.80; Order code CRMP/45

Complex Graphs and Networks

Fan Chung, *University of California at San Diego, La Jolla, CA*, and **Linyuan Lu**, *University of South Carolina, Columbia, SC*

CBMS Regional Conference Series in Mathematics, Number 107; 2006; 264 pages; Softcover; ISBN: 978-0-8218-3657-6; List US$57; SALE US$34.20; All individuals US$45.60; Order code CBMS/107

Frames and Operator Theory in Analysis and Signal Processing

David R. Larson, *Texas A&M University, College Station, TX*, **Peter Massopust**, *Technische Universität München, Munich, Germany*, **Zuhair Nashed**, *University of Central Florida, Orlando, FL*, **Minh Chuong Nguyen**, *Vietnamese Academy of Science and Technology, Hanoi, Vietnam*, **Manos Papadakis**, *University of Houston, TX*, and **Ahmed Zayed**, *DePaul University, Chicago, IL*, Editors

Contemporary Mathematics, Volume 451; 2008; 291 pages; Softcover; ISBN: 978-0-8218-4144-0; List US$92; SALE US$55.20; Order code CONM/451

Data Structures, Near Neighbor Searches, and Methodology: Fifth and Sixth DIMACS Implementation Challenges

Applied Mathematics

Michael H. Goldwasser, *Loyola University of Chicago, IL*, **David S. Johnson**, *AT&T Bell Laboratories, Florham Park, NJ*, and **Catherine C. McGeoch**, *Amherst College, MA*, Editors

Co-published with the Center for Discrete Mathematics and Theoretical Computer Science beginning with Volume 8. Volumes 1–7 were co-published with the Association for Computer Machinery (ACM).

DIMACS: Series in Discrete Mathematics and Theoretical Computer Science, Volume 59; 2002; 256 pages; Hardcover; ISBN: 978-0-8218-2892-2; List US$91; SALE US$54.60; Order code DIMACS/59

Discrete Methods in Epidemiology

Applied Mathematics

James Abello, *DIMACS, Piscataway, NJ, and ask.com Research, Piscataway, NJ*, and **Graham Cormode**, *DIMACS, Piscataway, NJ, and Bell Laboratories, Murray Hill, NJ*, Editors

Co-published with the Center for Discrete Mathematics and Theoretical Computer Science beginning with Volume 8. Volumes 1–7 were co-published with the Association for Computer Machinery (ACM).

DIMACS: Series in Discrete Mathematics and Theoretical Computer Science, Volume 70; 2006; 260 pages; Softcover; ISBN: 978-0-8218-4379-6; List US$92; SALE US$55.20; Order code DIMACS/70.S

Algebraic Coding Theory and Information Theory

Applied Mathematics

A. Ashikhmin, *Bell Labs, Lucent Technologies, Murray Hill, NJ*, and **A. Barg**, *University of Maryland, College Park, MD*, Editors

Co-published with the Center for Discrete Mathematics and Theoretical Computer Science beginning with Volume 8. Volumes 1–7 were co-published with the Association for Computer Machinery (ACM).

DIMACS: Series in Discrete Mathematics and Theoretical Computer Science, Volume 68; 2005; 177 pages; Hardcover; ISBN: 978-0-8218-3626-2; List US$87; SALE US$69.60; Order code DIMACS/68

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Photographs by E. David Luria, photographer, and Annette Emerson (AMS).

See photo key on page 555

Privacy in a World of Electronic Data: Whom Should You Trust?

We live in an electronic world, one that has transformed mathematics and mathematical communication. More then ever we are accustomed to sharing what we know and communicating about our work in an open and uninhibited fashion. But our personal data also appear online and in data banks wherever and whenever we act or are acted upon. Many express concerns about the theft of our personal information, but we do little to protect ourselves. Unlike the old world of physical banks where we once placed our money both for security and interest, the new world of electronic data banks holds something much more valuable—our very lives—and our personal information moves in ways that our money never did.

Mobile telephone operators track our every movement, whom we call, and who calls us. Electronic medical records that describe our ailments and treatments, including our hospital billing records, is accessible to a variety of others, such as government agencies overseeing Medicare and Medicaid. Credit card companies and banks record and store the details of financial transactions. Local authorities post information on the value of the property we own. Data warehouses amass personal information on us, details of what we buy, to whom we owe money, for whom we work, what we earn, etc. States and local authorities gather considerable information on us as part of voter registration—such as name, address, telephone number, date of birth, gender, and party affiliation—and voter lists are widely available in electronic form.

We voluntarily share our personal information, our pictures, and our links to our friends and family with social networking sites such as Facebook. That information and more—all the pages you subscribe to—is shared with other vendors and, more often than not, is searchable by others on the Web. Google and other search engine companies track the pages we visit and many of our online activities. Even the words we write—whether in email or in our professional papers—are available and searchable online. Many of the specialized online services we use draw on these data.

There are formal methods for linking data across many of these seemingly separate electronic spheres, using Social Security numbers and other personal identifiers, although often with substantial error. It is no wonder that many ask, "Is privacy dead?" My answer is, "not quite", but you will have to be vigilant if you want to protect your information.

In almost every sphere there are rules, supposedly to allow us to protect our data from misuse. But as with Facebook, these are ever-changing and usually too complicated for most people to bother with. Unfortunately, once others have our data, whether they be individuals, private businesses, or even government agencies, we cannot retract the information or restrict how it will be used. Whom should you trust? What can you and should you do to protect your privacy?

Many laws and regulations require businesses and organizations to share their privacy policies with you. The Health Insurance Portability and Accountability Act of 1996 (HIPAA), which sets standards for the protection of electronic medical records, also has rules requiring disclosure by doctors and other medical organizations regarding how your information can be used and with whom it can be shared. Read their documents!

It is important to post your research papers online, but think before you post personal information online, whether on Facebook or in some other forum. Do you want your photos and other images tagged and linked to other databases? What would your employer or a future employer think about information posted in your blog? Read the privacy rules and invoke the options they offer for restricting how your data can be used. Make a conscious choice to surrender your privacy, because the decision is irrevocable.

Some government agencies can request our data from other entities, and there are laws to regulate what happens when such data may be disclosed. For example, the Right to Financial Privacy Act of 1978 requires United States federal government agencies to provide individuals with a notice and an opportunity to object before a bank or other specified institution can disclose personal financial information. There are many exceptions to this stricture, however, including for the Internal Revenue Service (IRS) and for cases in which individuals are suspected of illegal behavior, especially terrorist activities.

There are many government agencies whom you can trust with your data and who go to great lengths to prevent the disclosure of your information, both to those inside government and to those outside. For example, the IRS and the U.S. Census Bureau have strict rules to protect the information they gather, and they may release individual information only in nonidentifiable form. Other government agencies, in the United States and abroad, gather data required by law and go to similar lengths before releasing public-use microdata files for research use. How to protect individuals' information has become a major area of research in different parts of the mathematical sciences.

The data deluge has opened up many new avenues for research in mathematics and statistics, including research into methods for protecting individuals' personal information. Proving guarantees of formal privacy requires serious mathematics, and getting methods to scale for databases in our new electronic online world involves innovative mathematical computation. Along with the pursuit of such research, the mathematical and statistical communities should raise awareness of the need to protect other people's privacy. Perhaps the place to begin is with our own data, with ourselves.

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Letters to the Editor

Word Problems

In a recent article ("Modeling the journey from elementary word problems to mathematical research"), Chris Sangwin aptly describes how word problems help developing mathematical aptitudes. His quote from Pólya, "the most important single task of mathematical instruction in the secondary school is to teach the setting up of equations to solve word problems", appears controversial but can be seen as an incentive for teaching how to solve problems via equations with *awareness* (about strengths, limitations, pitfalls, style, etc.). Equations delegate substantial parts of reasoning (especially the tricky ones) to symbolic calculations. However, this requires some mathematical literacy.

Therefore, and since diversity enhances understanding, it is helpful to consider methods suitable in primary school. These can serve later as a *sanity check* when learning to work with equations (before reversing the roles). An example is Sangwin's Example 4:

"A dog starts in pursuit of a hare at a distance of thirty of his own leaps from her. He takes five leaps while she takes six, but covers as much ground in two as she in three. In how many leaps of each will the hare be caught?"

Setting up an equation requires "careful work on the part of the student." Here follows a third-grade-level solution.

The dog advances 5 dog leaps whenever the hare advances 6 hare leaps, which is 4 dog leaps. So, the dog gains 1 dog leap with every 5 leaps. To annihilate the initial distance of 30 dog leaps, the dog must make 5×30 or 150 leaps. The hare has then made 6×30 or 180 leaps.

Sangwin also notes that "problems involving rates are particularly difficult." In an interview (*Notices of the AMS*, April 1997), Vladimir Arnol'd mentions such a problem he solved at age 12:

"Two old women started at sunrise and each walked at constant velocity. One went from A to B and the other from B to A. They met at noon and, continuing with no stop, arrived respectively at B at 4 p.m. and at A at 9 p.m. At what time was the sunrise on this day?"

In secondary school, one would set up an equation. In primary school, many learn the *rule of three*, a systematic method from the sixth century BCE for dealing with rates without using symbols, which (with just one nonstandard step) also yields the solution.

Another invaluable sanity check for equations is dimensional analysis.

—Raymond Boute
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(Received December 17, 2011)

Committee on Education Offers Qualified Endorsement of Common Core Standards

The Committee on Education recognizes and commends the efforts of those who produced the Common Core State Standards in Mathematics. We see substantial benefits to be gained from a successful implementation of these standards. One would be an approximate synchronization of mathematics learning in schools across the country, which has obvious advantages in a mobile society. More important, we endorse the goals of focus, logic, coherence, thoroughness, and grade-to-grade continuity that have guided the writing of the standards. We acknowledge the concern of some mathematicians that an attempt to adhere to these standards without a sufficiently large corps of mathematically knowledgeable teachers will not be beneficial, and in fact could do significant harm. Well-qualified teachers are absolutely essential in implementing ambitious standards like these. The adoption of the Common Core State Standards in Mathematics by more than forty states affords mathematicians the opportunity and the responsibility to involve ourselves in efforts to strengthen the mathematical knowledge of both pre-service and in-service teachers.

—David Wright
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(Received December 28, 2011)

Response to Quinn

I found particularly useful Frank Quinn's article "A revolution in mathematics? What really happened a century ago and why it matters today", published in the January issue of the *Notice*s. I have also read Dr. Tevian Dray's justified criticism (in the same issue of *Notices*) of a previous work authored by Quinn ["A science-of-learning approach to mathematics education", *Notices*, October 2011]. However, there is something particularly important that should be said about Quinn's "A Revolution in mathematics?…" I think that any discussion about mathematical education today, in particular about teaching strategies of proofs, should take into account the modernist transformation of mathematics that took place between 1890 and 1930. F. Quinn's recent article does a very useful service to the mathematical community: it merges Jeremy Gray's very insightful contribution from his 2008 monograph *Plato's Ghost* (one of the most useful books ever written in the history of mathematics) with the needs of contemporary pre-college education. F. Quinn states that the main point of his article "is not that a revolution occurred, but that there are penalties for not being aware of it." The matter is sensitive today for the mathematical community, as the topic borders on the main dilemma of the calculus wars. Quinn spells it out clearly when he mentions that "the precollege-education community was, and remains, antagonistic to the new methodology." Perhaps integrating the study of history of mathematics in our contemporary teaching of undergraduate courses could be particularly useful to close, even partially, this gap. For example, I found it extremely useful to embed Jeremy Gray's description of the modernist transformation of mathematics into the Foundation of Geometry course that I teach for math major undergraduate students who are planning a career in education. I am using the very well written *The Foundations of Geometry*, by Gerard Venema, as the regular textbook, but I additionally present in detail the evolution of various axiomatic systems in plane geometry. One can present the chronological sequence and important developments of various axiomatic systems (D. Hilbert, O. Veblen, G. D. Birkhoff) as an evolution within the ample change of modernist revolution. Quinn's recent article points out how describing such a historical evolution could make a difference and matter a lot for contemporary pre-college education, as well as for the K–12 education community.

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(Received January 17, 2012)

Response to Quinn

In response to the article by Dr. F. Quinn in the January 2012 *Notices*: You may be right in what you say. However, one thing I know is that mathematics is some unknown process taking place in the human brain. Mathematics is not floating around in spacetime. At least I haven't been struck and knocked down by a flying theorem lately. If I only understood what the brain is doing when it thinks 1 + 1 = 2, I would think I'm in heaven. Better than that would be to understand how the brain gets from one step in one of your "core math" proofs to the next step in a proof with a finite number of steps. Is it some sort of quantum jump between energy levels or does the brain "slide" in some manner between the steps? But best of all would be to understand what is going on in your brain when you think that your "core math" is superior to the old-fashioned math done by us old-time intuitionists. Maybe one of these "hostile" scientists will tell me how the brain uses experience to modify the strengths of the NMDA synapses. Then I could model the brain and perhaps come to an understanding of mathematics by running special cases on a computer using floating point arithmetic?

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(Received January 25, 2012)

Reformers Operating in a Vacuum

In his February 2012 *Doceamus* piece, Alan H. Schoenfeld provided some fascinating anecdotes concerning the current state of mathematics pseudo-education, and then reminisced about teaching precalculus at the University of Rochester. In the last paragraph, he proposed "that we revise the entire curriculum." Unfortunately, as is the case with many self-styled math reformers, Schoenfeld continues to operate in a complete historical vacuum. For example, with respect to precalculus, he could have pointed out how our textbooks have degenerated.

When I was in the twelfth grade, we used the textbook by William E. Kline, Robert O. Oesterle, Leroy M. Willson, *Foundations of Advanced Mathematics*, The American Book Company, New York (1959), 519 pp., which is described at: http://mathforum.org/kb/thread.jspa?forumID=206&threadID=478525.

In the late 1960s this was replaced by Mary P. Dolciani's (et al.), *Modern Introductory Analysis*, Houghton Mifflin (1964), 651 pp., which was influenced by the new-math strand developed by the School Mathematics Study Group and is described briefly at: http://mathforum.org/kb/thread.jspa?forumID=206&threadID=1728344&messageID=6179351.

When my son was in the eleventh grade, he used Merilyn Ryan's (et al.), *Advanced Mathematics: A Precalculus Approach*, Prentice Hall (1993), 946 pp., which was based on the above and included all the empty slogans of the NCTM "Standards".

Four years ago I received unsolicited desk copies of the previous editions of the following 1,000-page doorstops by Michael Sullivan and Michael Sullivan III:

http://www.amazon.com/Precalculus-Concepts-Functions-Approach-Trigonometry/dp/0321644875/

http://www.amazon.com/Precalculus-Concepts-Functions-Triangle-Trigonometry/dp/0321645081/.

I was appalled to see two distinct doorstops for the two approaches to trigonometry, whose equivalence was demonstrated in a few pages of my high school textbook. As long as we continue to operate in a historical vacuum, and continue to produce bloated doorstops, the pseudo-education of American students will continue unabated.

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Correction

Authors Barbara and Robert Reys ("Supporting the next generation of 'stewards' in mathematics education", *Notices*, February 2012) were incorrectly identified as being Curators' Professors of Mathematics at the University of Missouri–Columbia. They are, in fact, Curators' Professors in the Department of Learning, Teaching, and Curriculum at the University of Missouri–Columbia.

—Sandy Frost

Memories of Vladimir Arnold

Boris Khesin and Serge Tabachnikov, Coordinating Editors

Vladimir Arnold, an eminent mathematician of our time, passed away on June 3, 2010, nine days before his seventy-third birthday. This article, along with one in the previous issue of the *Notices*, touches on his outstanding personality and his great contribution to mathematics.

Dmitry Fuchs

Dima Arnold in My Life

Unfortunately, I have never been Arnold's student, although as a mathematician, I owe him a lot. He was just two years older than I, and according to the University records, the time distance between us was still less: when I was admitted to the Moscow State university as a freshman, he was a sophomore. We knew each other but did not communicate much. Once, I invited him to participate in a ski hiking trip (we used to travel during the winter breaks in the almost unpopulated northern Russia), but he said that Kolmogorov wanted him to stay in Moscow during the break: they were going to work together. I decided that he was arrogant and never repeated the invitation.

Then he became very famous. Kolmogorov announced that his nineteen year-old student Dima Arnold had completed the solution of Hilbert's 13th problem: every continuous function of three or more variables is a superposition of continuous functions of two variables. Dima presented a two-hour talk at a weekly meeting of the Moscow Mathematical Society; it was very uncommon for the society to have such a young speaker. Everybody admired him, and he certainly deserved that. Still there was something that kept me at a distance from him.

V. Arnold, drawing, 1968.

I belonged to a tiny group of students, led by Sergei Novikov, which studied algebraic topology. Just a decade before, Pontryagin's seminar in Moscow was a true center of the world of topology, but then Cartan's seminar in Paris claimed the leadership, algebraic topology became more algebraic, and the rulers of Moscow mathematics pronounced topology dead. Our friends tried to convince us to drop all these exact sequences and commutative diagrams and do something reasonable, like functional analysis or PDE or probability. However, we were stubborn. We even tried to create something like a topological school, and, already being a graduate student, I delivered a course of lectures in algebraic topology. The lectures were attended by several undergraduates, and we were happy to play this game.

Then something incredible happened. One day I found the lecture room filled beyond capacity; I even had to look for a bigger room. My audience

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DOI: http://dx.doi.org/10.1090/noti813

At Otepya, Estonia.

had become diverse: undergraduates, graduate students, professors. This change had a very clear reason: the Atiyah-Singer index theorem.

The problem of finding a topological formula for the index of an elliptic operator belonged to Gelfand. Our PDE people studied indexes a lot, and they had good results. It was not a disaster for them that the final formula was found by somebody else: their works were respectfully cited by Atiyah, Singer, and their followers. The trouble was that the formula stated, “the index is equal to” and then something which they could not understand. People rushed to study topology, and my modest course turned out to be the only place to do that.

And to my great surprise, I noticed Dima Arnold in the crowd.

I must say that Dima never belonged to any crowd. Certainly the reason for his presence did not lie in any particular formula. Simply, he had never dismissed topology as nonsense, but neither had he been aware of my lectures. When he learned of their existence, he appeared. That was all. He never missed a lecture.

One day we met in a long line at the student canteen. “Listen,” he said, “can you explain to me what a spectral sequence is?” I began uttering the usual words: a complex, a filtration, differentials, adjoint groups, etc. He frowned and then said, “Thus, there is something invariant [‘invariant’ in his language meant ‘deserving of consideration’] in all this stuff, and this is the spectral sequence, right?” I thought for a moment and said, yes. At this moment we got our meals, and our conversation changed its direction.

Evidently spectral sequences were not for Arnold. Nonetheless, there is such a thing as Arnold’s spectral sequence [9], a humble object in the world of his discoveries, resembling the asteroid Vladarnolda in the solar system (the stability of which he proved approximately at the time of our conversation in the canteen), named after him. When I say that he could not appreciate spectral sequences, I mean that he in general had a strong dislike for unnecessary technicalities, and technicalities were often unnecessary to him because of his extremely deep understanding. By the way, this attitude toward impressive but unnecessary tricks extended beyond mathematics. Years later we spent a week or so with friends at a ski resort in Armenia. We showed each other different turns and slidings, but Dima obviously was not interested. He said that the slope was not too steep, and he simply went straight from the top to the bottom, where he somehow managed to stop. I was surprised: there was a stone hedge in the middle of the slope that you needed to go around. Dima said modestly, “You know, at this place my speed is so high that I simply pull my legs up and jump over the hedge.” I could not believe it, so I waited at a safe distance from the hedge and watched him doing that. It was more impressive than all our maneuvers taken together. Whatever he did—mathematics, skiing, biking—he preferred not to learn how to do it but just to do it in the most natural way, and he did everything superlatively well.

I do not remember how it came about that I began attending his Tuesday seminar. Probably he asked me to explain some topological work there, then I had to participate in some discussion, and then I could not imagine my life without spending two hours every Tuesday evening in a small room on the fourteenth floor of the main building of the MSU. Works of Arnold, his numerous students, and other selected people were presented at the seminar, and Dima insisted that every word of every talk be clear to everybody in the audience. My role there was well established: I had to resolve any topology-related difficulty. Some of my friends said that at Arnold’s seminar I was a “cold topologist”. Certainly, a non-Russian-speaker cannot understand this, so let me explain. In many Russian cities there were “cold shoemakers” in the streets who could provide an urgent repair to your footwear. They sat in their booths, usually with no heating (this is why they were “cold”), and shouted, “Heels!…Soles!….” So I appeared as if sitting in a cold booth and yelling, “Cohomology rings!…Homotopy groups!…Characteristic classes!….”

In my capacity as cold topologist, I even had to publish two short articles. One was called “On the Maslov–Arnold characteristic classes,” and the other one had an amusing history. One day Dima approached me before a talk at the Moscow Mathematical Society and asked whether I could compute the cohomology of the pure braid group (“colored braid group” in Russian); he needed it urgently. I requested a description of the classifying space, and the calculation was ready at the end of the talk. It turned out that the (integral) cohomology ring was isomorphic to a subring of the ring of differential forms on the classifying manifold. He suggested that I write a note, but I refused: for a topologist it was just an exercise; it could be interesting only in conjunction with an application to something else. (I knew that Dima was thinking of Hilbert's 13th problem in its algebraic form: the possibility of solving a general equation of degree 7 not in radicals, but in algebraic functions of two variables.) I suggested that he write an article and mention my modest contribution in an appropriate place. He did [2]. But, a couple of months later, he needed the cohomology of the classical Artin's braid group. This was more difficult and took me several days to complete the calculation. I did it only modulo 2, but I calculated a full ring structure and also the action of the Steenrod squares. (The integral cohomology was later calculated independently by F. Vainshtein, V. Goryunov, and F. Cohen and still later Graeme Segal proved that the classifying space of the infinite braid group was homologically equivalent to $\Omega^2 S^3$.) I phoned Dima and explained the results. First he requested that I give a talk at the seminar (Next Tuesday! That is tomorrow!), and then he decidedly refused to do what we had agreed upon for pure braids: to write an article and mention my participation where appropriate. After a brief argument, we arrived at a compromise: I publish an article about the cohomology of the braid group without any mentioning of Hilbert's problem, and he publishes an article where this cohomology is applied to superpositions of algebraic functions. When we met the next day, his article was fully written and mine had not even been started. But his article contained a reference to mine and hence the title of the latter. I could delay no longer, and the two articles were published in the same volume of *Functional Analysis* [5], [14]. Since the articles in *Functional Analysis* were arranged alphabetically, his article was the first, and mine was the last. But this was not the end of the story. A cover-to-cover translation of *Functional Analysis* was published by an American publisher. The braid group in Russian is called группа кос; the word кос is simply the genitive of коса, a braid, but the American translators thought that КОС was a Russian equivalent of COS, and the English translation of my article was attributed to a mysterious *cosine group*. I do not know how many English-speaking readers of the journal tried to guess what the cosine group was.

As a permanent participant of Arnold's seminar, I had an opportunity to give talks on my works not explicitly related to the main directions of the seminar. I gave a brief account of my work with Gelfand on the cohomology of infinite-dimensional Lie algebras, of characteristic classes of foliations. These things did not interest Dima much, although he himself had a work on

Summer School "Contemporary Mathematics" at Dubna, near Moscow, 2006.

similar things [3]. He always considered algebra and topology as something auxiliary. Once I heard him saying respectfully, "Siegel's case, this is a true analysis," and this sounded like "true mathematics". Whatever he did, his unbelievably deep understanding of analysis was always his main instrument.

One more story of a similar kind. In 1982 John Milnor, who briefly visited Moscow, delivered a talk at Arnold's seminar on a very recent (and not yet published) work of D. Bennequin on a new invariant in the theory of Legendrian knots in contact 3-manifolds. The main result of Bennequin stated that the "Bennequin number" (now justly called the "Thurston-Bennequin number") of a topologically unknotted Legendrian knot in the standard contact space must be strictly negative. For an illustration, Milnor showed an example of a Legendrian trefoil with the Bennequin number $+1$. Arnold said that at last he had seen a convincing proof that the trefoil is a topologically nontrivial knot. Certainly, this was a joke: Bennequin's proof at that time did not look convincing, and the nontriviality of the trefoil has a popular proof understandable to middle school students (via the tricolorability invariant). But for Dima only an analytic proof could be fully convincing.

When I joined the Arnold seminar, it had just acquired the name of "the seminar on singularities of smooth maps". In the mid-1960s, Arnold was fascinated by work of John Mather on singularities. People could not understand this. Allegedly, Pontryagin said: "We can always remove complicated singularities of a smooth map by a small perturbation; it is sufficient to study the generic case." But singularities appear in families of smooth maps; you cannot remove them, insisted Dima. Some people mocked his affection for singularity theory. There is a short story of Stanislav Lem (a Polish science fiction writer) in which robots that could experience human emotions were manufactured. One of these robots felt an immense joy when he solved quadratic equations—just like you, Dima!

Dima smiled at such jokes but continued studying singularities.

The results of Arnold and his students in this area were very deep and diverse. He classified all singularities that appear in generic families depending on no more than 14 parameters and studied their moduli varieties and discriminants. He discovered the relations of the theory to symplectic, contact, and differential geometry. It had deep applications in topology (Vassiliev's invariants of knots), differential equations, and classical mechanics.

More or less at the same time, a widely popularized version of the singularity theory emerged under the colorful name of the theory of catastrophes. It was promoted by two remarkable topologists, R. Thom and E. C. Zeeman. "The most catastrophic feature of the theory of catastrophes is a full absence of references to the works of H. Whitney," Dima wrote in one of his books. Indeed, mathematically, the theory of catastrophes was based on a classification of singularities of generic smooth maps of a plane onto a plane. The classification was fully done in 1955 by Whitney [15], but the founding fathers of catastrophe theory preferred to pretend that the works of Whitney never existed. Still, Dima made his contribution to the popularization of catastrophes: he wrote a short popular book under the title "*Theory of Catastrophes*". It was written in 1983 and then translated into a dozen languages.

In 1990 I moved to a different country, and we met only four or five times after that. The last time that I saw him was in spring 2007, when he visited California. We travelled together through the Napa and Sonoma Valleys; he was especially interested in visiting Jack London's grave. He spoke endlessly of his new (was it new?) passion for continued fractions, numerical functions, and numerical experimentation. I boasted that I taught a course of history of mathematics, and he immediately began testing my knowledge of the subject: Who proved the Euler theorem of polyhedra? Who proved the Stokes theorem? To his apparent displeasure I passed the exam. (He was especially surprised that I knew that Descartes proved the Euler theorem more than one hundred years before Euler. Why do you know that? I said that Efremovich told me this some thirty years before.) More than that, I knew something that he did not know: the Stokes theorem as it is stated in modern books, $\int_C d\varphi = \int_{\partial C} \varphi$, was first proved and published by the French mathematician E. Goursat (1917). We discussed a bit our further plans, and Dima said that whatever he plans, he always adds, as Leo Tolstoy did, ЕБЖ = если буду жив, "If I am alive." I said that I also never forget to add this, but apparently neither of us took it seriously. Anyhow, we never met again.

In a cavern, 2008.

My tale of Dima Arnold is becoming lengthy, although I feel that what I have said is a small fraction of what I could say about this tremendous personality. Still, the story would be incomplete if I did not mention something known to everybody who has ever communicated with him, if only occasionally: his universal knowledge of everything. Whatever the subject was—Chinese history, African geography, French literature, the sky full of stars (especially this: he could speak endlessly on every star in every constellation)—he demonstrated without effort a familiarity with the subject which exceeded and dwarfed everybody else's, and this, combined with his natural talent as a storyteller, made every meeting with him a memorable event. Some friends recollect a sight-seeing tour in Paris he gave a couple of months before his death. Obviously, no tourist agency ever had a guide of this quality. Instead of adding my own recollections, I finish my account with a translation of a letter I received from him a year after our last meeting and two years before his death.

Paris, March 26, 2008

Dear Mitya,

I have recently returned to Paris from Italy where I wandered, for three months, in karstic mountains working at ICPT (the International Center for Theoretical Physics) at Miramare, the estate of the Austrian prince Maximilian who was persuaded by Napoleon III to become the Emperor of Mexico (for which he was shot around 1867 as shown in the famous and blood-drenched picture of Edouard Manet).

I lived in the village of Sistiana, some 10 kilometers from Miramare in the direction of Venice. It was founded by the pope Sixtus, the same one who gave names to both the chapel and Madonna. Passing the POKOPALIŠCE[1] (the cemetery) some 3 versts[2] to the North, I reached a deer path in a mountain pine grove. These deer do

[1] *This word has a notable similarity to Russian КОПАТЬ, to dig.*

[2] *ВЕРСТА is an old Russian measure of length, ≈ 1.1 km.*

not pay much attention to a small tin sign, DERŽAVNAYA MEŽA[3] (the state border). After that it is Slovenia to which I ran, following the deer. But at the next sign, PERICOLO, the deer refused to go any farther. The local people (whose language is closer to Russian than Ukrainian or Bulgarian) explained to me that the sign is a warning that the nearby caves have not been demined. And they were mined during the FIRST world war when my deer path was called SENTIERA DIGUERRA and was a front line (described by Hemingway in "A Farewell to Arms").

I did not go down to these particular caves, but every day I visited tens of them, of which some (but not all) were shown on a map (where they were called YAMA,[4] GROTTA, CAVA, CAVERNA, ABISSA, dependingly of the difficulty of the descent). All these caves look pretty much the same (*a colorful scheme is provided*): there is a hole on the mountain, a meter in size, and down go walls, of not even vertical but rather a negative slope. The depth of the mine is usually around 10 or 20 meters (but I descended to YAMA FIGOVICHEVA with the officially declared depth of 24 meters and to the half of the height, or rather the depth, of GROTTA TERNOVIZZA whose depth is marked as 32 meters and to which one cannot descend without a rope). At the bottom of the YAMA a diverging labyrinth of passages starts, of the lengths on the order of 100 meters. They go to lakes, stalactites, etc. Sometimes there is even a descent to the Timavo river (which flows about 50 kilometers at the depth 100 or 200 meters, depending on the height of hills above). Before this 50 kilometers it is a forest river resembling Moscow River at Nikolina Gora[5] with a charming Roman name of REKA.[6]

[3] *Both words belong to old Russian.*

[4] *Russian ЯМА means a gap.*

[5] *A village some 30 kilometers from Moscow where many remarkable Russian people (including Dima) used to spend their vacations.*

[6] *РЕКА is the Russian for a river.*

Ya. Eliashberg and V. Arnold, 1997.

This was a part of Jason's expedition (with argonauts). On his way back from Colchis (with the golden fleece) he sailed his ship Argo upstream Ister (Danube) and its tributaries to the Croatian peninsula named Capudistria (which is visible from my window at Sistiana), then they dragged the ship to REKA and, following Timavo, they reached to northernmost point of the Adriatic, where the Roman city of Aquileia was later built.

Near Aquileia, I discovered a goddess Methe, new to me, but this is a separate story. (She saves any drinker of drunkenness, however much he drank. Allegedly, she was the mother of Athena, and Jupiter ate her, since he was afraid that she would give birth to a son, and that this son would dethrone him, precisely as he himself had dethroned his father.) Aquileia is a Roman port of the first century, preserved as well as Pompeii, without any Vesuvius: simply Attila who destroyed the city left the port intact, including the canals, ships (which survive to our time), quays, knechts and basilicas (which became Christian in the IV century) with mosaics of 50 $m \times 100$ m in size, and absolutely everything as in Pompeii. No room to describe everything, I am just sending my best (Easter) wishes.

On June 3, I go to Moscow, there will be a conference dedicated to the centenary of LSP.[7]

Dima

[7] *Lev Semenovich Pontryagin*

Yakov Eliashberg

My Encounters with Vladimir Igorevich Arnold

My formation as a mathematician was greatly influenced by Vladimir Igorevich Arnold, though I never was his student and even lived in a different city. When I entered Leningrad University in 1964 as an undergraduate math student, Arnold was already a famous mathematician. By that time he had solved Hilbert's 13th problem and had written a series of papers which made him the "A" in the KAM theory. Arnold was also working as an editor of the publishing house Mir, where he organized and edited translations of several books and collections of papers not readily accessible in the USSR. One of these books, a collection of papers on singularities of differentiable mappings, was an eye opener for me.

The first time I met Arnold was in January 1969 at a Winter Mathematical School at Tsakhkadzor in Armenia. I was eager to tell him about some of my recently proved results concerning the topology of singularities. Later that year he invited me to give a talk at his famous Moscow seminar. I remember being extremely nervous going there. I could not sleep at all in the night train from Leningrad to Moscow, and I do not remember anything about the talk itself.

In 1972 Vladimir Igorevich was one of my Ph.D. dissertation referees or, as it was called, an "official opponent". I remember that on the day of my defense, I met him at 5 a.m. at the Moscow Train Station in Leningrad. He immediately told me that one of the lemmas in my thesis was wrong. It was a local lemma about the normal form of singularities, and I thought (and, frankly, still do) that the claim is obvious. I spent the next two hours trying to convince Vladimir Igorevich, and he finally conceded that probably the claim is correct, but still insisted that I did not really have the proof. A year later he wrote a paper devoted to the proof of that lemma and sent me a preprint with a note that now my dissertation is on firm ground.

After my Ph.D. defense I was sent to work at a newly organized university in Syktyvkar, the capital of Komi Republic in the north of Russia. In 1977 we organized there a conference on global analysis which attracted a stellar list of participants, including V. I. Arnold. During this conference I asked Arnold to give a lecture for our undergraduate students. He readily agreed and gave an extremely interesting lecture about stability of the inverse pendulum, and even made a demonstration prepared with the help of one of

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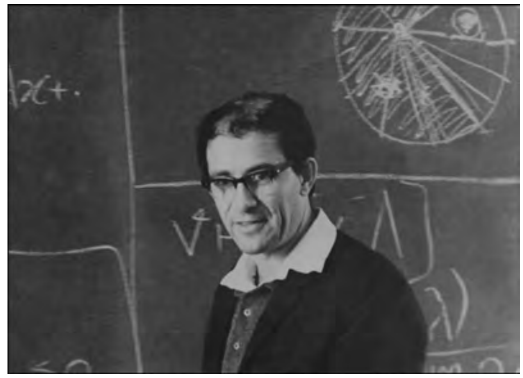


A talk at Syktyvkar, 1976.

our professors, Alesha Zhubr. Arnold had certain pedagogical methods to keep the audience awake. During his lectures he liked to make small mistakes, expecting students to notice and correct him. Apparently, this method worked quite well at the Moscow University. Following the same routine during his Syktyvkar lecture, he made an obvious computational error—something like forgetting the minus sign in the formula $(\cos x)' = -\sin x$—and expected somebody in the audience to correct him. No one did, and he had to continue with the computation, which, of course, went astray: the terms which were supposed to cancel did not. Very irritated, Arnold erased the blackboard and started the computation all over again, this time without any mistakes. After the lecture, he told me that the undergraduate students at Syktyvkar University are very bad. The next day, after my regular class, a few students came to me and asked how is it possible that such a famous mathematician is making mistakes in differentiating $\cos x$?

Whenever I happened to be in Moscow, which was not very often, Arnold usually invited me to visit the hospitable home he shared with his wife, Elya. When he moved to a new apartment in Yasenevo on the outskirts of Moscow, he told me over the phone how to get there. In particular, I was instructed to walk south when I got out of the metro station. When I got to that point it was a dark gray late winter afternoon, and it was quite a challenge to figure out in which direction I should go.

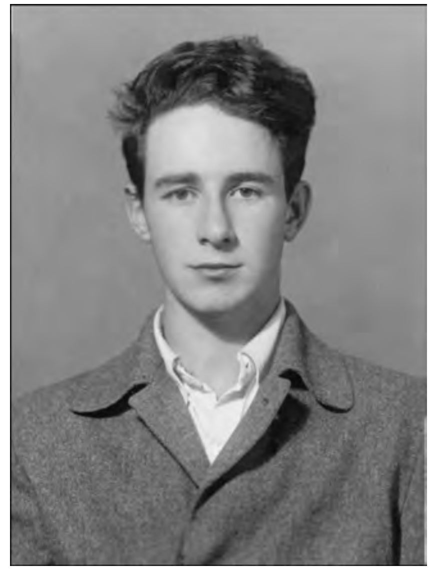
Once he ran a psychological test on me to determine which of my brain hemispheres is the dominant one. To his satisfaction, the test showed that it was the right one, which, according to Arnold, meant that I have a geometric rather than an algebraic way of thinking. During another visit, I was deeply honored when he told me that while he files most preprints systematically, I was among the few people who were assigned a personal folder.

Over the years I gave a number of talks at his seminar with variable success. The most disastrous was my last talk in 1985. Shortly before one of my trips to Moscow, Misha Gromov sent me a preliminary version of his now very famous paper "Pseudoholomorphic curves in symplectic geometry", which is one of the major foundational milestones of symplectic topology. I was extremely excited about this paper and thus volunteered to talk about it at Arnold's seminar. I think that I was at this moment the only person in the Soviet Union who had the paper. Arnold heard about Gromov's breakthrough but had not seen the paper yet. After a few minutes of my talk, Arnold interrupted me and requested that before continuing I should explain what is the main idea of the paper. This paper is full of new ideas and, in my opinion, it is quite subjective to say which one is **the** main one. I made several attempts to start from different points, but Arnold was never satisfied. Finally, towards the end of the two-hour long seminar, I said something which Arnold liked. "Why did you waste our time and did not start with this from the very beginning?", he demanded.

Vladimir Igorevich made two long visits to Stanford. During his first quarter-long visit Arnold was giving a lecture course, but he made it a rule for himself to go every morning for a long bike ride into the hills (called the Santa Cruz Mountains) surrounding Stanford. I have heard a lot of stories about Arnold's superhuman endurance and his extremely risky adventures, especially in his younger years. I can testify that at almost sixty years old, Arnold at Stanford was also very impressive. On a windy day after swimming in our cold Pacific Ocean, where the water temperature is usually around 13°C, he refused a towel. He had a very poor bike which was not especially suited for mountain biking. Yet he went with it everywhere, even over the roads whose parts were destroyed by a mudslide and where he had to climb clutching the tree roots, hauling his bike on his back. During one of these trips, Vladimir Igorevich met a mountain lion. He described this encounter in one of his short stories. Both Arnold and the lion were apparently equally impressed with the meeting. Many years later, during his second visit to Stanford, Arnold again went to the same place hoping to meet the mountain lion. Amazingly, the lion waited for him there! I am also fond of hiking in those hills, yet neither I nor any of people I know ever met a mountain lion there.

When he was leaving Stanford, Vladimir Igorevich gave me a present—a map of the local hills on which he had marked several interesting places that he had discovered, such as an abandoned apple farm or a walnut tree grove.

In between the two visits Arnold had a terrible bike accident in Paris which he barely survived. It was a great relief to see him active again when I met him in Paris two years later. He proudly told me that during this year he had written five books. "One of these books," he said, "is coauthored with two presidents. Can you guess with which ones?" I certainly could not guess that these were Vladimir Putin and George W. Bush.



Vladimir Arnold, 1957.

During his last visit to Stanford and Berkeley a year ago, Arnold gave two series of lectures: one for "Stanford professors", as he called it, and the other for the school-age children at Berkeley Math Circles. There is no telling which of these two groups of listeners Vladimir Igorevich preferred. He spent all his time preparing for his lectures for children and even wrote a book for them. Lectures at Stanford were an obvious distraction from that main activity. Each Stanford lecture he would usually start with a sentence like "What I am going to talk about now is known to most kindergarten children in Moscow, but for Stanford professors I do need to explain this." What followed was always fascinating and very interesting.

It is hard to come to terms that Vladimir Igorevich Arnold is no longer with us. It is certainly true, though commonplace to say, that Arnold was a great and extremely influential mathematician, that he created several mathematical schools, and that his vision and conjectures shaped a large part of modern mathematics. But, besides all that, he was a catalyst for the mathematical community. He hated and always fought mediocrity everywhere. With his extreme and sometimes intentionally outrageous claims, he kept everybody on guard, not allowing us to comfortably fall asleep.

His departure is also painful to me because there are several unfulfilled mathematical promises which I made to him but never had time to finish. Though it is too late, I will do it now as a priority.

Yulij Ilyashenko

V. I. Arnold, As I Have Seen Him

A student, visiting his schoolmaster in math, the famous and severe Morozkin. A radiant slim youth, almost a boy. This was Arnold as I first saw him, more than fifty years ago.

A graduate student (in 1960), conducting tutorials in honors calculus (taught to freshmen at Mekhmat, the Department of Mechanics and Mathematics of the Moscow State University). There was a permanent kind of smile on his face, his eyes were sparkling, and when he looked at you, a wave of good will would come forth.

From 1968 to 1986 I had the privilege of working with Arnold at the same section of Mekhmat, called the "Division of Differential Equations". It was shaped by Petrovski and chaired by him until his premature death in 1973. When Arnold joined the division, it was full of the best experts in differential equations, partial and ordinary. Besides Arnold and Petrovski, the faculty of the division included stars of the elder generation (who were then in their thirties and forties): Landis, Oleinik, Vishik, as well as brilliant mathematicians of Arnold's generation: Egorov, Kondratiev, Kruzhkov, and others.

The first glorious results of Arnold are described in other papers in this collection. Let me turn to differential equations, a subject whose development I have been closely following. Needless to say, these are personal remarks, not a complete history.

In 1965 Arnold came back from France, where he spent almost a year. From there he brought a keen interest in the newborn singularity theory, of which he became one of the founding fathers. He also brought the philosophy of general position invented by René Thom, which became sort of a compass in Arnold's investigations in differential equations and bifurcation theory.

In the form that Arnold gave to it, this philosophy claimed that one should first investigate objects in general position, then the simplest degenerations, together with their unfoldings. It makes no sense to study degenerations of higher codimension until those of smaller codimension have been investigated.

In 1970 he published a short paper [6], in which a strategy for developing any kind of local theory based on the above philosophy was suggested. He also defined algebraically solvable local problems. He started to call them "trivial", but later stopped doing that. "Let us forget the overloaded term," he once told me about this word. In the same paper he also stated that the problem of distinguishing center and focus is trivial. Bruno challenged this statement, and I proved that the center-focus problem is algebraically unsolvable (1972).

Also in 1970 Arnold proved that the problem of Lyapunov stability is algebraically unsolvable. He constructed a 3-parameter family in the space of high-order jets, where the boundary of stability is nonalgebraic. In the same paper he wrote: "One may expect that the Lyapunov stability, having lost algebraicity and no more restricted by anything, may present some pathologies on the set theoretic-level…." He also suggested that the problem may be algorithmically unsolvable. This conjecture is still open. In the mid-1970s it turned out that a nonalgebraic boundary of Lyapunov stability occurs in unfoldings of degenerations of codimension three in the phase spaces of dimension four. This was discovered by Shnol' and Khazin, who investigated the stability problem in the spirit of Arnold and studied all the degenerate cases up to codimension three.

In 1969 Bruno defended his famous doctoral thesis about analytic normal forms of differential equations near singular points. One of his results is the so-called Bruno condition: a sufficient condition for the germ of a map to be analytically equivalent to its linear part. In dimension one, Yoccoz proved the necessity of this condition (1987); this result was rewarded by a Fields Medal, which he got in 1994. So the problem is still a focus of interest in the mathematics community. But let us get back to the late 1960s. In his review of the Bruno thesis, Arnold wrote: "The existing proofs of the divergence [of normalizing series] are based on computations of the growth of coefficients and do not explain its nature (in the same sense as the computation of the coefficients of the series $\arctan z$ does not explain the divergence of this series for $|z| > 1$, although it proves this divergence)." Following this idea, Arnold tried to find a geometric explanation of the divergence of normalizing series when the denominators are too small. He predicted an effect which he later called "materialization of resonances". An "almost resonant" germ of a vector field that gives rise to "exceedingly small denominators" is close to a countable number of resonant germs. Under the unfolding of any such germ, an invariant manifold bifurcates from a union of coordinate planes and remains in a small neighborhood of the singular point of this almost resonant germ. These invariant manifolds, which constitute a countable number of "materialized resonances", accumulate to the singular point and prevent the linearization.

A. Pyartli, a student of Arnold, justified this heuristic description in his thesis in the early 1970s for vector fields with planar saddles. He continued the investigation and in 1976 found an invariant cylinder, a materialization of resonances for a germ of a planar map. Then he asked Arnold, "Why does

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With students of Moscow Mathematics Boarding School, 1960s.

such a cylinder prevent the linearization?" Why, indeed?! Arnold himself started thinking about the problem and came to the theory of normal forms for neighborhoods of embedded elliptic curves. An overview of this theory is given in his book [12]. As usual, this new path was paved by the followers of Arnold: Pyartli, myself, Saveliev, Sedykh, and others.

Arnold's approach to the local bifurcation theory produced a genuine revolution. In the late 1960s he suggested to his students two problems: to prove a reduction principle that excludes excessive "hyperbolic variables" from any local bifurcation problem and to study the first really difficult bifurcation problem in codimension two. The first problem was solved by A. Shoshitaishvili, the second one by R. Bogdanov. "It was not by chance that I launched two different people in two directions simultaneously," Arnold later said to me. Arnold was especially proud that Bogdanov proved the uniqueness of the limit cycle that occurs under the perturbation of a generic cuspidal singular point. F. Takens investigated independently the same codimension two bifurcation as Bogdanov; it is now named the "Bogdanov-Takens" bifurcation.

In [8] Arnold described the new approach to the theory and listed all problems that occur in the study of local bifurcations of singular points of vector fields in codimension two. This was a long-standing program. J. Guckenheimer and N. Gavrilov made important contributions to its development; final solutions were obtained by H. Zoladec (in the mid-1980s), again under the (nonofficial) supervision of Arnold.

In the mid-1970s Arnold himself considered another local bifurcation problem in codimension two, the one for periodic orbits. He discovered strong resonances in the problem and predicted all possible unfoldings occurring in generic perturbations of the Poincaré maps with these resonances (1977). There were four of them. The first case was reduced to Bogdanov-Takens; two other cases were investigated by E. Horosov (1979), a graduate student of Arnold, in his Ph.D. thesis. The fourth case, the famous resonance 1 : 4, was investigated by A. Neishtadt, F. Berezovskaya, A. Khibnik (influenced by Arnold), and B. Krauskopf, a student of Takens. The problem that remains unsolved for bifurcations of codimension two is the existence of very narrow chaotic domains in the parameter and phase spaces.

Later local bifurcations of codimension three were investigated by Dumortier, Roussarie, Sotomayor, and others. The bifurcation diagrams and the phase portraits became more and more complicated. It became clear that it is hopeless to get a complete picture in codimension four. The new part of the bifurcation theory started by Arnold and his school seems to be completed by now. What is described above is a very small part of the new domains that were opened in mathematics by Arnold.

One should not forget that Arnold also inspired many discoveries in oral communications, while no trace of this influence is left in his publications. For instance, he discovered "hidden dynamics" in various problems of singularity theory. This means that a classification problem for singularities often gives rise, in a nonevident way, to a classification problem for special local maps. Thus, he inspired the solution by S. Voronin (1982) of the local classification problem for singularities of envelopes for families of planar curves and the discovery of quite unexpected Ecalle-Voronin moduli of the analytic classification of parabolic fixed points (1981).

Arnold suggested a sketch of the proof of analytic unsolvability of the Lyapunov stability problem (Ilyashenko, 1976). Only later did I understand that, honestly speaking, it should have been a joint work.

In 1980 he pointed out that our joint work with A. Chetaev on an estimate of the Hausdorff dimension of attractors might be applied to the 2D Navier-Stokes equation. This gave rise to an explicit estimate of the Hausdorff dimension of these attractors (Ilyashenko, 1982–83), a first step in the subject later developed by O. Ladyzhenskaya and M. Vishik with his school.

This is only my personal experience, a minor part of the great panorama of Arnold's influence on contemporary mathematics. He had a very strong feeling of mathematical beauty, and his mathematics was at the same time poetry and art. From my youth, I considered Arnold as a Pushkin in mathematics. At present, Pushkin is a beloved treasure of the Russian culture, but during his life, he was not at all treated as a treasure.

V. Arnold and Yu. Ilyashenko, 1997.

The same is true for Arnold. His life in Russia before perestroika was in no way a bed of roses. I remember very well how we young admirers of Arnold expected in 1974 that he would be awarded the Fields Medal at the ICM at Vancouver. He did not receive it, and the rumor was that Pontryagin, the head of the Soviet National Mathematics Committee, at the discussion of the future awards said, “I do not know the works of such a mathematician.” For sure, it could not have been the personal attitude of Pontryagin only; it was actually the position of the Soviet government itself. Two medals instead of four were awarded that year. Much later, Arnold wrote that one of the others was intended for him, and then awarded to nobody.

In 1984 a very skillful baiting of Arnold was organized at Mekhmat. As a result, he had a serious hypertension attack. His election as a corresponding member of the Soviet Academy of Sciences stopped the baiting, but his enemies tried (though unsuccessfully) to renew it five years later.

In 1986 Arnold decided to quit Mekhmat and to move to the Steklov Institute. Yet he wanted to keep a half-time position of professor at Mekhmat. Only after considerable efforts did he get the desired half-time position. I tried to convince Arnold not to quit Mekhmat. I asked him, “Dima, who may say, following Louis XIV’s ‘L’etat s’est moi,’ Mekhmat is me?” “Well,” he answered, “I guess NN” (he named an influential party member at the department). “No, Dima, YOU are Mekhmat.” But he did not listen.

In 1994 he quit Mekhmat completely. He was offended. He taught a course and a seminar, and suddenly he was informed that this load was insufficient for the half-time position of professor, but only for a quarter-time position (a status that does not, in fact, exist). He spoke with the head of Mekhmat Human Resources. This was an aged woman who maintained her position from the communist times. “She screamed at me,” said Arnold with a sort of surprise. Then he resigned from the Moscow State University.

Needless to say, in such an environment the students of Arnold were not hired at Mekhmat. The only exceptions were N. Nekhoroshev and A. Koushnirenko, hired in the early 1970s, and much later A. Varchenko. I remember two other attempts, both unsuccessful. At the same time, the best of the best Mekhmat students asked Arnold to be their advisor. So, Mekhmat rejected the best of the best of its alumni. The same happened with students of Manin, Kirillov, Gelfand.…At the end of the 1980s, a critical mass of excellent mathematicians not involved in the official academic life had accumulated. Following a suggestion of N. N. Konstantinov, a well-known educator and organizer of mathematical olympiads, these mathematicians decided to create their own university. In 1991 a group of leading Russian mathematicians formed a council and established a new Independent University of Moscow, IUM. This group included the following members of the Russian Academy of Sciences: V. I. Arnold (chairman of the council), S. P. Novikov, Ya. G. Sinai, L. D. Faddeev; and the following professors: A. A. Beilinson, R. L. Dobrushin, B. A. Dubrovin, A. A. Kirillov, A. N. Rudakov, V. M. Tikhomirov, A. G. Khovanskii, M. A. Shubin. Professors P. Deligne and R. MacPherson of Princeton and MIT also played crucial roles in the founding of the Independent University.

Arnold was very enthusiastic about the new university, and in the first years of its existence he did a lot to shape its spirit and teaching style. Together with the first dean of the College of Mathematics of the IUM, A. Rudakov, Arnold thoroughly discussed the programs, and he himself taught a course on partial differential equations. Under his influence, the Independent University became one of the focal centers of Russian mathematical life.

In 1994 another educational institution, the Moscow Center of Continuous Mathematical Education (MCCME), was created. From the very beginning, Arnold was the head of the board of trustees of this center. The center, headed by I. Yashchenko, the director, became a very influential institution in Russian mathematical education and a powerful tool in the struggle against modern obscurantism. Arnold was one of the leaders of that struggle.

In 2005 Pierre Deligne, together with the IUM faculty, organized a contest for young Russian mathematicians. This contest was funded by Deligne from his Balzan Prize (and named after him) with the goal “to support Russian mathematics, struggling for survival.” The funds of the contest were strictly limited. In 2006 Arnold met D. Zimin, the head of “D. B. Zimin’s Charity Foundation Dynasty”, and convinced him to establish a similar “Dynasty contest”. Now the contest has become permanent, *Lord willing and the creek don’t rise,* as the proverb says. This is only one of the examples of the long-lasting influence of Arnold on Russian mathematical life.

Arnold’s talks were always special events. He began giving lectures at Mekhmat in September 1961 about the newborn theory later named KAM (Kolmogorov–Arnold–Moser). A rumor spread among the students that “Arnold has solved problems that Poincaré failed to solve.” His lectures were very fast and intense, yet they attracted the best students in the department. He repeated this course twice, in 1962–63 and in 1963–64.

After that he gave brilliant courses in theoretical mechanics, ordinary differential equations, supplementary chapters of ODE, singularity theory, geometric theory of PDE, and many others. All these courses gave rise to world-famous books, written by Arnold, sometimes with his students. In 1968 Arnold started teaching a course in ODE that became, in a sense, a course of his life. He taught it every year until the late eighties, except for sabbaticals.

Arnold completely changed the face of the discipline. His presentation was coordinate-free: all the constructions were invariant with respect to coordinate changes. “When you present material in coordinates,” he said, “you study your coordinate system, not the effect that you want to describe.” His language was quite different from that of the previous textbooks and courses: diffeomorphisms, phase flows, rectification of vector fields, exponentials of linear operators.…The language of pictures was even more important in his course than that of formulas. He always required a student to present the answer in both ways, a formula and a figure, and to explain the relation between them. He drastically renewed the problem sets for the course: propagation of rays in nonuniform media and geodesics on surfaces of revolution, phase portraits of the Newton equation with one degree of freedom, images of the unit square under linear phase flows—students were expected to draft all of these even without explicit calculations of the corresponding solutions. In the first years the course was difficult both for students and teaching assistants. Later on it smoothed out and became one of the highlights of the Mekhmat curriculum.

All his life V. I. Arnold was like a star that shines, sparkles, and produces new life around it.

V. Arnold and B. Khesin, Toronto, 1997.

Boris Khesin

On V. I. Arnold and Hydrodynamics

Back in the mid-1980s, Vladimir Igorevich once told us, his students, how different the notion of “being young” (and in particular, being a young mathematician) is in different societies. For instance, the Moscow Mathematical Society awards an annual prize to a young mathematician under thirty years of age. The Fields Medal, as is well known, recognizes outstanding young mathematicians whose age does not exceed forty in the year of the International Congress. Both of the above requirements are strictly enforced.

This can be compared with the Bourbaki group, which is comprised of young French mathematicians and which, reportedly, has an age bar of fifty. However, as Arnold elaborated the story, this limit is more flexible: upon reaching this age the Bourbaki member undergoes a “coconutization procedure”. The term is derived from a tradition of some barbaric tribe that allows its chief to carry out his duties until someone doubts his leadership abilities. Once the doubt arises, the chief is forced to climb to the top of a tall palm tree, and the whole tribe starts shaking it. If the chief is strong enough to get a good grip and survives the challenge, he is allowed to climb down and continue to lead the tribe until the next “reasonable doubt” in his leadership crosses someone’s mind. If his grip is weak and he falls down from the 20-meter-tall tree, he obviously needs to be replaced, and so the next tribe chief is chosen. This tree is usually a coconut palm, which gave the name to the coconutization procedure.

As far as the coconutization in the Bourbaki group is concerned, according to Arnold’s story, the unsuspecting member who reaches fifty is

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invited, as usual, to the next Bourbaki seminar. Somewhere in the middle of the talk, when most of the audience is already half asleep, the speaker, who is in on the game for that occasion, inserts some tedious half-a-page-long definition. It is at this very moment that the scrutinized ("coconutized") member is expected to interrupt the speaker by exclaiming something like, "But excuse me, only the empty set satisfies your definition!" If he does so, he has successfully passed the test and will remain a part of Bourbaki. If he misses this chance, nobody says a word, but he will probably not be invited to the meetings any longer.

Arnold finished this story by quoting someone's definition of youth in mathematics which he liked best: "A mathematician is young as long as he reads works other than his own!"

Soon after this "storytelling" occasion, Arnold's fiftieth anniversary was celebrated: in June 1987 his whole seminar went for a picnic in a suburb of Moscow. Among Arnold's presents were a "Return to Arnold" stamp to mark the reprints he gave to his students to work on, a mantle with a nicely decorated "swallowtail", one of low-dimensional singularities, and such. But, most importantly, he was presented with a poster containing a crossword on various notions from his many research domains. Most of the questions were rather intricate, which predictably did not prevent Arnold from easily cracking virtually everything. But one question remained unresolved: a five-letter word (in the English translation) for "A simple alternative of life". None of the ideas worked for quite some time. After a while, having made no progress on this question, Arnold pronounced sadly, "Now I myself have been coconutized...." But a second later he perked up, a bright mischievous expression on his face: "This is a PURSE!" (In addition to the pirate's alternative "Purse or Life", the crossword authors meant the term "purse" in singularity theory standing for the description of the bifurcation diagram of the real simple singularity D_4^+, also called hyperbolic umbilic—hence the hint on "simple" alternative.)

Arnold's interest in fluid dynamics can be traced back to his "younger years", whatever definition one is using for that purpose. His 1966 paper in the *Annales de l'Institut Fourier* had the effect of a bombshell. Now, over forty years later, virtually every paper related to the geometry of the hydrodynamical Euler equation or diffeomorphism groups cites Arnold's work on the starting pages. In the next four or five years Arnold laid out the foundations for the study of hydrodynamical stability and for the use of Hamiltonian methods there, described the topology of steady flows, etc.

New Haven, 1993.

Apparently Arnold's interest in hydrodynamics is rooted in Kolmogorov's turbulence study and started with the program outlined by Kolmogorov for his seminar in 1958-59. Kolmogorov conjectured stochastization in dynamical systems related to hydrodynamical PDEs as viscosity vanishes, which would imply the practical impossibility of long-term weather forecasts. Arnold's take on hydrodynamics was, however, completely different from Kolmogorov's and involved groups and topology.

The Euler equation of an ideal incompressible fluid filling a domain M in $\mathbb{R}^n$ is the evolution equation

$$\partial_t v + (v, \nabla)v = -\nabla p$$

on the fluid velocity field v, where this field is assumed to be divergence-free and tangent to the boundary of M (while the pressure p is defined uniquely modulo an additive constant by these conditions on v). In 1966 Arnold showed that this Euler equation can be regarded as the equation of the geodesic flow on the group $\mathrm{SDiff}(M)$ of volume-preserving diffeomorphisms of the domain M. The corresponding metric on this infinite-dimensional group is the right-invariant L^2 metric defined by the kinetic energy $E(v) = \frac{1}{2}\|v\|^2_{L^2(M)}$ of the fluid. (The analysis of Sobolev spaces related to this group-theoretic framework in incompressible fluid dynamics was later furnished by D. Ebin and J. Marsden.) Arnold's geometric view on hydrodynamics opened a multitude of different research directions:

Other groups and metrics. Many other evolution equations turned out to fit this universal approach suggested by Arnold, as they were found to describe geodesic flows on appropriate Lie groups with respect to one-sided invariant metrics. This shed new light on the corresponding configuration spaces and symmetries behind the relevant physical systems, and such geodesic equations are now

Vladimir Arnold with his wife, Elya, 1997.

called the Euler-Arnold equations. Here are several examples developed by many authors. The group $SO(3)$ with a left-invariant metric corresponds to the Euler top (this example appeared in the original paper by Arnold along with the hydrodynamical Euler equation). Similarly, the Kirchhoff equations for a rigid body dynamics in a fluid describe geodesics on the group $E(3) = SO(3) \ltimes \mathbb{R}^3$ of Euclidean motions of $\mathbb{R}^3$. In infinite dimensions, the group of circle diffeomorphisms $\mathrm{Diff}(S^1)$ with the right-invariant L^2-metric gives the inviscid Burgers equation, while the Virasoro group for three different metrics, L^2, H^1, and $\dot{H}^1$, produces respectively the Korteweg-de Vries, Camassa-Holm, and Hunter-Saxton equations, which are different integrable hydrodynamical approximations. The self-consistent magnetohydrodynamics describing simultaneous evolution of the fluid and magnetic field corresponds to dynamics on the semidirect product group $\mathrm{SDiff}(M) \ltimes \mathrm{SVect}(M)$ equipped with an L^2-type metric. Yet another interesting example, known as the Heisenberg chain or Landau–Lifschitz equation, corresponds to the gauge transformation group $C^\infty(S^1, SO(3))$ and H^{-1}-type metric. Teasing physicists, Arnold used to say that their gauge groups are too simple to serve as a model for hydrodynamics.

Arnold's stability and Hamiltonian methods in hydrodynamics. The geodesic property of the Euler hydrodynamical equation implied that it is Hamiltonian when considered on the dual of the Lie algebra of divergence-free vector fields. Arnold proposed using the corresponding Casimir functions, which are invariants of the flow vorticity, to study stability of steady fluid flows. Arnold's stability is now the main tool in the study of nonlinear stability of fluid motions and MHD flows. In particular, he proved that planar parallel flows with no inflection points in their velocity profiles are stable. (One should note that, for Hamiltonian systems, stability in linear approximation is always neutral and inconclusive about the stability in the corresponding nonlinear problem, so the result on a genuine Lyapunov stability of certain fluid flows was particularly rare and valuable.)

Study of fluid Lagrangian instability and curvatures of diffeomorphism groups. Negative sectional curvature on manifolds implies exponential divergence of geodesics on them. In the 1966 *Ann. Inst. Fourier* paper Arnold launched the first computations of curvatures for diffeomorphism groups. Negativity of most of such curvatures for the groups of volume diffeomorphisms suggested Lagrangian instability of the corresponding fluid flows. By applying this to the the atmospheric flows, he gave a qualitative explanation of unreliability of long-term weather forecasts (thus answering in his own way the problem posed by Kolmogorov in the 1950s). In particular, Arnold estimated that, due to exponential divergence of geodesics, in order to predict the weather two months in advance one must have initial data on the state of the Earth's atmosphere with five more digits of accuracy than that of the expected prediction. In practical terms this means that a dynamical weather forecast for such a long period is impossible.

The hydrodynamical Appendix 2 in the famous *Classical Mechanics* by Arnold,[8] where one can find the details of the above-mentioned calculation for the Earth's atmosphere, also contains one of Arnold's widely cited phrases: "We agree on a simplifying assumption that the earth has the shape of a torus," which is followed by his calculations for the group of area-preserving torus diffeomorphisms. It is remarkable that the later curvature calculations for the group of sphere diffeomorphisms (performed by A. Lukatskii) gave exactly the same order of magnitude and quantitative estimates for the curvature, and hence for the atmospheric flows, as Arnold's original computations for the torus!

Topology of steady flows. One of the most beautiful observations of Arnold (and one of the simplest —it could have belonged to Euler!) was the description of topology of stationary solutions of the 3D Euler equation. It turns out that for a "generic" steady solution the flow domain is fibered (away from a certain hypersurface) into invariant tori or annuli. The corresponding fluid motion on each torus is either periodic or quasiperiodic, while on each annulus it is periodic. This way a steady 3D flow looks like a completely

[8]*Speaking of writing, once I asked Arnold how he managed to make his books so easy to read. He replied: "To make sure that your books are read fast, you have to write them fast." His own writing speed was legendary. His book on invariants of plane curves in the AMS University Lecture series was reportedly written in less than two days. Once he pretended to complain: "I tried, but failed, to write more than 30 pages a day.…I mean to write in English; of course, in Russian, I can write much more!"*

integrable Hamiltonian system with two degrees of freedom.

The nongeneric steady flows include Beltrami fields (those collinear with their vorticity) and, in particular, the eigenfields for the *curl* operator on manifolds. The latter include the so-called ABC flows (for Arnold–Beltrami–Childress), the curl eigenfields on the 3D torus, which happen to have become a great model for various fast dynamo constructions.

Fast dynamo and magnetohydrodymanics. Arnold's interest in magnetohydrodynamics was to a large extent related to his acquaintance with Ya. Zeldovich and A. Sakharov. One of the results of their interaction at the seminars was the Arnold-Ruzmaikin-Sokolov-Zeldovich model of the fast dynamo on a 3D Riemannian manifold constructed from Arnold's cat map on a 2D torus. For a long time this was the only dynamo construction allowing complete analytical study for both zero and positive magnetic dissipation.

The asymptotic Hopf invariant. Finally, one of the gems of topological hydrodynamics is Arnold's 1974 study of the asymptotic Hopf invariant for a vector field. He proved that, for a divergence-free vector field v in a 3D simply connected manifold M, the field's helicity, $H(v) := \int_M (curl^{-1}v, v)\, d^3x$, is equal to the average linking number of all pairs of trajectories of v. This theorem simultaneously generalized the Hopf invariant from maps $S^3 \to S^2$ to arbitrary divergence-free vector fields in S^3, enriched K. Moffatt's result on the helicity of linked solid tori, described the topology behind the conservation law of the 3D Euler equation, and provided the topological obstruction to the energy relaxation of magnetic vector fields. This elegant theorem stimulated a tide of generalizations to higher-dimensional manifolds, to linking of foliations, to higher linkings, and to energy estimates via crossing numbers. In particular, there was substantial progress in the two directions suggested in the original 1974 paper: the topological invariance of the asymptotic Hopf numbers for a large class of systems was proved by J.-M. Gambaudo and É. Ghys, while the Sakharov–Zeldovich problem on whether one can make arbitrarily small the energy of the rotation field in a 3D ball by a volume-preserving diffeomorphism action was affirmatively solved by M. Freedman.

Virtually single-handedly Arnold spawned a new domain, now called topological fluid dynamics. His contribution to this area changed the whole paradigm of theoretical hydrodynamics by employing groups to study fluid flows. What doubles the awe is that this gem appeared almost at the same time with two other Arnold's foundational contributions—the KAM and singularity theories.

Vladimir Arnold lecturing.

Victor Vassiliev

Topology in Arnold's Work

Arnold worked comparatively little on topology for topology's sake. His topological studies were usually motivated by specific problems from other areas of mathematics and physics: algebraic geometry, dynamical systems, symplectic geometry, hydrodynamics, geometric and quantum optics. So the (very significant) place of topological studies in his work is well balanced with the (equally very significant) place and applications of topology in the entirety of contemporary mathematics.

The main achievement in a number of his works is a proper recognition and formulation of a topological result, allowing topologists to enter the area with their strong methods. A huge part of Arnold's work is contained not in his own articles but in well-formulated problems and hints that he gave to his students and other researchers; see especially [13]. So I will discuss below such Arnold hints as well and what followed from them.

Superpositions of Functions

The case of real functions: Kolmogorov-Arnold's theorem and Hilbert's 13th problem. This theorem states that every continuous function of $n > 2$ variables can be represented by a superposition of functions in 2 variables (and the superposition can be taken in a particular form). The first approach to this problem (based on the notion of the *Kronrod tree* of connected components of level sets) was found by Kolmogorov (1956),

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who did not, however, overcome some technical low-dimensional difficulties and proved only the same theorem with 2 replaced by 3. The final effort was made by (then-19-year-old) Arnold.

This theorem gives a negative solution to (probably the most natural exact understanding of) the following Hilbert 13th problem:

> …it is probable that the root of the equation of the seventh degree is a function of its coefficients which does not belong to this class of functions capable of nomographic construction, i.e., that it cannot be constructed by a finite number of insertions of functions of two arguments. In order to prove this, the proof would be necessary that the equation of the seventh degree
>
> $$t^7 + xt^3 + yt^2 + zt + 1 = 0 \tag{1}$$
>
> is not solvable with the help of any continuous functions of only two arguments.

A widespread belief concerning this problem is as follows: "with the help of functions" in its last sentence means that a continuous solution $t(x, y, z)$ of (1) should indeed be given by a function of the form described in the first one, i.e., by a superposition of continuous functions of two arguments. In this case the Kolmogorov-Arnold theorem would give a direct negative answer to this problem. Nevertheless, this understanding of Hilbert's question is probably erroneous, because (1) does not define any continuous function at all: *the multivalued function* $t(x, y, z)$ *defined by* (1) *does not have any continuous cross-section on the whole of* $\mathbb{R}^3_{(x,y,z)}$. Indeed, such *negative-valued* cross-sections do not already exist in a small neighborhood of the polynomial

$$t^7 - 14t^3 - 21t^2 - 7t + 1$$
$$\equiv (t+1)^3(t^4 - 3t^3 + 6t^2 - 10t + 1).$$

Such a neighborhood admits two *positive-valued* cross-sections, but they obviously cannot be continued to the polynomial $t^7 + 1$. So this direct understanding of the Hilbert problem could be correct only under the (quite improbable) conjecture that Hilbert has included in this problem the question whether (or was confident that) (1) defines a continuous function on the entire $\mathbb{R}^3$; in this case the problem would have a positive solution.

A more realistic assumption is that "with the help of continuous functions of two variables" means something more flexible, for example, that we can consider a triple of functions (χ, g_1, g_2) in x, y, z, defined by such superpositions, and represent our function $t(x, y, z)$ by g_1 in the area where $\chi > 0$ and by g_2 where $\chi \le 0$. However, in this case it is unclear why Hilbert did not believe that the desired representation (maybe with more functions χ_k and g_i) does exist for his particular function, which is piecewise analytic and certainly can be stratified by easy conditions into pieces with very simple behavior. The most realistic conjecture is that (like for many other problems) Hilbert wrote a slightly obscure sentence specifically to let the readers themselves formulate (and solve) the most interesting and actual exact statements: it is exactly what Kolmogorov and Arnold actually did.

Complex algebraic functions and braid cohomology. Hilbert's 13th problem, formally asking something about real continuous functions, is nevertheless evidently motivated by the study of superpositions of multivalued algebraic functions in complex variables. A dream problem in this area is to solve literally the same problem concerning such functions. Moreover, this problem was explicitly formulated in one of Hilbert's consequent works.

Arnold worked much on this problem, revising and reformulating the proof of the Ruffini-Abel theorem in topological terms of ramified coverings and their topological invariants and trying to extend it to superpositions of functions in more variables. Although the exact desired theorem was not proved, a byproduct of this attack was huge: among other topics, it contains the topological theory of generalized discriminants, homological theory of braid groups, and theory of plane arrangements. A particular result, the topological obstruction to the representation by *complete* superpositions of functions depending on few variables, was expressed in [5] in the terms of cohomology of braid groups. Indeed, the d-valued algebraic function $t(x_1, \dots, x_d)$ given by

$$t^d + x_1 t^{d-1} + \cdots + x_{d-1} t + x_d = 0 \tag{2}$$

defines a d-fold covering over the set $\mathbb{C}^d \setminus \Sigma$ of *nondiscriminant* points $(x_1, \dots, x_d)$ (i.e., of polynomials (2) for which all d values $t(x)$ are different). This covering defines (up to homotopy) a map from its base $\mathbb{C}^d \setminus \Sigma$ to the classifying space $K(S(d), 1)$ of all d-fold coverings, thus also a canonical map

$$H^*(K(S(d), 1) \to H^*(\mathbb{C}^d \setminus \Sigma). \tag{3}$$

If our algebraic function (2) is induced from another one, as in the definition of complete superpositions, then this cohomology map factorizes through the cohomology ring of some subset of the argument space of this new algebraic function. Hence the dimension of this space cannot be smaller than the highest dimension in which the map (3) is nontrivial.

This approach has strongly motivated the study of the cohomology ring of the space $\mathbb{C}^d \setminus \Sigma$ (which is the *classifying space of the d-braid group*) and, much more generally, of the following objects.

Discriminants and Their Complements

Given a space of geometric objects (say, functions, varieties, subvarieties, matrices, algebras, etc.), the *discriminant subset* in it consists of all degenerate (in some precise sense) objects: it may be the set of non-Morse functions or self-intersecting spatial curves, or degenerate (another version: having multiple eigenvalues) operators. Usually one studies the complementary space of nonsingular objects. However, Arnold's seminal reduction replaces the homological part of this study by that of discriminant spaces. Namely, in [4], Arnold exploits the Alexander isomorphism

$$H^i(\mathbb{C}^d \setminus \Sigma) \equiv \bar{H}_{2d-i-1}(\Sigma), \tag{4}$$

where $\bar{H}_*$ means the homology of the one-point compactification and $\mathbb{C}^d$ is considered to be the space of all complex polynomials (2) in one variable t. This reduction turned out to be extremely fruitful, because the set of nonsingular objects is usually open and does not carry any natural geometric structure. To study its topology, we often need to introduce some artificial structures on it, such as Morse functions, connections, families of vector fields or plane distributions, etc., which can have singularities helping us to calculate some topological invariants. On the other hand, the discriminant varieties are genuinely stratified sets (whose stratification corresponds to the hierarchy of singularity types); this stratification allows one to calculate various topological properties of these varieties and hence also of their complementary sets of generic objects. Already in [4] this approach has brought some progress, although the complete calculation of the group (4) was done only later by D. Fuchs for $\mathbb{Z}_2$-cohomology [14] and by F. Cohen and F. Vainshtein for integral cohomology.

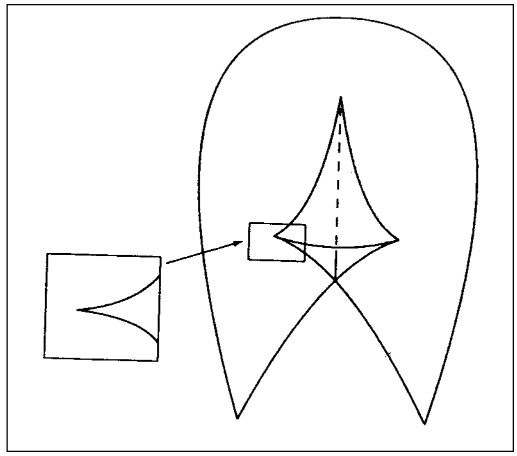
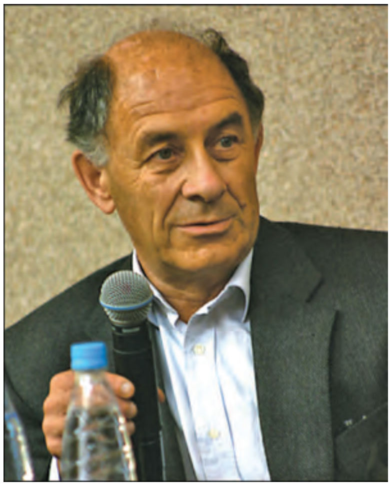


Figure 1. Stabilization of unfoldings.

Using the same approach, Arnold studied later many other spaces of nondegenerate objects, namely, spaces $P_d \setminus \Sigma_k$ of real degree d polynomials $\mathbb{R}^1 \to \mathbb{R}^1$ without roots of multiplicity $\geq k$, $k \geq 3$, spaces of functions $\mathbb{R}^1 \to \mathbb{R}^1$ (with a fixed behavior at infinity) also having no zeros of multiplicity $\geq k$ (1989), spaces of Hermitian operators with simple spectra (1995), spaces of generic (or generic Legendrian) plane curves (1994), etc.

Another very important idea of Arnold's in this area was his favorite *stabilization problem*, published first in 1976 and repeated many times in seminars; see problems 1975-19, 1980-15, 1985-7, 1985-22 in [13]. Formally speaking, the Alexander duality theorem is a finite-dimensional result. Also, all spaces of objects in which Arnold's approach originally led to more or less explicit results were finite-dimensional spaces considered as unfoldings of some particular objects. For example, the space $\mathbb{C}^d$ of complex polynomials (2) can be considered as an unfolding of the monomial t^d. When the degree d grows, the cohomology groups of spaces $\mathbb{C}^d \setminus \Sigma$ of nondiscriminant polynomials stabilize (to the cohomology of the infinite braid group), but it was quite difficult to trace the stabilization process in terms of the original calculations. Moreover, it was unclear what happens with similar stabilizations for objects more complex than just polynomials in one variable, how to deal with similar infinite-dimensional problems, and what is "the mother of all unfoldings". To attack this set of philosophical problems, Arnold formulated a very explicit sample problem. First, he noticed that the stabilization of cohomology groups such as (2) is natural: if we have two singular objects, one of which is "more singular" than the other, then the parameter space of the unfolding of the simpler object can be embedded into that of the more complicated one. This map sends one discriminant into the other, thus inducing the pull-back



Vladimir Igorevich Arnold.

map of cohomology groups of their complements. (For real polynomials t^3 and t^4 this embedding of parameter spaces of their unfoldings $t^3 + at + b$ and $t^4 + \alpha t^2 + \beta t + \gamma$ is shown in Figure 1. The discriminants drawn in this picture are the sets of polynomials having multiple roots.)

Arnold's respective problem was to determine the stable (under all such pull-back maps) cohomology groups of such complements of discriminants of isolated singularities of holomorphic functions in $\mathbb{C}^n$ (and to prove that they actually do stabilize; i.e., these stable cohomology groups are realized by such groups for some sufficiently complicated singularities). Solving this problem, I found in 1985 a method of calculating homology groups of discriminants that behaves nicely under the embeddings of unfoldings and thus gives an effective calculation of stable groups. Some elaborations and byproducts of this calculation method constitute a majority of my results on topology of discriminants, including my first works on knot theory. In the original problem on stable cohomology of complements of discriminants of holomorphic functions, this calculation gives us the following formula: the desired stable cohomology ring for singularities in n complex variables is equal to $H^*(\Omega^{2n}S^{2n+1})$, where Ω^k is the k-fold loop space.

Moreover, this Arnold problem not only dealt with the stabilization of particular finite-dimensional objects, but it also gave an approach to the study of actual infinite-dimensional function spaces.

Topology of Pure Braid Groups and Plane Arrangements

Together with the cohomology of the usual braid groups (2), Arnold also investigated the pure braid group, i.e., the fundamental group of the set of ordered collections of d distinct points in $\mathbb{C}^1$. The classifying space of this group is just the space $\mathbb{C}^d$ with all diagonal hyperplanes $\{x_i = x_j \text{ for } i \neq j\}$ removed. Arnold's calculation of its cohomology group [2] became a sample and a starting point of numerous generalizations and initiated the so-called theory of plane arrangements. The Arnold identity

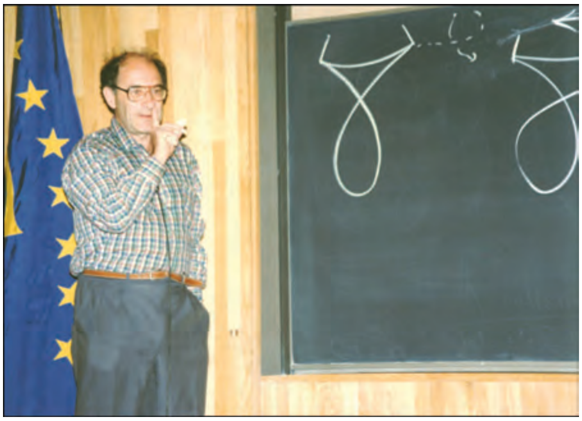
$$\omega_{ij} \wedge \omega_{jk} + \omega_{jk} \wedge \omega_{ki} + \omega_{ki} \wedge \omega_{ij} = 0$$

for basic classes of this cohomology ring later became one of the main ingredients of Kontsevich's construction of the universal finite-type knot invariant.

Maslov Index, Lagrange and Legendre Cobordism

Lagrange manifolds are specific n-dimensional submanifolds of the symplectic space $\mathbb{R}^{2n}$ (or, more generally, of the cotangent bundle of an arbitrary manifold M^n). They occur in problems of geometric optics as the manifolds into which all rays of light considered in such a problem can be lifted without intersections, and in quantum optics as a first step in obtaining an asymptotic approximation of light diffusion. However, further steps of this asymptotic description impose some consistency condition: the composition of transition functions relating their expressions in neighboring local charts should define the identity operator when we go along a closed chain of such charts. This condition is best formulated in terms of a certain 1-cohomology class of the Lagrange manifold, its *Maslov index*. If the Lagrange manifold $L^n \subset T^*\mathbb{R}^n$ is generic, then this index can be defined as the intersection index with the singular locus of the projection $L^n \to \mathbb{R}^n$ to the "physical" configuration space. It is important for this definition that, for generic Lagrangian manifolds, this locus has a well-defined transversal orientation (so that crossing it, we can always say whether we are going to the positive or the negative side) and its singular points form a subset of dimension at most $n-3$ in L^n (so that all homologous curves have one and the same Maslov index). If L^n is orientable, then this index is even; the above self-consistency condition requires that the value of this index on any closed curve should be a multiple of 4. Arnold [1] related this index with the topology of the Lagrange Grassmann manifold of all Lagrangian planes in the symplectic $\mathbb{R}^{2n}$-space, i.e., of all planes that can be tangent to some Lagrange submanifolds in this space. This settles immediately various problems related to the invariance of the definition of the Maslov index, as well as to its stability under deformations of the Lagrange manifold.

In 1980 Arnold initiated the theory of *Lagrange and Legendre cobordisms* [11]. Light distribution in the area defines light distribution on its border: for instance, the reflected light on the wall is defined by the light in the entire room. This means that a Lagrange manifold in the cotangent bundle



The Bowen lectures, Berkeley, 1997.

of the room defines its Lagrange boundary, which is a Lagrange manifold in the cotangent bundle of the wall. The *Legendre manifolds* are known to us mainly as resolutions of wave fronts. The wave front evolving in space defines a wave front of bigger dimension in the space-time. The fronts in M^n corresponding to some instants T_1 and T_2 are obviously defined by the big front in $M^n \times [T_1, T_2]$; the way in which they are obtained from this big front can be generalized to the notion of the Legendre boundary. Notice that both Lagrange and Legendre boundaries of manifolds are not their boundaries and not even the subsets in the usual sense: they are obtained from these boundaries by symplectic and contact reductions.

Arnold introduced cobordism theories based on these boundary notions and calculated the 1-dimensional Lagrange and Legendre cobordism groups: they turned out to be isomorphic to $\mathbb{Z} \oplus \mathbb{R}$ and $\mathbb{Z}$, respectively. The $\mathbb{Z}$-term in both answers is defined by the Maslov index, the $\mathbb{R}$-invariant of the Lagrange cobordism is given by $\int pdq$. Later, Ya. Eliashberg and M. Audin, using the Gromov-Lees version of the Smale-Hirsch h-principle for Lagrange manifolds, reduced the calculation of Legendre cobordism groups in any dimension to the standard objects of the cobordism theory, namely, to homotopy groups of appropriate Thom spaces (over the stable Lagrange Grassmann manifold).

At the same time, in the beginning of 1980, Arnold asked me whether it is possible to extend the construction of the Maslov index to cohomology classes of higher dimensions, dual to more degenerate singular loci of the Lagrangian projection $L^n \to \mathbb{R}^n$ than just the entire singular set. The resulting cohomology classes were expected to be closely related to the higher cohomology classes of Lagrange Grassmannians and to give invariants of Lagrange and Legendre cobordisms. The answer was found soon: I managed to construct the desired characteristic classes in terms of the universal complex of singularity types. Later, this theory was nicely and strongly extended by M. Kazarian in terms of equivariant homology.

On the other hand, the work with 1-dimensional wave fronts led Arnold to many essential problems of contact geometry, such as the 4-cusps problem (see the photograph above). Solutions of these problems by Chekanov, Eliashberg, Pushkar′and others resulted in significant development of this area.

There are many other topological results in Arnold's works, including major breakthroughs in real algebraic geometry [7], [10]; Arnold's conjecture in symplectic topology; the asymptotic Hopf invariant; and the vanishing homology theory of boundary singularities. These topics are covered in other articles in this collection.

Helmut Hofer

Arnold and Symplectic Geometry

V. I. Arnold was a character and a larger-than-life figure. I never knew him extremely well, but we became closer over the years, and I learned to know him a little bit more from the private side. He could be very charming.

As a student I read Arnold's wonderful book *Mathematical Methods of Classical Mechanics* and was impressed by the ease with which he was able to bring across important ideas. I never expected to meet him in real life.

I met him for the first time when I was a tenure-track professor at Rutgers University and was visiting the Courant Institute. This was between 1986 and 1987, so around three years before the Berlin Wall and the iron curtain came down. The Courant Institute had worked hard to make it possible for Arnold to visit. I attended one of Arnold's lectures, which was remarkable in two ways: there was great mathematics and something one would not expect in a mathematics lecture. At some point he went into a tirade about how Western mathematicians were not giving proper credit to Russian mathematicians. Most people in the audience took it with some kind of amusement, but not all. Somebody sitting beside me mumbled something along the lines that we should have left him in Moscow.

A year or so later he attended parts of the symplectic year (1988) at MSRI in Berkeley. What I remember from his visit was that at some point he decided to swim in San Francisco Bay. One has to know that the locals do not consider this the best idea, since the currents are quite unpredictable. The story which was told at that time was that he almost drowned fighting the

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currents. I thought to myself, "That is a really interesting multidimensional character pushing the envelope." I recently asked about this of Richard Montgomery, who had an account of this story from Arnold himself. He had concluded from the description that Arnold had tried to swim from Marina Green to Marin (linked by the Golden Gate Bridge) during ebb tide and at some point, in his own words, "It felt like I hit a wall of current" and "had to turn back." The maximum ebb out of the San Francisco Bay can be over six knots. If he hadn't returned, he would have been swept at least a mile out to sea. Talking to Richard I also learned about another story. He and Arnold went kayaking in the bay. After an involuntary Eskimo roll, Arnold insisted on entering orthogonally into the path of an ongoing yacht race, with 40-foot yachts going full speed being unable to dodge a kayak. Richard still remembers his fear of going down in mathematical infamy as the guy who killed Arnold. As I said before, Arnold pushed the envelope in real life as he did in mathematics.

One year later, in 1989, I became a full professor at the Ruhr-Universität Bochum. Shortly afterwards the Berlin Wall came down, with dramatic changes in Eastern Europe. Soon a complete brain drain of the Soviet Union became a concern, and one day I found myself, together with my colleagues A. Huckleberry and V. Arnold, presiding over some research funding to allow Russian mathematicians to spend longer periods with a decent pay at Bochum. Arnold was very concerned, and I got to know him somewhat better. Professor Arnold became Dima.

Around 1994 I met him again; this time in Stanford. Dima, Yasha (Eliashberg), and I went looking for walnuts at the San Andreas Fault. I am sure it was Dima's idea. Knowing the "almost drowning version" of Dima's swimming expedition in San Francisco Bay, I had quite high expectations for the afternoon. However, there was no earthquake.

Around this point we started talking about mathematics, specifically symplectic topology. His opening bid was, "Helmut, you are using the wrong methods," referring to pseudoholomorphic curves, and I responded with, "I am sure you know something better. Make my day!" He liked to probe and enjoyed seeing people's reactions. I think I did well that day.

In 1998 he introduced my plenary lecture at the ICM in Berlin, and we had a friendly chat before the talk. The year before I had moved to the Courant Institute. He said, "Helmut, you should come back to Europe." I answered, "No, Dima, I love New York. But if it makes you feel better, consider me the agent of European culture in the U.S." I saw immediately that he liked this sentence. We talked about some more things which I rather thought would stay between us. Of course, I should have known better! He made it all part of his introduction and started by introducing me as the agent of European culture in the U.S., to the delight of many, but that was only the beginning; the rest is on video.

Dima had an amazing mathematical intuition and (which at this point shouldn't come as a surprise) was daring enough to make conjectures when others would not dare to stick their necks out.

There are quite a number of Arnold conjectures in symplectic geometry. However, there is one which even people outside of the field know and which was the initial driving force behind the development of symplectic geometry.

Arnold and Weinstein developed the modern language of symplectic geometry. This could, for example, be used to prove interesting perturbation results. However, there were no global results. Arnold was the one who raised these types of questions, and the Arnold conjecture I describe below is an example. Surprisingly, the breakthrough due to Conley and Zehnder came from outside the field.

In the following, I try to motivate the Arnold Conjecture. One can understand it as an analogy of the relationships between the Euler characteristic, the Hopf index formula, and the Lefschetz fixed point theorem. I haven't seen anything in his writings pointing out this analogy, but added it here as an intermediate step which helps to understand the conjecture better. Arnold describes a way of reasoning in Appendix 9 of his previously mentioned book. The Poincaré twist theorem can be seen as a special case of the two-dimensional torus case of his conjecture. The general case would be the generalization of the torus case to arbitrary closed symplectic manifolds. There is quite often a difference between the original thought process and the didactical cleaned version. From that point of view I regret that I never asked him how he arrived at his conjecture. The discussion below adds another point of view, constructing an analogy to a reasoning in topology. I very much believe that Arnold was aware of this analogy.

We start with a closed oriented manifold M and a vector field X. The Euler characteristic $\chi(M)$ is a classical topological invariant, which is a generalization of the original concept introduced for polyhedra by Euler and which was fully generalized later by Poincaré. If M is a smooth manifold, Hopf's index formula establishes a relationship between the zeros of a vector field assumed to be transversal to the zero section and the Euler characteristic of M:

$$\chi(M) = \sum_m i(X, m),$$

where $i(X, m) = \pm 1$ is the local index at a zero of X.

In Yosemite, California, 1989.

How can we generalize this? First we observe that a diffeomorphism can be viewed as a generalization of a vector field. Indeed, the collection of smooth vector fields can be viewed as the Lie algebra of the (Fréchet)-Lie group $\mathrm{Diff}(M)$, so as an infinitesimal version of the latter. It is, however, not true that the diffeomorphisms close to the identity are in the image of the group exponential map. This is a consequence of being only a Fréchet Lie group and a universal problem in dealing with various sorts of diffeomorphism groups. Let us make a conjecture, which will come out as first going from the infinitesimal to the local to gain some confidence. We fix as an auxiliary structure a Riemannian metric with associated Riemannian exponential map exp. Assume that Φ is a diffeomorphism which is close to the identity. Then we can write Φ in a unique way in the form

$$\Phi(m) = \exp_m(X(m)),$$

for a small vector field X. Tranversality of X to the zero-section is equivalent to Φ not having 1 in the spectrum of its linearizations at fixed points. Most importantly, the fixed points for Φ correspond to the zeros for X. Hence a generic diffeomorphism which is close to the identity has an algebraic fixed point count $\chi(M)$, where the sign is taken according to $\Phi'(m)$ being orientation preserving or not. We can now make the "daring conjecture" that this should hold for all generic diffeomorphisms isotopic to the identity. That turns out to be correct and is, of course, a special case of the Lefschetz fixed point formula.

What Arnold did in symplectic geometry is such a daring conjecture in a more complicated context. We start with a closed symplectic manifold (M, ω), and in analogy to the previous discussion we generalize the theory of functions on M rather than the theory of vector fields. If f is a smooth function with all critical points nondegenerate, then Morse theory says its number of critical points is at least the sum of the Betti numbers (for any coefficient field). Morse theory also tells us that the algebraic count of critical points is $\chi(M)$. Since we have a symplectic structure, we can associate to f a vector field X_f by the obvious formula

$$df = i_{X_f}\omega.$$

This is the so-called Hamiltonian vector field. Obviously we are now back to the first discussion. However, with the vector fields being more special, one would like a stronger statement for a certain class of diffeomorphisms. This particular class of diffeomorphisms should generalize functions as diffeomorphisms isotopic to the identity generalize vector fields.

Symplectic diffeomorphisms isotopic to the identity are not a good guess, since for T^2 with the standard symplectic form a small translation would give no fixed points at all. We could, however, look at all symplectic diffeomorphisms obtained as time-1 maps for the family of vector fields X_{f_t} for a smooth time-dependent family $f : [0,1] \times M \to \mathbb{R}$, with $f_t(x) := f(t,x)$. This produces the group of all Hamiltonian diffeomorphisms $\mathrm{Ham}(M, \omega)$. Indeed the collection of smooth maps can be viewed as the Lie algebra for $\mathrm{Ham}(M, \omega)$.

How can we go from the infinitesimal to the local, as we did in the previous discussion? A basic and not too difficult symplectic result is that the neighborhood of a Lagrangian submanifold of a symplectic manifold is symplectically isomorphic to a neighborhood of the zero-section in its cotangent bundle with the natural symplectic structure. Now comes a little trick which replaces the use of the exponential map associated to an auxiliary metric. We define $N = M \times M$ with the form $\tau = \omega \oplus (-\omega)$. Then the diagonal Δ_M is a Lagrange submanifold of N, and an open neighborhood of it looks like an open neighborhood of Δ_M in $T^*\Delta_M$. Every symplectic map that is sufficiently close to the identity has a graph which when viewed as a subset of $T^*\Delta_M$ is a graph over the zero-section, i.e., the graph of a one-form λ. An easy computation shows that the original diffeomorphism is symplectic if and only if λ is closed. It is Hamiltonian if and only if λ is exact:

$$\lambda = dg$$

for some smooth function. Hence the fixed points of a Hamiltonian diffeomorphism Φ correspond to the intersection of its graph with the zero-section and hence with the critical points of g. Now we are in the local situation, similarly as in the previous case. We conclude that a generic element in $\mathrm{Ham}(M, \omega)$ has at least as many fixed points as a smooth function has critical points if it is close enough to the identity map.

Knowing all this, Arnold makes the following daring conjecture (nondegenerate case, in my words).

Arnold Conjecture: A nondegenerate Hamiltonian diffeomorphism has at least as many fixed points as a Morse function has critical points.

It wouldn't be Dima if it actually was that straightforward. The most prominent statement "Arnold-style" of this conjecture is in his book *Mathematical Methods of Classical Mechanics*. In the Springer 1978 edition (being a translation of the 1974 Russian edition) it reads on page 419 (and this is a restatement of some published version of the conjecture in 1965):

> Thus we come to the following generalization of Poincaré's theorem:
>
> **Theorem:** Every symplectic diffeomorphism of a compact symplectic manifold, homologous to the identity, has at least as many fixed points as a smooth function on this manifold has critical points (at least if this diffeomorphism is not too far from the identity).

The symplectic community has been trying since 1965 to remove the parenthetical part of the statement. After tough times from 1965 to 1982, an enormously fruitful period started with the Conley-Zehnder theorem in 1982–83, proving the Arnold conjecture for the standard torus in any (even) dimension using Conley's index theory (a powerful version of variational methods). This was followed by Gromov's pseudoholomorphic curve theory coming from a quite different direction. At this point the highly flexible symplectic language becomes a real asset in the field. Finally, Floer combines the Conley-Zehnder viewpoint with that of Gromov, which is the starting point of Floer theory in 1987. As far as the Arnold conjecture is concerned, we understand so far a homological version of the nondegenerate case. A Luisternik-Shnirelman case (also conjectured by Arnold) is still wide open, though some partial results are known.

The development of symplectic geometry has been and still is a wonderful journey. Thanks, Dima!

Acknowledgments

The photographs are courtesy of the Arnold family archive, F. Aicardi, Ya. Eliashberg, E. Ferrand, B. and M. Khesin, J. Pöschel, M. Ratner, S. Tretyakova, and I. Zakharevich.

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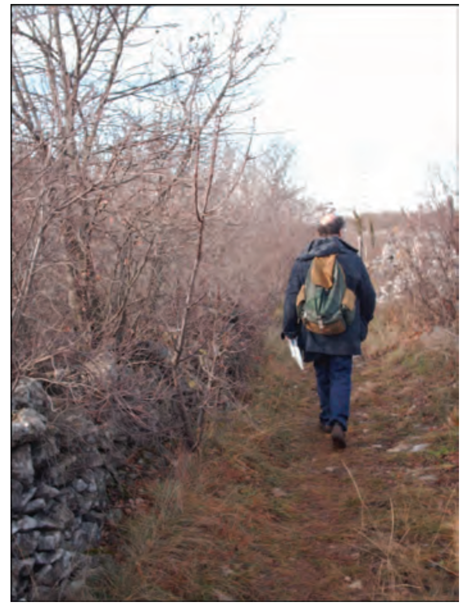

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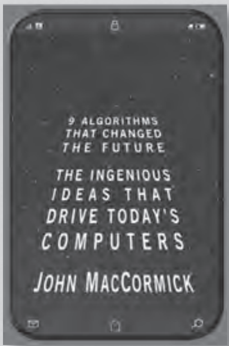

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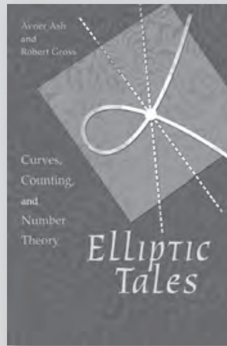

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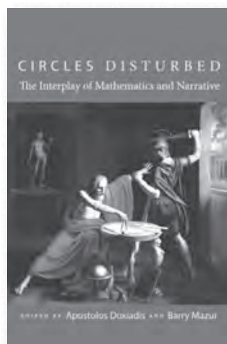

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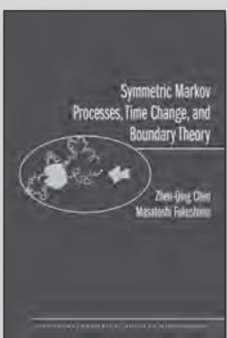

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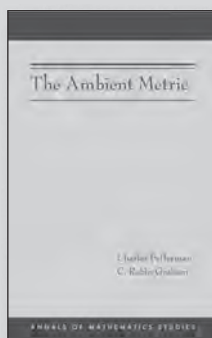

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Winifred Edgerton Merrill: "She Opened the Door"*

Susan E. Kelly and Sarah A. Rozner

Winifred Edgerton Merrill was the first American woman to receive a Ph.D. in mathematics. She received her degree from Columbia University in 1886 for her thesis, *Multiple Integrals (1) Their Geometrical Interpretation in Cartesian Geometry, in Trilinears and Triplanars, in Tangentials, in Quaternions, and in Modern Geometry; (2) Their Analytical Interpretations in the Theory of Equations, Using Determinants, Invariants and Covariants as Instruments in the Investigation.* This thesis presents geometrical representations of infinitesimals in several coordinate systems and uses theory involving the Jacobian to derive transformations between multiple integrals in various systems. After receiving her Ph.D. from Columbia, she continued to help women advance in a male-dominated society. She helped found Barnard College, a women's college affiliated with Columbia University, and she founded a girl's college preparatory school.[1]

Pre-Columbia

Winifred Haring Edgerton was born in Ripon, Wisconsin, on September 24, 1862, to parents Emmet and Clara (Cooper) Edgerton [38]. The 1860 Federal Census from Fond du Lac County, Wisconsin, lists the Edgerton family farming in Wisconsin with Winifred's maternal grandparents. It also lists a boy Eber Edgerton who was eight years old [39]. Eber was not listed on later census data from the family. By the 1870 census, the family is listed as living in New York City where her father had become a real estate broker [40, 41]. Family friends included James Russell Lowell, Oliver Wendell Holmes, and Thomas Bailey Aldrich, well-known writers of the time [21]. In Winifred's son's journal, he states that Winifred was educated by private tutors. The 1870 and 1880 census lists a woman living with the family who may have been such a tutor. An enjoyment of mathematics and astronomy blossomed, and her parents had an observatory built for her in one of their New Jersey homes [21, 40, 41].

Figure 1. Winifred Edgerton Merrill.

At age sixteen, Winifred decided to pursue a degree at Wellesley College, which had opened in 1875 as one of the first American colleges for women. In 1883 she graduated with honors and began teaching at Mrs. Sylvanus Reed's Boarding and Day School for Young Ladies in New York City [16, 38].

In 1883 Winifred also did independent calculations of the orbit of the Pons–Brooks comet based on data provided by the Harvard College observatory [21].[2] Winifred's interest in astronomy and her calculation on the Pons–Brooks comet soon

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* *This quote is from the legend to Columbia University's portrait of Merrill* [6].

[1] *To avoid confusion with last name changes and other family members, we will refer to Winifred Edgerton Merrill with her first name throughout most of the paper.*

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[2] *The Pons–Brooks comet was first discovered by Jean Louis Pons in 1812 and then again by accident in 1883 by*

led her to seek access to the telescope at Columbia University [8].

Columbia University

The history of education in the United States can provide a useful context for appreciating Winifred's pioneering role. Harvard was established as the first American college in 1636, and, by the turn of the nineteenth century, twenty-two colleges for men had been established in the United States. By 1830 fifty-six colleges existed in the country. In 1836 Mount Holyoke Female Seminary began to offer women a curriculum similar to that at men's institutions. Oberlin College became the first coeducational school in 1837. By the 1880s several state universities had provisions for co-education; a number of women's colleges opened, and in 1879 Harvard Annex (later named Radcliffe) opened for women [16]. The first American Ph.D. degrees were granted by Yale in 1861 [18]. In 1877 Boston University awarded the first American Ph.D. to a woman: Helen Magill earned a doctoral degree in Greek [15].

Columbia University was originally founded in 1754 as an all-male school called King's College. In 1876 the Sorosis, New York City's leading women's club, petitioned Columbia to admit women, and in 1879 Columbia's president, Frederick Barnard, a mathematician, began to push the board for coeducation at Columbia. Resistance came from board members who felt that women would be too frail and inferior to handle the rigorous academics. Some feared that the male students would be distracted or that, because of New York City's high population and parents' desire to keep daughters close to home, Columbia could be taken over by women. The board rejected coeducation at Columbia, but in 1883 they struck a compromise: Women would be barred from attending courses, but they would be given detailed syllabi and they would take exams. If the necessary exams were passed, the women would be awarded the appropriate degrees [12, 27].

At this time, women who did earn bachelor's degrees typically began teaching. After the Civil War, there was a demand for teachers and, financially, it was beneficial to hire women since they were only paid about one-third the salary of men [38].

It was in this setting that Winifred began her quest to continue her education at Columbia. Melvil Dewey, the former librarian at Wellesley and Columbia's librarian-in-chief, introduced her to President Barnard [12]. On January 5, 1884, and with Barnard's backing, she petitioned Columbia University for use of the school's telescope [8, 12]. Initially, her petition was rejected, but she was encouraged by President Barnard to personally meet with each member of the Board of Trustees. Reverend Morgan Dix, Dean of Students and a member of the Board, was a vocal opponent to women in higher education. However, Winifred was able to win him over, and he eventually became a friend and strong advocate for her [12, 21]. On February 4, 1884, the members of the Board of Trustees gave Winifred permission to use the school's telescope, and she was assigned to study astronomy and mathematics under Professor John Rees. The members of the board considered her to be an "exceptional case". The Board of Trustees decided that, since she wasn't specifically seeking admission to Columbia, and as there wasn't another facility in New York City where she could access a telescope, they would allow her to use the instruments at Columbia. Additionally, they considered the work she had done calculating the orbit of the Pons–Brooks comet and deemed her to be proficient in the subject matter. As part of her conditions to study, she was told "not to disturb the male students". She also was required to serve as a laboratory assistant to the director of the observatory [10]. This meant, in part, that she was to clean and care for the instruments in the laboratory [38].

Winifred's struggle to receive permission to use the telescope at Columbia was not her last hurdle. For, although she was not the first woman to attend classes at Columbia, it was then frowned upon for women to attend lectures. As a result, most women studied alone from the course text. In one of Winifred's courses, some men in the class asked the professor to choose one of the hardest textbooks of the time with hopes that she would fail since she would not be able to attend the lectures. This attempt backfired since she had already studied that text at Wellesley [38].

Another challenge she faced during her time at Columbia was the amount of time she spent in solitude. She was not supposed to interact much with the male students, she put in late hours of private study using the telescope, and she spent many hours cleaning and caring for instruments. However, like the other challenges she had faced, she found a way to overcome the loneliness. In her 1944 interview, she talked of her time alone in the observatory: "I had a great chest and in that chest I had twenty dolls. I was up there a great deal. I used to take them out... If I heard anyone coming I would sweep them into the chest" [17]. She later told her son that she brought the dolls out to keep her company [21]. When asked in the same

William Brooks. Based on the 1812 appearance, German astronomer Johann Encke worked to calculate the comet's orbit and period, which was found to be about seventy years [30].

interview if her parents supported her work, she answered, “In everything. Anything I wanted to do was all right. They apparently had confidence in me; I don’t see how they did. I was up there until 2 o’clock alone in that laboratory at night. Today, if it were my daughter, I would be on pins and needles” [17].

On June 7, 1886, just two-and-a-half years after Winifred entered Columbia, she completed her thesis. At the Board of Trustees meeting that day the following motion was made and passed unanimously: “That in consideration of the extraordinary excellence of the scientific work done by Miss Winifred Edgerton, as attested by the Professors who have had the superintendence of her course in practical Astronomy, and the Pure Mathematics in the Graduate Department, the degree of Doctor of Philosophy can be conferred upon Miss Edgerton cum laude” [8, 11].[3] Winifred Edgerton thus became the first American woman to receive her Ph.D. in mathematics and the first woman to graduate from Columbia University [12, 38]. The New York times reported the event: “... Nothing unusual occurred until Miss Winifred Edgerton, A.B., Wellesley College, came to the stage to accept her degree of Doctor of Philosophy, *cum laude*. She was greeted with a terrific round of applause which the gallant students in the body of the house kept up for fully two minutes. She was modestly dressed in a walking dress of dark brown stuff, trimmed with velvet of the same material, and wore a brown chip hat which had a pompon of white lace and feathers. She bore herself modestly and well in the face of the applause of the Professors and Trustees on the stage, and the slight flush on her face was perceptible only to those quite near her. When, with the coveted parchment, she turned to leave the stage, a huge basket and two bouquets were passed up to her. The half dozen young men who had received parchment with her seemed to have lost their sense of courtesy and politeness in spite of their titles, and did not offer to assist her. She would have been obliged to carry them from the stage had not the white-haired Professor Drisler hastened to her assistance and relieved her of her floral burden. Hearty applause and a Columbia cheer greeted this act of courtesy” [9].[4]

[3] *At the June 7, 1886, meeting of the Columbia Trustees, they also decided to admit women on equal footing as men, although no specific provisions as to whether or not women would study in the same classrooms were made* [7].

[4] *Winifred Edgerton was not the first American woman to earn her Ph.D. in mathematics. That honor goes to Christine Ladd Franklin, who earned her Ph.D. in mathematics in 1882 from Johns Hopkins University. Her work was published in 1883, but because she was a woman, Johns Hopkins did not award the degree until 1926* [14].

Thesis

Winifred’s 1886 doctoral thesis is titled *Multiple Integrals (1) Their Geometrical Interpretation in Cartesian Geometry, in Trilinears and Triplanars, in Tangentials, in Quaternions, and in Modern Geometry; (2) Their Analytical Interpretations in the Theory of Equations, Using Determinants, Invariants and Covariants as Instruments in the Investigation* [22]. In a 1937 interview, she noted her dissertation’s two parts, the first in mathematical astronomy, which included the orbit computation of the 1883 comet, and a second in pure mathematics [15]. And, although her official thesis on record at Columbia is in mathematics, she referred to it in a 1944 interview with Mr. Howson, as “... two practically, one for pure mathematics and one for astronomy...” [17].

A major part of this thesis explored infinitesimals in various systems from analytical geometry. The systems considered were Cartesian, oblique, polar, trilinear, triplanar, tangential, and quaternions. Winifred began by using a geometric approach to find the infinitesimals for length, area, and volume for these various systems. She stated that the presentation of the trilinear and triplanar coordinate systems was new. Then she presented the transformation method abridged from Bartholomew Price’s differential and integral calculus book for transformation of multiple integrals from one system to another [26]. Part of her original work thus included using this method for transformations from the Cartesian system to that of triplanars and tangential coordinates. She then examined the infinitesimals for area and volume obtained by the geometric approach and the analytical method for various systems and showed their equivalence. In addition, new work was done with transformations and the quaternion system [22].

In her thesis, Winifred beautifully sketched and explained the infinitesimals for length, area, and volume for the various coordinate systems listed above. When working with the oblique system (see Figure 2), where θ is the angle between the X and Y-axis, and θ' is the angle between Z-axis and the XY-plane, she showed that the infinitesimal for volume is $\sin(\theta)\sin(\theta')dX\,dY\,dZ$. As our Figure 3 (her Figure 5) shows, she examined the Triplanar System, where the coordinates for a point are determined by three intersecting planes. Distances α, β, γ determine the perpendicular Distances a point is from planes OBC, OAC, and OAB, respectively. Winifred related this system to the oblique system by letting ϕ be the angle between the normals to planes OAC and OBD and letting ϕ' be the angle between the plane containing the last two normals with the normal to the plane OAB. With this idea, the infinitesimal for volume was

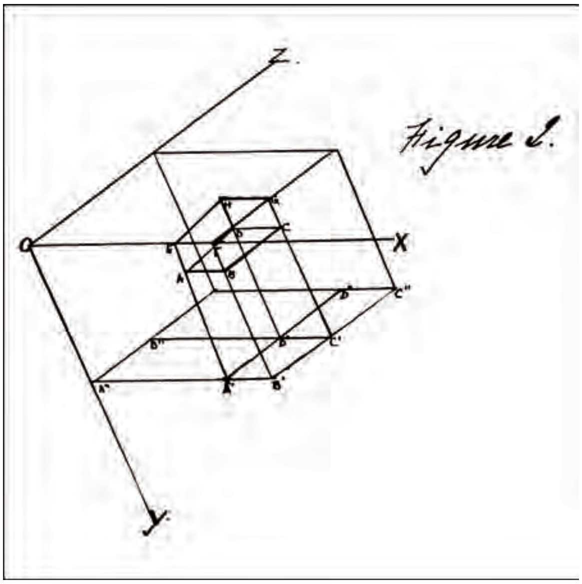


Figure 2. This thesis figure illustrates the geometry of the infinitesimal for volume in the oblique system.

found to be $\sin(\phi)\sin(\phi')d\alpha\, d\beta\, d\gamma$, similar to that for the oblique system.

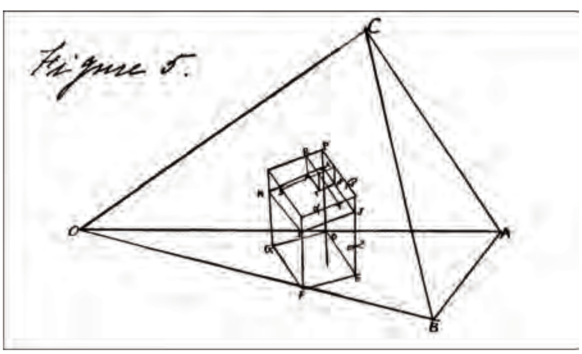


Figure 3. This thesis figure illustrates the geometry of the infinitesimal for volume in the triplanar system.

Winifred next presented a method from Price's book to transform integrals from one coordinate system to another. In Price's text, he first illustrated the idea with the special case of transforming a double integral in the Cartesian coordinates x and y to the polar coordinates r and θ. The two systems are related by

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta.$$

The differentials are then

$$dx = \cos(\theta)dr - r\sin(\theta)d\theta$$

and

$$dy = \sin(\theta)dr + r\cos(\theta)d\theta.$$

In the integral in the Cartesian system, x is held constant when integrating with respect to y. Thus, in the y-integral, one can consider

$$dx = 0 = \cos(\theta)dr - r\sin(\theta)d\theta.$$

This equation can be solved for dr and substituted into the expression for dy to yield $dy = \frac{r}{\cos(\theta)}d\theta$. This changes $dx\, dy$ to $\frac{r}{\cos(\theta)}dx\, d\theta$. With the same argument, θ can now be considered constant in an integral with x, and thus, dx becomes simply $\cos(\theta)dr$. This argument shows that $dx\, dy$ changes to $r\, dr\, d\theta$ in the transformation from the Cartesian system to the polar system [26].

Winifred presented the material from Price's book which generalizes this idea for a transformation from one n-dimensional system in $x_1, x_2, \ldots, x_n$ to a different n-dimensional system in $\eta_1, \eta_2, \ldots, \eta_n$. Similar to the relations between Cartesian and polar coordinates, equation array (8) of the thesis relates $x_1, x_2, \ldots, x_n$ in terms of $\eta_1, \eta_2, \ldots, \eta_n$ (see Figure 4). She thus obtained equations for the differentials of $x_1, x_2, \ldots, x_n$ in terms of the differentials of $\eta_1, \eta_2, \ldots, \eta_n$.

31.

On the Transformation of Multiple Integrals.

To resume (2),

$$\iint\cdots\int F(x_1, x_2, \ldots, x_n)\, dx_1\, dx_2 \ldots dx_n.$$

Let the new variables be $\eta_1 . \eta_2 . \eta_3 \ldots \eta_n$ and let us suppose that

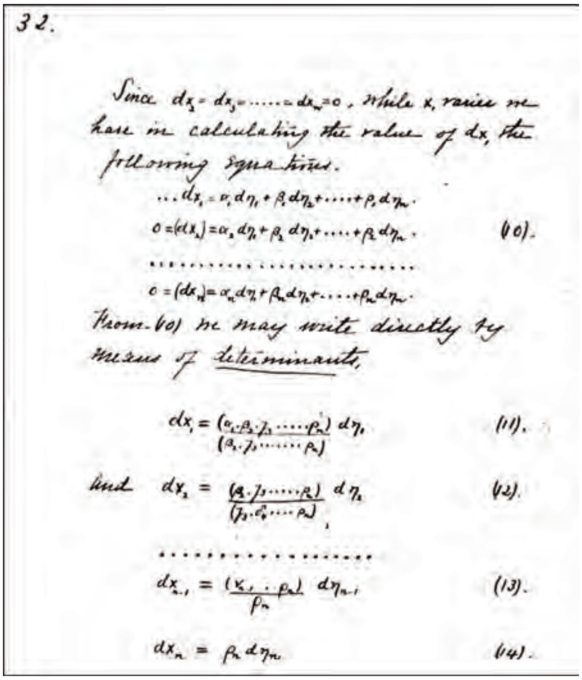
$$x_1 = f_1(\eta_1 . \eta_2 \ldots \eta_n)$$
$$x_2 = f_2(\eta_1 . \eta_2 \ldots \eta_n) \qquad (8).$$
$$\ldots\ldots\ldots$$
$$x_n = f_n(\eta_1 . \eta_2 \ldots \eta_n)$$

If then $\alpha_1, \beta_1, \ldots, \rho_1$ be the partial derivative of x_1 with respect to $\eta_1, \eta_2, \ldots \rho$; and $\alpha_2 . \beta_2 \ldots \rho_2$ of x_2 with respect to $\eta_1 . \eta_2 \ldots \eta_n$, and so on, we have

$$dx_1 = \alpha_1 d\eta_1 + \beta_1 d\eta_2 + \ldots + \rho_1 d\eta_n$$
$$dx_2 = \alpha_2 d\eta_1 + \beta_2 d\eta_2 + \ldots + \rho_2 d\eta_n$$
$$\ldots\ldots\ldots$$
$$dx_n = \alpha_n d\eta_1 + \beta_n d\eta_2 + \ldots + \rho_n d\eta_n$$

Figure 4. Page 31 of Winifred Edgerton's thesis.

Again, as was done in the Cartesian to polar transformation above, when dealing with an integral in x_1, one can consider $x_2, \ldots, x_n$ to all be constant, and thus expressions for $dx_2, \ldots, dx_n$ would all be zero when considering x_1. This gives equation array (10) of the thesis (see Figure 5).



32.

Since $dx_2 = dx_3 = \dots = dx_n = 0$, while x_1 varies we have in calculating the value of dx_1 the following equations.

$\dots dx_1 = \alpha_1 d\eta_1 + \beta_1 d\eta_2 + \dots + \rho_1 d\eta_n$.

$0 = (dx_2) = \alpha_2 d\eta_1 + \beta_2 d\eta_2 + \dots + \rho_2 d\eta_n$. (10).

$\dots\dots\dots$

$0 = (dx_n) = \alpha_n d\eta_1 + \beta_n d\eta_2 + \dots + \rho_n d\eta_n$.

From (10) we may write directly by means of determinants,

$dx_1 = \frac{(\alpha_1 \cdot \beta_2 \cdot \gamma_3 \cdots \rho_n)}{(\beta_2 \cdot \gamma_3 \cdots \rho_n)} d\eta_1$ (11).

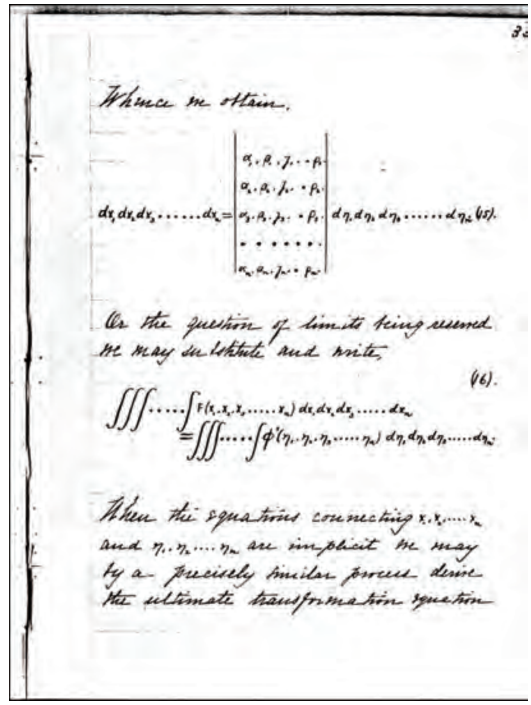
And $dx_2 = \frac{(\beta_2 \cdot \gamma_3 \cdots \rho_n)}{(\gamma_3 \cdot \delta_4 \cdots \rho_n)} d\eta_2$ (12).

$\dots\dots\dots$

$dx_{n-1} = \frac{(\kappa_{n-1} \cdot \rho_n)}{\rho_n} d\eta_{n-1}$ (13).

$dx_n = \rho_n d\eta_n$ (14).

Figure 5. Page 32 of thesis.



Whence we obtain,

$dx_1 dx_2 dx_3 \dots dx_n = \begin{vmatrix} \alpha_1 \cdot \beta_1 \cdot \gamma_1 \cdots \rho_1 \\ \alpha_2 \cdot \beta_2 \cdot \gamma_2 \cdots \rho_2 \\ \alpha_3 \cdot \beta_3 \cdot \gamma_3 \cdots \rho_3 \\ \dots\dots \\ \alpha_n \cdot \beta_n \cdot \gamma_n \cdots \rho_n \end{vmatrix} d\eta_1 d\eta_2 d\eta_3 \dots d\eta_n$ (15).

Or the question of limits being waived we may substitute and write,

(16).

$\iiint\cdots\int F(x_1, x_2, x_3, \dots x_n)\, dx_1 dx_2 dx_3 \dots dx_n = \iiint\cdots\int \phi(\eta_1, \eta_2, \eta_3, \dots \eta_n)\, d\eta_1 d\eta_2 d\eta_3 \dots d\eta_n$

When the equations connecting $x_1 x_2 \dots x_n$ and $\eta_1 \eta_2 \dots \eta_n$ are implicit we may by a precisely similar process derive the ultimate transformation equation

Figure 6. Page 33 of Winifred Edgerton's thesis.

Price cited Peacock's *A Treatise on Algebra* for a solution to this system of equations. He also cited a paper by Boole for the method demonstrated for transformations of multiple integrals [5, 25, 26]. When completing this work, one obtains thesis equation (15) (see Figure 6).

$$dx_1\, dx_2\, \cdots dx_n = \begin{vmatrix} \alpha_1 & \beta_1 & \cdots & \rho_1 \\ \alpha_2 & \beta_2 & \cdots & \rho_2 \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_n & \beta_n & \cdots & \rho_n \end{vmatrix} d\eta_1\, d\eta_2\, \cdots d\eta_n.$$

Here $\alpha_1, \beta_1, \dots, \rho_1$ are the partial derivatives of x_1 for $\eta_1, \eta_2, \dots, \eta_n$, respectively, $\alpha_2, \beta_2, \dots, \rho_2$ are the partial derivatives of x_2 for $\eta_1, \eta_2, \dots, \eta_n$, respectively, and the pattern continues.

Following Winifred's presentation of the transformation method, she demonstrated its application with transformations among several systems. She used this method to pass from the Cartesian system to the oblique system. She next used that work to pass from the Cartesian to the trilinear and triplanar systems. This last work with trilinear and triplanar systems, as stated earlier, was one of the new results in her thesis. This work involved obtaining relations between the Cartesian and oblique systems and the oblique and triplanar systems to obtain the needed equation arrays for the Cartesian and triplanar systems [22].

Finally, she presented the area and volume infinitesimal elements from the geometric and analytical techniques for the various systems discussed in her thesis. For some systems, such as the triplanars, the written forms varied, depending on the equations she used to do the analytical work. She then went on to show that the corresponding forms from both the geometrical and analytical methods were equivalent [22]. Her thesis continued with applications to quaternions and various other issues. This paper presents what the authors believed would be of most interest to the majority of readers.

Post Columbia

After her graduation, Winifred continued teaching at Mrs. Sylvanus Reed's School, where she became the vice principal [15, 38]. She was offered a professorship position at Wellesley, but she declined the offer to preserve a more traditional role.[5] In May of 1887, she wrote to Morgan Dix about her decision, saying that "this life which now opens before me is brighter than the stars". In September of 1887, she married Frederick Merrill, a graduate of Columbia's School of Mines [12, 21].

Having proven herself at Columbia, Winifred Edgerton Merrill was invited in 1888 to serve on a five-member committee to found Barnard College,

[5] *Wellesley is referred to in several published references, but it should be noted that, in her son Hamilton's journal, he transcribed a letter his mother wrote to Morgan Dix on May 29, 1887, which refers to "declining the position in Northampton (Smith)"* [21].

which was to be an affiliate of Columbia specifically for women. She worked on drafting the proposal for the new college, but later resigned from the committee [12]. In her 1944 interview she stated, "We had our meetings in downtown. My husband objected very much. He thought it was entirely improper for me to go to a man's office downtown. I had to resign from the Committee" [17].

Frederick Merrill completed his Ph.D. with honors at Columbia in 1890 [1]. In October of that year, Frederick Merrill was appointed as assistant state geologist of New York. Frederick and Winifred and their first child, Louise (born June 3, 1888), then moved to Albany, where their first son, Hamilton, was born on December 21, 1890. In that same month, Frederick was appointed assistant director of the New York State Museum, where he became the director in 1894.[6] A second daughter, Winifred, was born on July 21, 1897, and a second son, Edgerton was born April 21, 1901 [21]. In Albany, Winifred was invited to serve on the school board. In regards to this invitation, Winifred stated, "My husband did not speak to me for two days. He was born in New York and had these ideas of what was proper for women to do" [17].

The Merrills valued both intellect and social prominence. They entertained many well-known guests. One of Winifred's favorite guests was educator Booker T. Washington, founder of Tuskegee Institute [21].[7] They traveled for work related to Frederick's position, although Winifred traveled less as their family grew.

The Merrills' social standing was greatly aided by two large inheritances. Frederick's mother and an uncle each bequeathed $100,000 to the Merrills. To put this into perspective, as the state geologist, Frederick annually earned $3,500 [21]. At this same time, the average annual salary for the country was under $500 [35]. Hamilton wrote that it was evident his family had money because each time the family moved, they moved to a more prestigious residence with more hired help. The finest of these homes was the King Mansion on Washington Avenue in Albany. The three-story brownstone even included a bowling alley. In 1894 the family also purchased a 360-acre farm in Altamont, New York, where they would summer. Their home there was the former Kushaqua Hotel, a large Victorian structure [20, 21]. During this time, Hamilton wrote of the family's most famous acquaintances, Teddy Roosevelt and his family, and how the two families' children became familiar playmates. He wrote of an amusing story: "At Altamont we were occasionally visited by the Roosevelts who were all great outdoor enthusiasts and came hiking out our way during the summer. One day, I remember, both Mother and Louise were down with the mumps when Father and I spotted the entire Roosevelt tribe, led by Teddy himself, marching up the road on a hike. We ran downstairs and shooed the Governor and his family away without so much as a Hello" [21].

In 1902, only one year after the birth of Winifred and Frederick Merrill's fourth child, the family began having financial difficulties. Hamilton Merrill blamed his mother for throwing "lavish parties, spending frivolously, and making bad investments". These financial difficulties led to bankruptcy. Bankruptcy laws of the time required that Frederick Merrill resign his position as state geologist. The financial difficulties also led to the Merrills separating in 1904. Hamilton Merrill noted

Postcard from the Archives of the La Salette Missionaries, 85 New Park Ave., Hartford CT.

Figure 7. Summer home of the Merrills[8] and former Kushaqua Hotel. (In 1924, the property was purchased by the La Salette Missionaries. The mansion burned in 1946 [20]).

Mamaroneck Village Historical Society.

Figure 8. Page 49 of the 1921–1922 Oaksmere courses book.

[6]*Frederick Merrill directed New York State's scientific exhibits at the 1893 Chicago World's Fair and the 1904 St. Louis World's Fair* [36].

[7]*Tuskegee Normal School was founded on July 4, 1881, as an institution for African American students. Booker T. Washington, a former slave, was their first teacher and was the principal of the school until his death in 1915* [33].

[8]*The summer mansion of the Merrills came later.*

Lucile Godbold Papers, J. Drake Edens Library Archives, Columbia College.

Figure 9. 1922 U.S. Women's Track Team.

that his mother was a "brilliant, forceful woman with greatness in her. She had a tremendous inner urge. After she gave up her career for her family, this urge found an outlet in four children and intense social activities" [21].

After their separation, Frederick Merrill moved west for work. He stayed in contact with his family and died in 1916. Winifred became the principal at Anne Brown's School for Girls in New York City and attempted to furnish and rent apartments [21].

In 1906 she founded Oaksmere School for Girls in New Rochelle, New York, which was also known as Mrs. Merrill's School. She had no capital of her own to begin a school; however, she was able to assemble many men of means as benefactors. Oaksmere, a college preparatory school, offered a variety of courses and catered to patrons who could freely spend; its tuition in 1921 was $2,400 with an additional cost of $215 for uniforms. In 1914 the school was relocated to Mamaroneck, New York [21, 24].

Due to Oaksmere's success in New York, a Paris branch opened in 1921 [24].[9] Both schools thrived for some time before financial difficulties were encountered. Winifred was once again responsible for spending lavishly. For example, during the construction of a swimming pool, nearly $10,000 was raised but approximately $30,000 was spent. As a result of these financial difficulties, Oaksmere was forced to close in 1928 [21].

Some of the lavish spending for high quality facilities may have been carried out to maximize women's abilities to participate fully in athletics. The Mamaroneck campus had a variety of athletic facilities, including a large outdoor track and an indoor swimming pool [24]. When the first international track meet for women was held in Paris in 1922, some Oaksmere girls participated. Winifred helped sponsor the American team and served as a chaperone [32, 34]. The girls that attended Oaksmere had the opportunity to participate in track meets from 1921 through 1924 [31].

In addition to work in education, Winifred continued her scholarly work, publishing a book titled *Musical Autograms* with her future son-in-law Robert Russell Bennett, who later became an Oscar- and Emmy-award-winning composer and orchestrator. In the book they examined musical qualities in the signatures of famous people of her time. Bennett describes his first meeting with Winifred and the start of their work: "...Mrs. Winifred Edgerton Merrill needed a composer to help her work something out musically,... she asked me to take home a few pages of black dots spread out over musical staves and see what they suggested to me. Completely in the dark, I made notes out of all the black dots, chose certain things arbitrarily...". He went on to describe the created composition: "The only noticeable characteristic ... its tendency to sound Slavic in some spots." At a later meeting, Winifred told him she had taken the words *Russian National Anthem* written in Russian and translated them mathematically. Bennett wrote how Winifred would take words written on a music staff or place a staff over words. He stated, "... the black dots being at every dot or intersection of lines, one at each extremity of a straight line and three to express every simple curve". In their book, they studied the musical qualities of the signatures of twenty famous men of the time, including then-president Wilson, ex-presidents Taft and Roosevelt, and John Philip Sousa. Bennett wrote of Winifred, "It was her hope that a person's signature, written across a musical staff, would furnish a melodic line expressive of that person's character, and possibly his or her mood at the time of signing" [4] Five of these autograms have been performed and made available on the Web by pianist Phillip Sear [29].

Winifred was also involved in politics. She spoke publicly against prohibition in the campaign to repeal the Eighteenth Amendment [13].

In 1933 Columbia honored Winifred in a ceremony attended by more than 200 guests. The school hung her portrait within Philosophy Hall. The portrait, a joint gift of the Wellesley class of 1883 and the Columbia Women's Graduate Club, bears the legend "She Opened the Door" [6].

From 1928 through 1948, Winifred was the librarian at the Barbizon Hotel in New York City [12, 21]. This was a twenty-three-story hotel established in the 1920s to provide women a safe residential option when leaving home to seek

[9] *Prior to the Paris school, Winifred and Oaksmere also funded a Paris ambulance during World War I* [37].

Figure 10. Portrait of Winifred Edgerton Merrill.

professional careers [3]. She died in Fairfield, Connecticut, on September 6, 1951, at age eighty-eight [23].

Beaulah Amidon wrote in the *New York Times*, "Several years ago, … I had the privilege of a long conversation with Mrs. Merrill. … Though almost eighty years of age at the time, her eyes were on the future, and she was much more interested in the progress of women in business and the professions than in the old battle for their higher education, in which she played so notable a part." [2]

Conclusion

Winifred Edgerton Merrill pioneered a way for women to continue their education. Instead of losing hope when Columbia initially denied her access to the school's telescope, she patiently petitioned for it again. She became the first American woman to receive her Ph.D. in mathematics and helped give other women the chance to earn and receive degrees as well. She raised a family, and continued to help open doors to women by opening college preparatory schools for girls. These schools gave young women a background in a variety of academic areas as well as the opportunity to pursue athletics. Winifred Merrill helped women emerge and become more active in the male-dominated society in which she lived.

The authors would like to recognize the work of Judy Green and Jeanne LaDuke and the publication of their book, *Pioneering Women in American Mathematics: The Pre-1940 PhD's* [15]. The authors did most of their research prior to the publication of this book [19], but found it a wonderful reference when adding some final touches for this publication. Supplementary material to the book can be found on the American Mathematical Society hosted site `http://www.ams.org/publications/authors/books/postpub/hmath-34-PioneeringWomen.pdf`.

Appreciation is extended to Gloria Pritts and John Quadrini from the Mamaroneck Village Historical Society; George Miller and Bob Schuster at the Ripon Wisconsin Historical Society; David Hill and Jennifer Ulrich from Columbia University Archives; George Ferencz from the Music Department of the University of Wisconsin-Whitewater; Rosalind Rosenberg from Barnard College; Wilma Slaight, Inger Raahauge Nielsen, and Jean Berry from Wellesley College Archives; Veronica Cunningham from the Emma Willard School; and Fr. Ron Gagne from the La Salette Missionaries. Special thanks are given to Liz and Kean McDonald, who sent us a copy of Hamilton Merrill's personal family journal. Kelly would like to thank the referees of this article for their constructive feedback and sources for additional material for the final paper. She also thanks Tracy Helixon for feedback on the final version of the paper. Kelly would finally like to recognize the excellent website on *Biographies of Women Mathematicians* at Agnes Scott College. It was a casual reading of this site that sparked the interest for this project. For anyone we have missed, we apologize.

Susan Kelly dedicates this paper to her two sons, Kyle and Ryan.

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Mathematics and Home Schooling

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Empowered by the belief that an educated citizenry made for a strong nation, colonial governments as early as 1642 mandated compulsory education for school-age children (Hiatt 1944). Numerate citizens were needed for strong commerce and successful farming, and the governments saw their education as an important objective.

Today parents can choose from public schools, magnet schools, charter schools, private schools, and parochial schools. Despite the varied public and private opportunities available, over 1.5 million students are engaged in home school programs, some of whom will choose to pursue a postsecondary education. Given the increasing attention to national standards for the preparation of students, it is important to understand the climate for, and results of, home schooling insofar as mathematics is concerned.

In our analysis, we address the legal framework surrounding modern home schooling, noting variations in state regulations and curriculum options, with particular attention to mathematics. We then examine how well the structure can be said to prepare students for postsecondary education and whether there are legal remedies if it does not.

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State Mandated Education

Motivated generally by religiosity, ensuring literate children was a high priority among founding colonists (Gaither 2008). Toward that goal children were taught in the home or in small groups in a religious setting, where in addition to learning how to run a household, farm, or small business, they learned to read, primarily from the Bible. A shift away from home schooling first came in 1635 with the establishment of the first public school in Boston. In 1642 Massachusetts passed a law requiring parents to teach their children how to read and write; the statute gave the state the authority to provide education if the need arose. A second measure, known as the Old Deluder Satan Law or the General School Law of 1647 (Martin 1894), required towns with more than fifty families to hire a teacher and with more than one hundred families to support a grammar school. Massachusetts thus fostered literacy, which in turn meant the population could read and understand the Bible and undertake civic and economic responsibilities or, more colorfully, stay out of the hands of Satan. The costs of running the school and paying the teacher were placed upon the adults responsible for the children to be educated. The passage of this law marks the beginning of compulsory education in the United States. Through the country's history, a basic principle of federalism, namely that the federal government would have limited constitutionally defined powers with other governmental functions reserved to the states, has guided the development of education. In composing the Bill of Rights, the founding fathers notably did not address education; the Tenth Amendment states:

> The powers not delegated to the United States by the Constitution, nor prohibited by it to the States, are reserved to the States respectively, or to the people.

As colonies and territories became states and drew up their own constitutions, each came to include a clause assuming for the state the responsibilities for education. Some of these clauses were limited to providing a basic level of education to assure effective civil and economic participation, while others were broader and more prescriptive as to how this was to be achieved. For example, Vermont in 1827 was the first state to require that educators hold a teaching certificate (Cubberly 1919); other states specified subjects to be taught, often including mathematics.

Although the responsibility for education is clearly assigned to the states, in practice education in the United States has always been a local operation, generally with schools organized into districts by city or county. As the country expanded, so also did enthusiasm for inspiring a sense of civic responsibility through education. Over time home schooling a child was no longer widely prevalent; complementary to the constitutionally mandated state responsibility for education, compulsory attendance laws were instituted. The laws generally authorized private (including religious) school enrollment as meeting these requirements, sometimes prescribing that the education they provide should be equivalent to that in the public schools. Nineteenth- and early twentieth-century litigation upheld the right of the state to carry out its mandate through these laws, including prohibiting home schooling to the extent that the parents who chose to home school were sometimes pursued by the courts on the charge of truancy (Gaither 2008).

In State v. Bailey (1901) the Indiana Supreme Court declared that the natural rights of the parent with regard to the custody and control of his children are subordinate to the power of the state, so that no parent can be said to have the right to deprive his child of the advantages of public education. On the other hand, the lower court in Indiana, in State v. Peterman (1904), found it sufficient to employ a qualified teacher to teach children in the home under the private school provision of the compulsory attendance law, the court declaring that a school is any place where instruction is imparted to the young, a finding echoed by the Illinois Supreme Court half a century later in People v. Levisen (1950).

But the attitude of courts was not always so tolerant. Even conceding the defendant's claim that he was qualified to teach all grades and all subjects taught in the public schools, in Washington v. Counort (1912) the Washington Supreme Court held that teaching his children at home did not qualify as attending a private school and thus home schooling failed to conform to the compulsory attendance law. Directly addressing home schooling again, the Supreme Court of New Hampshire in Hoyt v. Daniels (1919) declared that being schooled at home was not schooling in a private school, the only alternative to public school permitted in the state's school attendance law. Nearly ninety years later, a lower court in California came to the same conclusion (In re Rachel 2008), but in the climate of the political power of the home schooling movement, that decision was overturned in Jonathan v. Superior Court of Los Angeles County (2008), with the court declaring that home schooling was considered to be a private school and could be forbidden only on safety grounds, for example to prevent child abuse (In the matter of William AA, 2005).

An Oregon case is often cited as supporting home schooling, although actually the freedom to run a private school was the issue. The state's compulsory attendance statute required enrollment in the public school system. In fact, although its stated intent included ensuring full opportunity for immigrant children to assimilate as well as mandating integration of children from all economic strata in an effort to achieve equality of educational opportunities, many viewed the law as an anti-Catholic, anti-immigrant measure, no matter how compelling the argument for universal exposure to common values and mingling with diverse populations might be. The U.S. Supreme Court in Pierce v. Society of Sisters (1925) was careful to make clear the state's right and responsibility to regulate education, although not to the extent that the Oregon law contemplated. Although often cited as upholding the freedom of religion clause of the First Amendment, the decision concerned itself more with the business interests of the Society of Sisters school.

A few years earlier, inspired by an excess of nationalistic fervor, Nebraska had passed a law forbidding teaching in a language other than English to students who had not completed the eighth grade. Once again, the Supreme Court (Meyer v. Nebraska 1923) strongly endorsed the right of the state to regulate education as a part of its mandate to provide for it, but found on Fourteenth Amendment equal protection grounds that the Nebraska law was arbitrary and served no rational purpose. Upholding the right of parents to direct the education of their children, at the same time the Court clearly did not contemplate home schooling, asserting, "Education of the young is only possible in schools conducted by especially qualified persons who devote themselves thereto."

Another case often cited for limiting state regulation of school attendance actually also strongly emphasized the state's responsibility to assure the education of children. In balancing the right of parental control with the rights of the state, the U.S. Supreme Court in Wisconsin v. Yoder (1972) allowed an exemption from Wisconsin's compulsory attendance law for the Amish community only because it believed that the very nature of the religion would be undermined with exposure of its young people to the worldly culture of education beyond the eighth-grade level. If the state's purpose in education is to prepare children for life, asserted the Court, the limiting nature of education espoused by the Amish was adequate for their separate agrarian way of life. Attempts by other religious groups to claim an "Amish exemption" for their educational practices or lack thereof have not been well received by the courts (Fellowship Baptist Church v. Benton (1987), Johnson v. Charles City Community Schools (1985)).

The Rise of Home Schooling

We have seen something of the ups and downs of litigation, but it is ironic that the state where public education originated helped set modern legal precedence for home schooling as an education option with the 1978 decision in Perchemlides v. Frizzle (1978). The Perchemlides family had home schooled their older children while living in Boston. After moving to Amherst, they sent their youngest son to the public school system for second grade, where he appeared to regress intellectually and socially. The family opted to continue their son's education at home and filed the required notice with the school district that the family intended to home school. The school district denied the request for four reasons: (a) the parents were not teachers; (b) the curriculum outlined by the family was not linked to the child's current developmental level; (c) the curriculum did not provide for interaction with other groups of children; and d) the child's difficulties were the result of his earlier education, not of the current public school setting. The district superintendent brought a charge of truancy against the parents when they chose to keep their son at home. The court ruled that parents were competent to teach their children and that as long as the curriculum covered those subjects mandated by state law on the mandated time schedule and there was a level of accountability in place, home schooling was a legal option for education in Massachusetts. The case established that home schooling was protected as a state constitutional right subject only to state regulation that must be essential to providing an adequate education. This turnaround in litigation concerning home schooling marked the rise not only of home schooling as an educational philosophy but as a political movement. Although not a decision with precedential value, the cause of the Perchemlides family, particularly their resistance to home inspection visits, was widely embraced by the home schooling community nationally.

Taking a somewhat different approach, the federal court in Jeffery v. O'Donnell (1988) ruled that the Pennsylvania compulsory attendance laws in place were constitutionally vague in the requirement that the curriculum must be "satisfactory" and that a parent must be "properly qualified" and thus constituted a threat to First Amendment freedoms. The aftermath, however, has not been less vague requirements but rather essentially no requirements at all.

Teacher Certification

As home schooling became more popular, particularly among religious conservatives, courts in different states adopted varying approaches to the practice. Some states focused on teacher certification. In Florida v. Buckner (1985), the Florida Court of Appeal declared that the school attendance statute clearly prohibited an unqualified parent from teaching a child at home under the guise of being a private school. On the other hand, in Delconte v. North Carolina (1985), the North Carolina Supreme Court declared that absent a clear legislative intent, the relevant statute could not be interpreted to prohibit home schooling, but it did not rule whether in fact the state could constitutionally prohibit it were the statute more carefully drafted. The court in Blackwelder v. Safnauer (1988) upheld a statute requiring instruction in home schools to be "substantially equivalent" to that in public schools and to be given by "competent" instructors. Additional rulings upheld a teaching certification requirement for home schooling (Clonlara, Inc. v. Runkel 1989; Hanson v. Cushman 1980; Jernigan v. State 1982; People v. DeJonge 1993). However, it is not clear how similar cases would fare today. In particular, between 1982 and 1992 more than twenty states repealed their teacher certification requirements for home schooling (Dwyer 1994).

All states require public school teachers to hold a teaching certificate, granted after completing a regime of college courses accredited by the state and often including an exam or series of exams covering basic subject matter. In general the curriculum for certification requires that teachers learn about childhood development and psychology and that secondary school teachers be immersed in courses that allow for subject specialization. The No Child Left Behind Act (NCLB) required states to ensure that education should come from a "highly qualified" person, described thusly:

> To be deemed highly qualified, teachers must have: 1) a bachelor's degree, 2) full state certification or licensure, and 3) prove that they know each subject they teach (NCLB 2002).

All indications are that the NCLB successor federal legislation will have similar provisions. Private schools are generally exempted from teacher certification requirements, and home schooling is specifically exempted from all provisions of the NCLB. In the states where legal criteria for home school providers are listed, generally the qualifications are broadly described and include some or all of the following: parents must be minimally competent, to have taken an education class, to have a GED, to have completed a bachelor's degree. Some states require that homeschoolers successfully complete an interview or periodic meetings with a public school representative, requirements not easily nor uniformly enforced.

In any case, federal legislation is likely to take a narrower view of education than do many homeschoolers.[1] The NCLB does not mention virtue, morality, or religious conviction as a purpose of education but speaks mainly of quantifiable academic assessment and achievement, exempting home schooling from any accountability for such standards. Should less controversial standards be adopted, home schooling is still likely to be exempt. Establishing the basic right to home school seems beyond effective challenge in the current climate. Uniform enforcement of compulsory education laws is difficult in a home school situation, particularly with public education strapped for funding for the public schools themselves. State regulations or standards to be met are often vague, and overseers generally represent the very school system the homeschoolers seek to escape. Moreover, there are generally no health and safety safeguards, such as compulsory vaccinations, in place. Not only is there a fierce lobby in support of home schooling, but given the inadequacy of many public schools with overcrowding, high failure rates, and confusing standards enforced by tests of questionable value, forcing those currently home schooled into public education might be a cure worse than the disease. However, "disease" does not describe the nature of much home schooling that is an option that is clearly valuable for many children, particularly those with special needs. Nonetheless, there are concerns centered on the acknowledged government obligation to assure education of children for full civic and economic participation.

To further our discussion, we consider the question, Who is a home school student in the United States?, and then we examine two aspects of these concerns: the adequacy of home schooling from a subject matter perspective and the potential for gender discrimination.

Who Chooses to Home School?

To provide a definition of a home-schooled student, we choose to follow the definition used by The National Center for Education Statistics (NCES), which states:

> Students were considered to be home schooled if their parents reported them being schooled at home instead of a public or private school, if their enrollment in public or private schools did not exceed 25 hours a week, and if they were not being home schooled solely because of a temporary illness. (NCES 2001)

The results of the NCES 1999 study found a small percentage (1.7%) of all school-age children, less than one million, were home schooled, and approximately 20 percent of those students took advantage of services offered at the public school. Services available included curriculum materials, texts, meetings for parents of students, and extracurricular activities (NCES 2003). Follow-up studies conducted by NCES show that by 2003 that number was increasing and by 2007 there were in excess of 1.5 million students home schooled (NCES 2009). Recent reporting from National Home Education Research Institute (NHERI) estimates the number of students in home schooling to be more than two million (`www.hslada.org`).

Data show that families who choose home schooling tend to be white, non-Hispanic, two-parent households with only one parent working. Religious beliefs are only one component of why parents report choosing to home school; data in the study also suggest that parents feel they can give their children a better education at home, that the curriculum offered in schools is not challenging their child, and that the school environment is poor (NCES 2001).

Although there are no reliable national statistics, it is the case that many homeschoolers supplement what is taught individually in the home with group study and opportunities for field trips as well as participation in public and private school activities. For the most part the involvement has been in such areas as sports (there are also home school leagues in some sports in some regions) and music. Entitlement to such participation is not clear, although there has been legislation introduced in several states to ensure it under certain conditions.

Curriculum Concerns

The largest group of homeschoolers consists of Christian evangelicals whose opposition to enrolling their children in public schools is based on two beliefs. One is their desire to avoid the schools' secular nature, in particular their children's potential exposure to and mingling with those whose religious and cultural background differs from their own. They may also object to particular subject matter, such as evolution or the teaching of the validity of and need for respect for different cultures. It even has been said that the teaching of probability should be avoided, since it evolved

[1] *See, e.g., Friendship Baptist Church, 620 F. Supp at 316: "Tests primarily determine knowledge of content of the subject matter. They do not test other aspects of education necessary to prepare a student for life in today's society."*

from and is associated with gambling. The highly effective opposition to regulation of home schooling is led by this group through the Home School Legal Defense Association and its lobbying arm, the Congressional Action Program (Stevens 2001).

Other homeschoolers are primarily concerned with the inadequacy of the public schools in meeting the needs of their children, especially those with special needs at either end of the learning spectrum. It is the first group that engenders the concerns cited earlier.

In the 1980s there was a spate of litigation concerning the material covered in public schools. General lack of success was, no doubt, bound to lead to opting out of the system. In Mozert v. Hawkins County Board of Education (1987) plaintiffs objected to requiring the reading of certain textbooks, claiming that it was a burden on the free exercise of religion. Since the First Amendment guarantees this fundamental right, the court chose to adopt an intermediate standard (between rational and strict scrutiny): Do defendants have a compelling interest in requiring all students in grades 1 to 8 to read the basic series of books of a specific publisher? The objections were, not surprisingly, based on the treatment of evolution, but also on what was perceived as endorsement of magic. Although plaintiffs' claim was a lack of balance, testimony at trial also revealed objections to teaching tolerance of religious views other than their own. Finding that public schools serve the purpose of teaching fundamental values "essential to a democratic society," the court concluded that reading the texts was not a burden on freedom of religion and hence that it need not decide whether the compelling interest standard was met, although a concurring opinion asserted that such a state interest would prevail even if there were to be a such a burden. Lacking the ability to tailor the public school curriculum to their views has been a motivating factor in the rise of home schooling. Few states mandate that home schooling cover topics such as tolerance for the religion and culture of others, but many homeschoolers fear any attempt to proscribe curriculum as a threat to their values.

There has been very little research on how well home-schooled students who choose to attend college do or what areas of study they choose. Scores on SATs are generally above the average of those of students as a whole, but the home-schooled students who take such exams are a relatively small, self-selected group.

Home Schooling Requirements by State

As noted, although until the 1980s most states actually prohibited home schooling, it is now the case that every state permits home schooling subject to varying degrees of regulation as an option under compulsory attendance laws. While ten states do not even require home schooling parents to notify the state of their intent to home school, currently all states have legislated some rules for providing home schooling, with the stringency of the requirements varying widely, as illustrated by the following examples (`www.hslda.org`).

Pennsylvania apparently demands much from home school families. In addition to submitting an affidavit of intent to home school, applicants must assure that topics are taught in English, provide immunization records, develop an instructional plan, and maintain a portfolio of accomplishments covering state requirements, including English and mathematics. At the end of each academic year, portfolios are evaluated by experts from the state education authority. However, home school teachers simply must hold the equivalent of a high school diploma. In addition, students in grades 3, 5, and 8 are required to participate in the Pennsylvania System of School Assessment, a statewide requirement for public school students. Enforcement of these requirements appears to be at best sporadic and superficial.

North Carolina requires home-schooled students to maintain immunization records and to keep attendance records. Students are not required to follow a state prescribed curriculum, but they must submit to annual standardized testing in several topics, including English and mathematics, and instructors must hold the equivalent of a high school diploma. For follow-up, the state's rights are limited to simply inspecting test scores.

Nebraska has very little regulation with regard to the policies and procedures home schools must follow. Teachers do not need to meet any qualifications, unless the family hires outside tutors. The state does not require visitation or testing, because it claims it cannot apply the standards for such uniformly across the state. Parents, under oath, swear to provide sequential instruction in several subjects, including language arts and mathematics.

California requires instruction to be in English and attendance records must be kept. Subjects to be studied include English; additional topics may be chosen from the topics covered in the public schools. Mathematics is required only during grades 1 through 6. No standardized testing is required.

Looking at the states as a whole, we find that currently twenty-eight states and the District of Columbia use "mathematics" for a topic to be learned by those who are home schooled, although content is not specified. North Dakota and Pennsylvania are two states that detail the number of mathematics credits and mention algebra as required study. Seven states note that subjects covered should be comparable to those in the curriculum of the public school system. Three other states only require that students learn arithmetic, and no mention is made of higher mathematics. In Vermont the statutory language for required topics notes that students should study the use of numbers. Although the state of Oklahoma strongly recommends the study of mathematics, it does not require it by law. Only three states impose on home-schooled students, by law, high school graduation requirements that include mathematics.

The lack of consistent mathematics requirements and assessment for home-schooled students by some states may seem surprising given the current climate of assessment of educational achievement required by the public school system. NCLB mandated that all students who attend institutions that receive federal subsidies must measure and demonstrate improvement in mathematics and reading ability in the students they educate. In detailing requirements for assessment, NCLB states:

> Nothing in this section shall be construed to affect home schools, whether or not a home school is treated as a home school or a private school under State law, nor shall any home schooled student be required to participate in any assessment referenced or authorized under this section (NCLB 2002).

In essence, the home-schooled student does not participate in assessments required by public schools to ensure federal funding. However, twenty-four states require students to be assessed either by standardized exams for math and English or in the form of a personal evaluation or student portfolio review by the school district. How this is enforced is not generally prescribed nor systemically recorded.

Contrasting with the "hands-off" approach to home schooling, at the national level there is a push for the development of a common core standard for language arts and mathematics in the K–12 curriculum. The goal of the core standards is to better prepare students for a college education as well as to compete successfully in a global economy. This state-led effort is not a push for national standards but rather an effort designed to give the fifty states a clear common guideline to discuss what is expected in terms of curriculum and student success in two subjects that are often the focus of assessment. The goal of the standards is to make instruction consistent as well as to provide comparable assessment regardless of geography (`www.corestandards.org`). States reserve the right to adopt or reject the standards developed.

Undoubtedly, state adoption of the core curriculum standards will result in changes and challenges to educational law and curriculum materials. It is unclear how adopting the standards will affect the home school community; however, the Home School Legal Defense Association (HSLDA) suggests that adopting a common core is simply a step toward nationalizing education, a move toward which they strongly oppose (`www.hslda.org`).

Gender Equity

The major issue we address here is, of course, the adequacy of home schooling with respect to mathematics, especially in light of the limited requirements coupled with apparently nonexistent supervision in most states. There seems little if any prospect of the new impetus for national standards through the core standards or the successor to the NCLB legislation to alter the current situation. Moreover, a concern that has received little attention is the possibility that a substantial portion of the home schooling community may not be providing equal education to girls, in particular in mathematics and science. One commonly used series of texts advises girls who may be good in mathematics not to dream of becoming engineers or scientists but rather to consider how their talent might be used to assist their future husbands (Dwyer 1994).

The traditional American inclination to avoid any federal involvement in education has eroded over the years, tied to federal funding of specific programs but also through enforcement of anti-discrimination laws. In public schools and in private schools that receive federal funds and in some cases even when they do not, constitutional protections (see, e.g., Runyon v. McCrary 1970), federal laws, and some state laws mandate non-discrimination; however, the major statute, Title IX of the Education Act of 1972, permits exemptions on the basis of religious tenets. In any state action, including education, the Fourteenth Amendment and the due process clause of the U.S. Constitution also provide a basis for assuring equal treatment. Once private actors take on what is a fundamental state function such as education, then they too are bound by constitutional provisions, in particular when the state affirmatively invests parents with the state responsibility for education. Hence one can argue that home schooling parents must provide the same education for their daughters as for their sons. In Norwood v. Harrison (1973), the state's provision of textbooks to schools with racially discriminatory admission policies was found impermissible as inducing, encouraging, or promoting private persons to accomplish what is constitutionally forbidden to the state. However, since the parents are the "state actors", the doctrine affords only intrafamily equity and not interfamily equity (Yuracko 2008).

The concept of homeschoolers as state actors might also be useful if one argues that state constitutional and statutory provisions for education mandate the responsibility of the state to assure at least a basic minimal education. Does not the equal protection clause impose a lower limit on state regulation of education (Yuracko 2008, p. 180)? For instance, one could argue that to be a full actor in today's technological world, home-schooled students need computer training and courses in statistics and even calculus (not to mention evolution and other science). The "Amish exemption" from providing an education adequate for the modern world has not in the past been available to those who argue that they do not want their children exposed to concepts that are in conflict with their beliefs, but in the present context of the political strength of the home school movement it is not surprising no one is really challenging the adequacy of home schooling for the state responsibility of full civic and economic participation.

The "state actor" doctrine is not available for every function of government undertaken by private actors—for example, parochial schools may teach religion—but in a responsibility as fundamental as basic education it might be held to apply (as it has to privately operated prisons). And if states can be sanctioned for failing to provide the basic minimum (Abbott v. Burke 1990), why not sanction homeschoolers? Probably insisting on the equivalent of public school education, however defined, is not possible given the finding of San Antonio v. Rodriguez (1973) that while a minimum standard must be met, equity is not required. But can homeschoolers be held to some minimum standard of equitable mathematical training for boys and girls?

The focus on obligations for education of citizens is generally on states. Federal education laws like the NCLB Act and its likely successor have specific statements that they do not authorize any federal control over home schools. However, it has been argued that the "Guarantee Clause of the Fourteenth Amendment authorizes and obligates Congress to ensure a meaningful floor of educational opportunity throughout the nation. The argument focuses on the Amendment's opening words, the guarantee of national citizenship. This guarantee does more than designate a legal status. Together with Section 5 [which assigns to Congress the power to enforce the amendment], it obligates the national government to secure the full membership, effective participation, and equal dignity of all citizens in the national community" (Liu 2006).

But whether the issue is that home schooling provides an adequate mathematical education to meet the responsibility that the state has ceded to homeschoolers or that girls are receiving an inferior education to that of their brothers, an underlying concern is that the children themselves have no legal or practical control over the decision whether they receive the state proffered benefit of a public school education or some other, possibly inferior, form of schooling. The tradition of parental control and, in some cases, free exercise of religion infringes on their children's equal protection rights (Dwyer 1994). In a case involving the right of undocumented children to public education, the Supreme Court found a Texas law excluding them to be a denial of equal protection (Plyler v. Doe 1982). The Court found that the exclusion served no rational state interest, much less a compelling interest. The situation of the children was obvious, and they were represented in the litigation by a guardian, but who might learn of and then bring a legal challenge in the case of inadequate home schooling? Courts are reluctant to confer representation on outside advocates in parent-child conflicts.

Were litigation to occur, religious beliefs of the parents, while sufficient for the Amish when it comes to exemption from high school education, would be unlikely to prevail in the case of basic minimal education. Although an inferior education may not be as life-threatening as is denial of certain medical procedures such as blood transfusions (Jehovah's Witnesses v. King County Hospital 1968), religious objections are unlikely to withstand the scrutiny given to bypassing the state's responsibility, assumed by parents, for basic education as preparation for life.

In the contexts of ensuring the right to vote or to obtain an abortion, courts have held that not only may states not block access but must at a minimum take steps to prevent private interference with these rights (Ex parte Yarbrough, 110 U.S. 651 (1884), Planned Parenthood of Central Missouri v. Danforth 1976). Of course, in the case of home schooling, the interference with the right to a fundamental education comes from parents, and how the state can protect the rights of the home-schooled children is problematic. Permitting third party complaints on behalf of the child or establishing judicial procedures (as in the abortion context) to allow children themselves to challenge the adequacy of their parents' choices is more likely to engender fierce opposition than public support in light of the political power of the home schooling movement. However, it is likely that the majority of responsible and committed homeschoolers would also argue for minimum standards somehow to be ensured, albeit in a nonintrusive manner.

In the case of inferior education for girls, such enunciated fundamentalist beliefs as "sexual equality denies God's word" and failure of a wife to accept a subordinate, obedient role in the home means "the doors are wide open to Satan" may well exclude girls from the level of mathematics known to be a critical filter for many careers and confine them to low-paying, servile occupations if employed outside the home (Yuracko 2008, p. 156). In *So Much More: The Remarkable Influence of*

Visionary Daughters on the Kingdom of God, popular in the Christian home schooling community, authors Anne Sofia Botkin and Elizabeth Botkin claim that college is dangerous for young women because it diverts them from their God-ordained role as helpmeets for their fathers and husbands (quoted in Yuracko 2008, p. 157). Stacy McDonald (2005) supplies even more explicit guidance for potentially discriminatory education: A girl's education "should be focused on assisting her future husband as his valuable helpmate, not on becoming her 'own person'." Girls are counseled to "[r]emember that a strong desire to be a doctor or a seeming by-God-given talent in mathematics is not an indication of God's will for you to have a career in medicine or engineering. Sometimes God gives us talents and strengths for the specific purpose of helping our future husbands in their calling." (Quoted in Yuracko 2008, p. 157, note 168.)

When the United States sued Virginia for violating the equal protection clause of the Fourteenth Amendment by denying women admission to the Virginia Military Institute, the Supreme Court, reversing the decision of the federal appeals court, declared "[S]uch sex classifications may not be used as they once were, to create or perpetuate the legal, social, and economic inferiority of women" (United States v. Virginia at 534). The Court went on to say that the state's important interests in education, in order to be constitutional, must undermine sex hierarchy and never reinforce hierarchy or promote sex stereotypes that foster hierarchy. Nonetheless, private single-sex higher education continues without constitutional challenge. However, in the case of basic education for children, the state has taken on the responsibility for its provision but has delegated its authority to parents who home school their children, thus making the parents state actors and subject to the equal protection requirement of the Fourteenth Amendment. Also relevant to the fundamental nature of education and the state's responsibility for it is the contrast between Griffin v. County School Board of Prince Edward County (1964), where the Supreme Court refused to countenance the closing of the public schools to avoid integration, and Palmer v. Thompson (1971), where closing the public swimming pools was permitted.

Conclusion

McMullen (2002) offers a remedy for the deficiencies such as those identified above: fair-minded, fairly minimal regulation aimed at the minority of bad actors in the home schooling community. She proposes that those who wish to home school be required to file an application with the local school district, to be approved automatically if the name, address, and proof of vaccination are in order (and the parents have not been convicted of child abuse). Age-appropriate competence testing in reading and mathematics would be mandatory in order to maintain home school status, preferably exams like the Iowa Basic Skills Test, which is hard to teach to. There should be independent monitoring of home schooling, since local school personnel could be said to have a conflict of interest. Teacher certification or more detailed regulation would be difficult to institute and not necessarily a solution, leaving only the ultimate remedy of litigation for the very few children who may be being deprived of basic rights.

What can be done to assure adequate training in mathematics and sciences specifically for home-schooled girls? In particular, who has the responsibility and the ability to secure legal protection for them? It can be argued that the state constitutional assumption of education as a state function implies that there is in fact recourse when parents take on the functions of the state.

Will concern for children's rights eventually swing the pendulum of home schooling back to universal substantial regulation if not to outright prohibition? This seems unlikely in the current political climate despite the attention being given to education as a national resource and to the necessity to provide a larger, more diverse, and better-trained STEM workforce if America's global position is not to decline.

Whether home schooling contributes to the goal of better-educated students on the whole or proportionally to the number of students in nationally important STEM disciplines remains to be determined.

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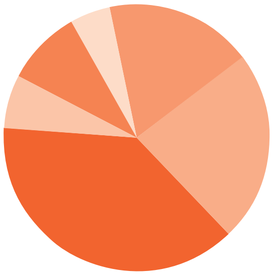


Preliminary Report on the 2010-2011 New Doctoral Recipients

Richard Cleary, James W. Maxwell, and Colleen Rose

This report presents a statistical profile of recipients of doctoral degrees awarded by departments in the mathematical sciences at universities in the United States during the period July 1, 2010, through June 30, 2011. All information in the report was provided over the summer and early fall of 2011 by the departments that awarded the degrees. The report includes a preliminary analysis of the fall 2011 employment plans of 2010–2011 doctoral recipients and a demographic profile summarizing characteristics of citizenship status, sex, and racial/ethnic group. This preliminary report will be updated by the Report on the 2010–2011 New Doctorates to reflect subsequent reports of additional 2010-2011doctoral recipients from the departments that did not respond in time for this report, along with additional information provided by the doctoral recipients themselves. A list of the nonresponding departments is on page 530.

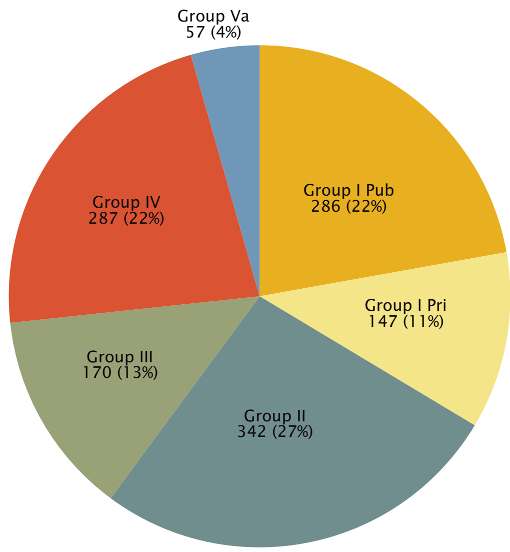
Detailed information, including tables which traditionally appeared in this report, is available on the AMS website at `www.ams.org/annual-survey/`.

Doctoral Degrees Awarded

The preliminary count for new Ph.D.s awarded is 1,289. This is down 186 from last year's preliminary count of 1,475. However, based on the data collected it appears the number of Ph.D.s being awarded is still increasing. The 206 departments that have responded so far this year report awarding 1,289 doctoral degrees. Last year the same set of departments reported awarding 1,225, a 5.2% increase. This strongly suggests that when the reports from the 96 departments that have yet to respond are included, the final count of 2010-2011 newdocs will exceed the record setting figure of 1,632 reported last year. (See page 299 for a list of departments still to respond.)

26% (336) of the new Ph.D.s had a dissertation in statistics/biostatistics, followed by algebra/number theory and applied mathematics with 16% (211) and 15% (198), respectively.

Figure A.1: Number and Percentage of Degrees Awarded by Department Groupings

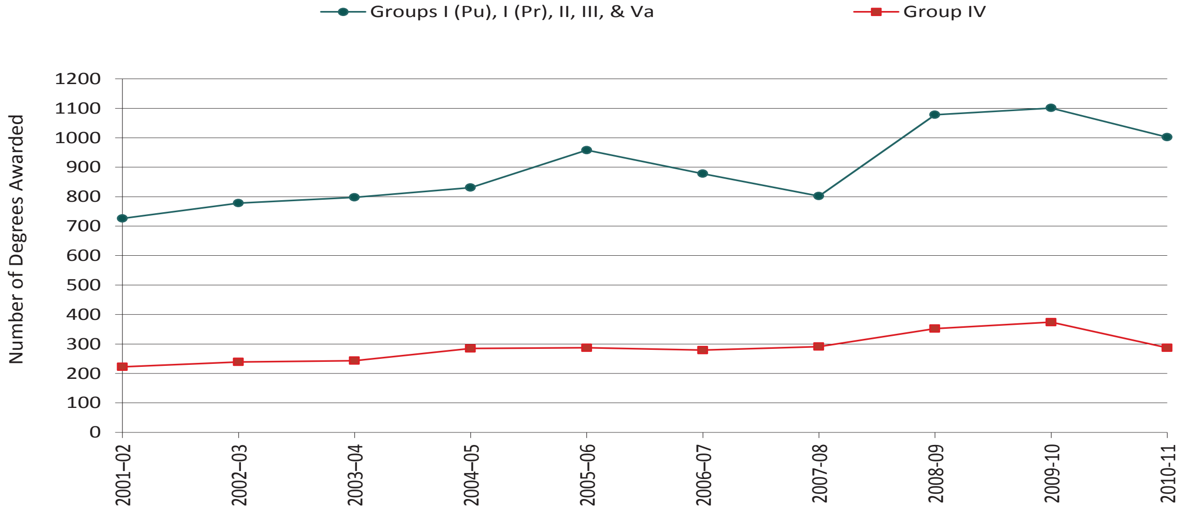


Total Degrees Awarded: 1,289

Richard Cleary is a professor in the Department of Mathematical Sciences at Bentley University. James W. Maxwell is AMS associate executive director for special projects. Colleen A. Rose is AMS survey analyst.

Doctoral Degrees Awarded

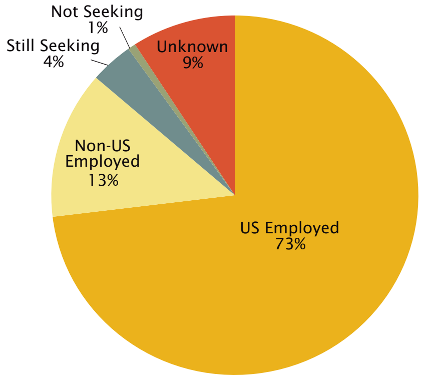
Figure A.2: New Doctoral Degrees Awarded by Combined Groups, Preliminary Counts



Employment of New Doctoral Recipients

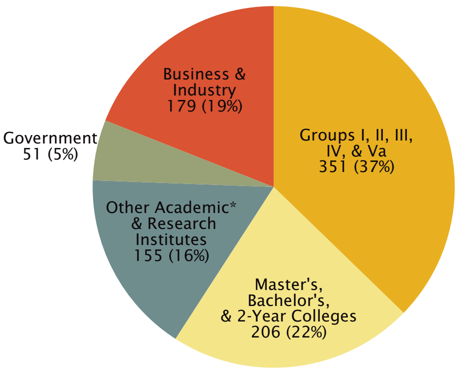
The overall preliminary unemployment rate is 4.9%, down from 9.9% last year. The employment plans are known for 1,172 of the 1,289 new doctoral recipients. The number of new doctoral recipients employed in the U.S. is 942, down 66 from last year's preliminary number. Employment in the U.S. decreased in all categories except "Master's, Bachelor's, & 2-Year Colleges" and "Business and Industry" which increased 3% and 2%, respectively. The number of new Ph.D.s taking positions in government has dropped to 51 this year. Academic hiring of new doctoral recipients decreased to 712, compared to 768 last year.

Figure E.1: Employment Status



- 9% of new Ph.D.s are working at the institution which granted their degree, the same as last year.
- 54% (511) of those employed in the U.S. are U.S. citizens, up from 53% last year.
- 75% (432) of non-U.S. citizens known to have employment are employed in the U.S.; the remaining 99 non-U.S. citizens are either employed outside of the U.S. or unemployed.
- 13% of new Ph.D.s are employed outside of the U.S. compared to 11% last year.

Figure E.2: U.S. Employed by Type of Employer



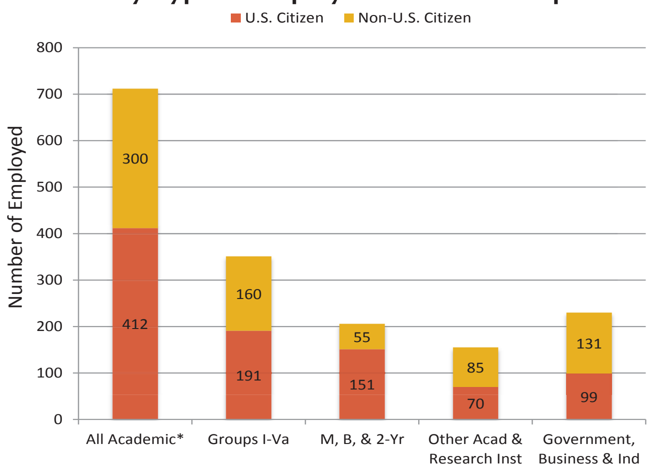
Total U.S. Employed: 942

*Other Academic consists of departments outside the mathematical sciences including numerous medical related units.

- Positions in Business & Industry increased slightly for new recipients from Groups I–II, and decreased slightly for new recipients from Groups III–Va.

Employment of New Doctoral Recipients

Figure E.3: Employment in the U.S. by Type of Employer and Citizenship



*Includes Groups I-Va, M, B, 2-Yr, other academic and research institutes/nonprofit.

Looking at U.S. citizens whose employment status is known:

- 86% (521) are employed in the U.S., of these:
  - 38% are employed in Ph.D.-granting departments
  - 44% are employed in all other academic positions
  - 19% are employed in government, business and industry positions

Figure E.4: New Ph.D.s Hired into Postdocs by Department Groupings

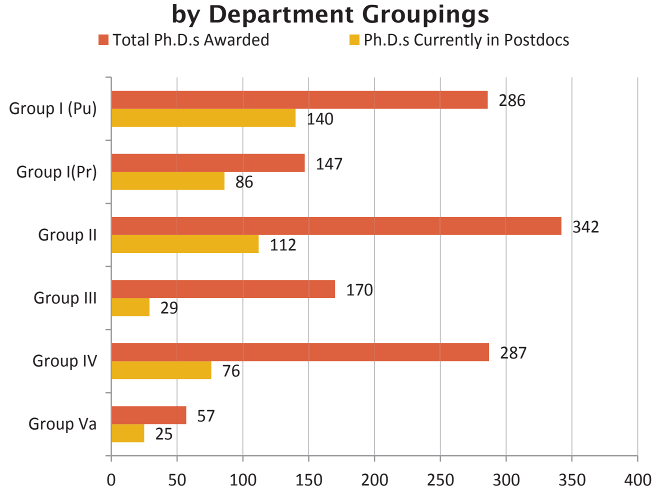
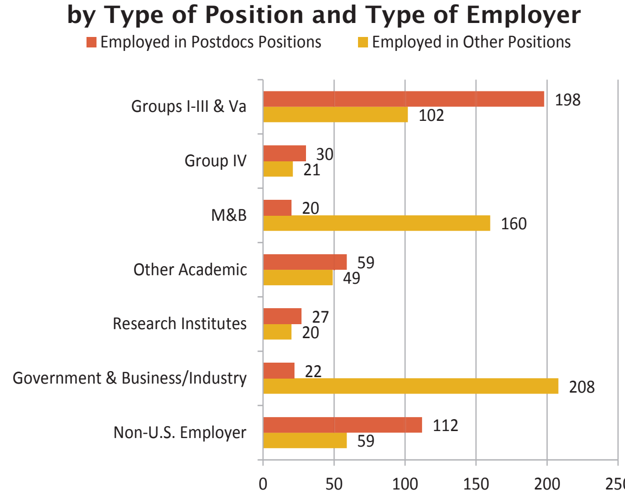


Figure E.5: New Ph.D. Employment by Type of Position and Type of Employer

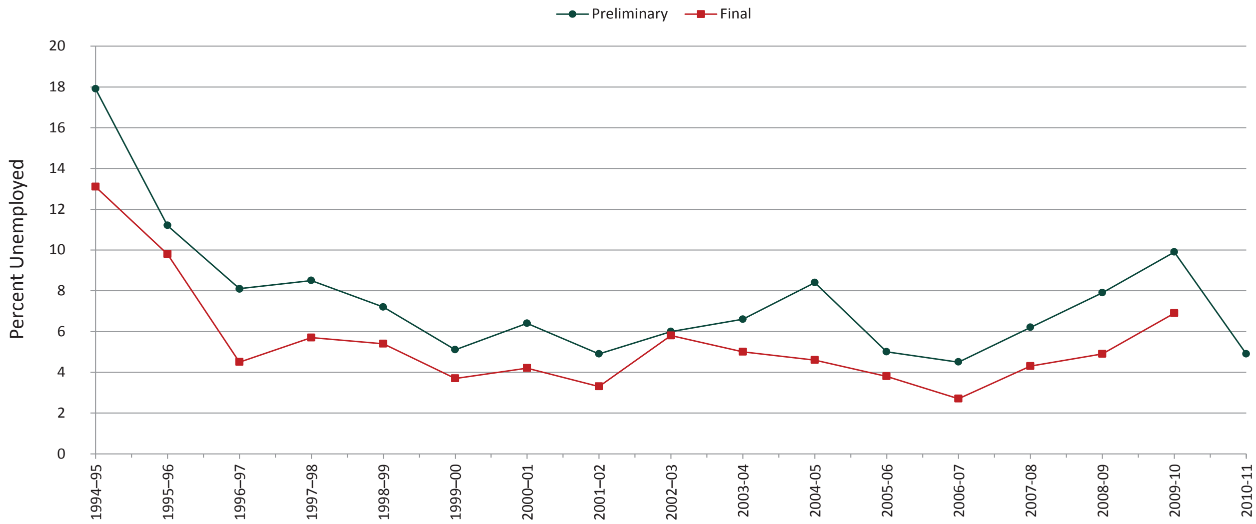


- 36% (468) of the new Ph.D.s are reported to be in postdoc positions.
- 24% of the new Ph.D.s in postdoc positions are employed outside the U.S.
- 47% of the new Ph.D.s having U.S. academic employment are in postdocs; the same as last year.
- 65% of the new Ph.D.s employed in Groups I-Va are in postdoc positions, 32% of these postdocs received their Ph.D.s from Group I (Pu) institutions.
- 59% of the new Ph.D.s in Group I (Pr) are employed in postdocs, while only 17% of new Ph.D.s awarded by Group III are in postdocs; last year figures were 55% and 13%, respectively.

- Unemployment among those whose employment status is known is 4.9%, down from 9.9% for fall 2010.
- Group I (Pu) reported highest unemployment at 8.5%.
- Group I (Pr) reported the lowest unemployment at 2.8%.
- 6.4% of U.S. citizens are unemployed, compared to 10.3% in fall 2010.
- 3.1% of non-U.S. citizens are unemployed; the rates by visa status are 3.6% for those with a temporary visa and no non-U.S. citizens with a permanent visa were reported as unemployed.

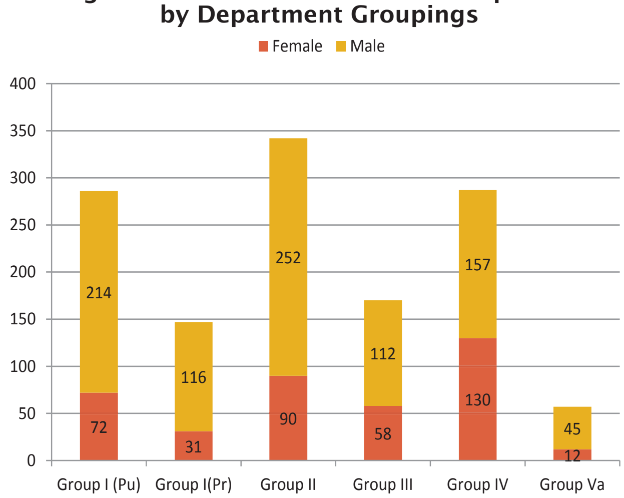
Employment of New Doctoral Recipients

Figure E.6: Percentage of New Doctoral Recipients Unemployed 1994-2011



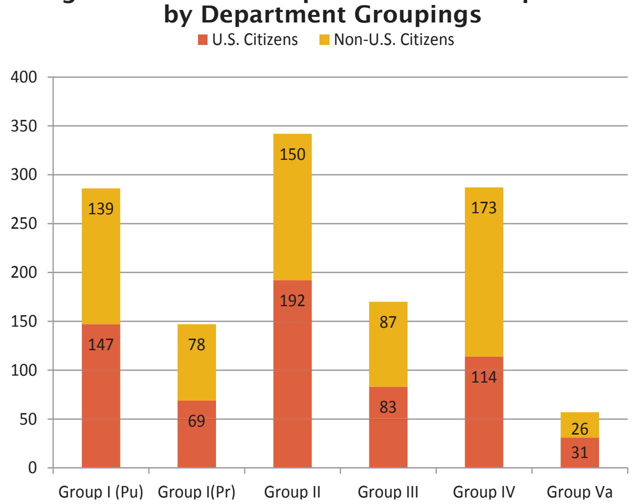
Demographics of New Doctoral Recipients

Figure D.1: Gender of Doctoral Recipients by Department Groupings



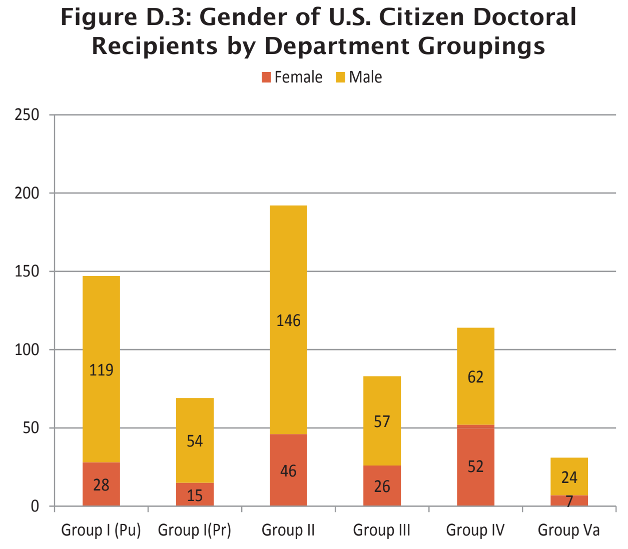
- Females account for 30% (393) of the 1,289 Ph.D.s; last year's figure was 32%.

Figure D.2: Citizenship of Doctoral Recipients by Department Groupings

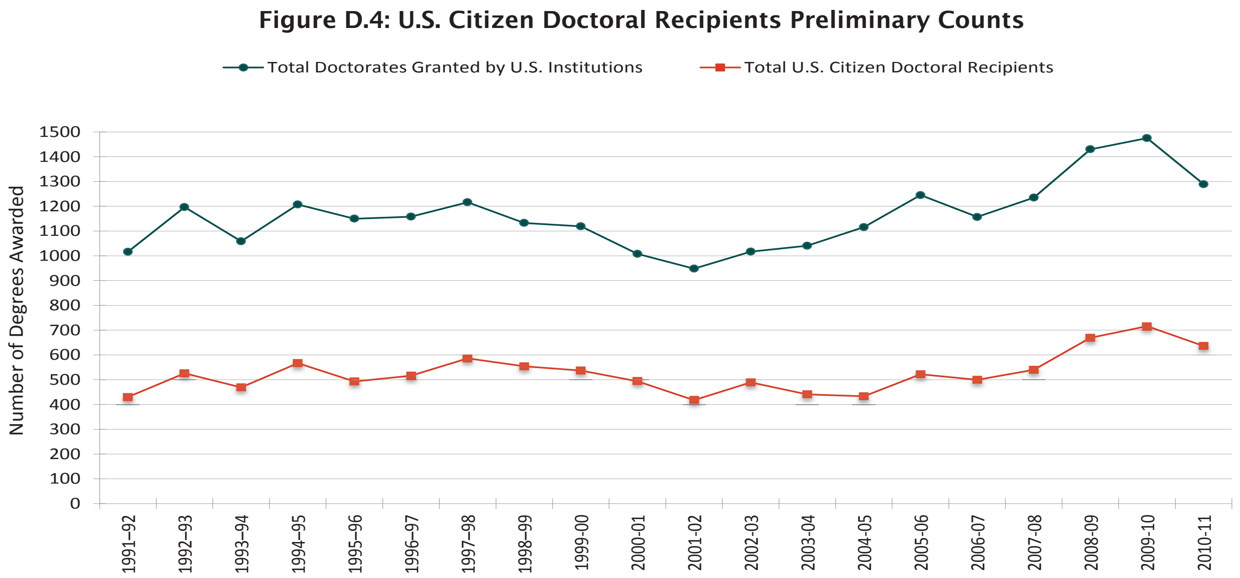


- 49% (636) of Ph.D.s are U.S. citizens; last year's figure was also 49%.

Demographics of New Doctoral Recipients



- 27% (174) of the U.S. citizens are female; last year's figure was 30%.
- Among the U.S. citizens: 4 are American Indian or Alaska Native, 32 are Asian, 21 are Black or African American, 11 are Hispanic or Latino, 3 are Native Hawaiian or Other Pacific Islander, 552 are White, and 12 are of unknown race/ethnicity.



Female New Doctoral Recipients

For the first time in three years, the proportion of female new doctoral recipients has dropped slightly to 30% (393), based on preliminary counts. For 2008 – 2010 this proportion was 32% and the number of females receiving Ph.D.s increased from 388 in 2008 to 472 in 2010. The unemployment rate for females is 4%, compared to 5.4% for males and 4.9% overall.

Figure F.1: Females as a Percentage of New Doctoral Recipients Produced by and Hired by Department Groupings

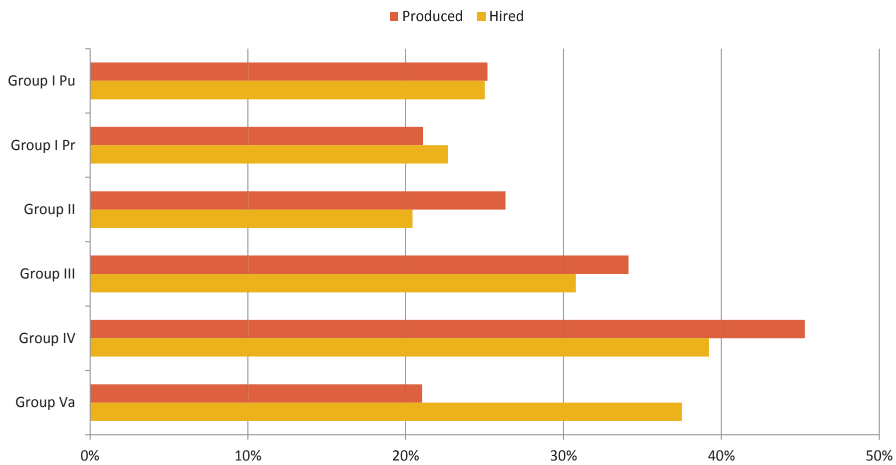
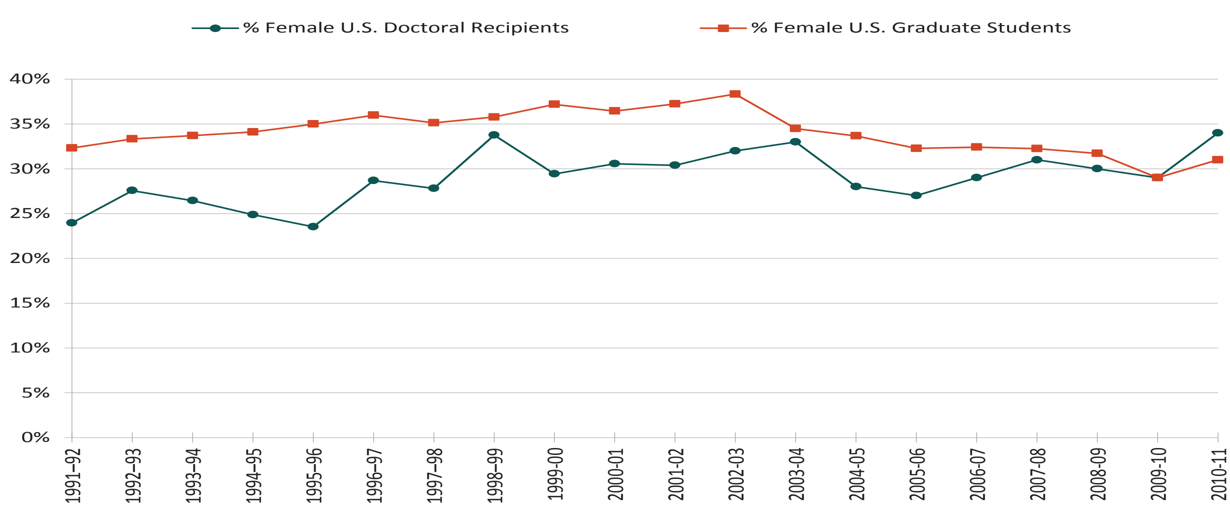


Table F.1: Number of Female New Doctoral Recipients Produced by and Hired by Department Groupings

	Females Produced	Females Hired
Group I Pu	72	18
Group I Pr	31	17
Group II	90	19
Group III	58	16
Group IV	130	20
Group Va	12	3

- 36% of those hired by Group B were women (down from 44% last year) and 36% of those hired by Group M were women (down from 43% last year).
- 26% of those reporting having postdoc positions (468) are women.
- 60% of the women employed in Groups I-Va are in postdoc positions.

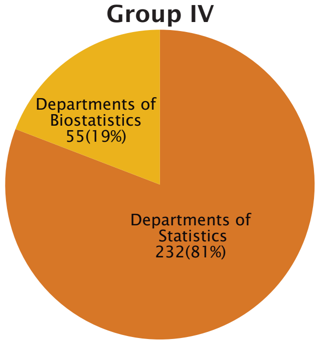
Figure F.2: Females as a Percentage of U.S. Citizen Doctoral Recipients



Ph.D.s Awarded in Group IV (Statistics/Biostatistics)

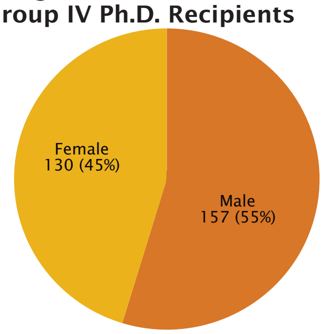
This section contains information about new doctoral recipients in Group IV. Group IV produced 287 new doctorates, of which all but 4 had dissertations in statistics/biostatistics. This is a 6% decrease over the number reported for fall 2010 of 374. In addition, Groups I–III and Va combined had 53 Ph.D. recipients with dissertations in statistics. In Group IV, 114 (40%) of the new doctoral recipients are U.S. citizens (while in the other groups 52% are U.S. citizens). While the unemployment rate for new Ph.D.s with dissertations in statistics or probability has decreased to 3.4%, the unemployment among the Group IV new Ph.D.s is 2.9%.

Figure S.1: Ph.D.s Awarded in Group IV



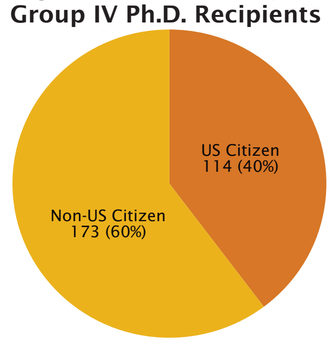
- 22% of all Ph.D.s awarded were in Group IV.
- Females account for 43% of statistics and 56% of biostatistics Ph.D.s awarded.

Figure S.2: Gender of Group IV Ph.D. Recipients



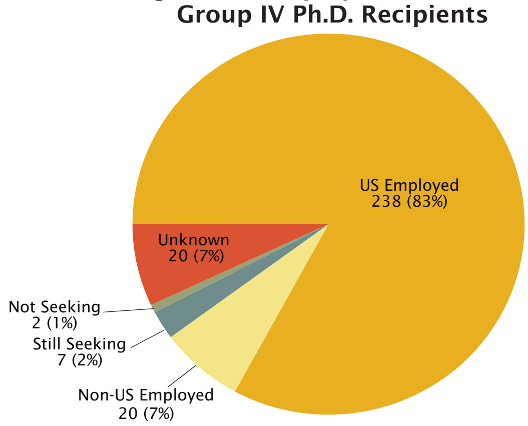
- Females accounted for 45% of the 287 Ph.D.s in statistics/ biostatistics, compared to all other groups combined, where 26% (263) are female.

Figure S.3: Citizenship of Group IV Ph.D. Recipients



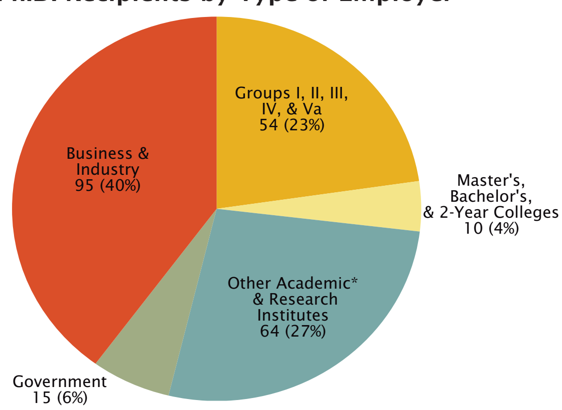
- 46% of of Group IV U.S. citizens are females, while in all other groups 23% are females.

Figure S.4: Employment Status of Group IV Ph.D. Recipients



- 2.9% of Group IV Ph.D.s are unemployed compared to 5.6% among all other groups. This is down from 4.2% last year.
- Unemployment among new Ph.D.s with dissertations in statistics/probability is 3.1%, down from 6%. Among all other dissertation groupings 4.6% are unemployed.

Figure S.5: U.S. Employed Group IV Ph.D. Recipients by Type of Employer



*Other Academic consists of departments outside the mathematical sciences including numerous medical related units.

- 33% of Group IV Ph.D.s are employed in Business/Industry, compared to 12% in all other groups.
- 39% of those hired by Group IV were females, compared to 24% in all other groups.

Other Information

Survey Response Rates

Doctorates Granted
Departmental Response Rates

Group I (Pu)	22 of 25 including	0 with no degrees
Group I (Pr)	14 of 23 including	0 with no degrees
Group II	49 of 56 including	1 with no degrees
Group III	57 of 82 including	9 with no degrees
Group IV	74 of 93 including	3 with no degrees
Statistics	33 of 58 including	2 with no degrees
Biostatistics	17 of 35 including	1 with no degrees
Group Va	16 of 24 including	1 with no degrees

Previous Annual Survey Reports

The 2009 First, Second, and Third Annual Survey Reports were published in the *Notices of the AMS* in the February, August, and November 2009 issues, respectively. These reports and earlier reports, as well as a wealth of other information from these surveys, are available on the AMS website at www.ams.org/annual-survey/survey-reports.

Acknowledgements

The Annual Survey attempts to provide an accurate appraisal and analysis of various aspects of the academic mathematical sciences scene for the use and benefit of the community and for filling the information needs of the professional organizations. Every year, college and university departments in the United States are invited to respond. The Annual Survey relies heavily on the conscientious efforts of the dedicated staff members of these departments for the quality of its information. On behalf of the Data Committee and the Annual Survey Staff, we thank the many secretarial and administrative staff members in the mathematical sciences departments for their cooperation and assistance in responding to the survey questionnaires.

Other Sources of Data

Visit the AMS website at www.ams.org/annual-survey/other-sources for a listing of additional sources of data on the mathematical sciences.

Group Descriptions

Group I is composed of 48 departments with scores in the 3.00–5.00 range. Group I Public and Group I Private are Group I departments at public institutions and private institutions, respectively.

Group II is composed of 56 departments with scores in the 2.00–2.99 range.

Group III contains the remaining U.S. departments reporting a doctoral program, including a number of departments not included in the 1995 ranking of program faculty.

Group IV contains U.S. departments (or programs) of statistics, biostatistics, and biometrics reporting a doctoral program.

Group Va is applied mathematics/applied science.

Group M contains U.S. departments granting a master's degree as the highest graduate degree.

Group B contains U.S. departments granting a baccalaureate degree only.

Listings of the actual departments which compose these groups are available on the AMS website at www.ams.org/annual-survey/groups_des.

About the Annual Survey

The Annual Survey series, begun in 1957 by the American Mathematical Society, is currently under the direction of the Data Committee, a joint committee of the American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society of Industrial and Applied Mathematics. The current members of this committee are Pam Arroway, Richard Cleary (chair), Steven R. Dunbar, Susan Geller, Abbe H. Herzig, Ellen Kirkman, Joanna Mitro, James W. Maxwell (ex officio), Bart S. Ng, Douglas Ravanel, and Marie Vitulli. The committee is assisted by AMS survey analyst Colleen A. Rose. In addition, the Annual Survey is sponsored by the Institute of Mathematical Statistics. Comments or suggestions regarding this Survey Report may be directed to the committee.

Doctoral Degrees Not Yet Reported

The following mathematical sciences, statistics, biostatistics, and applied mathematics departments have not yet responded with their doctoral degrees awarded. Every effort will be made to collect this information for inclusion in the New Doctoral Recipients Report which will be published in the August 2012 issue of *Notices of the AMS.*

Departments yet to respond can obtain copies of the Doctorates Granted survey forms on the AMS website at `www.ams.org/annual-survey/surveyforms`, by sending email to `ams-survey@ams.org`, or by calling 1-800-321-4267, ext. 4189.

Group I (Public)

Graduate Center, City University of New York
University of California, Los Angeles
University of Minnesota-Twin Cities

Group I (Private)

Brandeis University
California Institute of Technology
Johns Hopkins University, Baltimore
New York University, Courant Institute
Princeton University
Rensselaer Polytechnic Institute
Stanford University
University of Notre Dame
Yale University

Group II

Clemson University
Dartmouth College
Polytechnic Institute of New York University
Texas Tech University
University of California, Riverside
University of Miami

Group III

Bryn Mawr College
College of William & Mary
Colorado School of Mines
Delaware State University
Drexel University
George Mason University
Indiana University-Purdue University Indianapolis
Marquette University
Michigan Technical University
Missouri University of Science and Technoogy
Northern Illinois University
Oakland University
Oklahoma State University
Texas Christian University
University of Arkansas at Fayetteville
University of Central Florida
University of Delaware
University of Kansas
University of Missouri-Kansas City
University of Missouri-St. Louis
University of Nevada
University of New Mexico
University of North Carolina at Greensboro
University of Toledo
University of Vermont
Utah State University

Group IV (Statistics)

Baylor University
Carnegie Mellon University
Colorado State University
Cornell University
George Washington University
Harvard University
Kansas State University
Michigan State University
New York University, Stern School of Business
North Dakota State University, Fargo
Northwestern University
Rutgers University-New Brunswick
University of Alabama-Tuscaloosa
University of California, Berkeley
University of California, Los Angeles
University of California, Riverside
University of California, Santa Barbara
University of Florida
University of Iowa
University of Kentucky
University of Michigan
University of Missouri-Columbia
University of North Carolina at Chapel Hill
Western Michigan University
Yale University

Group IV (Biostatistics)

Boston University School of Public Health
Case Western Reserve University
Columbia University
Cornell University
Louisiana State University, Health Science Center
Medical College of Wisconsin
The University of Albany, SUNY
Medical University of South Carolina
Tulane University
University of California, Los Angeles
University of Cincinnati, Medical College
University of Colorado, Denver
University of Massachusetts, Amherst
University of Michigan
University of North Carolina at Chapel Hill
University of Pittsburgh
University of South Carolina
Virginia Commonwealth University

Group Va (Applied Mathematics)

Arizona State University
Illinois Institute of Technology
Louisiana Technology University
Princeton University
State University of New York at Stony Brook
University of California-Merced
University of Louisville
University of Texas at Austin

WHAT IS...

Data Mining

Mauro Maggioni

Data collected from a variety of sources has been accumulating rapidly. Many fields of science have gone from being data-starved to being data-rich and needing to learn how to cope with large data sets. The rising tide of data also directly affects our daily lives, in which computers surrounding us use data-crunching algorithms to help us in tasks ranging from finding the quickest route to our destination considering current traffic conditions to automatically tagging our faces in pictures; from updating in near real time the prices of sale items to suggesting the next movie we might want to watch.

The general aim of *data mining* is to find useful and interpretable patterns in data. The term can encompass many diverse methods and therefore means different things to different people. Here we discuss some aspects of data mining potentially of interest to a broad audience of mathematicians.

Assume a sample data point x_i (e.g., a picture) may be cast in the form of a long vector of numbers (e.g., the pixel intensities in an image): we represent it as a point in $\mathbb{R}^D$. Two types of related goals exist. One is to detect patterns in this set of points, and the other is to predict a function on the data: given a training set $(x_i, f(x_i))_i$, we want to predict f at points outside the training set. In the case of text documents or webpages, we might want to automatically label each document as belonging to an area of research; in the case of pictures, we might want to recognize faces; when suggesting the next movie to watch given past ratings of movies by a viewer, f consists of ratings of unseen movies. Typically, x_i is noisy (e.g., noisy pixel values), and so is $f(x_i)$ (e.g., mislabeled samples in the training set).

Of course mathematicians have long concerned themselves with high-dimensional problems. One example is studying solutions of PDEs as functions in infinite-dimensional function spaces and performing efficient computations by projecting the problem onto low-dimensional subspaces (via discretizations, finite elements, or operator compression) so that the reduced problem may be numerically solved on a computer. In the case of solutions of a PDE, the model for the data is specified: a lot of information about the PDE is known, and that information is exploited to predict the properties of the data and to construct low-dimensional projections. For the digital data discussed above, however, typically we have little information and poor models. We may start with crude models, measure their fitness to the data and predictive ability, and, those being not satisfactory, improve the models. This is one of the key processes in statistical modeling and data mining. It is not unlike what an applied mathematician does when modeling a complex physical system: he may start with simplifying assumptions to construct a "tractable" model, derive consequences of such a model (e.g., properties of the solutions) analytically and/or with simulations, and compare the results to the properties exhibited by the real-world physical system. New measurements and real-world simulations may be performed, and the fitness of the model reassessed and improved as needed for the next round of validation. While physics drives the modeling in applied mathematics, a new type of intuition, built on experiences in the world of high-dimensional data sets rather than in the world of physics, drives the intuition of the

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mathematician set to analyze high-dimensional data sets, where "tractable" models are geometric or statistical models with a small number of parameters.

One of the reasons for focusing on reducing the dimension is to enable computations, but a fundamental motivation is the so-called *curse of dimensionality*. One of its manifestations arises in the approximation of a 1-Lipschitz function on the unit cube, $f : [0,1]^D \to \mathbb{R}$ satisfying $|f(x) - f(y)| \leq ||x - y||$ for $x, y \in [0,1]^D$. To achieve uniform error ϵ, given samples $(x_i, f(x_i))$, in general one needs at least one sample in each cube of side ϵ, for a total of ϵ^{-D} samples, which is too large even for, say, $\epsilon = 10^{-1}$ and $D = 100$ (a rather small dimension in applications). A common assumption is that either the samples x_i lie on a low-dimensional subset of $[0,1]^D$ and/or f is not simply Lipschitz but has a smoothness that is suitably large, depending on D (see references in [3]). Taking the former route, one assumes that the data lies on a low-dimensional subset in the high-dimensional ambient space, such as a low-dimensional hyperplane or unions thereof, or low-dimensional manifolds or rougher sets. Research problems require ideas from different areas of mathematics, including geometry, geometric measure theory, topology, and graph theory, with their tools for studying manifolds or rougher sets; probability and geometric functional analysis for studying random samples and measures in high dimensions; harmonic analysis and approximation theory, with their ideas of multiscale analysis and function approximation; and numerical analysis, because we need efficient algorithms to analyze real-world data.

As a concrete example, consider the following construction. Given n points $\{x_i\}_{i=1}^n \subset \mathbb{R}^D$ and $\epsilon > 0$, construct $W_{ij} = \exp(-\frac{||x_i - x_j||^2}{2\epsilon})$, $D_{ii} = \sum_j W_{ij}$, and the Laplacian matrix $L = I - D^{-\frac{1}{2}} W D^{-\frac{1}{2}}$ on the weighted graph G with vertices $\{x_i\}$ and edges weighted by W. When x_i is sampled from a manifold $\mathcal{M}$ and n tends to infinity, L approximates (in a suitable sense) the Laplace-Beltrami operator on $\mathcal{M}$ [2], which is a completely intrinsic object. The random walk on G, with transition matrix $P = D^{-1}W$, approximates Brownian motion on $\mathcal{M}$. Consider, for a time $t > 0$, the so-called diffusion distance $d_t(x, y) := ||P^t(x, \cdot) - P^t(y, \cdot)||_{L^2(G)}$ (see [2]). This distance is particularly useful for capturing clusters/groupings in the data, which are regions of fast diffusion connected by bottlenecks that slow diffusion. Let $1 = \lambda_0 \geq \lambda_1 \geq \cdots$ be the eigenvalues of P and φ_i be the corresponding eigenvectors (φ_0, when G is a web graph, is related to Google's pagerank). Consider a diffusion map Φ_d^t that embeds the graph in Euclidean space, where $\Phi_d^t(x) := (\sqrt{\lambda_1^t}\varphi_1(x), \ldots, \sqrt{\lambda_d^t}\varphi_d(x))$, for some $t > 0$ [2]. One can show that the Euclidean distance between $\Phi_d^t(x)$ and $\Phi_d^t(y)$ approximates $d_t(x, y)$, the diffusion distance at time scale t between x and y on the graph G.

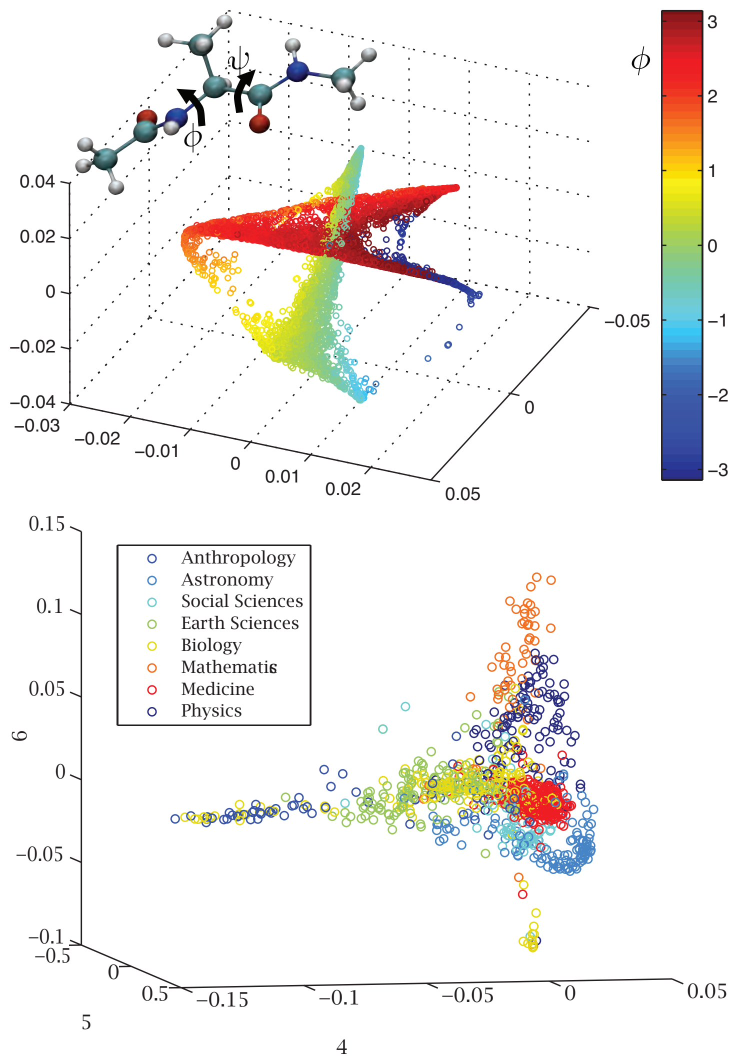

Figure 1. Top: Diffusion map embedding of the set of configurations of a small biomolecule (alanine dipeptide) from its 36-dimensional state space. The color is one of the dihedral angles φ, ψ of the molecule, known to be essential to the dynamics [4]. This is a physical system where (approximate) equations of motion are known, but their structure is too complicated and the state space too high-dimensional to be amenable to analysis. Bottom: Diffusion map of a data set consisting of 1161 *Science News* articles, each modeled by a 1153-dimensional vector of word frequencies, embedded in a low-dimensional space with diffusion maps, as described in the text and in [2].

In Figure 1 we apply this technique to two completely different data sets. The first one is a set of configurations of a small peptide, obtained by a molecular dynamics simulation: a point $x_i \in \mathbb{R}^{12\times 3}$ contains the coordinates in $\mathbb{R}^3$ of the 12 atoms in the alanine dipeptide molecule (represented as an inset in Figure 1). The forces between the atoms in the molecule constrain the trajectories to lie close to low-dimensional sets in the 36-dimensional state space. In Figure 1 we apply the construction above[1] and represent the diffusion map embedding of the configurations collected [4]. The second one is a set of text documents (articles from *Science News*), each represented as a $\mathbb{R}^{1153}$ vector whose kth coordinate is the frequency of the kth word in a 1153-word dictionary. The diffusion embedding in low dimensions reveals even lower-dimensional geometric structures, which turn out to be useful for understanding the dynamics of the peptide considered in the first data set and for automatically clustering documents by topic in the case of the second data set. Ideas from probability (random samples), harmonic analysis (Laplacian), and geometry (manifolds) come together in these types of constructions.

This is only the beginning of one of many research avenues explored in the last few years. Many other exciting opportunities exist, for example the study of stochastic dynamic networks, where a sample is a network and multiple samples are collected in time: quantifying and modeling change requires introducing sensible and robust metrics between graphs.

Further reading: [5, 3, 1] and the references therein.

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[1] *We use here a slightly different definition of the weight matrix W, which uses distances between molecular configurations up to rigid affine transformations, instead of Euclidean distances.*

Applications of Near Sets

Jim Peters and Som Naimpally

Near sets are disjoint sets that resemble each other. Resemblance is determined by considering set descriptions defined by feature vectors (n-dimensional vectors of numerical features that represent characteristics of objects such as digital image pixels). Near sets are useful in solving problems based on human perception [44, 76, 49, 51, 56] that arise in areas such as image analysis [52, 14, 41, 48, 17, 18], image processing [41], face recognition [13], ethology [63], as well as engineering and science problems [53, 63, 44, 19, 17, 18].

As an illustration of the degree of nearness between two sets, consider an example of the Henry color model for varying degrees of nearness between sets [17, §4.3]. The two pairs of ovals in Figures 1 and 2 contain colored segments. Each segment in the figures corresponds to an equivalence class where all pixels in the class have matching descriptions, i.e., pixels with matching colors. Thus, the ovals in Figure 1 are closer (more near) to each other in terms of their descriptions than the ovals in Figure 2. It is the purpose of this article to give a bird's-eye view of recent developments in the study of the nearness of sets.

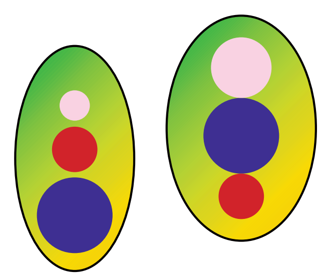

Figure 1. Descriptively, very near sets.

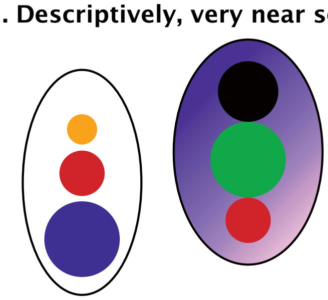

Figure 2. Descriptively, minimally near sets.

Brief History of Nearness

It has been observed that the simple concept of *nearness* unifies various concepts of topological structures [21] inasmuch as the category **Near** of all nearness spaces and nearness-preserving maps contains categories $\mathbf{Top}_s$ (symmetric topological spaces and continuous maps [3]), **Prox** (proximity spaces and δ-maps [8, 66]), **Unif** (uniform spaces and uniformly continuous maps [75, 71]), and **Cont** (contiguity spaces and contiguity maps [23]) as embedded full subcategories [21, 57]. The notion of nearness in mathematics and the more general notion of resemblance can be traced back to J. H. Poincaré, who introduced sets of similar sensations (nascent tolerance classes) to represent the results of G. T. Fechner's sensation sensitivity

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experiments [9] and a framework for the study of resemblance in representative spaces as models of what he termed *physical continua* [61, 58, 59].

The elements of a physical continuum (pc) are sets of sensations. The notion of a pc and various representative spaces (tactile, visual, motor spaces) were introduced by Poincaré in an 1894 article on the mathematical continuum [61], an 1895 article on space and geometry [58], and a compendious 1902 book on science and hypothesis [59] followed by a number of elaborations, e.g., [60]. The 1893 and 1895 articles on continua (Pt. 1, ch. II) as well as representative spaces and geometry (Pt. 2, ch. IV) are included as chapters in [59]. Later, F. Riesz introduced the concept of proximity or nearness of pairs of sets at the ICM in Rome in 1908 (ICM 1908) [64].

During the 1960s E. C. Zeeman introduced tolerance spaces in modeling visual perception [78]. A. B. Sossinsky observed in 1986 [67] that the main idea underlying tolerance space theory comes from Poincaré, especially [58] (Poincaré was not mentioned by Zeeman). In 2002, Z. Pawlak and J. Peters considered an informal approach to the perception of the nearness of physical objects, such as snowflakes, that was not limited to spatial nearness [42]. In 2006, a formal approach to the descriptive nearness of objects was considered by J. Peters, A. Skowron, and J. Stepaniuk [54, 55] in the context of proximity spaces [40, 35, 38, 22]. In 2007, descriptively near sets were introduced by J. Peters [46, 45], followed by the introduction of tolerance near sets [43, 47].

Nearness of Sets

The adjective *near* in the context of near sets is used to denote the fact that observed feature value differences of distinct objects are small enough to be considered indistinguishable, i.e., within some tolerance. The exact idea of closeness or "resemblance" or of "being within tolerance" is universal enough to appear, quite naturally, in almost any mathematical setting (see, e.g., [65]). It is especially natural in mathematical applications: practical problems, more often than not, deal with approximate input data and only require viable results with a tolerable level of error [67].

Frigyes Riesz, 1880–1956

The words *near* and *far* are used in daily life and it was an incisive suggestion of F. Riesz [64] to make these intuitive concepts rigorous. He introduced the concept of nearness of pairs of sets at the ICM 1908. This concept is useful in simplifying teaching calculus and advanced calculus. For example, the passage from an intuitive definition of continuity of a function at a point to its rigorous epsilon-delta definition is sometimes difficult for teachers to explain and for students to understand. Intuitively, continuity can be explained using nearness language, i.e., a function $f : \mathbb{R} \to \mathbb{R}$ is continuous at a point c, provided points $\{x\}$ near c go into points $\{f(x)\}$ near $f(c)$. Using Riesz's idea, this definition can be made more precise and its contrapositive is the familiar epsilon-delta definition.

Bringing *near* into the discussion of continuity makes the transition simpler from the intuitive level of continuity to the rigorous level of continuity. This approach has been successfully used in the classroom (see, e.g., [6, 22, 38]). This approach can also be used in teaching general topology [35, 39], function spaces, hyperspaces, lattices of closed sets, point-free geometries [22, 5], analysis and topology [38, 35, 5]. Point-free topology focuses on open sets rather than points of a space and deals with lattices of open sets called *frames* and their homomorphisms [57, p. 234ff]. Riesz's idea was to establish a natural framework for defining accumulation points that are derived from enchained sets. To formulate the notion of an accumulation point, Riesz proposed an axiomatization of the closeness between sets called *enchainment*, which is proximity *ante litteram* [5, §I]. Riesz chose enchainment as a vehicle for topology. In doing so, he shifted the focus from the closeness of points and sets (as in F. Hausdorff [15, 16] and K. Kuratowski [27, 28, 29]) to closeness between sets (as in [72, 38]).

Since 1908 a number of mathematicians, such as Efremovič, Smirnov, Leader, Čech, and others, have developed the theory of proximity spaces (see, e.g., [8, 66, 30, 72, 40, 69, 10, 21, 12], nicely summarized in [5]). As per the dictum of A. Einstein, "the significant problems we face cannot be solved by the same level of thinking that created them", problems in topology and analysis can be solved and generalized by the use of proximity that is at a higher level than topology [38]. Here are two examples.

(1) **Proximal Wallman compactification**. Using ultrafilters, one gets Wallman compactification [73], which is usually given as an exercise in topology texts. H. Wallman uses the family wX of *all* closed ultrafilters of X and assigns a topology on wX (known as Wallman topology) that makes wX compact. X is embedded in wX via the map that takes a point $x \in X$ to the closed ultrafilter $\mathcal{L}_x$ containing $\{x\}$ (for details, see [73, Theorem 2, p. 120] and [24, §5.R]). Thus, Wallman gets just one T_1 compactification of any T_1 space. By replacing *intersection* with *near* and using the resulting bunches and clusters of S. Leader, we get infinitely many compactifications, including

all Hausdorff compactifications. For details, see [38, §3.1, p. 39ff] and [39, §9.6].

(2) **Taimanov extension of continuous functions**. A. D. Taimanov [68] gave necessary and sufficient conditions for the existence of extensions of continuous functions from dense subspaces of topological spaces when the range is compact Hausdorff. That led to various generalizations using special techniques (see, e.g., [11, 36]). The generalized Taimanov theorem obtained via a proximity (nearness) relation includes as special cases all results on this topic. For the details, see [38, §1.3, p. 16ff] and [39, §1.16].

Using Mozzochi's results on symmetric generalized uniformity, Gagrat and Naimpally characterized developable spaces as those which have compatible upper semi-continuous semimetrics [10]. This result was used by Domiaty and Laback in a study of semi-metric spaces in general relativity [7]. This puts S. Hawking's approach to general relativity on a more general mathematical foundation (see [38, §15, p. 163ff] and [35]).

Various Near Sets

From a spatial point of view, nearness (*aka* proximity) is considered a generalization of set intersection. For disjoint sets, a form of nearness set intersection is defined in terms of a set of objects (extracted from disjoint sets) that have similar features within some tolerance (see, e.g., [74, §3]). For example, the ovals in Figure 1 are considered near each other, since these ovals contain pairs of classes that display matching (visually indistinguishable) colors. Next, we give some examples to motivate the theory.

Metric Proximity

In a metric space with a metric d, the metric proximity (denoted δ) is defined as follows. Two sets A and B are near (i.e., $A\ \delta\ B$) if and only if $d(A,B) = \inf\{d(a,b) : a \in A, b \in B\} = 0$. This form of metric proximity was introduced by E. Čech [72, §18.A.2], which Čech writes in terms of a *proximity* induced by d [72, §25.A.4] in a seminar on topology given in Brno between May 1936 and November 1939. Metric proximity provides a motivation for the axioms of a proximity space, where there may not even be a metric.

Topological or Fine Proximity

Let cl$E$ denote the *closure* of A such that

$$x \text{ is in the closure of } E \Leftrightarrow \{x\}\ \delta\ E.$$

Put another way, x lies in the closure of a set E, provided that there are points of E as near as we please to x [4, §1.6, p. 22]. In any topological space, there is an associated fine proximity (denoted δ_0). Two sets A and B are *finely* near if and only if their closures intersect, i.e.,

$$A\ \delta_0\ B \ \Leftrightarrow \mathrm{cl}A \cap \mathrm{cl}B \neq \emptyset.$$

It is easy to see that if δ is a metric proximity, then

$$A\ \delta_0\ B \Leftrightarrow \mathrm{cl}A \cap \mathrm{cl}B \neq \emptyset \Rightarrow \mathrm{cl}A\ \delta\ \mathrm{cl}B \Rightarrow A\ \delta\ B.$$

Proximity Space Axioms

Every proximity induces a unique topology that arises from the nearness of points to sets. On the other hand, a topology may have many associated proximities. Metric proximity and fine proximity in a topological space provide a motivation for the following axioms satisfied by all proximity spaces:

(**Prox**. 1) A and B are near sets implies they are not empty,

(**Prox**. 2) A is near B implies B is near A (symmetry),

(**Prox**. 3) A and B intersect implies A and B are near sets,

(**Prox**. 4) A is near $(B \cup C)$ if and only if A is near B or A is near C.

Most of the literature in topology uses an additional axiom that is a vestigial form of the triangle inequality [34, 40]:

(**Prox**. 5) If A is far from B, there is an $E \subset X$ such that A is far from E and B is far from $X - E$.

Quasi-Proximity

Dropping (**Prox**. 2) (symmetry) gives rise to a quasi-proximity relation [26, §2.5, p. 262ff] and (**Prox**. 4) becomes

(**qProx**. 4) $(B \cup C)$ near A if and only if B is near A or C is near A and A is near $(B \cup C)$ implies A is near B or A is near C.

It has been observed by H.-P. Künzi [26] that the topology $\tau(\delta)$ induced by the quasi-proximity δ on X arises from the closure $\mathrm{cl}_{\tau(\delta)}$ defined by

$$x \in \mathrm{cl}_{\tau(\delta)}A \Leftrightarrow x\ \delta\ A.$$

Alexandroff Spaces and Quasi-Proximity

An Alexandroff space is a topological space such that every point has a minimal neighborhood or, equivalently, an Alexandroff space has a unique minimal base [2]. A topological space is Alexandroff if and only if the intersection of every family of open sets is open. These spaces were introduced by P. Alexandroff in 1937 [1]. Let X be a topological space, and let $A, B \in \mathcal{P}(X)$. In terms of quasi-proximity δ, F. G. Arenas obtained the following result. $A\ \delta\ B$ if and only if $A \cap \mathrm{cl}(B) \neq \emptyset$ is a quasi-proximity compatible with the topology [2, Theorem 4.3]. Also observe that every finite topological space is Alexandroff. Starting in the 1990s, Alexandroff spaces were found to be important in the study of digital topology (see, e.g., [20, 25]).

For Alexandroff spaces considered in the context of near sets, see [76, 77].

Adding Proximity Space Axioms

In the study or use of a proximity space in a problem, an additional appropriate axiom is added relative to the application. For example, in studying the nearness of digital images, one can view an image X as a set of points with distinguishing features such as entropy or color or gray level intensity and introduce some form of tolerance relation that determines image tolerance classes. Let X, Y denote a pair of images and let $\phi : X \to \Re$ be a real-valued function representing an image feature such as an average gray level of subimages. Put $\varepsilon \in [0, \infty)$, and let $x \in X, y \in Y$ denote subimages. Then introduce the description-based tolerance relation $\simeq_{\phi,\varepsilon}$, i.e.,

$$\simeq_{\phi,\varepsilon} = \{(x, y) \in X \times Y : |\phi(x) - \phi(y)| < \varepsilon\}.$$

This leads to

(**Prox**. 6) A and B are near sets $\iff$ there are $x \in A, y \in B$, such that $x \simeq_{\phi,\varepsilon} y$.

If a pair of nonempty sets A, B satisfy (**Prox**. 6), then A and B are termed *tolerance near sets*. Such sets provide a basis for a quantitative approach to evaluating the similarity of objects without requiring object descriptions to be exact (see, e.g., [17]).

Nearness of Pictures

The concept of *nearness* enters as soon as one starts studying digital images (see, e.g., [62, 37, 52, 41, 47]). The digital image of a photograph should resemble, as accurately as possible, the original subject, i.e., an image should be globally close to its source. Since proximity deals with global properties, it is appropriate for this study. The quality of a digital image depends on proximity and this proximity is more general than the one obtained from a metric.

We note here that a digital image of a landscape is made up of a very limited number of points (depending on the sensory array of a camera), whereas the original landscape in a visual field contains many more points than its corresponding digital image. However, from the point of view of *perception*, they are near, depending on the tolerance we choose rather crudely in comparing visual field segments of a real scene with digital image patches (sets of scattered pixels). To make such comparisons work, the requirement that the image should appear as precise as possible as the original is relaxed. And the *precise-match* requirement is replaced by a *similarity* requirement so that a digital image should only remind us (within some tolerance) of the original scene.

Then, for example, a cartoon in a newspaper of a person may be considered near, if parts of a cartoon are similar or if it resembles an original scene. For instance, in Figure 3, if the feature we consider is behavior (e.g., braiding hair), the mother is perceptually near the daughter, since both are braiding hair. And, in Figure 3, the drawing of the mother braiding her daughter's hair is near a familiar scene where a mother is caring for her daughter's hair. This suggests that whether two objects are near or not depends on what is needed.

Figure 3. Chain Reaction, Punch, 1869.

Sufficient Nearness of Sets

The notion of *sufficiently near* appears in N. Bourbaki [4, §2, p. 19] in defining an open set, i.e., a set A is open if and only if for each $x \in A$, all points *sufficiently near* x belong to A.

Moreover, a property holds for all points sufficiently near $x \in A$, provided the property holds for all points in the neighborhood of x. Set F_1 in Figure 4 is an example of an open set represented by a dotted boundary. In fact, sets F_2, F_3 are also examples of open sets (i.e., open neighborhoods of the point x). Bourbaki's original view of *sufficiently near* (denoted δ_ε) is now extended to a relaxed view of the nearness of nonempty sets. This form of proximity relation is useful in considering, for example, a relaxed form of metric proximity in relation to ε-collars of sets [5, §2.2], especially in approach space theory [31, 33, 32, 70, 51, 56], as well as in considering the nearness of pictures.

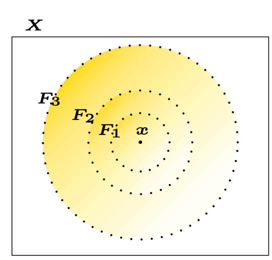

Figure 4. Near open sets.

Let $\varepsilon \in (0, \infty]$. Nonempty sets A, B are considered *sufficiently near* each other if and only if

$$A \, \delta_\varepsilon \, B \Leftrightarrow \inf\{d(a, b) : a \in A, b \in B\} < \varepsilon.$$

Figure 5. A Bit Far, *Punch*, 1845.

Otherwise, sets A, B are *remote* (denoted $\underline{\delta}_\varepsilon$), i.e., *sufficiently apart* or *far* from each other, provided

$$A \ \underline{\delta}_\varepsilon \ B \Leftrightarrow \inf\{d(a,b) : a \in A, b \in B\} \geq \varepsilon.$$

In keeping with the proximity space approach, an axiom is added to cover sufficient nearness. This leads to axiom (**Prox**. 7).

(**Prox**. 7) A and B are sufficiently near sets $\iff$ $\inf\{d(a,b) : a \in A, b \in B\} < \varepsilon$.

In a more general setting, nearness and apartness are considered relative to the gap between collections $\mathcal{A}, \mathcal{B} \in \mathcal{P}^2(X)$ in an approach space [50, 51, 56]. The choice of a particular value of ε is application dependent and is typically determined by a domain expert.

Far Apart and Near Pictures

Apart from the fact that the knight is far from his horse[1] in Figure 5 and the picture of the hairdressers in Figure 3 can be viewed as either near (if we consider the gray-level intensities of the pixels in the two pictures) or far apart (if we consider the behaviors represented by the two pictures). Description-based nearness or apartness between sets depends on the features we select for comparison.

Let $\phi : X \to \mathbb{R}$ be a probe function that extracts a feature value from a picture element. Let $A_i, i \in \{1,2,3,4,5\}$ denote sets of picture elements in A_1 (Figure 1 (ovals)), A_2 (Figure 2 (ovals)), A_3 (Figure 3 (hairdressers)), A_4 (Figure 4 (concentric neighborhoods of point x)), and A_5 (Figure 5 (knight)), respectively, define the description-based sufficient nearness relation $\delta_{\varepsilon,\phi}$ by

$$A \ \delta_{\varepsilon,\phi} \ B \Leftrightarrow \inf\{d(\phi(a),\phi(b)) : a \in A, b \in B\} < \varepsilon,$$

where $d(\phi(a),\phi(b))$ is the standard distance between feature values $\phi(a), \phi(b)$. For Examples 1 and 2 below assume $\phi(x)$ returns the gray level intensity of picture element x and assume $\varepsilon = 25$ (almost black, i.e., almost zero light intensity on a scale from 0 (black) to 255 (white)). It is easy to verify the following examples of near and far sets.

(**Ex**. 1) $A_1 \ \delta_{\varepsilon,\phi} \ A_2$, i.e., A_1 is near A_2, since the greatest lower bound of the differences will be close to zero because the intensities of the darker oval pixels are almost equal.

(**Ex**. 2) $F_1 \ \delta_{\varepsilon,\phi} \ F_2$, i.e., neighborhood F_1 is near neighborhood F_2 in A_4, since the pixels in F_1 are common to both neighborhoods.

(**Ex**. 3) For this example, let $X = A_3 \cup A_5, \varepsilon = 0.5$ and define

$$\phi(x) = \begin{cases} 1, & \text{if } x \in X \text{ portrays a hairdressing behavior,} \\ 0, & \text{otherwise.} \end{cases}$$

$A_3 \ \underline{\delta}_{\varepsilon,\phi} \ A_5$, i.e., A_3 (hairdressers) is far from A_5 (knight), since the behaviors represented by the two pictures are different. If we assume $A = A_3, B = A_5$, then $\inf\{d(\phi(a),\phi(b)) : a \in A, b \in B\} = 1$.

[1] *The picture of the knight being lowered onto his horse appears in vol. IX, 1845,* Punch.

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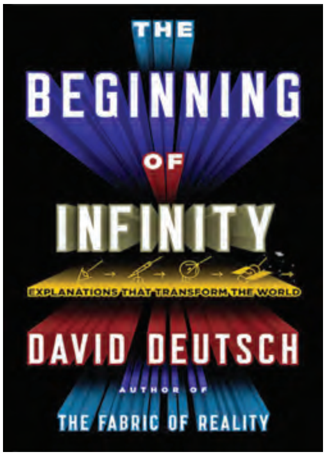
Book Review



The Beginning of Infinity: Explanations That Transform the World

Reviewed by William G. Faris

The Beginning of Infinity: Explanations That Transform the World
David Deutsch
Viking Penguin, New York, 2011
US$30.00, 496 pages
ISBN: 978-0-670-02275-5

The Cosmic Scheme of Things

A number of recent popular scientific books treat a wide range of scientific and philosophical topics, from quantum physics to evolution to the nature of explanation. A previous book by the physicist David Deutsch [2] touched on all these themes. His new work, *The Beginning of Infinity*, is even more ambitious. The goal is to tie these topics together to form a unified world view. The central idea is the revolutionary impact of our ability to make good scientific explanations.

In books written by physicists one expects to find awe of the universe and delight that we have come to understand something about it. There might also be an implicit assumption that people are not particularly significant in the cosmic scheme of things. Deutsch makes the contrary claim (p. 45):

> People *are* significant in the cosmic scheme of things.

He presents the following example. The atomic composition of the universe consists mainly of the light elements hydrogen and helium. Consider a heavy element such as gold. Where can it be found, and where did it come from? There are two very different sources. One is the transmutation of elements in the interior of an exploding supernova. Gold, among other elements, is created and then distributed throughout the universe. A small fraction winds up on planets like Earth, where it can be mined from rock. The other source is intelligent beings who have an explanation of atomic and nuclear matter and are able to transmute other metals into gold in a particle accelerator. So gold is created either in extraordinary violent stellar explosions or through the application of scientific insight.

In the claim there is a potential ambiguity in the use of the word "people". Deutsch characterizes them (p. 416) as "creative, universal explainers". This might well include residents of other planets, which leads to a related point. Most physical effects diminish with distance. However, Deutsch argues (p. 275):

> There is only one known phenomenon which, if it ever occurred, would have effects that did not fall off with distance, and that is the creation of a certain type of knowledge, namely a beginning of infinity. Indeed, knowledge can aim itself at a target, travel vast distances having scarcely any effect, and then utterly transform the destination.

Deutsch is a physicist who believes that matter is strictly governed by the laws of physics. However, he is no reductionist. He believes that atoms are real, but also that abstractions are real. One level of abstraction is knowledge, which he defines as "information which, when it is physically embodied in a suitable environment, tends to cause itself to remain so" (p. 130). This includes both knowledge embedded in DNA and ideas in human brains. Biological knowledge is nonexplanatory, but human knowledge includes explanations that solve

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unexpected problems. The gold example shows that an explanation can have physical effects just as real as those created by an exploding star.

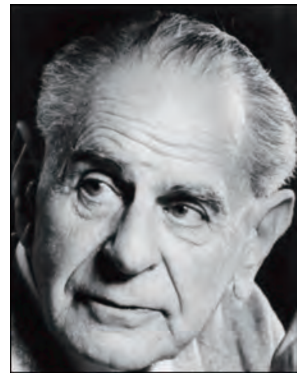
Deutsch's book is also a manifesto in praise of the notion of progress. His claim is that a new kind of progress began with the Enlightenment. In the best case the growth of knowledge may proceed indefinitely in the future without bound. This is the "beginning of infinity" of the title and of the quotation above. The argument has several threads. It begins with a definite position on philosophy of science, centered on an argument that a notion of "good explanation" of reality is the key to progress. It includes a theory of cultural evolution, which explains obstacles to progress and the lucky circumstance that swept them away. There is extensive discussion of the anomalous status of quantum theory, which on some accounts is incompatible with a realistic world view. The argument connects views on topics as varied as mathematical reality, voting systems, aesthetics, and sustainability. The author rejects sustainability as an aspiration or as a constraint on planning; he favors an open-ended journey of creation and exploration.

Good Explanations

Deutsch attributes recent progress to the discovery of how to create good explanations. A "good explanation" is defined as an explanation "that is hard to vary while still accounting for what it purports to account for" (p. 30). This philosophy is inspired by the writings of Karl Popper. In his influential works, Popper argued against empiricism, the notion that we derive all our knowledge from sensory experience. There is no such thing as raw experience; scientific observation is always theory-laden. Furthermore, there is no principle of inductive reasoning that says that patterns of the past will be repeated in the future. In Popper's view the path to scientific progress is to make conjectures and then test them. Deutsch insists that even explanations that are testable are not enough; they should be good explanations in the sense that he defines.

His version of philosophy of science has two complementary strands. One is realism, the notion that there is a physical world about which it is possible to obtain knowledge. The other is fallibilism, the doctrine that there is no sure path to justify the knowledge that has been obtained at any stage. These together support the metaphor of a knowledge base of good explanations that is increasing but bounded above by a reality that is never fully knowable.

One useful feature of the book is the explicit definitions of philosophical terms, explaining how they are used in his system. Thus "instrumentalism" is defined as the "misconception that science cannot describe reality, only predict outcomes of observations" (p. 31). As is clear from this example, definitions often come with a judgment.



Karl Popper: philosopher of "theory-laden" science.

Deutsch's main work as a physicist has been on quantum computing, and it is natural that he would test his ideas in the framework of quantum theory. This leads to a problem. Quantum theory is the accepted framework for understanding properties of matter, viewed as consisting of constituents on the molecular, atomic, and subatomic scale. It explains such properties of matter as density and strength and conductivity and color. It also underlies the deeper understanding of the chemical bond. It is so pervasive in fundamental physics that it is difficult to see how to make the smallest modification to it without destroying the whole edifice.

The problem is that the formulation of quantum theory is abstract and mathematical, and, as usually presented, it also has a peculiar dual character. There is a law of time evolution given by the Schrödinger equation: For every time interval there is a transformation U obtained by solving this equation. It maps the initial state of a system to the final state of a system. (The notation U indicates a *unitary* linear transformation.)

There is another law of time evolution, a random transformation R that determines the result of the measurement. Various possibilities for this transformation may occur, according to certain calculated probabilities. The state change under this transformation is called *reduction* or *collapse*.

The origin of the transformation R is that it describes an intervention from the outside. The system is in a certain state. A physicist sets up a measuring apparatus and performs the experiment. The observed outcome occurs. The new state after the experiment is determined by the particular R that corresponds to the experimental outcome. If the experiment is repeated under identical conditions, then the outcomes vary, but the frequencies are predicted by the calculated probabilities.

Such instrumentalist accounts have a long and complicated history. An early version is sometimes referred to as the "Copenhagen interpretation", since it stems from ideas of Niels Bohr and his school. (See [8] for a critical account.) Deutsch will have none of this. For him instrumentalism is defeat; the only acceptable account is realistic. Here is his explicit definition (or dismissal) of Bohr's approach (p. 324):

> *Copenhagen interpretation*
> Niels Bohr's combination of instrumentalism, anthropocentrism and studied ambiguity, used to avoid understanding quantum theory as being about reality.

If the instrumentalist account is not acceptable, then what are the alternatives? One is to develop a theory that includes the deterministic U time evolution but also has additional structure. This structure should introduce randomness in some way; perhaps the reduction transformation R will emerge as a consequence. The other way is to develop a theory with only the deterministic time evolution U. Such a theory must explain why the outcomes of experiments on quantum systems appear random.

The final section of this review gives a more detailed discussion of these alternatives. It will come as no surprise that Deutsch prefers a realistic version of quantum theory with only the dynamical time evolution given by U. This seems the happiest outcome, but we shall see the price he must pay.

Cultural Evolution

Many of the ideas in this book are related to evolutionary theory. In biological evolution genes are replicated, and they change through variation and selection. In cultural evolution the analog of a gene is a "meme". (The term was coined by Richard Dawkins in 1976.) Memes are replicated, and they also change through variation and selection.

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Richard Dawkins: evolutionary theorist.

The copying mechanism is different for genes and memes. A gene exists in a physical form as DNA that may be copied intact through several generations without ever being expressed as behavior. It acts much like a computer program. A meme can be copied only if it is enacted. In fact, a meme has two forms. It may be an idea in a brain. This idea may provoke a behavioral embodiment of the meme, such as action or body language or speech. The idea in the next recipient brain has to be guessed from the observed behavior. The successful meme variant is the one that changes the behavior of its holders in such a way as to make itself best at displacing other memes from the population.

Deutsch contrasts two ways that a meme may successfully replicate itself. A meme may survive as a meme of conformity, because it is never questioned. It relies on disabling the recipients' critical facilities. When such memes dominate, the result is a static society in which dangerous dysfunctional memes are suppressed. The other possibility is a dynamic society. In such a society memes are replicated in a rapidly changing environment. This can take place in a culture of criticism, because the memes embody new truths.

The transition from a static society to a dynamic society depends on an almost accidental shift in how meme transmission is employed. A meme cannot be simply copied from behavior. There has to be a mental capacity to infer the idea from the behavior. This creative ability developed in service to the task of replicating memes of conformity. It was hijacked to the task of creating new knowledge.

Deutsch considers the transition to a society where there is deliberate creation of new knowledge to begin with the Enlightenment. There is no precise date, but the founding of the Royal Society in 1660 is a landmark. There may have been previous transitions that did not survive. Deutsch calls such a period a "mini-enlightenment". He speculates that two such mini-enlightenments may have occurred. One was at the time of the Athenian advances in political freedom and openness to new ideas in philosophy and science. Another was when Florence became a center of creativity in art, accompanied by advances in science, philosophy, and technology. In both cases the initial spark was extinguished. This has major enduring consequences (pp. 220–221):

> The inhabitants of Florence in 1494 or Athens in 404 BCE could be forgiven for concluding that optimism just isn't factually true. For they know nothing of such things as the reach of explanations or the power of science or even laws of nature as we understand them, let alone the moral and technological progress that was to follow when *the* Enlightenment got under way. At the moment of defeat, it must have seemed at least plausible to the formerly optimistic Athenians that the Spartans might be right, and to the formerly optimistic Florentines that Savonarola might be. Like every other destruction of optimism, whether in a whole civilization or in a single individual, these must have been unspeakable catastrophes for those who had dared to expect progress. But we should feel more than sympathy

> for these people. We should take it personally. For if any of those earlier experiments in optimism had succeeded, our species would be exploring the stars by now, and you and I would be immortal.

As we have seen, Deutsch's book combines various threads to create a vision of continuing progress in enlightenment and human flourishing. It is a unified portrait that explains scientific advance and puts it in a moral framework. The vision is supported by a great number of assertions, some of them extravagant. Take, for instance, his claim that "everything that is not forbidden by laws of nature is achievable, given the right knowledge" (p. 76). Here is a supporting argument (p. 56):

> ...every putative physical transformation, to be performed in a given time with given resources or under any other conditions, is either:
> — impossible because it is forbidden by laws of nature; or
> — achievable, given the right knowledge.
>
> That momentous dichotomy exists because if there were transformations that technology could never achieve regardless of what knowledge was brought to bear, then this fact would itself be a testable regularity in nature. But all regularities in nature have explanations, so the explanation of that regularity would itself be a law of nature, or a consequence of one. And so, again, everything that is not forbidden by laws of nature is achievable, given the right knowledge.

A passage like this is tough to decipher.

The question of whether to believe every detail may be beside the point. A manifesto is not a scientific or philosophical treatise; it is an outline of a world view that one can try on for comfort. Some readers will find satisfaction in a systematic view of the world that is compatible with notions of science and progress. Others may find it too simple or too optimistic. In the latter case they are violating yet another of Deutsch's maxims (p. 212):

The Principle of Optimism

All evils are caused by insufficient knowledge.

So much the worse for them.

Quantum Theory

From here on this review concentrates on quantum theory and is more technical. The formulation of quantum theory centers on the wave function, a quantity that is difficult to interpret in terms of a realistic world view. In compensation the theory has an elegant mathematical structure. What follows is a brief review of this structure, followed by a discussion of three possibilities for interpretation. (Deutsch would not agree that interpretation is an issue, since he sees only one reasonable way to think of the theory.)

The state of a quantum system at a given moment in time is described by a complex valued wave function $\psi(x)$. This depends on the positions of all the particles in the system. For instance, say that there are N particles (no spin or statistics). The position of particle i at fixed time is described by a coordinate $\mathbf{x}_i$ in three-dimensional space. The wave function depends on $x = (\mathbf{x}_1, \dots, \mathbf{x}_N)$, which ranges over configuration space of dimension $3N$. This is already an incredible picture of nature: everything is related to everything else through the wave function. In particular, the joint probability density for the system of N particles is proportional to

$$\rho(x) = |\psi(x)|^2. \tag{1}$$

It is possible for the different particles to be highly correlated, even when they are widely separated in space.

The change in the state over a given time interval is ordinarily given by a transformation U. This transformation is deterministic, and it is computed by solving the Schrödinger equation. This is a complicated linear partial differential equation, and much of theoretical physics consists of attempts to solve it for some particular system.[1]

Since the Schrödinger equation is linear, the corresponding transformations U are also linear. This naturally leads to a more abstract point of view. The characteristic feature of vectors is that it is possible to take linear combinations of vectors to produce new vectors. In particular, if ψ_1 and ψ_2 are vectors, then the sum $\psi_1 + \psi_2$ is also a vector. Also, every scalar multiple of a vector is a vector. Since wave functions share these properties, there is a convention of calling a wave function a vector. This in turn leads to the use of geometrical

[1] *In some accounts the complete description of the physical state of a quantum system involves more than its wave function; it brings in additional structure, namely the actual positions of the particles. This leads to the question of how to specify the time evolution of particle positions. This issue is not treated in orthodox quantum mechanics, but Bohmian mechanics and Féynes-Nelson stochastic mechanics each specify a possible continuous time evolution for particle positions. The reviewer thanks Sheldon Goldstein for comments on this topic.*

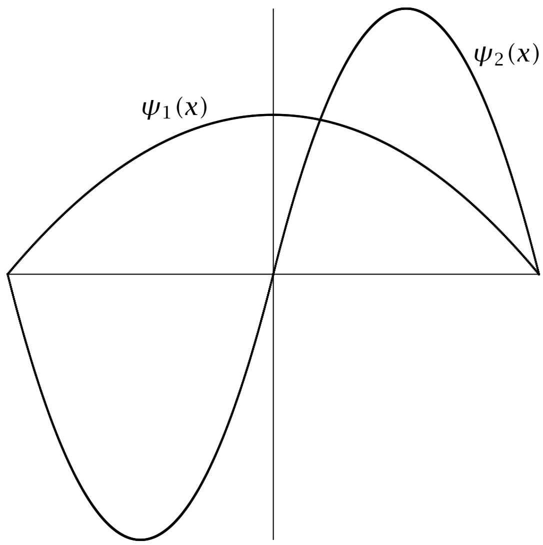

Figure 1. Example of orthogonal functions.

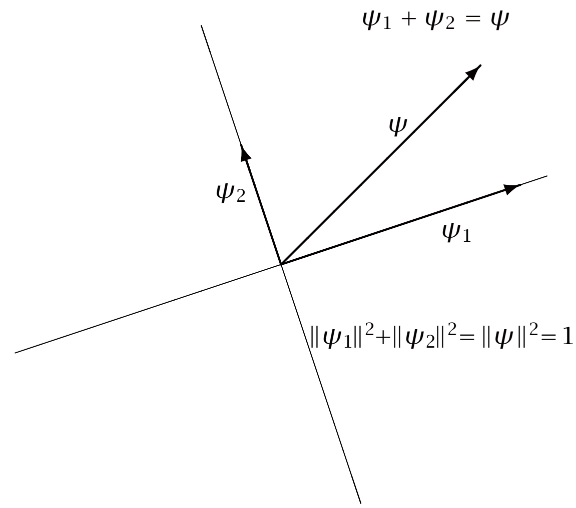

Figure 2. Probability = Pythagoras.

analogies. (The fact that the scalars are complex numbers is only a minor problem.)

Each pair of vectors ψ_1 and ψ_2 has a scalar product $\langle\psi_1, \psi_2\rangle$. This too has an analog for wave functions; the scalar product is defined by the definite integral

$$\langle\psi_1, \psi_2\rangle = \int \overline{\psi_1(x)}\psi_2(x)\,dx. \tag{2}$$

The space of all wave functions is regarded as a vector space of functions, in fact, as a Hilbert space. Each vector ψ in the Hilbert space has a norm (length) $\|\psi\|$ which is determined by the usual formula $\|\psi\|^2 = \langle\psi, \psi\rangle$ for the square of the norm. In the case of wave functions the explicit expression is

$$\|\psi\|^2 = \int |\psi(x)|^2\,dx. \tag{3}$$

Vectors ψ_1 and ψ_2 are orthogonal (perpendicular) if their scalar product has the value zero, that is, $\langle\psi_1, \psi_2\rangle = 0$. (Figure 1 shows an example of orthogonal wave functions. In this example $\psi_1(x)$ is even and $\psi_2(x)$ is odd, so their product has integral zero.) With these definitions various notions of geometry have attractive generalizations. Suppose, for instance, that there is a sequence of orthogonal vectors ψ_j with sum ψ, so

$$\psi = \sum_j \psi_j. \tag{4}$$

In this situation the theorem of Pythagoras takes the form

$$\|\psi\|^2 = \sum_j \|\psi_j\|^2. \tag{5}$$

If $\|\psi\|^2 = 1$, then the theorem of Pythagoras has the special form

$$1 = \sum_j \|\psi_j\|^2. \tag{6}$$

These give positive numbers that sum to one, just what is needed for probability. In fact, this is the standard framework for the interpretation of quantum mechanics. For simplicity consider an observable quantity with discrete values, such as the energy of a bound system. The possible values are indexed by j. The observable quantity determines a decomposition of the Hilbert space into orthogonal subspaces.[2] The state vector is a vector ψ of unit length. It is expressed as a sum (4) of the projected vectors. Different observable quantities define different decompositions. For each observable quantity there are probabilities given by the terms in (6).

This geometrical picture of quantum mechanics, abstract and beautiful, is immensely appealing. The reviewer is tempted to summarize it in a slogan:

Probability = Pythagoras

To see such a surprising and satisfying connection (Figure 2) is to be seduced by perfection. However, as we shall see, there are reasons to resist its allure.

[2] *The mathematical structure is an orthogonal direct sum decomposition $\mathcal{H} = \bigoplus_j \mathcal{H}_j$ of the Hilbert space. There are corresponding real numbers λ_j that are possible values for the observable quantity. These data determine a self-adjoint operator A whose action on each $\mathcal{H}_j$ is multiplication by λ_j. If ψ is the state vector and ψ_j is the orthogonal projection of ψ onto $\mathcal{H}_j$, then the expected value of this quantity has the elegant expression $\sum_j \lambda_j \|\psi_j\|^2 = \langle\psi, A\psi\rangle$.*

The trouble begins with a counterexample. Consider a quantum system. Each possible decomposition (4) defines probabilities. It is tempting to think of these probabilities as describing outcomes associated with this system. This is not a consistent view. An early argument to this effect was given by John Bell. A more recent example of Lucian Hardy [5] makes the same point in an even more convincing way. Something else is needed to select a particular decomposition relevant to a given situation. See the appendix for a brief account of Hardy's argument.

What is it that determines which decomposition is relevant? If in a given situation probabilities are to predict frequencies, then which probabilities are to be used, and which are to be discarded? Any serious account of quantum mechanics must consider this problem. There are various ways to address it, but in the reviewer's view they fall into three general classes:

- Instrumentalism
- Quantum theory with additional structure
- Pure quantum theory

The *instrumentalist account* of quantum theory emphasizes its ability to make experimental predictions. This solves the problem, because the outcomes emerge as a result of the particular experimental setup. Deutsch summarizes the usual rule for such predictions (p. 307):

> With hindsight, we can state the rule of thumb like this: whenever a measurement is made, all the histories but one cease to exist. The surviving one is chosen at random, with the probability of each possible outcome being equal to the total measure of all the histories in which that outcome occurs.

He then describes the adoption of the instrumentalist interpretation (p. 307):

> At that point, disaster struck. Instead of trying to improve and integrate these two powerful but slightly flawed explanatory theories [of Schrödinger and Heisenberg], and to explain why the rule of thumb worked, most of the theoretical-physics community retreated rapidly and with remarkable docility into instrumentalism. If the predictions work, they reasoned, then why worry about the explanation? So they tried to regard quantum theory as being *nothing but* a set of rules of thumb for predicting the observed outcomes of experiments, saying nothing (else) about reality. This move is still popular today, and is known to its critics (and even to some of its proponents) as the 'shut-up-and-calculate interpretation of quantum theory'.

In an instrumentalist account the relevant decomposition into orthogonal subspaces is determined by the measurement one performs on the system. This leads to the nontrivial problem: how does one characterize measurement? There is no universally accepted solution. The notion of "measurement" is defined with varying degrees of precision. In some versions a measurement is said to necessarily involve interactions on a macroscopic scale. This is in spite of the fact that the macroscopic world is supposed to be made of atoms.

The mathematical formulation ignores most of this detail. The particular measurement is specified by an orthogonal decomposition, which then determines a decomposition of the state vector ψ as a sum of components ψ_j as in (4). For each j there is a reduction operator R_j. The effect of reduction on the state vector ψ is another vector $\chi_j = R_j\psi$. In some accounts it is required that $\chi_j = \psi_j$, but this is unnecessarily restrictive. All that is needed is that $\|\chi_j\|^2 = \|\psi_j\|^2$. The particular j that is used is random; its probability is given by the squared norm $\|\psi_j\|^2$. Strictly speaking, the vector χ_j is not a state vector, since it does not have norm one. However, it may be multiplied by a scalar to give a normalized state vector, which could be taken as a new state of the system.

In general, a probability describes the statistics of what happens when an experiment is repeated many times. But what happens in a particular experiment? As always in probability, when the experiment is performed, a particular outcome value j' occurs. In this instrumentalist version of quantum mechanics there is an additional postulate: the new reduced wave function is $\chi_{j'} = R_{j'}\psi$. No mechanism is given.

Another direction is *quantum theory with additional structure*. One ingredient might be to try to model the experimental apparatus along with the system of interest. There can also be an explicit mechanism for introducing randomness. Some of the ideas go back to von Neumann; others have been elaborated by various authors. There is no agreement on details. Here is a brief account of a possible view, taken from [4]. It is not universally accepted; in particular, Deutsch presumably would be appalled at the ad hoc introduction of randomness.

The complete description involves the system of interest together with an environment, forming what one might call the total system. The system of interest might be a particle, an atom, or a molecule. To make the description concrete, it will be called an atomic system. The environment could be an experimental apparatus or, more generally, a larger system of some complexity. For brevity call it the apparatus. The combined system is described by a wave function $\Psi(x, y)$, where x describes the positions of particles in the atomic system, and y describes the positions of particles in the apparatus. Typically there is no natural way for the wave function for the combined system to determine a wave function for the atomic subsystem.

One situation where it is meaningful to have a wave function $\psi(x)$ for the atomic subsystem is when the total system wave function is of the product form

$$\Psi_0(x, y) = \psi(x)\phi_0(y). \tag{7}$$

Suppose that this is an initial state that has no immediate interaction between the atomic system and apparatus. This means that the wave function is nonzero only if the $\mathbf{x}_i$ are so far away from the $\mathbf{y}_j$ that the interaction is negligible. The atoms are headed toward the apparatus, but they are not there yet. As long as the combined wave function has this form, the dynamics of the atomic subsystem is described by the Schrödinger time evolution U for the atomic system alone.

Now the system is to interact with the apparatus. The starting point is a decomposition of the wave function $\psi(x)$ of the atomic system as a sum

$$\psi(x) = \sum_j \psi_j(x). \tag{8}$$

The measurement itself is accomplished by the unitary dynamics of the combined system. This deterministic transformation must be appropriate to the decomposition (8) in the following sense. It should map each wave function $\psi_j(x)\phi_0(y)$ to a new wave function $\chi_j(x)\phi_j(y)$ for which the new environment wave function $\phi_j(y)$ factor also depends on j. These wave functions $\phi_j(y)$ should be normalized and form an orthogonal family. This implies that the atomic wave function normalization is preserved, in the sense that $\|\chi_j\|^2 = \|\psi_j\|^2$. For such a transformation to exist, there must be a physical interaction of a suitable type—for instance, electric or magnetic—between the atomic system and apparatus.

Courtesy of Mark Everett.

Hugh Everett: originator of many-worlds quantum theory.

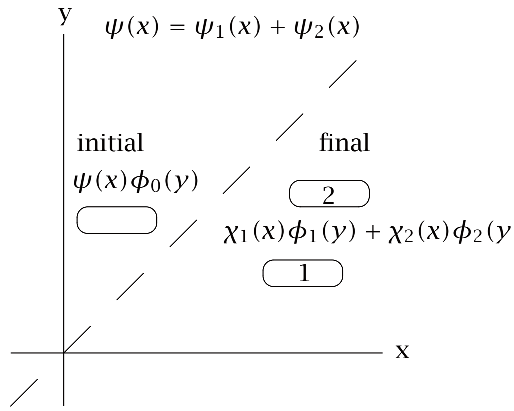

Figure 3. Support of system-apparatus wave function.

Suppose that there is such an interaction. Since the transformation is linear, it maps Ψ_0 to a state Ψ in which the atomic system is coupled to the states of the apparatus. The new wave function is

$$\Psi(x, y) = \sum_j \chi_j(x)\phi_j(y). \tag{9}$$

One can think of $\chi_j(x)$ as the result of applying a reduction operator R_j to the state $\psi(x)$. The probability associated with such an atomic wave function is $\|\chi_j\|^2 = \|\psi_j\|^2$.

The apparatus wave functions $\phi_j(y)$ are functions of the apparatus configuration $y = (\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_M)$, where M is the number of particles in the apparatus. When M is very large it is plausible that the apparatus wave functions $\phi_j(y)$ with various indices j are macroscopically different. Such effects have been studied under the name *quantum decoherence*. (See [7] for a recent survey.) In the present account the condition that the apparatus states are macroscopically different is interpreted to mean that the wave functions $\phi_j(y)$ with different j are supported on subsets with negligible overlap in apparatus configuration space (Figure 3). The corresponding atomic wave functions $\chi_j(x)$ are the result of dynamical interaction. Each of them is a candidate for the result of the reduction process.

In order to have an actual result, additional structure is required. This is obtained by introducing a new dynamics with random outcome. One of the indices is randomly selected. Say that this is the j' index. Then the corresponding reduced wave function of the atomic system is the $\chi_{j'}(x)$ given by applying $R_{j'}$ to $\psi(x)$. This reduction process does not contradict the unitary dynamics for the atomic subsystem. The wave function for the atomic subsystem is not even defined while the atomic system and apparatus are interacting; it is only defined before the interaction and after the interaction is over. Suppose that for a subsequent time interval a decomposition (9) with nonoverlapping $\phi_j(y)$ persists and there is no immediate interaction between the atomic system and apparatus. Then over this interval of time the wave function of the atomic subsystem continues to be defined, and the deterministic dynamics U describes its time evolution.

The reader will notice that the above account is complicated and artificial. However, it is at least consistent. This is because it is a consequence of a variant of quantum theory [3] that is itself known to be consistent.

The final possibility is *pure quantum theory*. Almost everyone agrees on the deterministic dynamics given by the Schrödinger equation. Perhaps this is all that is needed. This idea is the genesis of the many-worlds theory (see [6] for a recent survey). This theory began with hints by Schrödinger himself as early as 1926 (see [1] for a modern account) and was developed in more detail by Everett in 1957. The rough idea is that when there are macroscopically separated wave functions $\phi_j(y)$, each of these describes a world with its own history. We live in and experience only one such world, but they are all equally real. There is no miraculous reduction of the wave function and no randomness. The apparent randomness is perhaps due to the effect that we experience a history that is typical and hence appears random. Or there may be some other mechanism.

Deutsch is an enthusiastic proponent of such a theory. It does everything he wants, avoiding instrumentalism and ad hoc introduction of randomness. He refers to the resulting picture of physics as the "multiverse," and he is willing to draw the consequences (p. 294):

> ...there exist histories in which any given person, alive in our history at any time, is killed soon afterwards by cancer. There exist other histories in which the course of a battle, or a war, is changed by such an event, or by a lightning bolt at exactly the right place and time, or by any of countless other unlikely, 'random' events.

It is not the case that everything is permitted.

> A great deal of fiction is therefore close to a fact somewhere in the multiverse. But not all fiction. For instance, there are no histories in which my stories of the transporter malfunction are true, because they require different laws of physics. Nor are there histories in which the fundamental constants of nature such as the speed of light or the charge on an electron are different.

As with much of what Deutsch writes, the question is not so much whether to believe it as whether to march under his banner.

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Appendix: The Hardy Example

In quantum mechanics each observable defines a decomposition of the Hilbert space into orthogonal subspaces. Each state vector ψ (length one) is then written as a sum of projections ψ_j onto these subspaces. The probability corresponding to j is $\|\psi_j\|^2$. The Hardy example shows that these probabilities cannot simultaneously predict outcomes for all the observables together.

First consider a single particle. Suppose that for such a particle there are two distinct observable quantities D and U. Each can have either of two values, 1 or 0. The event that $D = 1$ is denoted d, and the event that $D = 0$ is denoted $\bar{d}$. Similarly, the event that $U = 1$ is denoted u, and the event that $U = 0$ is denoted $\bar{u}$.

Now consider two particles, perhaps widely separated in space. For the first particle one can observe either D or U, and for the second particle one can observe either D or U. This gives four possibilities for observation. There are four corresponding decompositions of the state vector describing the two particles:

$$\psi = \psi_{dd} + \psi_{d\bar{d}} + \psi_{\bar{d}d} + \psi_{\bar{d}\bar{d}}, \tag{1}$$
$$\psi = \psi_{du} + \psi_{d\bar{u}} + \psi_{\bar{d}u} + \psi_{\bar{d}\bar{u}}, \tag{2}$$
$$\psi = \psi_{ud} + \psi_{u\bar{d}} + \psi_{\bar{u}d} + \psi_{\bar{u}\bar{d}}, \tag{3}$$
$$\psi = \psi_{uu} + \psi_{u\bar{u}} + \psi_{\bar{u}u} + \psi_{\bar{u}\bar{u}}. \tag{4}$$

Furthermore, Hardy constructs the quantities D and U and the state ψ in such a way that

$$\psi_{dd} \neq 0, \tag{5}$$
$$\psi_{d\bar{u}} = 0, \tag{6}$$
$$\psi_{\bar{u}d} = 0, \tag{7}$$
$$\psi_{uu} = 0. \tag{8}$$

For fixed ψ he chooses the parameters that define D and U to satisfy the last three of these equations. He then optimizes the parameters that specify ψ to maximize the probability $\|\psi_{dd}\|^2$. The maximum value works out to be $\frac{1}{2}(5\sqrt{5} - 11)$, which is about 9 percent.

Suppose that in a given physical situation the observables all have values. According to the first equation above, the probability of dd is greater than zero. So it is possible that the outcome is d for the first particle and d for the second particle. Suppose that this is the case. The second equation says that d for the first particle implies u for the second particle, and the third equation says that d for the second particle implies u for the first particle. It follows that the outcome is u for the first particle and u for the second particle. However, the fourth equation says that this is impossible.

The conclusion is that in a given physical situation the observables cannot all have values. The usual explanation for this is that measurement in quantum mechanics does not reveal preexisting values but in effect creates the values. The four decompositions correspond to four different experiments, and each decomposition provides the correct probabilities for the result of the corresponding experiment. For a given experiment, only one of the four decompositions is relevant to determining what actually happens. The other three decompositions give mathematical probabilities that are not relevant to this context.

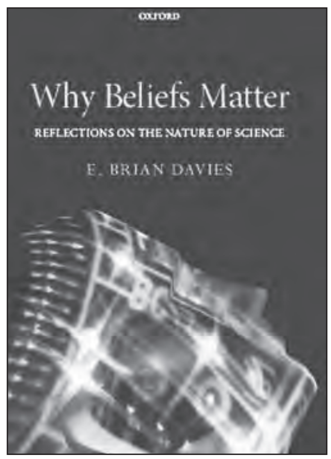
Why Beliefs Matter: Reflections on the Nature of Science

Reviewed by Gerald B. Folland

Why Beliefs Matter: Reflections on the Nature of Science
E. Brian Davies
Oxford University Press, 2010
US$45.00, 256 pages
ISBN 978-0-19-958620-2

Brian Davies is a distinguished mathematician with a long list of publications in operator theory and related areas, but in recent years he has increasingly devoted his attention to the philosophy of mathematics and science. The book under review, according to the preface, began as a response to the reductionist view of the world espoused by some physicists, but it encompasses a good deal more than that.

It begins with a cogent account of the development of the scientific worldview from Copernicus to Newton, along with the more recent commentaries of people such as Karl Popper and Paul Feyerabend. There follows a chapter on the interactions between human intelligence, human culture, and the scientific understanding of the world. Its main point is a defense of the pluralist, or antireductionist, position that we obtain the fullest understanding of the phenomena around us by being flexible in adopting multiple points of view. The third chapter, on the nature of mathematics, is primarily a critique of Platonism in mathematics; I shall say more about it below. Chapter four, cryptically titled "Sense and nonsense", begins with a good summary of the Standard Model of elementary particle physics but then moves on to some speculative



areas on the border between physics and science fiction: multiverses, wormholes, and simulated universes. The final chapter is on science and religion, but it includes some material on religious issues (primarily Christian and more specifically Anglican) that have little to do with science.

As the preceding thumbnail description suggests, in this book one will find intelligent and readable discussions of many scientific, mathematical, and cultural issues; there is intellectual pleasure to be found in many places here. On the larger scale, though, one may find that Davies's meandering line of thought leads to a certain lack of focus. Readers should not expect to have arrived at any profound philosophical conclusions at the end or indeed to have received any striking new answer to the issue posed in the title.

Davies buttresses his discussion of various issues with numerous quotes from the people whose ideas he considers, but he sometimes interprets these quotes in a more doggedly literal way than they were intended to bear. Let me cite one notable example. Davies has serious problems with the peculiarities of quantum mechanics; on p. 35 he calls it "a subject so difficult that even the experts do not claim to understand it at an intuitive level," and similar assertions are found in many other places in the book. In support of this position he quotes

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Richard Feynman: “It is safe to say that nobody understands quantum mechanics.” This quote is offered in complete seriousness, but it seems obvious to me that Feynman had a twinkle in his eye when he came out with that bit of false modesty. What he meant—what is undeniably true—is that nobody can explain quantum mechanics in a way that is consistent with our everyday intuitions about the behavior of macroscopic objects. But that just means that, if one wants to understand the submicroscopic world, one has to retrain and refine one’s intuition until it feels at home with the way things really work. I don’t think this is much different from the process by which mathematicians train their minds to think about geometric phenomena that don’t fit into our three-dimensional world: nobody can really see the locus of the equation $z^6+w^6=1$ in complex 2-space, but that doesn’t mean that nobody can understand algebraic geometry. In another essay [1, p. 10] Feynman returned to this point in a way that is more indicative of his true feelings: “Please don’t turn yourself off because you can’t believe Nature is so strange. Just hear me out, and I hope you will be as delighted as I am when we are through.”

For that matter, one doesn’t have to go to quantum mechanics to find counterintuitive phenomena. A spinning gyroscope—a toy top, say—will remain balanced on the tip of its axis as long as it remains spinning. I understand why this works in the sense that I have faith in the law of conservation of angular momentum and know how to calculate its consequences. But that doesn’t stop my untutored intuition from thinking that the damn thing ought to tip over.

Davies’s critique of Platonism is based on what one might call strong Platonism: that mathematical objects have an objective existence that is independent of the people who study them, in an ideal realm that is outside of space and time. There are mathematicians who would agree wholeheartedly with this proposition; Alain Connes is one of them, according to some statements of his quoted by Davies, and so was G. H. Hardy [3, §22]. But it is quite possible to speak of mathematical objects in a Platonic way, as a convenient and useful method of expressing one’s thoughts, without making a serious ontological commitment to the Platonic world. (In the same way, a physicist who describes an ultimate theory of elementary particles as “knowing the mind of God” is not necessarily presupposing the existence of God.) I talk about the real number system as a concrete entity that I know well, but if asked whether I really believe in its existence as a specific object in some ideal universe, I back off. I wish I could answer in the affirmative, but the utter failure of generations of set theorists to shed significant light on the continuum hypothesis saps my confidence to do so. I would rather echo Laplace’s response when someone asked why he never mentioned God in his treatise on celestial mechanics: I have no need of that hypothesis.

Among the alternatives to Platonism, Davies pays considerable attention to constructivism, as it has played a substantial role in shaping his own thought. On the other hand, he gives short shrift to formalism. He presents a quote from Edward Nelson summarizing Nelson’s formalistic views, expresses perplexity at Nelson’s statement that “the theorems are not *about* anything,” and then quickly moves on to other matters. I think this is unfortunate. Formalism deserves to be taken more seriously because, as I have pointed out elsewhere [2], it is the attitude of most scientists who use mathematics in their work. (I say “attitude” rather than “philosophy” because most appliers of mathematics are even less inclined than mathematicians to philosophize about the mathematics they use.)

Nelson’s statement requires a little exegesis. What he is really saying (and I have heard him say precisely this) is that *mathematics is all syntax*, no semantics; in other words, (pure) mathematical statements are not about any *particular* thing. Hilbert expressed this point with his dictum that the theorems of Euclidean geometry should remain valid if one replaces points, lines, and planes with tables, chairs, and beer mugs. In a more sober vein, in calculus one can replace x, $f(x)$, and $f'(x)$ by horizontal coordinate, vertical coordinate, and slope; or by time, position, and velocity; or by wave number, frequency, and group velocity; or by price, profit, and marginal profit; or… well, you get the idea. Appliers of mathematics are perfectly happy with the idea that mathematics is all syntax because they can supply the semantics from their own disciplines. On the other hand, I think that a large part of the appeal of Platonism for mathematicians is that thinking about pure syntax is difficult, and it becomes easier if one has something for the syntax to hang onto: hence our insistence on describing mathematical objects as specific sets and our tendency to speak of these sets as concrete objects. In other words, Platonism is of service more as a psychological tool than as a philosophical position.

In fact, I think the whole question of whether mathematics is a matter of discovery or invention —that is, of exploring a Platonic realm or creating new concepts with the human mind—is more interesting as a psychological question, a question about the human activity of doing mathematics, than as a philosophical one. It is this aspect of the matter that Barry Mazur primarily addresses in his book [4] and his essay [5], which are both highly recommended reading. One could hardly put it better than this:

> On the days when the world of mathematics seems unpermissive, with its gem-hard exigencies, we all become

fervid Platonists (mathematical objects are “out there”, waiting to be discovered—or not) and mathematics is all *discovery*. And on days when we see someone who, Viète-like, seemingly by willpower alone, extends the range of our mathematical intuition, the freeness and open permissiveness of mathematical invention dazzle us, and mathematics is all *invention*. [4, p. 70]

But this is a statement about how we mathematicians think about what we do rather than about the ultimate nature of mathematical reality, whatever that might mean.

In any case, when someone has put a new mathematical concept into the realm of discourse, it is “out there” for the rest of us to explore (if you like) or add our own inventions to (if you prefer). With this realization, one can enjoy the interplay between discovery and invention without committing oneself to an exclusive belief in one or the other and leave the question of whether groups and topological spaces were already “out there” before Galois and Hausdorff to the metaphysicians. I suppose that Davies would say that I am advocating pluralism, but the need to label all one’s thoughts in terms of various “isms” is a disease of philosophers. The unexamined life may not be worth living, but the unlabeled one enjoys the gift of freedom.

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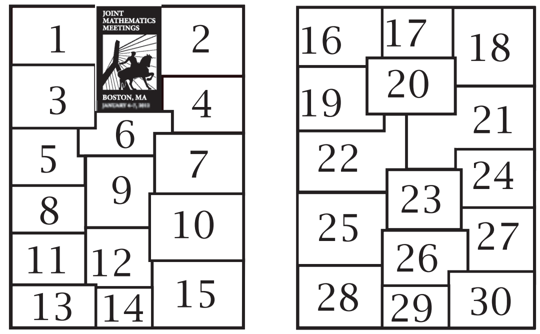

1. View of Boston from Hynes Convention Center.
2. AMS Employment Center.
3. AMS Booth in the Exhibits Hall.
4. Ivo Babuška, winner of the Steele Prize for Lifetime Achievement.
5. Fan Wei, winner of the AWM Alice T. Schafer Prize.
6. Email Center.
7. Ribbon cutting ceremony to open JMM Exhibits.
8. Prize Ceremony reception.
9. AMS Booth in the Exhibits Hall.
10. Mathematical Art Exhibit.
11. MAA Booth in the Exhibits Hall.
12. AMS Colloquium Lecturer Edward Frenkel.
13. Who Wants to Be a Mathematician contestants.
14. In the Networking Center.
15. Math games.
16. Networking Center.
17. William McCallum, winner of the AMS Award for Distinguished Public Service.
18. Prize Ceremony reception.
19. William Thurston, winner of the AMS Steele Prize for Seminal Contribution to Research.
20. Mathematical Art Exhibit.
21. In the Exhibits Hall.
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23. Shayam Narayanan (left), winner of 2012 Who Want to Be a Mathematician national contest.
24. Who Want to Be a Mathematician host Mike Breen and contestants.
25. Meeting and greeting.
26. Bonnie Gold, winner of the AWM Louise Hay Award.
27. John Pardon, winner of the AMS-MAA-SIAM Morgan Prize.
28. Joseph Dauben receiving the AMS Whiteman Prize from AMS President Eric Friedlander.
29. Dana MacKenzie, winner of the JPBM Communications Award.
30. Exhibits Hall.

(Math) Teachers Are the Key

Irwin Kra

Math education is hot. The venerable *New York Times* published an op-ed piece by S. Garfunkel and D. Mumford in August of 2011 (`http://www.nytimes.com/2011/08/25/opinion/how-to-fix-our-math-education.html?_r=2&ref=contributors`) that, on first reading appears to be right on the mark. However, because it omits several key issues, their proposal invites many negative consequences and would ultimately lead to further delays in solving the problems besetting our troubled educational enterprise. A lively follow-up discussion on the *New York Times* blog for students (`http://learning.blogs.nytimes.com/2011/08/26/do-we-need-a-new-way-to-teach-math/?scp=1&sq=Mumford&st=cse`) produced about twenty responses by September 19—roughly three to one in disagreement with the authors.

Garfunkel and Mumford write, "There is widespread alarm in the United States about the state of our math education. The anxiety can be traced to the poor performance of American students on various international tests … [T]his worry, however, is based on the assumption that there is a single established body of mathematical skills that everyone needs to know to be prepared for twenty-first-century careers. This assumption is wrong. The truth is that different sets of math skills are useful for different careers, and our math education should be changed to reflect this fact."

There is apparently very little to disagree with in the claim. But it completely misses the urgency in addressing the crisis facing us. The "worry" is based mostly on the poor learning and hence poor performance of our high school students who are just not ready at graduation to either succeed in the current job market or in college programs. Garfunkel and Mumford propose major curriculum revisions, in effect to make the curriculum more practical and more relevant to the marketplace. A properly revised curriculum certainly makes sense for many, and perhaps even most, students. Innovative teachers, and not just in high school (see, for example, K. Stroyan's piece in this column—*Notices*, Volume 58, Number 8), already use many of the ideas (for example, introducing real-life examples) they propose. But here is the rub: there are not enough knowledgeable, talented math teachers in our public, and many private, high schools. Many math classes are taught by people with scant knowledge, and little love, of mathematics. We need to emphasize the need for content knowledge for our teachers. With only superficial understanding of math, one can teach applications of mathematics to neither the real world nor to astrophysics. With the minimal requirements for mathematics knowledge that qualify for most high school teacher certification today, one cannot truly teach mathematics at all. Even more worrisome, the problem begins much earlier, probably in kindergarten, with teachers uneasy with mathematics projecting their uneasiness onto students. No one will ever tell you "English was my worst subject," but many elementary school teachers are unashamed to admit to the claim that "studying math gave me nightmares."

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Any major mathematics curriculum reform, especially the drastic and controversial one proposed by Garfunkel and Mumford, requires a long time to test and implement. Less radical solutions to improve learning are needed now. Improving math teacher quality is a crucial step. Training and selecting better teachers is independent of curriculum revisions. Studies [1] have shown that improved teacher quality can only lead to improved student learning and performance. Unfortunately, the self-evident fact that you need mathematics content knowledge, in addition to pedagogical skills and a personality suited for the classroom, to teach mathematics is not supported, especially on economic grounds, by all studies. Attracting and keeping better teachers in classrooms must involve improving their working conditions, and salaries are a big part of the equation.

Before implementation in the classroom, major curriculum changes require new texts and a thorough reorientation of teacher training programs—a long-term process involving years of testing and experimentation. The complications in designing the right math curricula for this, or any, century are enormous. The resulting timeline for meaningful action is totally unacceptable to serious education reformers, particularly because there are short-term remedies with predictable and positive results.

Another major shortcoming of the Garfunkel and Mumford proposals is that they ignore the concept of student tracking. This emotional and pedagogical issue is guaranteed to generate controversy since the suggested curriculum would certainly not be appropriate for all. With a tracking system, how do you ensure the ability to move from one track to another—we cannot possibly suggest an educational system where the courses taken at the age of twelve completely disqualify students from many choices at age twenty-one. Moreover, it is hard to define what practical mathematics needed by everyone should be. While the plumber might never have to solve a quadratic equation, the town clerk with a building application to review might. And, in view of past and almost certain future nuclear power plant disasters, every voter needs to have some understanding of the concept of decay and half-life.

Where in the curriculum do we address fundamental logical thinking? Traditionally, this has been done in the mathematics curriculum and it probably needs to stay in. Yet, a curriculum narrowly focused on applications would hardly address the core lessons of logic and reasoning. Moreover, in a narrow and applied curriculum, where would we address the intrinsic beauty and poetry in mathematics and other sciences? I believe that these qualities can only be taught by someone with an appreciation for those aspects of the subject. A solid argument can be made for exposing all students to basic and fundamental mathematical concepts, methods, and reasoning. No one knows what the future will demand of our scientists and mechanics. It makes sense to expose everyone to as vast a mathematical landscape as possible.

Educational research and long-term plans for reform are urgently needed, but they cannot be an excuse for delaying some important actions that will certainly do no damage and will very likely improve our enterprise. Improving teacher quality and dramatically raising the bar for content knowledge across the primary and secondary educational establishment is an indispensable good. It has been endorsed by school administrations, by unions, and by parents.[1] The only thing lacking is the political will to make it happen. If we do not take immediate steps to improve math education—and teacher quality is key here—we will, before long, be a second-rate industrial nation with an undereducated work force. Programs to produce and keep better-trained and better-performing teachers require a nontrivial investment of talent and dollars, money we cannot afford not to spend. The issue is not just money. Too much has been spent on bad and misguided approaches to solving our problems. Not enough attention has been paid to the content knowledge of teachers and in preparing them, and not enough attention is paid to developing the necessary pedagogical skill. The National Science Foundation has produced a video on its website [2] that documents part of the problem and illustrates the work of D. Ball.

To quote Garfunkel and Mumford once more: "Today, American high schools offer a sequence of algebra, geometry, more algebra, pre-calculus, and calculus (or a 'reform' version in which these topics are interwoven). This has been codified by the Common Core State Standards, recently adopted by more than forty states. This highly abstract curriculum is simply not the best way to prepare a vast majority of high school students for life." Again, I agree almost completely, but the codified standards, not perfect instruments by any means, represent a step forward. Build on them. Do not tear them down, and always keep in mind that good teachers are the key. To put good math teachers in the classroom, society must expect more from them, must pay them more, and must give them the support and respect they deserve.

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[2] NSF special report, `http://www.nsf.gov/news/special_reports/math/classroom.jsp`.

[1]R. Weingarten, *president of the American Federation of Teachers, is a member of the* MfA *Board, an organization that supports these goals.*

The Future of Mathematical Publishing

Michael G. Cowling

Where are mathematics journals going? As an editor of an Australian Mathematical Society journal, I have serious concerns about the future. Many national mathematical societies derive most of their income from publishing and spend it on supporting mathematics, including activities that cannot be funded by research grants but are important to the long-term future of the discipline, such as promoting mathematics in schools. Research mathematicians therefore need to understand the trends, if not to fight the battles.

Journal prices are rising for several reasons, including production costs, and affect us all. Increasingly, national mathematical societies are handing over their inexpensive journals to commercial publishers, who raise prices. In part, this happens because libraries prefer to subscribe to packages of journals, and journals published by small organizations see their subscriptions drop to the point where they become uneconomical; further, there are efficiencies of scale in merging publishing operations. A ray of hope here is that a number of editorial boards of well-known journals have resigned in the past few years in protest at the pricing policies of the publisher and have set up alternative, less expensive journals. The blogosphere is full of information about these resignations (see, e.g., [1]). But the cost problem remains, and commercial publishers do not want us to discuss it. Indeed, one publisher sued the American Mathematical Society some years ago because the Society published figures on journal costs that the publisher found unflattering.

My biggest current concern is the trend to tie funding to "publication quality", often measured by "impact". The Excellence for Research in Australia evaluation looked at "research outputs" from departments across the country using (in many cases) impact factors to make decisions about quality; in a number of European countries, universities are girding their loins, or have already done so, for similar processes.

I present two of my worries about ranking journals. First, rankings are often used by libraries to decide which subscriptions to cancel when times are tough and so help determine which journals fail. As I have already pointed out, national mathematical societies derive much of their income from publishing and spend it on supporting mathematics. Inevitably, not all these journals can be in the top tiers, so mathematical society journals will fail and support for mathematics will decrease where this happens. Many mathematicians have a conflict of interest here: the best papers in national mathematical society journals are often by local mathematicians, so mathematicians in countries with lower ranked journals are torn between publishing in other journals, to support their personal interests, and at home, to support national interests.

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Next, impact factors can be and are manipulated in various well-documented ways. But we are all caught up in the citation game. In two randomly chosen issues of *Annals of Mathematics*, the average number of references per paper (omitting appendices and corrections) in 1985 was 16.5, and in 2005 it was 32.2.

Let me now discuss some more mundane editorial problems.

For many editors, plagiarism is a worsening problem. There are a few egregious plagiarists in mathematics, about whom readers can find information elsewhere (see, e.g., [2]). But there are more subtle problems, such as "trivial generalization", where very minor modifications lead to notionally different articles. Another is "self-plagiarism" (which I do here, see [3]). For example, some authors publish essentially the same article in two different languages; sometimes authors find a new technique, turn it into a number of almost identical papers, and submit them simultaneously to different journals; many papers have introductions that are almost identical to introductions of previous papers by the same authors. What is legitimate? In any case, when many institutions reward their staff according to the number of papers produced, it is hardly surprising that some mathematicians cut corners to produce more.

The first step in tackling plagiarism must be a clear statement of what it is because different cultures have different ideas about this. The American Mathematical Society website includes some ethical guidelines, but these appear in English only, and some non-English-speaking mathematicians may have trouble understanding them; further, they do not treat self-plagiarism. I would argue that the International Mathematical Union should draw up definitions of plagiarism and self-plagiarism and translate them into many languages so that we have an agreed statement of what is not acceptable. Mathematics is such that dictionary definitions are not adequate.

Timeliness is a major issue for authors. We all agree (in principle) that it is important to referee papers punctually, but delays between submission and publication are also caused by backlogs. In an ideal world, all journals would provide this information on their webpages, with backlog information by area when they have quotas for different areas.

Although refereeing is an important part of our responsibilities, as we move into the brave new world of Research Assessment Exercises, it seems that "the system" does not give credit for refereeing. If we believe that refereeing (and other editorial work) is important to upholding standards in the research community, we should argue this point.

Refereeing is a job without rewards, which brings growing frustrations. Many referees report that "this is the nth time that I have seen this paper." The report then continues in one of two ways: in one case, the authors have already received a rejection and a referee's report elsewhere but have not even considered the corrections suggested by the referee; rather, they have simply resubmitted the paper to another journal. In the other case, the referee continues: "I have made suggestions to improve the paper m times before, and finally it seems that another few iterations will produce something publishable." In other words, some authors rely on referees to correct their mathematics and proofread the text; the final paper owes as much to referees as to the author. This is also plagiarism, but it goes unpunished. What can be done about it?

My final editorial lament is that TEX has not reduced the cost of producing journals because many authors use it horribly. To cope with this, some journals retype all papers that are sent to them: this is as expensive as dealing with a traditional typescript, but, in this globalized era, it is often done by people with limited mathematics and English; the corrections for the worst proofs I ever received (from a commercial publisher) took six pages because of confusion between w and ω. The other solution is to adapt the TEX file—this is easy when the authors are competent, but it is more time-consuming than retyping ab initio otherwise. The mathematical community itself is to blame for this: we rarely teach mathematics students good TEX (or LATEX).

If the international mathematical community wants to make use of the savings that TEX could offer, then journals should reintroduce page charges for poorly written papers: there is good free information about TEX available online (see, e.g., [4]). According to the economic rationalists, financial incentives are the most effective way to get people to do things properly.

Surely the long-term health of mathematics relies at least in part on inexpensive journals, and so getting the mathematical community to educate itself would be a good step in this direction.

This article is based on an earlier article [3] in which I expand on some of the points here.

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Improvement in Survival of People Living with HIV/AIDS and Requirement for 1st- and 2nd-Line ART in India: A Mathematical Model

Arni S. R. Srinivasa Rao, Kurien Thomas, Sudhakar Kurapati, and Ramesh Bhat

Predictions on the future number of individuals who will require second-line antiretroviral therapy (ART) (among the individuals who are currently infected with HIV and future possible new infections) have not been generated since the initial estimates made by the World Health Organization for Treatment 2.0. As part of the preparatory activity to assist the National AIDS Control Organization (NACO) in India for the fourth phase of the National AIDS Control Program (NACP IV) (2012–2017), mathematical models were developed describing the survival dynamics in people living with HIV on first-line antiretroviral therapy (ART-1),[1] second-line antiretroviral therapy (ART-2),[2] and No ART in those who are not detected by the system. Since the launch of the free second-line antiretroviral therapy (ART-2) to people living with HIV/AIDS (PLHIV) in India during 2008 [1], there has been a slow recruitment into ART-2 until the end of 2010 (with an anticipated ART-2 number around 2,000). NACO, the apex body of the government of India, announced in November 2010 that it will provide ART-2 to all the first-line ART individuals who are currently enrolled either in government programs or in private centers [2] when they become eligible for the treatment.

As part of a model explaining the trends in AIDS-related mortality and incidence [3], we have constructed a submodel for understanding the survival trends in PLHIV receiving first-line and second-line ART during the next phase and beyond in India [4]. Model analysis reveals that from 2013 onwards the number of people eligible for ART-2 could gradually increase to around 100,000, with a concurrent decline in individuals who are eligible

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Sudhakar Kurapati is a senior HIV/AIDS advisor at the Centers for Disease Control and Prevention, U.S. Embassy, New Delhi. His email address is `sudhakark@in.cdc.gov`.
Ramesh Bhat is a director at WOne Management Systems, New Delhi. His email address is `ramesh@wonemanagement.com`.

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[1] *First-line therapy is given when the count of CD4/ml reaches below 350 in an individual who has been detected with HIV.*
[2] *Second-line antiretroviral therapy is given to HIV-infected individuals who have developed drug resistance to first-line therapy.*

for ART-1 (peak attained in 2012 at above one million) if case detection remains at current levels. We project that there will be 0.6 million PLHIV on ART-1 by the end of 2011. At the end of the NACP IV period (i.e., 2017), the number of eligible PLHIV on ART-1, ART-2, and No ART will be around 0.49 million, 96,000, and 91,000 respectively. Our submodel takes into consideration parameter values for the incubation period with and without ART from the references listed in the recent UN population perspectives [5]. Instead of two years [5] of mean survival after AIDS without ART, we have adopted four and a half years as the mean survival period without ART. Sensitivity analysis was performed on two key parameters: namely, recruitment rate of ART among people eligible for first-line, and rate of development of resistance to the first-line therapy (see Figure 1). The values in the legends in Figure 1 are annual recruitment rates into ART-1, and legends in Figure 2 are rates of development of resistance to the first-line ART. We have also generated model-based output based on two years mean survival after AIDS without ART for the period 2007–2017, which we have not presented in this short communication but have shared with senior officials at NACO. Care and support systems in the recognized government centers need strengthening in the wake of new ART-2 projections to facilitate accurate diagnosis of resistance. It is important to build on the success of the third National AIDS Control Program, NACP III (2007–2012), and address the emerging issues such as second-line ART during the fourth phase of the program. If the present level of recruitment to ART-1 is sustained during the next phase, there will be around 69,000 people who will be cumulatively eligible for ART-2 by the end of 2017. The model took into account the dynamic interaction of ART-1 and ART-2 on the pool of eligible PLHIV and the impact on survival within the PLHIV pool. Such modeling schemes are new to the literature; hence we could not draw any comparative conclusions on the number of second-line ART people during NACP IV. These results have implications not only for India but for other countries—for example, Brazil, Nigeria, Russia, etc.—which have successful first-line therapy and are initiating second-line therapy.

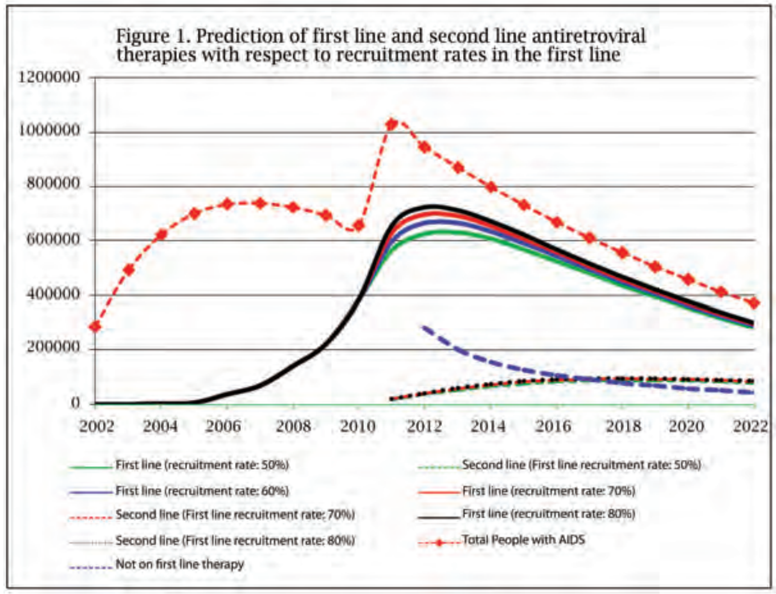

Figure 1.

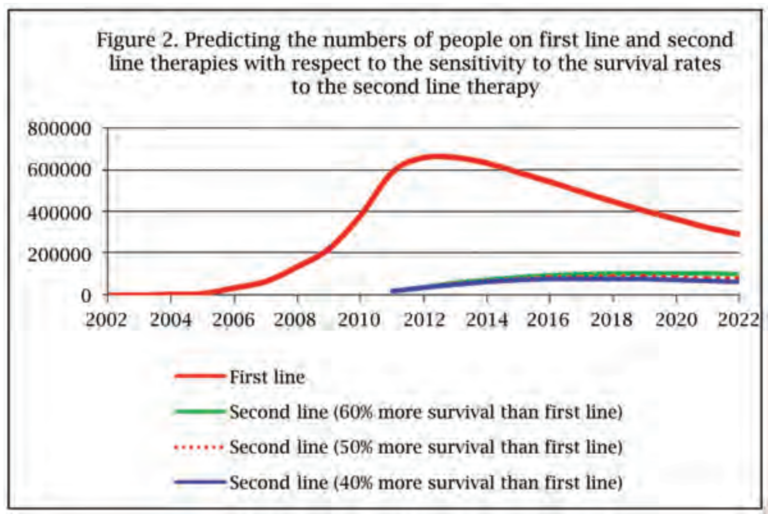

Figure 2.

Acknowledgments

The authors gratefully acknowledge the support and input received from the National AIDS Control Organization (NACO), Department of AIDS Control, Ministry of Health and Family Welfare, and the government of India and its senior officers during the course of this work. Professor Philip K. Maini, University of Oxford, commented on the original drafts, model, and parameters, and made helpful corrections in the earlier drafts. Comments and corrections from Professors Vidyanand Nanjundiah and Nilanjan V. Joshi, Indian Institute of Science, Bangalore, helped to improve clarity in earlier drafts. Dr. Cynthia Harper, Oxford, read the paper for English language before final revision and assured us that everything was all right. We are indebted for their valuable time.

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Table 1: Parameters.

Parameter	Description	Value	Reference
α_1	Mean incubation period during pre-ART period	9.2 years	1
δ_1	Mean survival period among AIDS without treatment	2, 4.5 years	2, Assumption
p_1	Annual rate of recruitment in ART-1	58.67 %	**
ρ	Annual rate of development of resistance to ART-1	4 %	Assumption
δ_2	Mean survival period among AIDS individuals with ART-1	9.5 years	3
δ_3	Mean survival period among AIDS individuals with ART-2	5.2 years	4
r	Annual growth rate of people living with HIV	-0.03	*

* Calculated as weighted rate of decline in last three years
** Calculated from model output and actual data

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4. A dynamical model was developed with flow of infected people to the AIDS compartment due to disease progression, and then a proportion of them were recruited to the ART-1 compartment. Due to resistance, a proportion of people on ART-1 will move to ART-2. The results were presented to government officials during July–August 2011. Modeling equations and descriptions appear in the appendix.
5. United Nations, Department of Economic and Social Affairs, Population Division (2006), World Population Prospects: The 2007 Revision.

Appendix

We have developed a mathematical model for predicting the number of people who are eligible for ART-1 and ART-2 and who will be on ART-1 and ART-2 beginning in 2011 in India. We carried out the modeling in two phases: in the first phase we estimated the number of individuals who developed AIDS after HIV infection and the number eligible to receive ART-1 after subtracting annual deaths before people were ready to be given therapy. The modeling equations for describing the process are:

$$\frac{dX_1}{dt} = rX_1 - \alpha_1 X_1, \quad \frac{dX_2}{dt} = \alpha_1 X_1 - \delta_1 X_2,$$
$$\frac{dX_3}{dt} = \alpha_1 X_0 - p_1 X_3 - \delta_1 X_3,$$
$$\frac{dY_1}{dt} = p_1 X_3 - \rho Y_1 - \delta_2 Y_1, \quad \frac{dY_2}{dt} = \rho Y_1 - \delta_3 Y_2.$$

Here X_1 is the number of infected people before developing AIDS, X_2 is the number of people who have full-blown AIDS in the scale-up period, X_3 is the number of people who have full-blown AIDS after removing annual recruited people on ART-1 and annual deaths without ART-1, Y_1 is the number of people on ART-1, and Y_2 is the effective number of people on ART-2. After we projected X_1 values for the predominantly No ART period, we used the model below for projections of the number of people on ART-1 and ART-2 during post ART scale-up years, i.e., from the middle of NACP III years and for the NACP IV period.

The parameter descriptions and values are provided in Table 1. Reciprocals of the parameter values are used in the model wherever the parameter has units in time.

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2012 Steele Prizes

The 2012 AMS Leroy P. Steele Prizes were presented at the 118th Annual Meeting of the AMS in Boston in January 2012. The Steele Prizes were awarded to Michael Aschbacher, Richard Lyons, Steve Smith, and Ronald Solomon for Mathematical Exposition; to William Thurston for a Seminal Contribution to Research; and to Ivo M. Babuška for Lifetime Achievement.

Mathematical Exposition: Michael Aschbacher, Richard Lyons, Steve Smith, and Ronald Solomon

Citation

The 2012 Leroy P. Steele Prize for Mathematical Exposition is awarded to Michael Aschbacher, Richard Lyons, Steve Smith, and Ronald Solomon for their work, *The Classification of Finite Simple Groups: Groups of Characteristic 2 Type,* Mathematical Surveys and Monographs, 172, American Mathematical Society, Providence, RI, 2011. In this paper, the authors, who have done foundational work in the classification of finite simple groups, offer to the general mathematical public an articulate and readable exposition of the classification of characteristic 2 type groups.

Biographical Sketches

Michael Aschbacher was born in Little Rock, Arkansas, in 1944. He received his undergraduate degree from Caltech in 1966 and his Ph.D. from the University of Wisconsin in 1969 under the direction of Richard Bruck. He was a postdoctoral fellow at the University of Illinois in 1969–70, and since then he has been at Caltech, where he is the Shaler Arthur Hanisch Professor of Mathematics. He received the Cole Prize in Algebra from the AMS in 1980 and the Rolf Schock Prize from the Royal Swedish Academy of Sciences in 2011. He was an invited speaker at the International Congress of Mathematicians in 1978 and a vice president of the AMS from 1996 to 1998. He is a member of the National Academy of Sciences and the American Academy of Arts and Sciences. Aschbacher's research focuses on the finite simple groups.

Richard Lyons earned his Ph.D. under the supervision of John G. Thompson at the University of Chicago, with a brief stop at the University of Cambridge. At Chicago he had further tutelage from Jon Alperin, Richard Brauer, George Glauberman, Marty Isaacs, and Leonard Scott. He had graduated from Harvard College and had been inspired by the high school teaching of Dr. Beryl E. Hunte. After a J. Willard Gibbs Instructorship at Yale, he joined the faculty of Rutgers, where he began a long-term collaboration with the late Danny Gorenstein and where he now serves in his fortieth year.

Stephen D. Smith is Professor Emeritus of Mathematics at the University of Illinois at Chicago. He received his S.B. from M.I.T. in 1970 and his D.Phil. in 1973 under the supervision of Graham Higman at Oxford (where he was a Rhodes Scholar). He was a Bateman Research Instructor at Caltech from 1973 to 1975 and then moved to UIC as assistant professor and later associate professor and professor. He has published mainly in finite group theory, with further interests in combinatorics, algebraic topology, and computer science. In addition to the book cited for this Steele Prize, which was written jointly with Aschbacher, Lyons, and Solomon, he has also published *The Classification of Quasithin Groups* with Michael Aschbacher, *Classifying Spaces of Sporadic Groups* with Dave Benson, and most recently *Subgroup Complexes.* Smith married Judith L. Baxter in 1980 and has two adult stepchildren.

Ronald Solomon got his love of words from his mother and his love of math from his high school teacher, Blossom Backal. He graduated from Queens College (CUNY) in 1968 and earned his Ph.D. at Yale University in 1971 under the supervision of Walter Feit. After a Dickson Instructorship at the University of Chicago and a year at Rutgers University, he joined the faculty of the Ohio State

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Michael Aschbacher

Richard Lyons

Steve Smith

Ronald Solomon

University in 1975. Since 1982 he has been a member of a team, with Danny Gorenstein and Richard Lyons, that wrote a series of volumes (Mathematical Surveys and Monographs, 40.1–40.6, American Mathematical Society, Providence, RI, 1994, 1996, 1998, 1999, 2002, 2005) which presents a substantial portion of the proof of the classification of the finite simple groups. Ron earned an Ohio State Distinguished Teaching Award in 1997 and the Levi L. Conant Prize of the AMS in 2006. He is grateful and proud to be the husband of Rose and the father of Ari and Michael.

Joint Response from Michael Aschbacher, Richard Lyons, Steve Smith, and Ronald Solomon

We are deeply grateful to the Society for honoring us with this Steele Prize for Mathematical Exposition. For decades Danny Gorenstein was the voice of the Classification Project, providing the community with a vivid narrative of our travails and accomplishments. Unfortunately, he departed this life before the task was completed and the tale fully told. Our book serves in part as a sequel to his 1983 volume, providing a detailed reader's guide to the major papers composing the second ("even") half of the Classification proof, but we have prefaced it with an outline and synopsis of the entire proof, updating Danny's references and giving our personal view of the entire enterprise. In writing a book it always helps to have a great story to tell, and few mathematical projects have played out on such an epic scale and reached such a gratifying culmination as the Classification of the Finite Simple Groups. We appreciate that, in awarding us this prize, the Society acknowledges the importance of this work.

Seminal Contribution to Research: William Thurston

Citation

The Leroy P. Steele Prize for Seminal Contribution to Research is awarded to William Thurston for his contributions to low dimensional topology, and in particular for a series of highly original papers, starting with "Hyperbolic structures on 3-manifolds. I. Deformation of acylindrical manifolds" (*Ann. of Math. (2)* **124** (1986), no. 2, 203–246), that revolutionized 3-manifold theory. These papers transformed the field from a subfield of combinatorial topology to a web of connections between topology, complex analysis, dynamical systems, and hyperbolic geometry. In addition, Thurston not only gave a complete conjectural picture of all compact 3-manifolds, but in these papers he proved his conjecture for a large class of examples, namely Haken manifolds, which include all compact 3-manifolds with nonempty boundary.

Biographical Sketch

William P. Thurston was born October 30, 1946, in Washington, DC, and he received his Ph.D. in mathematics from the University of California at Berkeley in 1972. He taught at the IAS (1972–73) and at MIT (1973–74) before joining the faculty of Princeton University in 1974. Professor Thurston returned to UC-Berkeley, this time as a faculty member, in 1991 and became director of MSRI in 1993. He then taught at UC-Davis from 1996 to 2003 and accepted a position at Cornell University in 2003, where he holds joint appointments in the Department of Mathematics and the Faculty of Computing and Information Science.

William Thurston

Professor Thurston held an Alfred P. Sloan Foundation Fellowship in 1974–1975; in 1976 he was awarded the AMS Oswald Veblen Geometry Prize for his work on foliations. In 1979 he became the second mathematician ever to receive the Alan T. Waterman Award, and in 1982 Professor

Thurston was awarded the Fields Medal. He is a member of the American Academy of Arts and Sciences and the National Academy of Sciences.

Response

I am deeply honored by this recognition from the American Mathematical Society. I have loved mathematics all my life. I felt very lucky when I discovered the mathematical community—local, national, and international—starting in graduate school. So the Steele Prize, with its long and distinguished history of honoring mathematicians whom I greatly admire, means a lot to me.

The work cited by the Steele Prize focuses on what I called the "geometrization conjecture". When I gradually realized the geometric beauty of 3-manifolds, it was as if a giant whirlwind, far bigger and far stronger than I, had swept me up and taken over my mathematical life. I couldn't escape (admittedly, I didn't even want to escape). At first I glimpsed only parts of the big picture, but little by little it came into focus and the mist blew away. I worked very hard and was able to prove the geometrization conjecture in many important cases, including, in some sense, "almost all" cases. I became completely convinced that the geometrization conjecture is true, but my approaches were extremely difficult, if not impossible, to push through.

I was ecstatic to be able to prove the geometrization conjecture in certain sweeping families of cases, but I was frustrated that my various methods seemed very difficult, if not impossible, to extend to all cases. I became completely convinced that the geometrization theorem was true, but it was frustrating not to have a complete proof. I was very pleased when Grigori Perelman, using very natural methods pioneered by Richard Hamilton (but foreign to my technical expertise) proved the geometrization conjecture in full generality.

I have been very lucky to have a long stream of wonderful students. They and others have built up a thriving mathematical community well versed in geometric structures on 3-manifolds, as well as other related structures on 3-manifolds, such as taut foliations, tight contact structures, etc. There are still many mysteries to solve in this area. I used to feel that there was certain knowledge and certain ways of thinking that were unique to me. It is very satisfying to have arrived at a stage where this is no longer true—lots of people have picked up on my ways of thought, and many people have proven theorems that I once tried and failed to prove.

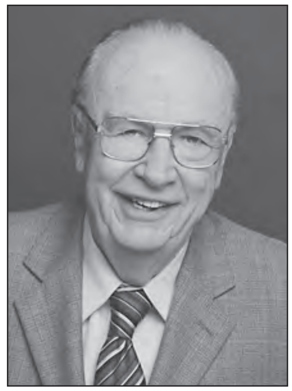
Lifetime Achievement: Ivo M. Babuška

Citation

The 2012 Steele Prize for Lifetime Achievement is awarded to Ivo M. Babuška for his many pioneering advances in the numerical solution of partial differential equations over the last half century.

In his work on finite element methods, Babuška has developed and applied mathematics in profound ways to develop, analyze, and validate algorithms which are crucial for computational science and engineering. In so doing, he has helped to define that field and has had a great impact on the modern world.

A constant characteristic of Babuška's work is the combination of deep and imaginative mathematical analysis with a constant concern for the practical implications of his work for engineering applications. In seminal work of the 1960s and 1970s, he established the mathematical foundations of the finite element method culminating in a monumental and highly influential treatise coauthored with Aziz in 1972. In this early work he established the essential role of stability of Galerkin methods and formulated the discrete inf-sup condition, later to be named the Babuška-Brezzi condition, and developed the approximation theory of finite element spaces. He also introduced many techniques of lasting importance, such as the imposition of Dirichlet boundary conditions through Lagrange multipliers and through penalties, analysis through mesh-dependent norms, the first studies of a posteriori error estimation, and the Babuška-Rheinboldt theory of adaptivity. The Babuška paradox for elastic plates, which shows strikingly that the deformation of a circular elastic plate is not well approximated by the deformation of even a nearly equal polygonal plate, has inspired important developments in mechanics, partial differential equations, and numerical methods.



Ivo Babuška

Babuška is an exceptionally productive author, collaborator, and mentor. He has published over 350 refereed journal articles and 26 books, has had nearly 150 coauthors, and has advised 40 Ph.D. students. An astounding feature of Babuška's work is how many themes he initiated [that] grew into large and active research fields. In the mid-1970s Babuška was among the pioneers of homogenization, which aims to capture the large-scale effects of fine-scale features of materials without resolving them. Later he developed generalized finite element methods with J. E. Osborn which sought to capture the influence of subgrid scale features in computational methods and anticipated a large and currently active branch of research in multiscale numerical methods. In the late 1980s

Babuška and collaborators developed the *p*-version of the finite element method, and later the *hp*-method, and developed an elaborate theory for understanding its convergence in the presence of singularities. His important work in dimensional reduction and hierarchical models also dates from this time. In the 1990s, he developed the partition-of-unity finite element method, which led to another large and active area of development on meshless methods. In this century he has led the way to the computation of partial differential equations with uncertain data and the booming field of uncertainty quantification.

Ivo M. Babuška is among the foremost numerical analysts of all time and a unique leader in applied mathematics. His many contributions have had a lasting impact on mathematics, engineering, science, and industry. The Steele Prize honors him for all of these achievements.

Biographical Sketch

Ivo Babuška was born 1926 in Prague, Czechoslovakia. He received his civil engineering degree (Ing.) and his Ph.D. degree from the Czech Technical University in Prague. After that he studied mathematics and received a Ph.D. (then called a Candidate of Science, C.Sc., degree as in the USSR) and then the doctorate degree (Doctor of Science, D.Sc.) in mathematics from the Czechoslovak Academy of Sciences. He worked in the Mathematical Institute of the Academy and received the Czechoslovak State Award for his scientific work in 1968.

In 1968 he came to the University of Maryland at College Park as a visiting scientist, where he then became a professor in the mathematics department. He retired from Maryland as Distinguished University Professor in 1995.

Since 1995 he has been a senior scientist of the Institute for Computational Engineering and Sciences and professor of aerospace engineering and engineering mechanics, holding the Robert Trull Chair in Engineering at the University of Texas at Austin. Now half retired, he is still working at the university.

Ivo Babuška has received various honors recognizing his contributions. He has been awarded five honorary doctorate degrees, was elected to the U.S. National Academy of Engineering, the European Academy of Sciences, the Engineering Academy of the Czech Republic, and the Learned Society of Czech Republic. He is a fellow of SIAM and ICAM. Asteroid 36060 was named Babuška. He also received various recognitions, including the Birkhoff Prize of SIAM and the AMS, the ICAM Congress Medal, and the Bolzano Medal.

Response from Ivo Babuška

I am deeply honored to receive the Steele Prize for Lifetime Achievement from the AMS because my work encompasses both mathematics and engineering applications and computations. I am very fortunate that many of my mathematical results are used widely in engineering and practical computations. It is very satisfactory and important to me that my mathematical results are appreciated by both the mathematical and engineering communities and [that] they are also used in practice. This was influenced by my combined education in engineering and mathematics and by my mentors, Professor Faltus in engineering, Professor E. Cech, the well-known topologist, and Professor F. Vycichlo, to whom I am very grateful. On this occasion, I would like to thank all my scientific collaborators and friends in the mathematics and engineering communities. I cannot list them all, so I will mention here only a very few: J. Osborn, J. Whiteman, J. T. Oden, and B. Szabo, and my students with whom I have enjoyed not only doing mathematics but also hiking, skiing, and various excursions and adventures.

Finally, I would like to thank very much the selection committee for this great honor.

About the Prize

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–06, and Birkhoff served in that capacity during 1925–26. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) Mathematical Exposition: for a book or substantial survey or expository research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field or a model of important research. Each Steele Prize carries a cash award of US$5,000.

Beginning with the 1994 prize, there has been a five-year cycle of fields for the Seminal Contribution to Research Award. For the 2012 prize, the field was geometry/topology. The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2012 prizes, the members of the selection committee were Peter S. Constantin, Yakov Eliashberg, John E. Fornaess, Irene M. Gamba, Barbara L. Keyfitz, Joel A. Smoller, Terence C. Tao, Akshay Venkatesh, and Lai-Sang Young. The list of previous recipients of the Steele Prize may be found on the AMS website at `http://www.ams.org/prizes-awards`.

— Elaine Kehoe

2012 Cole Prize in Algebra

ALEXANDER S. MERKURJEV received the 2012 AMS Frank Nelson Cole Prize in Algebra at the 118th Annual Meeting of the AMS in Boston in January 2012.

Citation

The 2012 Frank Nelson Cole Prize in Algebra is awarded to Alexander S. Merkurjev of the University of California, Los Angeles, for his work on the essential dimension of groups.

The essential dimension of a finite or of an algebraic group G is the smallest number of parameters needed to describe G-actions. For instance, if G is the symmetric group on n letters, this invariant counts the number of parameters needed to specify a field extension of degree n, which is the algebraic form of Hilbert's thirteenth problem.

Merkurjev's papers ("Canonical p-dimension of algebraic groups", with N. Karpenko, *Adv. Math.* **205** (2006), no. 2, 410–433; and "Essential dimension of finite p-groups", with N. Karpenko, *Invent. Math.* **172** (2008), no. 3, 491–508) introduce breakthrough new techniques to compute the essential dimension of p-groups. In his paper "Essential p-dimension of $\mathrm{PGL}(p^2)$" (*Jour. Amer. Math. Soc.* **23** (2010), no. 3, 693–712), which is a tour de force, Merkurjev calculates the essential dimension, localized at a prime p, of the group $\mathrm{PGL}\,(p^2)$, which is bound up with understanding the structure of division algebras of dimension p^4 over general fields.

Merkurjev's unique style combines strength, depth, clarity, and elegance, and his ideas have had broad influence on algebraists over the last three decades.

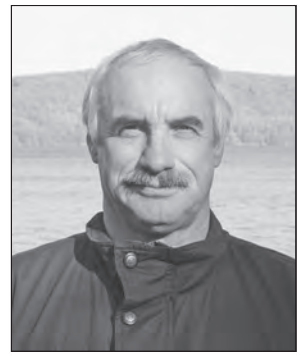
Biographical Sketch

Alexander Merkurjev was born on September 25, 1955, in St. Petersburg (Leningrad), Russia. In 1977 he graduated from St. Petersburg University, and he received his Ph.D. there in 1979 under the direction of Anatoly Yakovlev. In 1983 he earned the Doctor of Sciences degree from St. Petersburg University for the work "Norm residue homomorphism of degree two". In 1983 Merkurjev won the Young Mathematician Prize of the St. Petersburg Mathematical Society for his work on algebraic K-theory. In 1995 he was awarded the Humboldt Prize. In 1977 Merkurjev became a professor at St. Petersburg University. Since 1997 he has been a professor at UCLA.

Merkurjev's interests lie in algebraic K-theory, algebraic groups, algebraic theory of quadratic forms, and essential dimension. He was an invited speaker at the International Congress of Mathematicians (Berkeley, 1986). Twice he has delivered an invited address at the European Congress of Mathematics (1992, 1996), and he was a plenary speaker in 1996 (Budapest).

Response from Alexander S. Merkurjev

It is a great honor and great pleasure for me to receive the 2012 Frank Nelson Cole Prize in Algebra. I would like to thank the American Mathematical Society and the Selection Committee for awarding the prize to me.



Alexander S. Merkurjev

I am very grateful to my teacher, Andrei Suslin (he was awarded the Frank Nelson Cole Prize in Algebra in 2000). I also want to thank my parents, family, friends, and colleagues for their help and support over the years.

About the Prize

The Cole Prize in Algebra is awarded every three years for a notable paper in algebra published during the previous six years. The awarding of this prize alternates with the awarding of the Cole Prize in Number Theory, also given every three years. These prizes were established in 1928 to honor Frank Nelson Cole on the occasion of his retirement as secretary of the AMS after twenty-five years of service. He also served as editor-in-chief of the *Bulletin* for twenty-one years. The Cole Prize carries a cash award of US$5,000.

The Cole Prize in Algebra is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2012 prize, the members of the selection committee were Robert L. Griess, János Kollár, and Parimala Raman.

Previous recipients of the Cole Prize in Algebra are: L. E. Dickson (1928), A. Adrian Albert (1939), Oscar Zariski (1944), Richard Brauer (1949), Harish-Chandra (1954), Serge Lang (1960), Maxwell A. Rosenlicht (1960), Walter Feit and John G. Thompson (1965), John R. Stallings (1970), Richard G. Swan (1970), Hyman Bass (1975), Daniel G. Quillen (1975), Michael Aschbacher (1980), Melvin Hochster (1980), George Lusztig (1985), Shigefumi Mori (1990), Michel Raynaud and David Harbater (1995), Andrei Suslin (2000), Aise Johan de Jong (2000), Hiraku Nakajima (2003), János Kollár (2006), and Christopher Hacon and James McKernan (2009).

— Elaine Kehoe

DOI: http://dx.doi.org/10.1090/noti823

2012 Conant Prize

Persi Diaconis received the 2012 AMS Levi L. Conant Prize at the 118th Annual Meeting of the AMS in Boston in January 2012.

Citation

The Levi L. Conant Prize for 2012 is awarded to Persi Diaconis for his article, "The Markov chain Monte Carlo revolution" (*Bulletin Amer. Math. Soc.* **46** (2009), no. 2, 179–205).

This wonderful article is a lively and engaging overview of modern methods in probability and statistics and their applications. It opens with a fascinating real-life example: a prison psychologist turns up at Stanford University with encoded messages written by prisoners, and Marc Coram uses the Metropolis algorithm to decrypt them. From there, the article gets even more compelling!

Persi Diaconis

After a highly accessible description of Markov chains from first principles, Diaconis colorfully illustrates many of the applications and venues of these ideas. Along the way, he points to some very interesting mathematics and some fascinating open questions, especially about the running time in concrete situations of the Metropolis algorithm, which is a specific Monte Carlo method for constructing Markov chains. The article also highlights the use of spectral methods to deduce estimates for the length of the chain needed to achieve mixing.

The article is eminently readable, with amply illustrated applications to random permutations and random walks on Cayley graphs, which bring into the picture symmetric function theory, Schur functions, and Jack polynomials. Other examples relate to the connectedness of hard disc arrays, phase transitions in statistical mechanics, and population dynamics with immigration.

Diaconis entertains and educates us at every step of his journey, delightfully convincing us that Markov chains are everywhere. His voice shines through the writing, for example: "I clearly remember my first look at David Wilson's sample of a 2000×2000 Ising model at the critical temperature. I felt like someone seeing Mars for the first time through a telescope."

After providing helpful instructions for how "grown-up" mathematicians can begin to learn about this field, Diaconis concludes his tour with brief descriptions of connections to group representation theory, algebraic geometry, PDEs, chemistry, physics, biology, and computer science. He writes, "To someone working in my part of the world, asking about applications of Markov chain Monte Carlo (MCMC) is a little like asking about applications of the quadratic formula. The results are really used in every aspect of scientific inquiry." His article convinces us that this is so.

Biographical Sketch

Persi Diaconis graduated from New York's City College in 1971 and earned a Ph.D. in mathematical statistics from Harvard in 1974. He has taught at Stanford, Cornell, and Harvard. An early MacArthur winner, he is a member of the American Academy of Arts and Sciences, the U.S. National Academy, and the American Philosophical Society. He is always trying to play down his ten years as a professional magician.

Response from Persi Diaconis

As a regular reader of expository articles, I am thrilled that mine seemed useful. The *Bulletin* does a great service with these. While I have the chance, I want to point to two other recent *Bulletin* articles that I am proud of: "Patterns in eigenvalues" (my Gibbs Lecture, 2002) and "On adding a list of numbers (and other one-dependent determinantal processes)" (with A. Borodin and J. Fulman, 2009). I promise to keep at it. Thank you.

About the Prize

The Conant Prize is awarded annually to recognize an outstanding expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years. Established in 2001, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic University. The prize carries a cash award of US$1,000.

The Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2012 prize, the members of the selection committee were Jerry L. Bona, J. Brian Conrey, and Ronald M. Solomon.

Previous recipients of the Conant Prize are Carl Pomerance (2001); Elliott Lieb and Jakob Yngvason (2002); Nicholas Katz and Peter Sarnak (2003); Noam D. Elkies (2004); Allen Knutson and Terence Tao (2005); Ronald M. Solomon (2006); Jeffrey Weeks (2007); J. Brian Conrey, Shlomo Hoory, Nathan Linial, and Avi Wigderson (2008); John W. Morgan (2009); Bryna Kra (2010); and David Vogan (2011).

— *Elaine Kehoe*

DOI: http://dx.doi.org/10.1090/noti824

2012 Morgan Prize

JOHN PARDON received the 2012 AMS-MAA-SIAM Frank and Brennie Morgan Prize for Outstanding Research in Mathematics by an Undergraduate Student at the Joint Mathematics Meetings in Boston in January 2012. Receiving honorable mentions were HANNAH ALPERT and ELINA ROBEVA.

Citation

John Pardon has been named the recipient of the 2012 Morgan Prize for Outstanding Research by an Undergraduate Student for solving a problem on distortion of knots posed in 1983 by Mikhail Gromov. Demonstrating brilliant geometric understanding, John solved the problem by exhibiting a sequence of torus knots with distortions going to infinity. More precisely, given a smooth (or rectifiable) embedding of a knot K into 3-space, consider the ratios of the intrinsic and extrinsic distances between pairs of distinct points on the knot. The supremum of this ratio over all pairs is the distortion of the embedding. The distortion of a knot K is the infimum of the distortions of all rectifiable curves in the isotopy class of K. John's elegant proof was a beautiful mix of geometry and topology combined with some analytic arguments. John learned about this problem on his own (and in high school). According to his letters of recommendation, with this problem, no one had any idea how to get started; the key insight that cracked this problem is due to John. This paper appeared in the July issue (volume 174, number 1) of the *Annals of Mathematics.*

John has had five papers published, with another two submitted, one of which (with János Kollár) resulted from a conversation at a Phi Beta Kappa dinner at which Kollár asked John about a topology problem he had been posing to various topologists for about a year, without success. A week later, John sent Kollár an email with a solution to the problem, and they began working together, leading to the paper submitted in April. John's letters of recommendation describe him as very knowledgeable and insightful. John has given talks on his work at the Southeast Geometry Conference at the University of South Carolina, at geometry and topology seminars at City University of New York, Georgia Tech, University of Georgia, as well as at Princeton seminars and conferences.

DOI: http://dx.doi.org/10.1090/noti825

Biographical Sketch

John Pardon was raised in Chapel Hill, North Carolina, and began taking mathematics classes at Duke University while he was still in high school at Durham Academy. He was the valedictorian of Princeton's 2011 graduating class, majoring in mathematics. John was also a member of Princeton's winning team in an international Chinese-language debate, having taken Chinese throughout all four years at Princeton. A Phi Beta Kappa, he is also an accomplished cellist, twice winning the Princeton Sinfonia's annual concerto competition; he was a four-year member of the Sinfonia.

John Pardon

John's first paper in mathematics, "On the unfolding of simple closed curves", was submitted to the *Transactions of the American Mathematical Society* in January of his senior year in high school. It was also in high school that one of his favorite pastimes (reading mathematics papers online) introduced him to the problem on distortion of knots posed [by] Gromov, the solution of which brings to him the Morgan Prize. John has received numerous recognitions for his academic achievements; some of these are the Goldwater scholarship, two-time winner of Princeton's Shapiro Prize for academic excellence, and an NSF Graduate Research Scholarship to support his graduate studies at Stanford University, where he is currently.

Response from John Pardon

I am very honored to receive the 2012 Morgan Prize. I would like to thank the AMS, MAA, and SIAM for sponsoring the award, and Mrs. Frank Morgan for endowing it.

I am grateful to everyone who has taught me mathematics, especially my dad, for sharing their expertise and enthusiasm.

Thanks are due to David Gabai for helpful discussions about my work on knot distortion and to János Kollár for sharing and discussing topology problems with me.

Citation for Honorable Mention: Hannah Alpert

Hannah Alpert is recognized with an Honorable Mention for the 2012 Morgan Prize for Outstanding Research by an Undergraduate Student for a body of work consisting of six papers, five of which were published and one submitted prior to her graduation from the University of Chicago in June 2011. The first of these, in terms of her timeline of work, is a joint paper on topological graph theory on which she worked while in high school. Her coauthors on this paper point out that they sent this high school student the remaining cases in the proof that all six-colorable triangulations of the torus satisfy Grünbaum's conjecture, cases on which they were stuck. Hannah finished them off quickly, and this paper appeared in the *Journal of Graph Theory* early in 2010. An anonymous referee's comment on Hannah's paper "Rank numbers of grid graphs" (*Discrete Mathematics*, 2010) says, "The compilation of results forms arguably the best paper on the topic in the last decade." This is one of three professional-level papers she wrote in her 2009 REU at University of Minnesota Duluth.

Rather than exploit her novel approach to ranking numbers (her first paper of the summer of 2009) to obtain more results, Hannah asked for a different topic and successfully extended previous results on phase transitions in countable Abelian groups. She also provided the first results on phase transitions for uncountable Abelian groups and infinite nonabelian groups. She spoke on this work at the Combinatorial and Additive Number Theory conference, which is sponsored by the New York Number Theory Seminar. Hannah had two papers related to tournaments following the Lafayette College REU and a joint paper in *Discrete and Computational Geometry* as a result of the Willamette Valley REU the summer after her first year at the University of Chicago.

Biographical Sketch

Hannah Alpert grew up in Boulder, Colorado, attending Fairview High School. It was in high school that she began her mathematical research that led to a joint paper published in the *Journal of Graph Theory*. Hannah participated in the Hampshire College Summer Studies in Mathematics for three summers and was a MathPath camp counselor the summer before she entered the University of Chicago, from which she graduated in June 2011.

While an undergraduate at Chicago, Hannah participated in three REUs (Willamette Valley, University of Minnesota Duluth, and Lafayette College) and in each of these successfully solved posed problems, resulting in publications in *Discrete and Computational Geometry, Discrete Mathematics, Integers,* and *Archiv der Mathematik*. She also participated in the Budapest Semesters in Mathematics. Hannah was recognized at JMM 2009 with an MAA Undergraduate Poster Session Prize. She was awarded the Barry M. Goldwater Scholarship in 2009 and was a winner of the 2010 Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman. Hannah is in her first year of graduate work at MIT, where she is supported with an NSF Graduate Fellowship.

Response from Hannah Alpert

I am grateful to have been selected for Honorable Mention for the 2012 Morgan Prize. I would like to thank Sarah-Marie Belcastro, Josh Laison, Joe Gallian, Mel Nathanson, and Garth Isaak for the work they have done to facilitate my research.

Citation for Honorable Mention: Elina Robeva

Elina Robeva is recognized with an Honorable Mention for the 2012 Morgan Prize for Outstanding Research by an Undergraduate Student for her work with Sam Payne of Yale University on a new proof of the Brill-Noether theorem using tropical geometry. Elina began work on the deep and difficult mathematics of Brill-Noether theory during her sophomore year at Stanford; the coauthored paper "A tropical proof of the Brill-Noether theorem" has been recommended for publication in *Advances in Mathematics.* Elina's letters of recommendation say that without her persistence, independence, and insight this project would have ended far short of the ultimate goal of a new proof of the Brill-Noether theorem. It is noted that the Brill-Noether theorem is a remarkable result that has spawned an entire subfield of algebraic geometry and that the paper of which Elina is a coauthor may reasonably be the most important paper of the year in tropical geometry. Multiple definitive breakthroughs along the way to the new proof were due solely to Elina.

Prior to the Brill-Noether work, Elina proved an elegant formula for the optimal strategy in Bidding Hex (where players bid for the right to move, rather than taking turns). Her formula is beyond the computing capacities of contemporary machines; however, Elina developed and implemented a Monte Carlo approximation to this optimal strategy that is available online and is undefeated against human opponents. This work led to a joint paper, "Artificial intelligence for Bidding Hex", which appeared in the volume *Games of No Chance* in December 2008. Elina is referred to by her references as a mature and powerful research mathematician who is known for her attitude of seeking out challenges and working both hard and wisely. "The essential quality in a mathematician, the willingness to dive into a research problem and not be fearful, is something that Elina has developed at a young age."

Biographical Sketch

Elina Robeva was born in and grew up in Sofia, Bulgaria. Her interest in mathematics developed in middle school through competitions. By the time she graduated from high school, she had won two silver medals in the International Mathematical Olympiad, a gold medal in the Balkan Mathematical Olympiad, and various other awards from national and international competitions. Then she enrolled at Stanford, where she concentrated on theoretical mathematics and research. She graduated in June 2011 and was recognized with a Deans' Award for Academic Accomplishment and a Sterling Award for Scholastic Achievement. The article announcing these awards says that she "devoured the most challenging undergraduate and graduate mathematics courses at Stanford." She also achieved an honorable mention on the 2010 William Lowell Putnam examination and spent a summer at Facebook as a software engineer. Elina is now in her first year of the mathematics Ph.D. program at Harvard.

Response from Elina Robeva

I am very honored to have received this recognition, and I thank the AMS, MAA, and SIAM for selecting me for this award.

I would like to express my gratitude to the people who have had the most impact on my mathematical education thus far. I thank Ravi Vakil for the great support and advice and for all the times when he encouraged me to pursue various challenging mathematical tasks. I thank Sam Payne for being a wonderful research advisor and providing me with really interesting and engaging research problems. I also express my gratitude to Persi Diaconis for his great advice during my time at Stanford. I thank my high school teacher, Svetla Angelova, and the Bulgarian Academy of Sciences for the great preparation and opportunities to take part in mathematical competitions. Finally, I thank my mother, Rumyana Ivanova, for her unbounded love, support, and patience, which have continuously guided me during my education.

About the Prize

The Morgan Prize is awarded annually for outstanding research in mathematics by an undergraduate student (or students having submitted joint work). Students in Canada, Mexico, or the United States or its possessions are eligible for consideration for the prize. Established in 1995, the prize was endowed by Mrs. Frank (Brennie) Morgan of Allentown, Pennsylvania, and carries the name of her late husband. The prize is given jointly by the AMS, the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) and carries a cash award of US$1,200.

Recipients of the Morgan Prize are chosen by a joint AMS-MAA-SIAM selection committee. For the 2012 prize, the members of the selection committee were Colin C. Adams, Jill Dietz, Kathleen R. Fowler, Anna L. Mazzucato, Kannan Soundararajan, and Sergei Tabachnikov.

Previous recipients of the Morgan Prize are Kannan Soundararajan (1995), Manjul Bhargava (1996), Jade Vinson (1997), Daniel Biss (1998), Sean McLaughlin (1999), Jacob Lurie (2000), Ciprian Manolescu (2001), Joshua Greene (2002), Melanie Wood (2003), Reid Barton (2005), Jacob Fox (2006), Daniel Kane (2007), Nathan Kaplan (2008), Aaron Pixton (2009), Scott Duke Kominers (2010), and Maria Monks (2011).

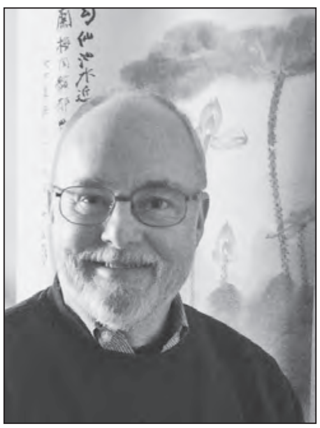
— Elaine Kehoe

2012 Whiteman Prize

JOSEPH WARREN DAUBEN received the 2012 AMS Albert Leon Whiteman Memorial Prize at the 118th Annual Meeting of the AMS in Boston in January 2012.

Citation

The American Mathematical Society is pleased to award the Albert Leon Whiteman Prize to Joseph Warren Dauben for his contributions to the history of Western and Chinese mathematics and for deepening and broadening the international mathematical community's awareness and understanding of its history and culture. "In truth mathematics can be called the Pleasure Garden of the myriad forms, the Erudite Ocean of the Hundred Schools of Philosophy," said the sixteenth-century mathematician Xu Guangqi. Joe Dauben's work illuminates his epigram.



Joseph W. Dauben

Dauben's first book, *Georg Cantor: His Mathematics and the Philosophy of the Infinite* (Harvard University Press, Cambridge, MA, 1979; reprinted, Princeton University Press, Princeton, 1990), is a clear, readable, detailed, richly textured history of Cantor's development of transfinite numbers and, at the same time, an insightful but nonreductive account of the complex and difficult personality that brought this revolution about. His second biography, *Abraham Robinson: The Creation of Nonstandard Analysis, A Personal and Mathematical Odyssey* (Princeton University Press, Princeton, 1995), is another near-impossible feat of scholarship and exposition. Dauben traces Robinson's nonstandard path from his birthplace in Germany to his death in New Haven, via Palestine, Paris, London, Toronto, Jerusalem, and Los Angeles, again intertwining the mathematician and his mathematics.

While writing *Abraham Robinson*, Dauben began a study of "Ten Classics of Ancient Chinese Mathematics", a 656 CE edition of texts taught throughout Chinese history, in order to relate them to early Western mathematics and make them understandable to the modern reader. "Chinese mathematics," his contribution to *The Mathematics of Egypt, Mesopotamia, China, India, and Islam* (Princeton University Press, Princeton, 2007), is the fruit of twenty years of scholarship and, at nearly 200 pages, is a book in its own right. Dauben studies the history of modern Chinese mathematics as well; see, for example, his chapter "Modern science emerges in China" in *Mathematics Unbound: The Evolution of an International Mathematical Research Community, 1800–1945* (History of Mathematics, 23, Karen Hunger Parshall and Adrian L. Rice, editors, American Mathematical Society, Providence, RI; London Mathematical Society, London, 2002). He is an elected Honorary Professor of the Institute for the History of Natural Science (a branch of the Chinese Academy of Sciences) and lectures there regularly.

In *Writing the History of Mathematics: Its Historical Development* (Sci. Networks Hist. Stud., 27, Birkhäuser, Basel, 2002), Dauben and coeditor Christoph Scriba engage historians of mathematics around the world to show how the practice of writing history of mathematics has varied from country to country and era to era and how this historiography is intertwined with the philosophical, scientific, and industrial demands of time and place. In the United States, the history of mathematics, once an excellent but eclectic collection of teaching tools and postretirement projects, has in the last decades become an integral component of the mathematical community, with well-attended sessions at major meetings. Joe Dauben has spurred this professionalism by his scholarly example and through his service to the profession, which includes organizing international workshops and symposia and editing *Historia Mathematica* for a decade.

Like his list of published articles, the list of Joe Dauben's honors is long. We are proud to add the Whiteman Prize to it.

Biographical Sketch

Joseph W. Dauben is Distinguished Professor of History and the History of Science at Herbert H. Lehman College and a member of the Ph.D. Program

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in History at the Graduate Center of the City University of New York. He is a fellow of the New York Academy of Sciences, a *membre effectif* of the International Academy of History of Science, a corresponding member of the German Academy of Sciences Leopoldina, and a member of the Society of Fellows of the American Academy in Rome. He has been editor of *Historia Mathematica*, an international journal for the history of mathematics, and chairman of the International Commission on the History of Mathematics. He is the author of *Georg Cantor, His Mathematics and Philosophy of the Infinite* (Harvard University Press, Cambridge, MA, 1979; reprinted, Princeton University Press, Princeton, 1990) and *Abraham Robinson: The Creation of Nonstandard Analysis, a Personal and Mathematical Odyssey* (Princeton University Press, Princeton, 1995), both of which have been translated into Chinese. Among his most recent publications is the monographic study of one of the most ancient works of Chinese mathematics, "*Suan shu shu.* A book on numbers and computations. English translation with commentary" (*Archive for History of Exact Sciences* **62** (2008), no. 2, 91–178). A graduate of Claremont McKenna College, magna cum laude, in mathematics and of Harvard University (A.M. and Ph.D.) in history of science, Dauben has been a member of the Institute for Advanced Study (Princeton) and Clare Hall (Cambridge University), where he was affiliated with the Needham Research Institute. He has been the recipient of a Guggenheim Fellowship, Senior NEH and ACLS Fellowships, and was named Outstanding Teacher of the Year at Lehman College in 1986. He is an honorary member of the Institute for History of Natural Sciences of the Chinese Academy of Sciences, where he was the Zhu Kezhen Visiting Professor in spring of 2005. In 2010 he was Visiting Research Professor at the Institute for Humanities and Social Sciences at National Chiao-Tung University in Hsinchu (Taiwan).

Response from Joseph Warren Dauben

Albert Leon Whiteman was a mathematician with a passion for number theory and an abiding interest in the history of mathematics. He was Hans Rademacher's first student at the University of Pennsylvania and a Benjamin Peirce instructor at Harvard before taking up a position in 1948 at the University of Southern California, where he spent the rest of his career. I might have been one of his graduate students, for USC was one of the universities where I was accepted to continue as a graduate student in mathematics after I had completed my undergraduate degree in 1966 at Claremont McKenna College in southern California. I had studied mathematics there with John Ferling, Granville Henry, and Janet Myhre and wrote my senior thesis, which later would prove to be especially useful, as it turned out, on nonstandard analysis. Ferling had himself been a student at USC and was the reason I had applied there as one of my choices for graduate school. But, having grown up in southern California, the offer Harvard made to pursue my doctorate there in history of science proved irresistible, and in the fall of 1966 I found myself on the East Coast, in Cambridge, where there was a lively, if small but in retrospect remarkable, concentration of historians of mathematics. The Department for History of Science, one of the first in the country, had just been established, and I was among the earliest groups to join the new department, along with my friend, colleague, and fellow historian of mathematics Wilbur Knorr. Wilbur and I were very fortunate to have been trained by, among others, John Murdoch and Judith Grabiner, who prepared us with tutorials for just the two of us for our oral examinations prior to working on our Ph.D.'s, which I did under the direction of Erwin Hiebert and Dirk Struik. While at Harvard, I was also fortunate to have worked with Richard Brauer, who took an interest in my thesis on Cantor and whose own memories of mathematics in Germany were always helpful to me in getting right the historical sense of the times in which Cantor had lived, especially his later years. If Brauer said something I had written sounded correct to him, I felt it must be pretty close to the mark. Later, another Harvard mathematician, Garrett Birkhoff, would likewise prove very helpful when the subject of my research turned to a biography of Abraham Robinson. Birkhoff had known Robinson and was interested in the historical implications of nonstandard analysis. Wim Luxumburg at Caltech and George Seligman at Yale were also colleagues and good friends of Robinson and likewise mathematicians to whom I am grateful for their reading of the history I was writing.

This is all by way of saying that, for historians of mathematics, our best audiences and most valuable collaborators are our mathematician colleagues. In the course of my career as a historian of mathematics, I have also learned the craft from many of my colleagues, who have been both inspirational and supportive. I was indeed fortunate to have several of what in Germany is called the *Doktorvater*, beginning with my mentor at Harvard, I. Bernard Cohen, whose own interest in history of mathematics included the work of Isaac Newton but also the great revolution in computers that led him to serve as IBM's chief historical consultant. That led to several summers when both Wilbur Knorr and I, among a number of graduate students, were employed to work for IBM on a massive history of computing database that Cohen was overseeing.

In Berlin, when I was there for a year doing archival research for my dissertation on Georg Cantor and the origins of transfinite set theory, Kurt R. Biermann, director of the Alexander von Humboldt

Forschungsstelle at the Deutsche Akademie der Wissenschaften in what was then East Berlin, took me under his wing, as did Herbert Meschkowski at the Freie Universität and Christoph Scriba at the Technische Universität in West Berlin. The German circle of historians of mathematics kindly invited me to their annual meetings at the Mathematisches Forschungsinstitut at Oberwolfach in the Black Forest, where Joseph Ehrenfried Hofmann, especially known for his research on Leibniz, had established the Problemgeschichte der Mathematik seminars. Over the years meetings there with Christoph Scriba, Menso Folkerts, Ivo Schneider, Eberhard Knobloch, Herbert Mehrtens, among many others, introduced me to the grand tradition of history of mathematics in Germany. Later Christoph Scriba and I would codirect, with Hans Wussing and Jeanne Peiffer, a group project for the International Commission of the History of Mathematics that resulted in a collaborative historiographic study bringing colleagues from literally all parts of the world together to write a history of the discipline. Also among those to whom I owe so much, beginning with the help and encouragement he offered me as a graduate student, is Ivor Grattan-Guinness, whose own early work on Georg Cantor proved extremely helpful to me, especially through the strategic suggestions Ivor was willing to offer at the time to a fledgling graduate student; but over the years since, his critical and editorial eye has always been welcome, as well as the various projects on which we have worked together.

In fact, throughout my career, working with mathematicians and historians in different parts of the world has been the most rewarding and inspirational aspect of collaborative research. This has certainly been true of the undertaking I began nearly twenty-five years ago, in 1988, when I was invited to spend six months in China under a program jointly sponsored by the Chinese Academy of Sciences and the U.S. National Academy of Sciences. I spent several months that spring in Beijing at the Institute for History of Natural Sciences, where I joined the seminar on history of Chinese mathematics being taught by Du Shiran. It was there that I met Guo Shuchun and Liu Dun, two historians of mathematics who, along with Lam Lay Yong in Singapore and Horng Wann-Sheng in Taiwan, have also been those from whom I have learned the history of Chinese mathematics. Wann-Sheng, in fact, not long after I returned to New York from Beijing, was the first of my Chinese graduate students, and the other joy of teaching is the opportunity it provides for working with especially able students. Although I have not had many graduate students, those with whom I have worked have been a pleasure to mentor, and in the process, I am convinced that I learn as much from them as I hope they have learned from working with me. The first student to find his way to New York for a second doctorate in history of science after having already completed a Ph.D. in mathematics was David Rowe, then Horng Wann- Sheng, and most recently, Xu Yibao. At the moment, I have yet another graduate student from Taiwan, Chang Ping-Ying, who is working on a history of the Suanxue (College of Mathematics) in the Qintianjian (Bureau of Astronomy) in the early Qing Dynasty, a reminder of the fact that in virtually all cultures in all parts of the world, mathematicians have significant roles to play, not just in the advancement of the theoretical understanding of the subject but in a wide variety of applications.

In addition to the very great extent to which the American Mathematical Society and the Mathematical Association of America have furthered the history of mathematics through the invited sessions on history, which usually run a full two days at the annual joint meetings (for which over the years I've served as co-organizer with, at various times, David Zitarelli, Karen Parshall, Victor Katz, Patti Hunter, and Deborah Kent), the History of Science Society in recent years has also shown increasing interest in the history of mathematics. There Karen Parshall and Albert Lewis have been instrumental, along with support offered by Harry Lucas, in establishing a Forum of the History of Mathematics that we hope will serve to increase further the prominence of history of mathematics at HSS annual meetings.

In retrospect, I have much to be thankful for in a career that has brought me into touch with mathematicians and historians the world over, many of whom I consider not just colleagues but good friends, including the students I have been privileged to have at the Graduate Center of the City University of New York. But it is my home institution, Herbert H. Lehman College, to which I must say a special thank you for the resources and encouragement it gives to faculty, especially for their research and participation in conferences and projects involving the larger academic community of scholars. It was Lehman that gave financial support and released time from teaching when Esther Phillips and I served as editors of *Historia Mathematica* following the sudden and unexpected death of its founding editor, Kenneth O. May. More recently, Lehman helped make possible a year at the Research Center for Humanities and Social Sciences at National Chiao-Tung University in Hsinchu, Taiwan. There, with few obligations apart from my own research, I spent all of 2010 completing a translation with critical commentary of the *Nine Chapters on the Art of Mathematics* (one of the *Ten Classics of Ancient Mathematics*). The *Nine Chapters*, including its commentaries, is the compendium of mathematics from ancient China on which I have been fortunate to work with my colleagues Guo Shuchun in Beijing and Xu Yibao in New York. This collaborative effort would not have been possible were it not for the joint support of the City University of New York, Chiao-Tung University in Taiwan, and the Institute for History of Natural Sciences in Beijing.

What sets the history of mathematics apart from the history of science generally is that it is not an arcane history about past theories that have been discarded, forgotten as failed attempts to understand the workings of nature, but instead the history of mathematics is a living history. In the nineteenth century Weierstrass recommended that his students read the classics of the past, for such works might well contain ideas and methods that could still prove useful, even inspirational, to current mathematical research on the frontiers of the subject. It is for this reason that mathematicians, more than any other practitioners among the sciences, have a very real interest in their history—not just to remember the past, but to use it. For the tangible support and visibility that awards like the Whiteman Memorial Prize provide for the subject, historians of mathematics can truly be grateful, and at this moment in particular, none more than I. To the AMS Council and to the members of the selection committee, I want to express my sincere appreciation, and on behalf of historians of mathematics everywhere, my heartfelt thanks as well to Mrs. Sally Whiteman for establishing this outstanding memorial for her husband, Albert Leon Whiteman.

About the Prize

The Whiteman Prize is awarded every three years to recognize notable exposition and exceptional scholarship in the history of mathematics. The prize was established in 1998 using funds donated by Mrs. Sally Whiteman in memory of her husband, the late Albert Leon Whiteman. The prize carries a cash award of US$5,000.

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—Elaine Kehoe

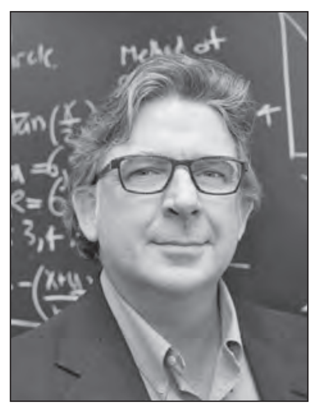
Award for Distinguished Public Service

William McCallum received the 2012 Award for Distinguished Public Service at the 118th Annual Meeting of the AMS in Boston in January 2012.

Citation

William McCallum is University Distinguished Professor and Head of the Mathematics Department at the University of Arizona. In recent years, McCallum has shown extraordinary energy in promoting improvement of mathematics education, and he has been almost ubiquitous in organizations devoted to mathematics education. He has served as chair of the Committee on Education of the AMS and as chair of CBMS. He is a member of the International Design Committee for the Klein Project, an effort of the International Commission on Mathematics Instruction and the International Mathematical Union to produce a set of narratives or "vignettes" about contemporary mathematics to educate and inspire today's high school teachers in the way that Felix Klein's lectures and books on "Elementary Mathematics from an Advanced Standpoint" did 100 years ago. He has also been Principal Investigator on a Mathematics and Science Partnership grant.

However, his most significant recent activities have also been the most distinctive. He is the founding Director of the Institute for Mathematics and Education (IME) at the University of Arizona and is currently director of its Advisory Committee. This Institute was founded explicitly on the principle that to deal effectively with issues of mathematics education requires communication and cooperation among teachers, mathematics education researchers, and mathematicians. In dozens of events over the past five years, many people from all three groups have met for mutually productive activities under the auspices of IME.



William McCallum

Most recently, he was one member of the three-person writing team selected by the Council of Chief State School Officers and the National Governors Association to orchestrate and execute the production of the Common Core State Math-

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What sets the history of mathematics apart from the history of science generally is that it is not an arcane history about past theories that have been discarded, forgotten as failed attempts to understand the workings of nature, but instead the history of mathematics is a living history. In the nineteenth century Weierstrass recommended that his students read the classics of the past, for such works might well contain ideas and methods that could still prove useful, even inspirational, to current mathematical research on the frontiers of the subject. It is for this reason that mathematicians, more than any other practitioners among the sciences, have a very real interest in their history—not just to remember the past, but to use it. For the tangible support and visibility that awards like the Whiteman Memorial Prize provide for the subject, historians of mathematics can truly be grateful, and at this moment in particular, none more than I. To the AMS Council and to the members of the selection committee, I want to express my sincere appreciation, and on behalf of historians of mathematics everywhere, my heartfelt thanks as well to Mrs. Sally Whiteman for establishing this outstanding memorial for her husband, Albert Leon Whiteman.

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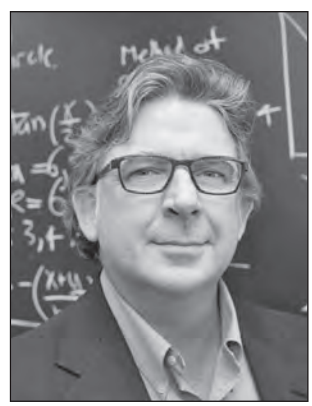
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ematics Standards. As such, he was the principal representative of the mathematics research community in the creation of the CCSS. The happy fact that so many mathematicians can read these standards with approval can be attributed in considerable part to his involvement.

Biographical Sketch

William McCallum was born in Sydney, Australia, and received his Ph.D. in mathematics from Harvard University in 1984 under the supervision of Barry Mazur. He has taught at the University of California, Berkeley, and the University of Arizona, where he is currently University Distinguished Professor. He is a founding member of the Harvard Calculus Consortium and has been a research fellow at the Mathematical Sciences Research Institute, the Institut des Haute Études Scientifiques, and the Institute for Advanced Studies. His honors include a Centennial Fellowship from the American Mathematical Society and a Director's Award for Distinguished Teaching Scholars from the National Science Foundation. In 2006 he founded the Institute for Mathematics and Education at the University of Arizona. His professional interests include arithmetical algebraic geometry and mathematics education.

Response from William McCallum

I am deeply honored to receive this award and accept it not only on my own behalf but also on behalf of the growing community of mathematicians who have chosen to dedicate their time and intellect to the scholarship of mathematics education. This community includes many previous recipients of this award and many others who deserve similar accolades. I am grateful for their leadership and inspiration. I am also grateful to the many mathematics educators and teachers with whom I have worked, both for their willingness to speak and for their willingness to listen as I explored their communities. With the Institute for Mathematics and Education I have tried to build a home where mathematicians, educators, and teachers can meet, collaborate, and learn from each other, as I myself have learned from all three groups. I was fortunate to be at the right place at the right time when the Common Core State Standards initiative came along so that I was able to put my learning to good use. The Common Core is the best chance we have had in a long time to improve school mathematics education in this country; I invite my colleagues in the research community to join the effort to make it succeed.

About the Award

The Award for Distinguished Public Service is presented every two years to a research mathematician who has made a distinguished contribution to the mathematics profession during the preceding five years. The purpose of the award is to encourage and recognize those individuals who contribute their time to public service activities in support of mathematics. The award carries a cash prize of US$4,000.

The Award for Distinguished Public Service is made by the AMS Council acting on the recommendation of the selection committee. For the 2012 award, the members of the selection committee were Richard A. Askey, C. H. Clemens, Roger E. Howe, Richard A. Tapia, and Sylvia M. Wiegand.

Previous recipients of the award are Kenneth M. Hoffman (1990), Harvey B. Keynes (1992), I. M. Singer (1993), D. J. Lewis (1995), Kenneth C. Millett (1998), Paul J. Sally Jr. (2000), Margaret H. Wright (2002), Richard Tapia (2004), Roger Howe (2006), Herbert Clemens (2008), and Carlos Castillo-Chavez (2010).

—Elaine Kehoe

2012 AMS–SIAM Birkhoff Prize

Bjorn Engquist received the 2012 AMS-SIAM George David Birkhoff Prize in Applied Mathematics at the Joint Mathematics Meetings in Boston in January 2012.

Citation

The 2012 George David Birkhoff Prize in Applied Mathematics is awarded to Bjorn Engquist for his contributions to a wide range of powerful computational methods over more than three decades. These include the numerical analysis of boundary conditions for wave propagation, which provided deep understanding about constructing accurate numerical schemes, efficient shock capturing schemes for nonlinear conservation laws which have found their way far beyond fluid mechanics into such disparate fields as image processing and materials, techniques for numerical homogenization, and methods for computing across multiple scales. His work blends mathematical analysis, modeling, and computation and has led to numerical tools with enormous impact across a broad

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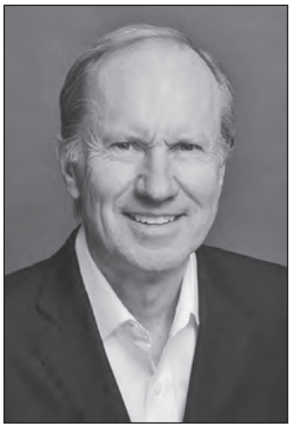
range of applications, including aerodynamics, acoustics, electromagnetism, computational fluid mechanics, and computational geoscience.

Biographical Sketch

Bjorn Engquist was born in Stockholm, Sweden, in 1945. He studied as an undergraduate and graduate student at Uppsala University, where he obtained his Ph.D. in 1975. After two years as a postdoc at Stanford University, he joined the faculty of the UCLA Department of Mathematics in 1978. He has been a professor at Uppsala University, the Royal Institute of Technology in Stockholm, and Princeton University. At Princeton he was also the director for the program in applied and computational mathematics. Since 2004 he has been the CAM Chair I Professor at the University of Texas at Austin and also director for the Center for Numerical Analysis at the Institute for Computational Engineering and Science. He has supervised thirty-three Ph.D. students. He was a speaker at the International Congresses of Mathematicians in 1982 and 1998, and he received the first SIAM Prize in Scientific Computing as well as the Celsius Medal, the Wallmark Prize, a Guggenheim Fellowship, and recently the Henrici Prize. He is a SIAM Fellow and member of the Royal Swedish Academy of Sciences and the Royal Swedish Academy of Engineering Sciences and a foreign member of the Norwegian Academy of Science and Letters.


Bjorn Engquist

Response from Bjorn Engquist

I am deeply honored and delighted to receive the 2012 George David Birkhoff Prize in Applied Mathematics. I greatly appreciate the citation and the recognition from the American Mathematical Society and the Society for Industrial and Applied Mathematics. I have always found computational science, which is at the interface between mathematics and applications, to be an exciting and a fruitful field for research. It is highly rewarding to see mathematical advances impact science and engineering. I am grateful to all collaborators throughout my career.

I thank my advisor, Heinz-Otto Kreiss, for his guidance and insight. I also thank many of my colleagues from the important early years, when I was fortunate to collaborate with Andrew Majda and Stanley Osher, to the present time working with Richard Tsai and Lexing Ying. I am also thankful for the inspiring interaction with my many excellent students, from whom I learned at least as much as they from me. Many thanks to Weinan E and Tom Hou from our time at UCLA and beyond and to Olof Runborg and Anna-Karin Tornberg from the time at the Royal Institute of Technology in Stockholm.

About the Prize

The Birkhoff Prize recognizes outstanding contributions to applied mathematics in the highest and broadest sense and is awarded every three years. Established in 1967, the prize was endowed by the family of George David Birkhoff (1884–1944), who served as AMS president during 1925–1926. The prize is given jointly by the AMS and the Society for Industrial and Applied Mathematics (SIAM). The prize carries a cash award of US$5,000.

The recipient of the Birkhoff Prize is chosen by a joint AMS-SIAM selection committee. For the 2012 prize, the members of the selection committee were Andrew J. Majda, James A. Sethian (chair), and Michael S. Waterman.

Previous recipients of the Birkhoff Prize are Jürgen K. Moser (1968), Fritz John (1973), James B. Serrin (1973), Garrett Birkhoff (1978), Mark Kac (1978), Clifford A. Truesdell (1978), Paul R. Garabedian (1983), Elliott H. Lieb (1988), Ivo Babuška (1994), S. R. S. Varadhan (1994), Paul H. Rabinowitz (1998), John N. Mather (2003), Charles S. Peskin (2003), Cathleen S. Morawetz (2006), and Joel Smoller (2009).

—Elaine Kehoe

Mathematics People

Zhan and Dubedat Awarded 2011 Salem Prize

The Salem Prize 2011 has been awarded to Dapeng Zhan of Michigan State University and Julien Dubedat of Columbia University for their outstanding work on the Schramm-Loewner evolutions (SLE), specifically for the proof of the reversibility and duality conjectures. The prize, in memory of Raphael Salem, is awarded yearly to young researchers for outstanding contributions to the field of analysis.

Previous winners of the Salem Prize include the following mathematicians: N. Varopoulos, R. Hunt, Y. Meyer, C. Fefferman, T. Körner, E. M. Nikišin, H. Montgomery, W. Beckner, M. R. Herman, S. B. Bočkarëv, B. E. Dahlberg, G. Pisier, S. Pichorides, P. Jones, A. B. Aleksandrov, J. Bourgain, C. Kenig, T. Wolff, N. G. Makarov, G. David, J. L. Journé, A. L. Vol'berg, J.-C. Yoccoz, S. V. Konyagin, C. McMullen, M. Shishikura, S. Treil, K. Astala, H. Eliasson, M. Lacey, C. Thiele, T. Wooley, F. Nazarov, T. Tao, O. Schramm, S. Smirnov, X. Tolsa, E. Lindenstrauss, K. Soundararajan, B. Green, A. Avila, S. Petermichl, A. Venkatesh, B. Klartag, A. Naor, and N. Anantharaman.

The prize committee consisted of J. Bourgain, C. Fefferman, P. Jones,N. Nikolski, G. Pisier, P. Sarnak, and J.-C. Yoccoz.

—Salem Prize Committee announcement

Nang Awarded Ramanujan Prize for Young Mathematicians from Developing Countries

Philibert Nang of the École Normale Supérieure, Laboratoire de Recherche en Mathématiques, Libreville, Gabon, has been named the winner of the 2011 Ramanujan Prize for Young Mathematicians from Developing Countries in recognition of his contributions to the algebraic theory of *D*-modules. According to the prize citation, he has created important classification theorems for equivariant algebraic *D*-modules, in terms of explicit algebraic invariants, and his results complement the insights obtained by others using perverse sheaves, thus shedding new light on the Riemann-Hilbert correspondence.

The prize is awarded jointly by the Abdus Salam International Centre for Theoretical Physics (ICTP), the Niels Henrik Abel Memorial Fund, and the International Mathematical Union (IMU). It is awarded annually to a researcher from a developing country who is less than forty-five years of age and has conducted outstanding research in a developing country. The prize is supported financially by the Niels Henrik Abel Memorial Fund and carries a US$15,000 cash award. The selection committee for the 2011 prize consisted of Lothar Göttsche (chair), Helge Holden, Maria Jose Pacifico, Vasudevan Srinivas, and Gang Tian.

—Ramanujan Prize Committee announcement

Wohlmuth Awarded Leibniz Prize

Barbara Wohlmuth of the University of Technology, Munich, has been awarded the 2012 Leibniz Prize in Mathematics. The prize citation states that she was honored for her research achievements in numerical analysis, which enable direct applications in scientific and engineering computing. A focus of her research is the numerics of partial differential equations, to which she has made key contributions, especially with her theoretical study of mortar domain decomposition methods. With this work, and with its translation into practical techniques, she has achieved an internationally leading role in her field. Wohlmuth's research demonstrates an extraordinarily deep theoretical understanding that also produces better computational methods, for example in solid and fluid mechanics.

The Leibniz Prize is awarded by the German Research Foundation (DFG) and carries a cash award of 2,500,000 euros (approximately US$3,300,000), which may be used for up to seven years for the recipient's scientific work.

—From a DFG announcement

Todorcevic Awarded CRM-Fields-PIMS Prize

Stevo Todorcevic of the University of Toronto has been awarded the 2012 CRM-Fields-PIMS Prize in mathematical sciences for work of striking originality and technical brilliance. The prize citation reads in part, "His contributions to set theory made him a world leader in this topic with a particular impact on combinatorial set theory and its connections with topology and analysis." He has made major contributions to the study of *S*- and *L*- spaces in topology, proved a remarkable classification theorem for transitive relations on the first uncountable ordinal, and made a deep study of compact subsets of the Baire class 1 functions, thus continuing work of Bourgain, Fremlin, Talagrand, and others in Banach space theory. Together with P. Larson he completed the solution of Katetov's old compact spaces metrization problem. Among the most striking recent accomplishments of Todorcevic (and coauthors) are major contributions to the von Neumann and Maharam problems on Boolean algebras; the theory of nonseparable Banach spaces, including the solution of an old problem of Davis and Johnson; the solution of a long-standing problem of Laver; and the development of a duality theory relating finite Ramsey theory and topological dynamics.

The prize is awarded by the Centre de Recherches Mathématiques (CRM), the Fields Institute, and the Pacific Institute for Mathematical Sciences (PIMS).

—From a CRM announcement

Gualtieri and Kim Awarded 2012 Aisenstadt Prize

Marco Gualtieri of the University of Toronto and Young-Heon Kim of the University of British Columbia have been awarded the 2012 André-Aisenstadt Prize of the Centre de Recherches Mathématiques (CRM). Gualtieri made essential contributions to the development of generalized complex geometry, an active area of research at the interface of complex geometry and symplectic geometry. Kim's most important contributions concern the fast-developing topic of optimal transportation.

—From a CRM announcement

Mathematics Opportunities

DMS Workforce Program in the Mathematical Sciences

The Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) welcomes proposals for the Workforce Program in the Mathematical Sciences. The long-range goal of the program is increasing the number of well-prepared U.S. citizens, nationals, and permanent residents who successfully pursue careers in the mathematical sciences and in other NSF-supported disciplines. Of primary interest are activities centered on education that broaden participation in the mathematical sciences through research involvement for trainees at the undergraduate through postdoctoral educational levels. The program is particularly interested in activities that improve recruitment and retention, educational breadth, and professional development.

The submission period for unsolicited proposals is **May 15–June 15, 2012.** For more information and a list of cognizant program directors, see the website `http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=503233`.

—From a DMS announcement

Project NExT: New Experiences in Teaching

Project NExT (New Experiences in Teaching) is a professional development program for new and recent Ph.D.'s in the mathematical sciences (including pure and applied mathematics, statistics, operations research, and mathematics education). It addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, and participating in professional activities. It also provides the participants with a network of peers and mentors as they assume these responsibilities. In 2012 about eighty faculty members from colleges and universities throughout the country will be selected to participate in a workshop preceding the Mathematical Association of America (MAA) summer meeting, in activities during the summer MAA meetings in 2012 and 2013 and the Joint Mathematics Meetings in January 2013, and in an electronic discussion network. Faculty for whom the 2012–2013 academic year will be the first or second year of full-time teaching (post-Ph.D.) at the college or university level are invited to apply to become Project NExT Fellows.

Applications for the 2012-2013 Fellowship year will be due **April 13, 2012**. For more information, see the Project NExT website, `http://archives.math.utk.edu/projnext/`, or contact Aparna Higgins, director, at Aparna.`Higgins@notes.udayton.edu`. Project NExT is a program of the MAA. It receives major funding from the Mary P. Dolciani Halloran Foundation and additional funding from the Educational Advancement Foundation, the American Mathematical Society, the American Statistical Association, the National Council of Teachers of Mathematics, the Association for Symbolic Logic, the W. H. Freeman Publishing Company, John Wiley & Sons, MAA Sections, and the Greater MAA Fund.

—Project NexT announcement

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—Project NexT announcement

Call for Proposals for 2013 NSF-CBMS Regional Conferences

To stimulate interest and activity in mathematical research, the National Science Foundation (NSF) intends to support up to seven NSF-CBMS Regional Research Conferences in 2013. A panel chosen by the Conference Board of the Mathematical Sciences will make the selections from among the submitted proposals.

Each five-day conference features a distinguished lecturer who delivers ten lectures on a topic of important current research in one sharply focused area of the mathematical sciences. The lecturer subsequently prepares an expository monograph based on these lectures, which is normally published as a part of a regional conference series. Depending on the conference topic, the monograph will be published by the American Mathematical Society, by the Society for Industrial and Applied Mathematics, or jointly by the American Statistical Association and the Institute of Mathematical Statistics.

Support is provided for about thirty participants at each conference, and both established researchers and interested newcomers, including postdoctoral fellows and graduate students, are invited to attend. The proposal due date is **April 13, 2012**. For further information on submitting a proposal, consult the CBMS website, http://www.cbmsweb.org/NSF/2013_call.htm, or contact: Conference Board of the Mathematical Sciences, 1529 Eighteenth Street, NW, Washington, DC 20036; telephone: 202-293-1170; fax: 202-293-3412.

—From a CBMS announcement

NSF-CBMS Regional Conferences, 2012

With funding from the National Science Foundation (NSF), the Conference Board of the Mathematical Sciences (CBMS) will hold nine NSF-CBMS Regional Research Conferences during the summer of 2012. These conferences are intended to stimulate interest and activity in mathematical research. Each five-day conference features a distinguished lecturer who delivers ten lectures on a topic of important current research in one sharply focused area of the mathematical sciences. The lecturer subsequently prepares an expository monograph based on these lectures.

Support for about thirty participants will be provided for each conference. Both established researchers and interested newcomers, including postdoctoral fellows and graduate students, are invited to attend. Information about an individual conference may be obtained by contacting the conference organizer. The conferences to be held in 2012 are as follows.

May 11–15, 2012: Topological and Algebraic Regularity Properties of Nuclear C^*-Algebras. Wilhelm Winter, lecturer. University of Louisiana, Lafayette. Organizers: Gary Birkenmeier, 337-482-6545, gfb1127@louisiana.edu; Nathanial Brown, 814-863-9095, nbrown@math.psu.edu; Daniel G. Davis, 337-482-5943, dgdavis@louisiana.edu; Thierry Giordano, 613-562-5864, giordano@uOttawa.ca; Ping Wong Ng, 337-482-5272, png@louisiana.edu; Leonel Robert, 337-482-6772, lrobert@louisiana.edu. Conference website: www.ucs.louisiana.edu/~pwn1677/cbms2012.html.

May 29–June 2, 2012: Mathematical Methods of Computed Tomography. Peter Kuchment, lecturer. University of Texas, Arlington. Organizers: Tuncay Aktosun, 817-272-1545, aktosun@uta.edu; and Gaik Ambartsoumian, 817-272-3384, gambarts@uta.edu. Conference website: omega.uta.edu/~aktosun/cbms2012.

June 4–8, 2012: Small Deviation Probabilities: Theory and Applications. Wenbo V. Li, lecturer. University of Alabama, Huntsville. Organizers: Dongsheng Wu, 256-824-6676, dongsheng.wu@uah.edu; and Kyle Siegrist, 256-824-6486, siegrist@math.uah.edu. Conference website: www.math.uah.edu/~cbms.

June 11–15, 2012: Finite Element Exterior Calculus. Douglas N. Arnold, lecturer. Brown University. Organizers: Alan Demlow, 859-257-6797, alan.demlow@uky.edu; Johnny Guzman, 401-863-6360, Johnny_Guzman@brown.edu; and Dmitriy Leykekhman, 860-405-9294, leykekhman@math.uconn.edu. Conference website: icerm.brown.edu/tw12-2-cbms.

June 18–22, 2012: Hodge Theory, Complex Geometry, and Representation Theory. Phillip A. Griffiths, lecturer. Texas Christian University. Organizers: Greg Friedman, 817-257-6343, g.friedman@tcu.edu; Robert S. Doran, 817-257-7335, r.doran@tcu.edu; and Scott Nollet, 817-257-6339, s.nollet@tcu.edu. Conference website: faculty.tcu.edu/gfriedman/CBMS2012/.

July 16–20, 2012: Unitary Representations of Reductive Groups. David Vogan, lecturer. University of Massachusetts, Boston. Organizer: Alfred Noel, 617-287-6458, alfred.noel@umb.edu. Conference website: www.math.umb.edu/CBMS2012.

July 23–27, 2012: Model Uncertainty and Multiplicity. James O. Berger, lecturer. University of California Santa Cruz. Organizers: Bruno Sanso, 831-459-1484, bruno@ams.ucsc.edu; Abel Rodriguez, 831-459-5278, abel@soe.ucsc.edu; and Yuefeng Wu, 831-459-5311, yuefeng@soe.ucsc.edu. Conference website: cbms-mum.soe.ucsc.edu/.

August 6–10, 2012: Statistical Climatology. Douglas W. Nychka, lecturer. University of Washington. Organizer: Peter Guttorp, 206-543-6774, peter@stat.washington.edu. Conference website: www.nrcse.washington.edu/statmos/nychka.html.

August 13–17, 2012: The Mathematics of the Social and Behavioral Sciences. Donald G. Saari, lecturer. West Chester University of Pennsylvania. Organizer: Michael Fisher, 610-430-4196, mfisher@wcupa.edu. Conference website: www.wcupa.edu/math/2012CBMS.

—From a CBMS announcement

AWM Gweneth Humphreys Award

The Association for Women in Mathematics (AWM) sponsors the Gweneth Humphreys Award to recognize outstanding mentorship activities. This prize will be awarded annually to a mathematics teacher (female or male) who has encouraged female undergraduate students to pursue mathematical careers and/or the study of mathematics at the graduate level. The recipient will receive a cash prize and honorary plaque and will be featured in an article in the AWM newsletter. The award is open to all regardless of nationality and citizenship. Nominees must be living at the time of their nomination.

The deadline for nominations is **April 30, 2012**. For details, see www.awm-math.org, telephone 703-934-0163, or email awm@awm-math.org.

—From an AWM announcement

NSF Integrative Graduate Education and Research Training

The Integrative Graduate Education and Research Training (IGERT) program was initiated by the National Science Foundation (NSF) to meet the challenges of educating Ph.D. scientists and engineers with the interdisciplinary backgrounds and the technical, professional, and personal skills needed for the career demands of the future. The program is intended to catalyze a cultural change in graduate education for students, faculty, and universities by establishing innovative models for graduate education in a fertile environment for collaborative research that transcends traditional disciplinary boundaries. It is also intended to facilitate greater diversity in student participation and to contribute to the development of a diverse, globally aware science and engineering workforce. Supported projects must be based on a multidisciplinary research theme and administered by a diverse group of investigators from U.S. Ph.D.-granting institutions with appropriate research and teaching interests and expertise.

The deadline for letters of intent is **May 1, 2012**; the deadline for full proposals is **July 2, 2012**. Full proposals may be sent by invitation only. Further information may be found at the website http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=12759.

—From an NSF announcement

NSF Scholarships in Science, Technology, Engineering, and Mathematics

The NSF Scholarships in Science, Technology, Engineering, and Mathematics (S-STEM) program provides institutions with funds for student scholarships to encourage and enable academically talented students demonstrating financial need to enter the STEM workforce or STEM graduate school following completion of an associate, baccalaureate, or graduate degree in fields of science, technology, engineering, or mathematics. Students to be awarded scholarships must demonstrate academic talent and financial need. S-STEM grants may be made for up to five years and provide individual scholarships of up to US$10,000 per year, depending on financial need. Proposals must be submitted by institutions, which are responsible for selecting the scholarship recipients. The deadline for full proposals is **August 14, 2012**. For more information, see the website http://www.nsf.gov/pubs/2012/nsf12529/nsf12529.htm.

—From an NSF announcement

Call for Nominations for the Ramanujan Prize for Young Mathematicians from Developing Countries

The Abdus Salam International Centre for Theoretical Physics (ICTP), the Niels Henrik Abel Memorial Fund, and the International Mathematical Union (IMU) seek nominations for the 2012 Ramanujan Prize for Young Mathematicians from Developing Countries. The prize is funded by the Niels Henrik Abel Memorial Fund. The prize is awarded annually to a researcher from a developing country who must be less than forty-five years of age on December 31 of the year of the award and who has conducted outstanding research in a developing country. Researchers working in any branch of the mathematical sciences are eligible. The prize carries a cash award of US$15,000. The winner will be invited to the ICTP to receive the prize and deliver a lecture. The prize is usually awarded to one person but may be shared equally among recipients who have contributed to the same body of work.

The selection committee will take into account not only the scientific quality of the research but also the background of the candidate and the environment in which the work was carried out. The deadline for receipt of nominations for the 2012 Prize is **April 1, 2012**. Please send nominations to math@ictp.it describing the work of the nominee in adequate detail. Nominations should include a CV and a list of publications, as well as a letter of recommendation. See the website http://prizes.ictp.it/Ramanujan/call-for-nominations-for-the-2012-prize.

—From an IMU announcement

Math in Moscow Scholarship Program

The Math in Moscow program at the Independent University of Moscow (IUM) was created in 2001 to provide foreign students (primarily from the U.S., Canada, and Europe) with a semester-long, mathematically intensive program of study in the Russian tradition of teaching mathematics, the main feature of which has always been the development of a creative approach to studying mathematics from the outset—the emphasis being on problem solving rather than memorizing theorems. Indeed, discovering mathematics under the guidance of an experienced teacher is the central principle of the IUM, and the Math in Moscow program emphasizes in-depth understanding of carefully selected material rather than broad surveys of large quantities of material. Even in the treatment of the most traditional subjects, students are helped to explore significant connections with contemporary research topics. The IUM is a small, elite institution of higher learning focusing primarily on mathematics and was founded in 1991 at the initiative of a group of well-known Russian research mathematicians, who now compose the Academic Council of the university. Today, the IUM is one of the leading mathematical centers in Russia. Most of the Math in Moscow program's teachers are internationally recognized research mathematicians, and all of them have considerable teaching experience in English, typically in the United States or Canada. All instruction is in English.

With funding from the National Science Foundation (NSF), the AMS awards five US$9,000 scholarships each semester to U.S. students to attend the Math in Moscow program. To be eligible for the scholarships, students must be either U.S. citizens or enrolled at a U.S. institution at the time they attend the Math in Moscow program. Students must apply separately to the IUM's Math in Moscow program and to the AMS Math in Moscow Scholarship program. Undergraduate or graduate mathematics or computer science majors may apply. The deadlines for applications for the scholarship program are **April 15, 2012**, for the fall 2012 semester and **September 15, 2012**, for the spring 2013 semester.

Information and application forms for Math in Moscow are available on the Web at `http://www.mccme.ru/mathinmoscow`, or by writing to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: `mim@mccme.ru`. Information and application forms for the AMS scholarships are available on the AMS website at `http://www.ams.org/programs/travel-grants/mimoscow`, or by writing to: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; e-mail `student-serv@ams.org`.

—AMS Membership and Programs Department

For Your Information

2012 Everett Pitcher Lectures

William P. Minicozzi II of Johns Hopkins University will deliver the 2012 Everett Pitcher Lectures, held April 16–20, 2012, on the campus of Lehigh University in Bethlehem, Pennsylvania. The title of his lecture series is "Singularities in mean curvature flow". The three lectures are titled "Geometric heat equations" (for a general audience), "Singularities and dynamics of mean curvature flow", and "Mean curvature flow near a singularity". The lectures, which are open to the public, are held in honor of Everett Pitcher, who was secretary of the AMS from 1967 until 1988. Pitcher served in the mathematics department at Lehigh from 1938 until 1978, when he retired as Distinguished Professor of Mathematics. He passed away in December 2006 at the age of ninety-four.

For further information contact the Everett Pitcher Lecture Series, Department of Mathematics, Lehigh University, Bethlehem, PA 18015; telephone 610-758-3731; website `http://www.lehigh.edu/~math/pitcher.html`.

—Huai-Dong Cao
Lehigh University

Who's That Mathematician? Photos from the Halmos Collection

Throughout 2012 the Mathematical Association of America is posting over 300 photos of mathematicians snapped by Paul Halmos (1916–2006) during his career—in the online feature "Who's That Mathematician? Images from the Paul R. Halmos Photograph Collection". You are invited to share what you know about the people, places, dates, and circumstances of each photo via an easy-to-use interactive discussion tool. To reach the archive, visit the page `http://mathdl.maa.org/mathDL/46/` and click on the link indicating the Halmos Photo Collection.

—Janet Beery, editor
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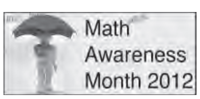
—Janet Beery, editor
MAA Convergence
University of Redlands

Inside the AMS

From the AMS Public Awareness Office

Selected Highlights of the 2012 Joint Mathematics Meetings. Over 7,200 mathematicians, exhibitors, and students came from thirty-six countries to the 2012 Joint Mathematics Meetings of the American Mathematical Society (AMS) and Mathematical Association of America (MAA) in Boston, Massachusetts, January 4–7. Read about some of the many highlights and see photo slideshows of the prizes and awards, invited addresses and sessions, Mathematical Art Exhibition, the sponsors and exhibits, AMS events, national *Who Wants to Be a Mathematician* game, JMM TV, media coverage, and more: `http://www.ams.org/meetings/national/jmm12-highlights`.

Mathematics Awareness Month April 2012. The theme for Mathematics Awareness Month 2012 is ***Mathematics, Statistics, and the Data Deluge***. See the website for the theme announcement, essay, related resources, and poster: `http://www.mathaware.org/`.



—Annette Emerson and Mike Breen
AMS Public Awareness Officers
`paoffice@ams.org`

Deaths of AMS Members

William B. Arveson, professor, University of California Berkeley, died on November 15, 2011. Born on November 22, 1934, he was a member of the Society for 46 years.

James E. Baumgartner, professor, Dartmouth College, died on December 28, 2011. Born on March 23, 1943, he was a member of the Society for 44 years.

Bogdan Choczewski, of Cracow, Poland, died on September 18, 2011. Born on July 12, 1935, he was a member of the Society for 27 years.

Douglas G. Dickson, of Ann Arbor, Michigan, died on July 30, 2011. Born on November 11, 1924, he was a member of the Society for 58 years.

Bryan V. Hearsey, professor, Lebanon Valley College, died on October 28, 2011. Born on August 2, 1942, he was a member of the Society for 42 years.

Lloyd K. Jackson, of Liverpool, New York, died on April 15, 2009. Born on August 25, 1922, he was a member of the Society for 58 years.

Thomas J. Mahar, of Annapolis, Maryland, died on November 26, 2011. Born on July 29, 1949, he was a member of the Society for 35 years.

Gyula I. Maurer, professor, Hungarian Academy of Science Mathematics Institute, died on January 8, 2012. Born on January 18, 1927, he was a member of the Society for 18 years.

Philip E. Miles, of Madison, Wisconsin, died on November 8, 2011. Born on December 27, 1929, he was a member of the Society for 54 years.

Jacques A. Mizrahi, of Washington, DC, died on May 30, 2011. Born on October 11, 1922, he was a member of the Society for 30 years.

Robert Osserman, professor, Mathematical Sciences Research Institute, died on November 30, 2011. Born on December 19, 1926, he was a member of the Society for 61 years.

Berthold Schweizer, of Oakland, California, died on August 19, 2010. Born on July 20, 1929, he was a member of the Society for 53 years.

Tonny A. Springer, professor, Utrecht University, died on December 7, 2011. Born on February 13, 1926, he was a member of the Society for 49 years.

James M. Veneri, of Grosse Pointe Woods, Michigan, died on July 20, 2011. Born on March 27, 1958, he was a member of the Society for 29 years.

Kaoru Wakana, of Saitama-Shi, Japan, died on June 26, 2010. Born on September 17, 1921, he was a member of the Society for 44 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.wustl.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

March 30, 2012: Applications for AMS-Simons Travel Grants for Early-Career Mathematicians. See `www.ams.org/programs/travel-grants/AMS-SimonsTG`.

March 31, 2012: Nominations for prizes of the Academy of Sciences for the Developing World (TWAS). Send nominations to TWAS Prizes, International Centre for Theoretical Physics (ICTP) Campus, Strada Costiera 11, 1-34151 Trieste, Italy; fax: 39 040 2240 7387; email: `prizes@twas.org`. See `http://www.twas.org/`.

April 9, 2012: Applications for Math for America San Diego site. See the website at `http://www.mathforamerica.org/`.

April 13, 2012: Applications for Project NExT: New Experiences in Teaching fellowships. See "Mathematics Opportunities" in this issue.

April 13, 2012: Proposals for 2013 NSF-CBMS Regional Conferences. See "Mathematics Opportunities" in this issue.

April 15, 2012: Applications for fall 2012 semester of Math in Moscow. See "Mathematics Opportunities" in this issue.

April 30, 2012: Nominations for AWM Gweneth Humphreys Award. See "Mathematics Opportunities" in this issue.

May 1, 2012: Letters of intent for NSF Integrative Graduate Education and Research Training (IGERT) program. See "Mathematics Opportunities" in this issue.

May 1, 2012: Applications for National Academies Research Associateship Programs. See `http://sites.nationalacademies.org/PGA/RAP/PGA_050491` or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email `rap@nas.edu`.

May 1, 2012: Applications for National Academies Christine Mirzayan Graduate Fellowship Program for fall 2012. See the website `http://sites.nationalacademies.org/PGA/policyfellows/index.htm` or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: `policyfellows@nas.edu`.

May 1, 2012, October 1, 2012: Applications for AWM Travel Grants.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

See http://www.awm-math.org/travelgrants.html#standard; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030; 703-934-0163; awm@awm-math.org.

May 15–June 15, 2012: Proposals for NSF DMS Workforce Program in the Mathematical Sciences. See "Mathematics Opportunities" in this issue.

July 2, 2012: Full proposals for NSF Integrative Graduate Education and Research Training (IGERT) program. See "Mathematics Opportunities" in this issue.

July 10, 2012: Full proposals for NSF Research Networks in the Mathematical Sciences. See http://www.nsf.gov/pubs/2010/nsf10584/nsf10584.htm?WT.mc_id=USNSF_25&WT.mc_ev=click.

August 1, 2012, November 1, 2012: Applications for National Academies Research Associateship Programs. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email rap@nas.edu.

August 14, 2012: Full proposals for NSF Scholarships in Science, Technology, Engineering, and Mathematics (S-STEM) program. See "Mathematics Opportunities" in this issue.

September 15, 2012: Applications for spring 2013 semester of Math in Moscow. See "Mathematics Opportunities" in this issue.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

The Adventure of Reason: Interplay between Philosophy of Mathematics and Mathematical Logic, 1900–1940, by Paolo Mancosu. Oxford University Press, January 2011. ISBN-13: 978-01995-465-34.

At Home with André and Simone Weil, by Sylvie Weil. (Translation of *Chez les Weils*, translated by Benjamin Ivry.) Northwestern University Press, October 2010. ISBN-13: 978-08101-270-43. (Reviewed May 2011.)

The Autonomy of Mathematical Knowledge: Hilbert's Program Revisited, by Curtis Franks. Cambridge University Press, December 2010. ISBN-13: 978-05211-838-95.

The Beginning of Infinity: Explanations That Transform the World, by David Deutsch. Viking Adult, July 2011. ISBN-13: 978-06700-227-55. (Reviewed in this issue.)

The Best Writing on Mathematics: 2010, edited by Mircea Pitici. Princeton University Press, December 2010. ISBN-13: 978-06911-484-10. (Reviewed November 2011.)

The Big Questions: Mathematics, by Tony Crilly. Quercus, April 2011. ISBN-13: 978-18491-624-01.

The Blind Spot: Science and the Crisis of Uncertainty, by William Byers. Princeton University Press, April 2011. ISBN-13:978-06911-468-43.

The Calculus Diaries: How Math Can Help You Lose Weight, Win in Vegas, and Survive a Zombie Apocalypse, by Jennifer Ouellette. Penguin, reprint edition, August 2010. ISBN-13: 978-01431-173-77.

The Calculus of Selfishness, by Karl Sigmund. Princeton University Press, January 2010. ISBN-13: 978-06911-427-53. (Reviewed January 2012.)

Chasing Shadows: Mathematics, Astronomy, and the Early History of Eclipse Reckoning, by Clemency Montelle. Johns Hopkins University Press, April 2011. ISBN-13: 978-08018-969-10. (Reviewed March 2012.)

Crafting by Concepts: Fiber Arts and Mathematics, by Sarah-Marie Belcastro and Carolyn Yackel. A K Peters/CRC Press, March 2011. ISBN-13: 978-15688-143-53.

**The Crest of the Peacock: Non-European Roots of Mathematics,* by George Gheverghese Joseph. Third edition. Princeton University Press, October 2010. ISBN: 978-0-691-13526-7.

Cycles of Time: An Extraordinary New View of the Universe, by Roger Penrose. Knopf, May 2011. ISBN-13: 978-03072-659-06.

Divine Machines: Leibniz and the Sciences of Life, by Justin E. H. Smith. Princeton University Press, May 2011. ISBN-13: 978-06911-417-87.

An Early History of Recursive Functions and Computability from Gödel to Turing, by Rod Adams. Docent Press, May 2011. ISBN-13: 978-09837-004-01.

Emmy Noether's Wonderful Theorem, by Dwight E. Neuenschwander. Johns Hopkins University Press, November 2010. ISBN-13: 978-08018-969-41.

The Evolution of Logic, by W. D. Hart. Cambridge University Press, August 2010. ISBN-13: 978-0-521-74772-1

**Experimental and Computational Mathematics: Selected Writings,* by Jonathan Borwein and Peter Borwein. PSIpress, 2011. ISBN: 978-19356-380-56.

**Excursions in the History of Mathematics,* by Israel Kleiner. Birkhäuser, 2012. ISBN-13: 978-08176-826-75.

Fascinating Mathematical People: Interviews and Memoirs, edited by Donald J. Albers and Gerald L. Alexanderson. Princeton University Press, October 2011. ISBN: 978-06911-482-98.

Gottfried Wilhelm Leibniz: The Polymath Who Brought Us Calculus, by M. B. W. Tent. A K Peters/CRC Press, October 2011. ISBN: 978-14398-922-20.

The History and Development of Nomography, by H. A. Evesham. Docent Press, December 2010. ISBN-13: 978-14564-796-26.

Hot X: Algebra Exposed, by Danica McKellar. Hudson Street Press, August 2010. ISBN-13: 978-15946-307-05.

In Pursuit of the Unknown: 17 Equations that Changed the World, by Ian Stewart. Basic Books, March 2012. ISBN-13: 978-04650-297-30.

**In Service to Mathematics: The Life and Work of Mina Rees,* by Amy Shell-Gellasch. Docent Press, December 2010. ISBN: 978-0-9837004-1-8.

The Information: A History, a Theory, a Flood, by James Gleick.

Pantheon, March 2011. ISBN-13: 978-03754-237-27.

Knots Unravelled: From String to Mathematics, by Meike Akveld and Andrew Jobbings. Arbelos, October 2011. ISBN: 978-09555-477-20.

Le Operazioni del Calcolo Logico, by Ernst Schröder. Original German version of Operationskreis des Logikkalkuls and Italian translation with commentary and annotations by Davide Bondoni. LED Online, 2010. ISBN-13: 978-88-7916-474-0.

**Lost in a Cave: Applying Graph Theory to Cave Exploration,* by Richard L. Breisch. National Speleological Society, January, 2012. ISBN: 978-1-879961-43-2.

**The Lost Millennium: History's Timetables Under Siege,* by Florin Diacu. Johns Hopkins University Press (second edition), November 2011. ISBN-13: 978-14214-028-88.

Magical Mathematics: The Mathematical Ideas that Animate Great Magic Tricks, by Persi Diaconis and Ron Graham. Princeton University Press, November 2011. ISBN: 978-06911-516-49.

Mathematics and Reality, by Mary Leng. Oxford University Press, June 2010. ISBN-13: 978-01992-807-97.

Mathematics Education for a New Era: Video Games as a Medium for Learning, by Keith Devlin. A K Peters/CRC Press, February 2011. ISBN-13: 978-1-56881-431-5.

The Mathematics of Life, by Ian Stewart. Basic Books, June 2011. ISBN-13: 978-04650-223-80. (Reviewed December 2011.)

Mathematics, Religion and Ethics: An Epistemological Study, by Salilesh Mukhopadhyay. Feasible Solution LLC, September 2010. ISBN: 978-1-4507-3558-2.

Mysteries of the Equilateral Triangle, by Brian J. McCartin. Hikari, August 2010. ISBN-13: 978-954-91999-5-6. Electronic copies available for free at `http://www.m-hikari.com/mccartin-2.pdf.`

NIST Handbook of Mathematical Functions, Cambridge University Press, Edited by Frank W. J. Olver, Daniel W. Lozier, Ronald F. Boisvert, and Charles W. Clark. Cambridge University Press, May 2010. ISBN-13: 978-05211-922-55 (hardback plus CD-ROM); ISBN-13: 978-05211-406-38 (paperback plus CD-ROM). (Reviewed September 2011.)

The Noether Theorems: Invariance and Conservation Laws in the Twentieth Century, by Yvette Kosmann-Schwarzbach. Springer, December 2010. ISBN-13: 978-03878-786-76.

Numbers: A Very Short Introduction, by Peter M. Higgins. Oxford University Press, February 2011. ISBN 978-0-19-958405-5. (Reviewed January 2012.)

One, Two, Three: Absolutely Elementary Mathematics [Hardcover] David Berlinski. Pantheon, May 2011. ISBN-13: 978-03754-233-38.

Origami Inspirations, by Meenakshi Mukerji. A K Peters, September 2010. ISBN-13: 978-1568815848.

The Perfect Swarm: The Science of Complexity in Everyday Life, by Len Fisher. Basic Books, March 2011 (paperback). ISBN-13: 978-04650-202-49.

The Philosophy of Mathematical Practice, Paolo Mancosu, Editor. Oxford University Press, December 2011. ISBN: 978-01996-401-02. (Reviewed March 2012.)

The Proof is in the Pudding: A Look at the Changing Nature of Mathematical Proof, by Steven G. Krantz. Springer, May 2011. ISBN: 978-03874-890-87.

Proofiness: The Dark Arts of Mathematical Deception, by Charles Seife. Viking, September 2010. ISBN-13: 978-06700-221-68.

The Quants: How a New Breed of Math Whizzes Conquered Wall Street and Nearly Destroyed It, by Scott Patterson. Crown Business, January 2011. ISBN-13: 978-03074-533-89. (Reviewed May 2011.)

Roads to Infinity: The Mathematics of Truth and Proof, by John C. Stillwell. A K Peters/CRC Press, July 2010. ISBN-13: 978-15688-146-67.

**Scientific Reflections: Selected Multidisciplinary Works,* by Richard Crandall. PSIpress, 2011. ISBN: 978-19356-380-87.

**Six Gems of Geometry,* by Thomas Reale. PSIpress, 2010. ISBN: 978-19356-380-25.

Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving, by Sanjoy Mahajan. MIT Press, March 2010. ISBN-13: 978-0-262-51429-3. (Reviewed August 2011.)

Survival Guide for Outsiders: How to Protect Yourself from Politicians, Experts, and Other Insiders, by Sherman Stein. BookSurge Publishing, February 2010. ISBN-13: 978-14392-532-74.

**Taking Sudoku Seriously: The Math Behind the World's Most Popular Pencil Puzzle,* by Jason Rosenhouse and Laura Taalman. Oxford University Press, January 2012. ISBN: 978-01997-565-68.

The Theory That Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy, by Sharon Bertsch McGrayne. Yale University Press, April 2011. ISBN-13: 978-03001-696-90.

Top Secret Rosies: The Female Computers of World War II. Video documentary, produced and directed by LeAnn Erickson. September 2010. Website: `http://www.topsecretrosies.com`. (Reviewed February 2012.)

Towards a Philosophy of Real Mathematics, by David Corfield. Oxford University Press, April 2003. ISBN-13: 0-521-81722-6. (Reviewed November 2011.)

Train Your Brain: A Year's Worth of Puzzles, by George Grätzer. A K Peters/CRC Press, April 2011. ISBN-13: 978-15688-171-01.

Viewpoints: Mathematical Perspective and Fractal Geometry in Art, by Marc Frantz and Annalisa Crannell. Princeton University Press, August 2011. ISBN-13: 978-06911-259-23.

**Vilim Feller, istaknuti hrvatsko-americki matematicar/William Feller, Distinguished Croatian-American Mathematician,* by Darko Zubrinic. Bilingual Croatian-English edition, Graphis, 2011. ISBN: 978-953-279-016-0.

Visual Thinking in Mathematics, by Marcus Giaquinto. Oxford University Press, July 2011. ISBN-13: 978-01995-755-34.

What's Luck Got to Do with It? The History, Mathematics and Psychology of the Gambler's Illusion, by Joseph Mazur. Princeton University Press, July 2010. ISBN: 978-0-691-13890-9. (Reviewed February 2012.)

Why Beliefs Matter: Reflections on the Nature of Science, by E. Brian Davies. Oxford University Press, June 2010. ISBN13: 978-01995-862-02. (Reviewed in this issue.)

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl.

The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

April 2012

* 6–8 **TORA II: The Second Texas-Oklahoma Representations and Automorphic Forms Conference**, Oklahoma State University, Stillwater, Oklahoma.
Description: Texas-Oklahoma Representations and Automorphic Forms (TORA) is a series of conferences hosted in rotation by Oklahoma State University, the University of Oklahoma, and the University of North Texas. The TORA meetings will bring together the automorphic forms and representation theory community of the South Central region to hear about recent research in automorphic forms and representation theory.
Information: `http://www.math.okstate.edu/~asgari/tora2.html`.

* 16–20 **Workshop on Convexity and Asymptotic Geometric Analysis**, Centre de recherches mathématiques, Université de Montréal, Pavillon André-Aisenstadt, 2920, Chemin de la tour, 5th floor, Montréal (Québec), H3T 1J4 Canada.
Description: Asymptotic and classical convexity theories are nowadays intertwined. Results of one field are used in the other with numerous applications. Among recent important developments are results of a geometric-probabilistic flavor on the volume distribution in convex bodies, central limit theorems for convex bodies and others, with close links to geometric inequalities and optimal transport. In fact, the geometric theory of convexity is extended to a larger category of (log-concave) measures. This point of view introduces, in particular, functional versions for many geometric inequalities, and also leads to solutions of some central problems of geometry and analysis.
Information: `http://www.crm.umontreal.ca/Convexity12/index_e.php`.

* 23–27 **Workshop on Geometric PDE**, Centre de recherches mathématiques, Université de Montréal, Pavillon André-Aisenstadt, 2920, Chemin de la tour, 5th floor, Montréal (Québec), H3T 1J4 Canada.
Description: Nonlinear Partial Differential Equations are of fundamental importance in studying geometric and topological questions. Combining geometric insight and analytic techniques, the subject of partial differential equations arising from such questions has developed immensely in recent years. Simultaneously, a new range of challenges emerged.
Information: `http://www.crm.umontreal.ca/2012/EDP12/index_e.php`.

May 2012

* 7–18 **CIMPA Research School - Senegal 2012**, University Cheikh Anta Diop of Dakar, Dakar, Senegal, Africa.
Description: CIMPA and the University Cheikh Anta Diop of Dakar (Senegal) are organizing a research school on Geometric Structures and Control Theory.
Courses: Mini-courses will be delivered by A. Banyaga (Penn State University, USA), U. Boscain (Ecole Polytechnique, Paris, France), F.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to `notices@ams.org` or `mathcal@ams.org`.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: `http://www.ams.org/`.

Dal'Bo (Universite Rennes 1, France), B. Jakubczy (Impan, Poland), W. Respondek (INSA of Rouen, France), Ph. Rukimbira (FIU, USA), A. Wade (Penn State University, USA).
Information: http://www.ucad.sn/cimpa.

* 13–14 **International Conference on Emerging Trends of Computer and Information Technology (ICETCIT 2012)**, Coimbatore, India.
Description: The purpose of the conference is to provide a common forum for researchers, scientists, and students from India and abroad to present their latest research findings, ideas, developments and applications in the broader areas of Information Systems.
Information: http://www.icetcit.com.

* 20–23 **The 5th Symposium on Analysis & PDEs**, Purdue University, West Lafayette, Indiana.
Description: The symposium will focus on recent developments in Partial Differential Equations which are at the forefront of current international research. The format of the meeting consists of two lecture series (four lectures each) together with hourly invited lectures and short contributed talks. Professor Caffarelli will speak on "Regularity theory for non-linear problems involving non-local diffusions". Professor Alice Chang will speak on "Non-linear PDEs in the study of conformal invariants". The overall conference theme ranges from geometry to population dynamics and social sciences. The goal of the meeting is to bring together researchers at different stages of their careers to summarize current results, exchange ideas towards the solution of open problems and also formulate new avenues of research. Graduate students will also be introduced to contemporary research problems.
Information: http://www.math.purdue.edu/~danielli/symposium12.

* 20–June 2 **The UGA VIGRE Algebraic Geometry Summer School and Conference**, University of Georgia, Athens, Georgia.
Description: The algebraic geometry group at the University of Georgia will organize a 2-week summer school on algebraic geometry. The following lecture series will be given. First week: 1. Curves and their moduli (lectures by Joe Harris and Ian Morrison), 2. Abelian varieties and their moduli (Gregory Sankaran), 3. Minimal model program (James McKernan). Second week: 1. Cycles (Patrick Brosnan), 2. Hodge Theory (Ron Donagi, to be confirmed), 3. Mirror Symmetry (Tony Pantev). There will be research conference in between.
Support: We have support for students and recent Ph.D.s. For information and to apply, visit: http://www.math.uga.edu/~dkrashen/agssp/index.html.

* 25–30 **Instabilities and Control of Excitable Networks: From macro- to nano-systems**, Moscow Institute of Physics and Technology, Dolgoprudny, Russia.
Description: The conference will be devoted to the problems of complex excitable network dynamics in physiology, biomedicine, physics, chemistry and social systems.
Topics: Instabilities in far from equilibrium excitable network dynamics, pattern formation in network-organized systems, control of threshold and kinetic cascade, avalanche-like phenomena, conceptual items and application to natural and social systems.
Goal: To advance interdisciplinary research and creative new cross-disciplinary links in Russia and abroad. We hope to bring together a diverse energetic group of researchers at different stages of their research careers, working in the field of self-organization in various systems, engineering of excitable biological tissues and control of excitable networks.
Information: http://icenet2012.net.

* 28–June 1 **International Conference on Applied Mathematics 2012: Modeling, Analysis and Computation**, City University of Hong Kong, 83 Tat Chee Avenue, Kowloon Tong, Hong Kong.
Description: In recent years, tremendous progress has been made in various areas of applied mathematics. This conference will provide a unique forum for exchanging ideas and in-depth discussions on different aspects of applied mathematics, including analysis and computational methods, and applications. This conference consists of plenary lectures, invited talks and contributed talks. During the conference, the second William Benter Prize in Applied Mathematics will be awarded, and the recipient will give a plenary lecture. The aim of the prize is to recognize outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, finance and engineering applications.
Information: http://www6.cityu.edu.hk/rcms/ICAM2012/index.html.

* 28–June 1 **Tenth Workshop on Interactions between Dynamical Systems and Partial Differential Equations (JISD2012)**, School of Mathematics and Statistics, Universitat Politècnica de Catalunya, Barcelona, Spain.
Topics: There will be four main courses, some seminars, communications, and posters. The four courses will be taught by M. Capinski (Geometric methods for invariant manifolds in dynamical systems), M. Gidea (Aubry Mather Theory from a topological viewpoint), H. Shahgholian (Obstacle type free boundaries (Theory and applications)), and E. Valdinoci ((Non) local phase transition equations), within the Master of Science in Advanced Mathematics and Mathematical Engineering (MAMME) of the UPC Graduate School.
Information: email: raquel.caparros@upc.edu; http://www.ma1.upc.edu/recerca/seminaris/JISD2012/indexjisd2012.html;

* 28–June 1 **Workshop on Quantum Many-Body Systems**, Centre de recherches mathématiques, Université de Montréal, Pavillon André-Aisenstadt, 2920, Chemin de la tour, 5th floor, Montréal (Québec), H3T 1J4 Canada.
Description: The workshop focuses on recent progress on the analysis of many-body systems in quantum mechanics and related topics.
Information: http://www.crm.umontreal.ca/Systems12/index_e.php.

* 29–31 **2012 International Conference on Electrical Engineering (ICEENG'8)**, Cairo.
Organizer: The Conference is organized to invite international delegates, to share their latest research findings on Electrical Engineering. ICEENG'8 is organized by Military Technical College (http://www.mtc.edu.eg).
Topics: Papers on original works are solicited on a variety of topics, including but not limited to the following symposium tracks: Communication systems, computer engineering, biomedical engineering, power engineering, circuits, signals and systems, electromagnetic fields and waves, electronic measurements and instrumentations, remote sensing and avionics, optoelectronics, guidance and control, radar systems, student session.
Submission: Please submit full papers as attached file (free .doc or .docx format) to the Conference email address: iceeng-8@mtc.edu.eg.
Deadline: Full Paper Submission Deadline (January 31, 2012).
Information: http://www.mtc.edu.eg/all-conf.htm.

June 2012

* 4–6 **International meeting on "Statistical Analysis: Theory and Applications" (JIASTA2012)**, Mohamed First University, Faculty of Science, Oujda, Morocco.
Objective: The main objective is to bring together reseachers, users of statistics and Ph.D students to discuss and exchange ideas and present new results in the fields of probabilities and statistics according to the proposed themes. The organizers hope to: Bring together researchers in the field of statistics and its uses, to review the recent progress of research in statistics, to enable Ph.D. students to present their research and discuss with other leading researchers.

Organizer: The laboratory of stochastic and deterministic modeling (LaMSD) and the CNRST associated research unit (URAC 04), at the Faculty of Science, Mohamed 1st University, Oujda.
Information: `http://sciences1.ump.ma/JIASTA2012/`.

* 4–6 **Workshop on Parameter Estimation for Dynamical Systems**, Eurandom, Eindhoven, The Netherlands.
Description: The workshop aims at providing a meeting place for researchers in the area of parameter estimation for deterministic and stochastic differential equations, who will review different methods used to tackle the problems arising in these fields, assess the achieved progress and identify future research directions.
Information: `http://www.few.vu.nl/~shota/peds2.php`.

* 4–8 **Workshop on Geometry of Eigenvalues and Eigenfunctions**, Centre de Recherches Mathématiques, Université de Montréal, Pavillon André-Aisenstadt, 2920, Chemin de la tour, 5th floor, Montréal (Québec), H3T 1J4 Canada
Description: Geometric spectral theory focuses on the properties of eigenvalues and eigenfunctions of the Laplacian and other differential operators defined on geometric objects, such as Riemannian manifolds and Euclidean domains. Many problems in the field are motivated by questions originating in the study of real life phenomena: quantum-mechanical effects, vibration of membranes and plates, oscillations of fluids, etc.
Information: `http://www.crm.umontreal.ca/Eigenvalues12/index_e.php`.

* 11–15 **International Workshop on Complex Analysis and its Applications**, Walchand College of Engineering, Sangli 416415, India.
Description: The objective of the workshop is to bring together groups of experts in complex analysis and its applications to discuss recent progress and open problems in this area and thus foster the interaction and collaboration between researchers in diverse fields. In this workshop not only pure but computational and algorithmic aspects will be emphasized.
Sessions: The workshop will consists of various short sessions (3-4 lectures each) in subjects of particular relevance in modern complex analysis such as: Geometric function theory, planar harmonic mappings, quasiconformal mappings, potential theory.
Invited Speakers: Stephan Ruscheweyh (Germany), Roger Barnard (Texas Tech), Ilpo Laine (Finland), Teodor Bulboaca (Romania), C. S. Aravinda (TIFR Bangalore), S. R. Kulkarni (Pune), Indrajit Lahiri (Kolkatta), A. Swaminathan (I.I.T. Roorkee), Gautam Bharali (IISc Bangalore).
Information: `http://www.walchandsangli.ac.in`.

* 22–23 **2nd International Conference of Information Systems, Computer Engineering & Application (ICISCEA 2012)**, Kathmandu, Nepal.
Description: ICISCEA 2012 aims at bringing together researchers, engineers and practitioners interested on Information Systems, Computer Engineering and its Application.
Information: `http://www.cfisws.com`.

* 25–29 **Conference on Differential and Difference Equations and Applications 2012 (CDDEA2012)**, Terchova (close to Zilina), Slovak Republic.
Description: The Department of Mathematics, University of Zilina, Slovakia, in cooperation with Faculty of Electrical Engineering and Communication, Brno University of Technology, Czech Republic, Poznań; University of Technology, Poland, Kyiv National Economic University, Ukraine organize a traditional conferences on differential and difference equations and their applications.
Sections: The following sections are planned: Ordinary differential equations, functional differential equations, partial differential equations, stochastic differential equations, difference equations and dynamic equations on time scales, numerical methods in differential and difference equations. The programme will consist of invited lectures, contributing talks and poster session.
Information: `http://fpv.uniza.sk/cddea2012`.

* 25–July 6 **Seminaire de Mathematiques Superieures 2012: Probabilistic Combinatorics**, University of Montreal, Montreal, Canada.
Description: One of the cornerstones of the probabilistic approach to solving combinatorial problems is the following guiding principle: information about global structure can be obtained through local analysis. This principle is ubiquitous in probabilistic combinatorics. It arises in problems ranging from graph colouring, to Markov chain mixing times, to Szemerédi's regularity lemma and its applications, to the theory of influences. The 2012 Séminaire de Mathématiques Supérieures brings together experts in probabilistic combinatorics from around the world, to explain cutting edge research which in one way or another exhibits this principle.
Information: `http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-/event/Wm9214`.

* 26–28 **The 11th International Workshop on Dynamical Systems and Applications**, Cankaya University, Ankara, Turkey.
Description: These workshops constitute the annual meetings of the series of dynamical systems seminars traditionally organized at Middle East Technical University throughout each academic year.
Theme: Of this coming workshop will be Fractional Differential Equations and Dynamic Equations with Applications. However, talks are not restricted to these subjects only.
Information: `http://tmdankara.org.tr/dsw11/`.

July 2012

* 1–21 **IAS/PCMI Summer 2012: Geometric Group Theory**, IAS/Park City Mathematics Institute, Park City, Utah.
Description: Some mobility between the Research in Mathematics and Graduate Summer School programs is expected and encouraged, but interested candidates should read the guidelines carefully and apply to the one program best suited to their field of study and experience. Postdoctoral scholars who are working in the field of Geometric Group Theory should apply to the Research Program in Mathematics, not to the Graduate Summer School. Graduate students who are beyond their basic courses and recent Ph.D.'s in all fields of mathematics are encouraged to apply to the Graduate Summer School. Funding will go primarily to graduate students. Postdoctoral scholars not working in the field of Geometric Group Theory should also apply, but should be within four years of receipt of their Ph.D.
Information: `http://www.msri.org/web/msri/scientific/workshops/summer-graduate-workshops/show/-/event/Wm9215`.

* 2–6 **Algebra and Topology: Methods, Computation, and Science (ATMCS 2012)**, International Centre for Mathematical Sciences, Edinburgh, United Kingdom.
Description: The goal of the meeting is to provide a forum for discussion of applications of topological methods in science and engineering. The following speakers have confirmed their participation. J.D. Boissonnat (INRIA Sophia Antipolis) R. Van de Weijgaert (Groningen) N. Linial (Hebrew University, Jerusalem) S. Weinberger (University of Chicago) S. Smale (City University of Hong Kong) H. Edelsbrunner (IST, Austria) E. Goubault (Commissariat à l'énergie atomique, Paris), S. Krishnan (University of Pennsylvania), M. Kahle (The Ohio State University), L. Guibas (Stanford University), R. Macpherson (IAS Princeton) (tentative), A. Szymczak (Colorado School of Mines), P. Skraba/M. Vejdemo-Johansson (Ljubljana/St. Andrews), Y. Mileyko (Duke University). There will be an opportunity for contributed talks.
Information: `http://www.icms.org.uk/workshop.php?id=205`.

* 3–6 **International Symposium on Asymptotic Methods in Stochastics, in Honour of Miklós Csörgő's work on the occasion of his 80th birthday**, Carleton University, Ottawa, Canada.
Description: In 2012 Professor Miklós Csörgő will be celebrating his 80th birthday, as well as his 50 years of continuous publications in

Stochastics (Probability and Statistics) since 1962. On this occasion there will be an International Symposium on Asymptotic Methods in Stochastics. The symposium is part of our ongoing series of workshops at Carleton University, organized with annually varying themes in diverse areas of probability and stochastic processes. The symposium will honour 50 years of Professor Miklós Csörgő's research in probability theory, stochastic processes and mathematical statistics. The topics of the conference can be broadly described as asymptotic methods in stochastics.
Information: http://www.fields.utoronto.ca/programs/scientific/12-13/stochastics/.

* 9–11 **15th Galway Topology Colloquium**, University of Oxford, Oxford, United Kingdom.
Description: The Galway Topology Colloquium was created in 1997 in Galway with the intention of encouraging graduate students studying Analytic and Set Theoretic Topology to present their findings to a professional, supportive, and inquisitive audience. It is also the established annual arena to bring together British and Irish analytic and set-theoretic topologists to collaborate on their research. Its particular focus is on the active participation of all who attend to promote a spirit of training, learning and communicating. The atmosphere is informal and graduate students are especially encouraged to participate.
Information: email: GalwayTopologyColloquium15@gmail.com; http://www.maths.ox.ac.uk/events/conferences/galway15.

* 9–13 **Algorithmic Number Theory Symposium (ANTS-X)**, University of California, San Diego, California.
Description: The ANTS meetings, held biannually since 1994, are the premier international forum for new research in computational number theory. They are devoted to algorithmic aspects of number theory, including elementary number theory, algebraic number theory, analytic number theory, geometry of numbers, arithmetic algebraic geometry, finite fields, and cryptography.
Information: http://math.ucsd.edu/~kedlaya/ants10/.

* 15–20 **XXII Brazilian Algebra Meeting**, Salvador, in the State of Bahia, Brazil.
Description: This is the largest biennial meeting in Brazil, and one of the most important in Latin America, entirely devoted to Algebra and related topics. Its main objective is to provide an opportunity for researchers and students to exchange ideas, to communicate and discuss research findings in all branches of Algebra. In order to bring together leading experts and new researchers, such as graduate students, post-docs and young faculty members, the meeting will offer plenary talks, mini-courses (in diverse levels), as well as sections of communications and posters. We will be celebrating the 40th anniversary of this event, affectionately called Escola de Álgebra by the Brazilians.
Information: http://www.algebra2012.ufba.br.

* 16–18 **Recent Developments in Statistical Multiscale Methods**, University of Goettingen, Germany.
Description: Statistical Multiscale Methods have been developed rapidly during the last decades and provide nowadays an indispensable tool for a variety of applications. The aim of this workshop is to provide an up to date overview of the different aspects of this emerging field. This includes on the one hand a rigorous mathematical treatment of statistical multiscale estimators such as risk bound analysis, deviation inequalities and their statistical applications, e.g., the construction of confidence sets. On the other hand, algorithmic and computational issues will be discussed as well, and their performance in various areas of applications, ranging from signal processing, time series analysis and financial statistics to computational genetics and bio-imaging.
Local Organization: Axel Munk, Klaus Frick, Thomas Rippl.
Organizer: This conference is organized by the German-Swiss research group FOR 916 "Statistical Regularization and Qualitative Constraints" and is sponsored by the German Science Foundation.
Information: http://www.stochastik.math.uni-goettingen.de/for916/SMM2012.

* 16–21 **West Coast Algebraic Topology Summer School**, Stanford University, Stanford, California.
Description: This summer school is aimed at graduate students and post-docs, though all are welcome. The scientific goal is to provide a modern introduction to algebraic K-theory, with the aim of bringing participants to the research frontier. In particular, the workshop will focus on the algebraic K-theory spectrum, trace methods for computing, localization phenomena, and applications to the topology of manifolds. Some participants will be preparing and giving lectures which have been planned in advance by the scientific committee. The number of lectures each day will be limited, and the the additional time used for activities to complement lectures.
Scientfic Committee: Andrew Blumberg, Bill Dwyer, Teena Gerhardt, John Klein, and Mike Mandell.
Information: http://noether.uoregon.edu/~dps/wcatss2012/.

* 29–August 3 **XVIII EBT: 18th Brazilian Topology Meeting**, Circuito das Aguas, in the State of Sao Paulo, Brazil.
Description: The Brazilian Topology Meeting (Encontro Brasileiro de Topologia) is held bi-annually in various locations in Brazil. It is a five-day meeting and includes three mini-courses : elementary (intended for students), intermediate, and advanced. The talks usually cover a wide variety of areas within topology and geometry, and applications.
Information: http://www.dm.ufscar.br/~ebt2012/.

August 2012

* 6–10 **Geometric Structures and Representation Varieties NSF Research Network Senior Retreat**, University of Illinois Urbana-Champaign, Urbana, Illinois.
Description: The first GEAR Network Retreat will be held August 6–10, 2012. The retreat is designed is to build bridges between mathematicians working in areas relating to Geometric structures and representation varieties. Each of the five days of the retreat will feature one of the following themes: Higgs bundles, geometric structures and Teichmüller spaces, dynamics on moduli spaces, special representations and geometric structures, hyperbolic 3-manifolds. There will be two 50-minute lectures in the morning, with the goal of surveying the state-of-the-art, followed by 3 or 4 shorter research lectures in the afternoon. A tentative list of survey speakers includes R. Canary (Michigan), D. Cooper (UCSB), D. Dumas (UIC), N. Dunfield (UIUC), O. Garcia-Prada (CSIC, Madrid), F. Labourie (Paris-Sud), T. Pantev (Penn), J. Smillie (Cornell), A. Wienhard (Princeton) and A. Wilkinson (Chicago).
Information: http://gear.math.illinois.edu/programs/SeniorRetreat2012.html.

* 6–10 **ICERM Topical Workshop: Bridging Scales in Computational Polymer Chemistry**, Institute for Computational and Experimental Research in Mathematics (ICERM), Providence, Rhode Island.
Description: Many important advances in material and biomedical science will come from controlling the chemical properties and nanoscale morphology of polymer mixtures. Predicting the longtime continuum-level properties of such complex systems poses a canonical computational challenge due to the disparate length and time scales separating the molecular description from the macroscopic behavior, particularly the evolution of morphology. This workshop focuses on four overlapping approaches to bridging this gap: Accelerated Molecular Methods, Coarse-Graining of Molecular Dynamics, Computational Approaches to Self-Consistent Mean Field, and Coupled Molecular and Continuum-Variational models. The goal is to spur the development of hybrid computational methods with the capacity to identify and characterize the rare events and the driving forces which steer the

systems towards equilibrium, and connect the burgeoning growth in parallel-computation techniques for particle-based systems with recently developed classes of continuum models.

Information: email: nicole_henrichs@icerm.brown.edu; http://icerm.brown.edu/tw12-4-bscpc.

* 13–17 **2012 CBMS-NSF Conference: Mathematics of the Social and Behavioral Sciences**, West Chester University, West Chester, Pennsylvania.

Lecturer: Donald G. Saari.

Description: Emerging areas of mathematical interest are coming from the social and behavioral sciences. What makes these concerns, which are motivated by issues in economics, sociology, political science, and psychology, of particular mathematical interest is that many standard mathematical tools were developed in response to questions from the physical and engineering sciences, which means that often they are not appropriate to analyze concerns from the social and behavioral sciences. The precision of differential equations, for instance, can lead to misleading conclusions for the qualitative types of issues that arise in the social sciences; many questions about aggregation rules, such as voting rules and price models in economics, have not been answered. In other words, to make advances in the mathematical social and behavioral sciences, there is a need to develop different forms of mathematical approaches. The theme of this series is to introduce and describe a portion of them.

Information: http://www.wcupa.edu/math/2012CBMS.

* 20–24 **The Fourth Geometry Meeting dedicated to the centenary of A. D. Alexandrov**, The Euler International Mathematical Institute, Saint-Petersburg, Russia.

Description: The Euler International Mathematical Institute is organizing the fourth Geometry Meeting dedicated to the centenary of A. D. Alexandrov. The Meeting is to be held at the Euler International Mathematical Institute and the Steklov Institute (St. Petersburg, Russia).

Topics: Differential Geometry, PDE in Geometry, Geometric Analysis, Singular spaces, Applications of Geometry, Polyhedra.

Information: http://www.pdmi.ras.ru/EIMI/2012/A100/index.html.

September 2012

* 1–3 **13th International Pure Mathematics Conference**, Quaid-i-Azam University, Islamabad, Pakistan.

Description: The 13th international conference in the series of Pure Mathematics Conferences that take place in Islamabad every year in August/September. It is a thematic conference on Algebra, Geometry, and Analysis held under the auspices of the Pakistan Mathematical Society (http://www.pakms.org.pk) and Algebra Forum (http://www.algebraforum.org.pk).

Support: There will be free housing for foreign participants. Some travel grants are available for foreign speakers. Several free recreational trips will be organized in and around Islamabad introducing the unique local and multi-ethnic culture.

Registration: Please fill in the on-line registration form at http://www.pmc.org.pk and find more information therein. The conference is convened by Professor Dr. Qaiser Mushtaq (Department of Mathematics, Quaid-i-Azam University, Islamabad, Pakistan, president@pakms.org.pk).

Information: http://www.pakms.org.pk, www.pmc.org.pk.

* 3–8 **XVII Geometrical Seminar**, Zlatibor, Serbia Hotel "Ratko Mitrović".

Conference Topics: Differential geometry, topology, lie groups, mathematical physics, discrete geometry, integrable systems, visualization, as well as other subjects related to the main themes are welcome.

Organizers: Faculty of Mathematics, University of Belgrade, Belgrade, Serbia; Mathematical Institute of the Serbian Academy of Sciences and Arts Belgrade, Serbia.

Co-organizer: Bogolubov Laboratory of Geometrical Methods Mathematical Physics, Moscow.

Deadlines: Registrations: June 1, 2012. Abstracts: July 1, 2012.

Hotel reservation: By June 1, 2012. Contact person: Miroslava Antic; email: geometricalseminar@matf.bg.ac.rs.

Information: http://poincare.matf.bg.ac.rs/~geometricalseminar/.

* 4–9 **MADEA 2012 International Conference on Mathematical Analysis, Differential Equations and Their Applications**, Mersin University, Mersin, Turkey.

Description: This is the sixth Turkish-Ukrainian Mathematical conference which will be held in Mersin-Turkey. The first conference was held August 26–30, 2003, in National Juriy Fedkovich University of Chernivtsi (Chernivtsi, Ukraine), The second was held September 7–11, 2004, in Mersin University (Mersin, Turkey). The third was held September 18–23, 2006, in Uzhgorod National University (Uzhgorod, Ukraine). The fourth was held September 12–15, 2008, at Eastern Mediterranean University (Famagusta, North Cyprus). The fifth was held September 15–20, 2010, at Sunny Beach, Bulgaria. The conference format includes the plenary lectures and the section sessions.

Languages: English, Russian and Turkish.

Information: email: mkucukaslan@mersin.edu.tr or mkkaslan@gmail.com; http://madea2012.mersin.edu.tr/.

* 17–19 **IMA Conference on Mathematics of Medical Devices and Surgical Procedures**, University of London, United Kingdom.

Description: The conference programme will include keynote speakers drawn from both clinical and mathematical communities, along with contributed presentations and poster sessions. The programme will also include breakout sessions in certain topics as well as refreshment breaks for informal discussions. Social events include a drinks reception and a conference dinner.

Topics: The topics that will be discussed will broadly include cardiovascular devices, medical imaging, ophthalmology, cell biology, disease transmission, orthopaedic, advanced simulations, as well as health in ageing.

Information: http://www.ima.org.uk/conferences/conferences_calendar/maths_of_medical_devices_&_surgical_procedures.cfm.

* 17–21 **Summer School on New Trends in Harmonic Analysis, Fractional Operator Theory, and Image Analysis**, Inzell, Germany.

Description: Recently, focus has been placed on the detection and classification of singularities in images possessing fractional or fractal characteristics. Therefore, investigations center on fractional operator-like bases acting as multiscale versions of derivatives. This new methodology needs to firmly embed into the existing classical concepts of harmonic analysis, and the relations to image analysis have to be established and unified. To achieve the greatest possible synergy between the areas of harmonic analysis, fractional operator theory and image analysis, we have invited seven highly renowned researchers from these fields. The goal of the summer school is to bring together young researchers and a distinguished group of scientists whose lectures are intended to establish new and exciting directions for future investigations.

Information: http://www-m6.ma.tum.de/Lehrstuhl/Inzell2012.

* 24–27 **56th Annual Meeting of the Australian Mathematical Society**, University of Ballarat, Mt Helen Campus, Victoria, Australia.

Plenary Speakers: Henning Haahr Andersen (Aarhus University), Michel Brion (Université Joseph Fourier, Grenoble), Sidney A. Morris (University of Ballarat), Mary Myerscough (University of Sydney), Assaf Naor (Courant Institute of Mathematical Sciences, New York University), Narutaka Ozawa (Research Institute for Mathematical

Sciences, Kyoto University), Aidan Sims (University of Wollongong), Kate Smith-Miles (Monash University), Fedor Sukochev (University of New South Wales), Benar F. Svaiter (Institute of Pure and Applied Mathematics, Rio de Janeiro), Neil Trudinger (Australian National University).

Information: `http://www.ballarat.edu.au/austms2012`.

October 2012

* 9–11 **Algerian-Turkish International days on Mathematics 2012, "ATIM'2012"**, Badji Mokhtar Annaba University, Annaba, Algeria.

Aim: Of this conference is to provide a platform for scientific experts in mathematics to present their recent works, exchange ideas and new methods in this important area and to bring together mathematicians to improve collaboration between local and international participants. We are looking forward to meeting you in Annaba at ATIM'2012.

Organizers: Jointly organized by the Laboratory of Advanced Materials, Badji Mokhtar Annaba University and Fatih University, Istanbul, Turkey.

Information: `http://www.univ-annaba.org/ATIM2012/`.

* 24–26 **International Conference in Number Theory and Applications 2012 (ICNA 2012)**, Department of Mathematics, Faculty of Science, Kasetsart University, Bangkok, Thailand.

Aim: Providing a forum for researchers, teachers, students and people interested in Number Theory and Applications to present, exchange and get in touch with one another in a relaxed atmosphere. The academic program of the conference consists of invited lectures by leading mathematicians in the field of Number Theory and Applications and sessions for contributed talks, which will be included in a special issue of the *East-West Journal of Mathematics* after a peer review process. There will also be a social banquet (included in the registration fee) and a sight-seeing tour (not included in the registration fee).

Information: `http://maths.sci.ku.ac.th/icna2012`.

November 2012

* 1–2 **Central and Eastern European Software Engineering Conference in Russia 2012**, Digital October Center, Moscow, Russia.

Description: Organized since 2005, CEE-SECR is the key annual software event in Central and Eastern Europe that is regularly attended by about 800 participants from local industry. The conference was initially positioned as a Russian event; however, it attracted speakers from 20 countries and regular attendees from even more places, so in 2009 the conference was repositioned as a CEE event. The conference employs a double-blind review process producing a high quality program with acceptance rate about 35%.

Keynote speakers: From previous conferences includes Jeff Sutherland, Bertrand Meyer, Bjarne Stroustrup, Thomas Erl, Grady Booch, Ivar Jacobson, Erich Gamma, Michael Cusumano, Larry Constantine, Lars Bak, Michael Fagan, Bill Hefley, Rick Kazman, Yuri Gurevich, Steve Masters, Mark Paulk and other software thought leaders as well as VP-s and Technical Fellows of major high-tech corporations.

Information: `http://www.cee-secr.org`.

December 2012

* 22–24 **The International Congress on Science and Technology**, Allahabad, U.P., India.

Focus: The conference has the focus on the current trends on frontier topics of the science and technology (Applied Engineering) subjects. The ICST conferences serve as good platforms for our members and the entire science and technological community to meet with each other and to exchange ideas.

Organizer: The CWS, a non-profit society for the scientists and the technocrats.

Deadline: Submission of abstracts with full-length paper to `complexgeometry18@yahoo.com` with a cc: to `ss123a@rediffmail.com`: July 25, 2012.

Information: `http://sites.google.com/site/intcongressonsciandtech/`.

January 2013

* 6–8 **ACM-SIAM Symposium on Discrete Algorithms (SODA13)**, Astor Crowne Plaza Hotel, New Orleans, Louisiana.

Description: This symposium focuses on research topics related to efficient algorithms and data structures for discrete problems. In addition to the design of such methods and structures, the scope also includes their use, performance analysis, and the mathematical problems related to their development or limitations. Performance analyses may be analytical or experimental and may address worst-case or expected-case performance. Studies can be theoretical or based on data sets that have arisen in practice and may address methodological issues involved in performance analysis.

Information: `http://www.siam.org/meetings/da13/`.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

April 2013

* 13–14 **3rd IIMA International Conference on Advanced Data Analysis, Business Analytics and Intelligence**, Indian Institute of Management, Ahmedabad, India.

Description: Indian Institute of Management Ahmedabad is happy to announce the 3rd international conference dedicated to advanced data analysis, business analytics and business intelligence. The objectives of the conference are to facilitate sharing of: a) Research based knowledge related to advanced data analysis, business analytics and business intelligence among academicians and practitioners, b) Case studies and novel business applications of tools and techniques of advanced data analysis, business analytics and business intelligence among academicians and practitioners. Papers are invited from academicians and practitioners on any topic mentioned in the list of conference topics and related areas. Applications, case studies, review and discussion papers on these topics and related areas are also welcome.

Information: `http://www.iimahd.ernet.in/icadabai2013/`.

September 2013

* 11–13 **14th IMA Conference on Mathematics of Surfaces**, University of Birmingham, United Kingdom.

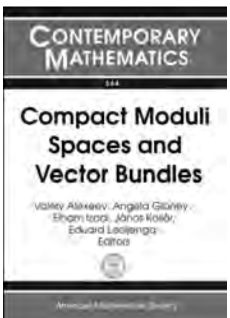
Description: Computer-based methods for the capture, construction, representation, fitting, interrogation and manipulation of complicated surfaces have led to a wide interest in, and need for, the mathematics of surfaces and related curves. Many applications require the use of surface descriptions, especially in such fields as computer aided design and manufacturing, computer graphics and computer vision. The description of surfaces is also of interest in geographic information systems, multimedia, and many other areas of science and medicine. This diversity and the wide range of applicability of the subject have already enabled the IMA to hold thirteen very successful international conferences in the Mathematics of Surfaces series. Several international authorities are being invited to present papers. The Institute of Mathematics and its Applications is a not-for-profit organisation registered as a charity in the UK.

Information: `http://www.ima.org.uk/conferences/conferences_calendar/14th_mathematics_of_surfaces.cfm`.

New Publications Offered by the AMS

To subscribe to email notification of new AMS publications, please go to http://www.ams.org/bookstore-email.

Algebra and Algebraic Geometry



Compact Moduli Spaces and Vector Bundles

Valery Alexeev, **Angela Gibney**, and **Elham Izadi**, *University of Georgia, Athens, GA*, **János Kollár**, *Princeton University, NJ*, and **Eduard Looijenga**, *Universiteit Utrecht, The Netherlands*, Editors

This book contains the proceedings of the conference on Compact Moduli and Vector Bundles, held from October 21–24, 2010, at the University of Georgia.

This book is a mix of survey papers and original research articles on two related subjects: Compact Moduli spaces of algebraic varieties, including of higher-dimensional stable varieties and pairs, and Vector Bundles on such compact moduli spaces, including the conformal block bundles. These bundles originated in the 1970s in physics; the celebrated Verlinde formula computes their ranks.

Among the surveys are those that examine compact moduli spaces of surfaces of general type and others that concern the GIT constructions of log canonical models of moduli of stable curves.

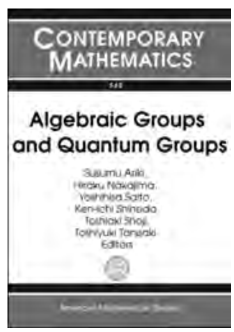
The original research articles include, among others, papers on a formula for the Chern classes of conformal classes of conformal block bundles on the moduli spaces of stable curves, on Looijenga's conjectures, on algebraic and tropical Brill–Noether theory, on Green's conjecture, on rigid curves on moduli of curves, and on Steiner surfaces.

This item will also be of interest to those working in geometry and topology.

Contents: P. Hacking, Compact moduli spaces of surfaces of general type; **A.-M. Castravet** and **J. Tevelev**, Rigid curves on $\overline{M}_{0,n}$ and arithmetic breaks; **L. Caporaso**, Algebraic and combinatorial Brill-Noether theory; **J. Alper** and **D. Hyeon**, GIT constructions of log canonical models of $\overline{M}_g$; **S. Casalaina-Martin**, **D. Jensen**, and **R. Laza**, The geometry of the ball quotient model of the moduli space of genus four curves; **E. Arbarello** and **G. Mondello**, Two remarks on the Weierstrass flag; **N. Fakhruddin**, Chern classes of conformal blocks; **V. Balaji** and **J. Kollár**, Restrictions of stable bundles; **P. Belkale**, Orthogonal bundles, theta characteristics and symplectic strange duality; **S. J. Kovács**, The splitting principle and singularities; **S. Mukai**, Igusa quartic and Steiner surfaces; **M. Aprodu** and **G. Farkas**, Green's conjecture for general covers; **B. Hassett** and **Y. Tschinkel**, Spaces of sections of quadric surface fibrations over curves.

Contemporary Mathematics, Volume 564

March 2012, approximately 256 pages, Softcover, ISBN: 978-0-8218-6899-7, 2010 *Mathematics Subject Classification:* 14D22, 14D20, 14H10, 14H60, 14J10, **AMS members US$71.20**, List US$89, Order code CONM/564



Algebraic Groups and Quantum Groups

Susumu Ariki, *Osaka University, Japan*, **Hiraku Nakajima**, *Kyoto University, Japan*, **Yoshihisa Saito**, *University of Tokyo, Japan*, **Ken-ichi Shinoda**, *Sophia University, Tokyo, Japan*, **Toshiaki Shoji**, *Nagoya University, Japan*, and **Toshiyuki Tanisaki**, *Osaka City University, Japan*, Editors

This volume contains the proceedings of the tenth international conference on Representation Theory of Algebraic Groups and Quantum Groups, held August 2–6, 2010, at Nagoya University, Nagoya, Japan.

The survey articles and original papers contained in this volume offer a comprehensive view of current developments in the field. Among others reflecting recent trends, one central theme is research on representations in the affine case. In three articles, the authors study representations of W-algebras and affine Lie algebras at the critical level, and three other articles are related to crystals in the affine case, that is, Mirkovic–Vilonen polytopes for affine type A and Kerov-Kirillov-Reshetikhin type bijection for affine type E_6.

Other contributions cover a variety of topics such as modular representation theory of finite groups of Lie type, quantum queer super Lie algebras, Khovanov's arc algebra, Hecke algebras and cyclotomic q-Schur algebras, G_1T-Verma modules for reductive algebraic groups, equivariant K-theory of quantum vector bundles, and the cluster algebra.

This book is suitable for graduate students and researchers interested in geometric and combinatorial representation theory and other related fields.

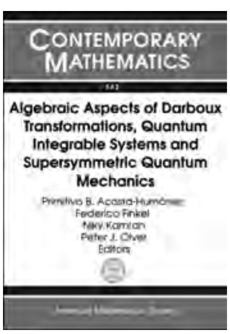
This item will also be of interest to those working in mathematical physics.

Contents: T. Arakawa, W-algebras at the critical level; **R. Bezrukavnikov** and **Q. Lin**, Highest weight modules at the critical level and noncommutative Springer resolution; **J. Brundan**, An orthogonal form for level two Hecke algebras with applications; **P. Fiebig**, On the restricted projective objects in the affine category $\mathcal{O}$ at the critical level; **M. Geck**, Remarks on modular representations of finite groups of Lie type in non-defining characteristic; **J. H. Jung** and **S.-J. Kang**, Quantum queer superalgebras; **M. Kaneda**, Homomorphisms between neighboring G_1T-Verma modules; **G. I. Lehrer** and **R. B. Zhang**, Quantum group actions on rings and equivariant K-theory; **S. Naito**, **D. Sagaki**, and **Y. Saito**, Toward Berenstein-Zelevinsky data in affine type A, part I: Construction of the affine analogs; **S. Naito**, **D. Sagaki**, and **Y. Saito**, Toward Berenstein-zelevinsky data in affine type A, part II: Explicit description; **T. Nakanishi** and **A. Zelevinsky**, On tropical dualities in cluster algebras; **M. Okado** and **N. Sano**, KKR type bijection for the exceptional affine algebra $E_6^{(1)}$; **T. Shoji** and **N. Xi**, Iwahori's question for affine Hecke algebras; **K. Wada**, On Weyl modules of cyclotomic q-Schur algebras.

Contemporary Mathematics, Volume 565

March 2012, 286 pages, Softcover, ISBN: 978-0-8218-5317-7, LC 2011050433, 2010 *Mathematics Subject Classification:* 05E10, 16Exx, 17Bxx, 20Cxx, 20Gxx, 81Rxx, **AMS members US$79.20**, List US$99, Order code CONM/565

Analysis



Algebraic Aspects of Darboux Transformations, Quantum Integrable Systems and Supersymmetric Quantum Mechanics

Primitivo B. Acosta-Humánez, *Universidad del Norte, Barranquilla, Colombia*, **Federico Finkel**, *Universidad Complutense de Madrid, Spain*, **Niky Kamran**, *McGill University, Montreal, Quebec, Canada*, and **Peter J. Olver**, *University of Minnesota, Minneapolis, MN*, Editors

This volume represents the 2010 Jairo Charris Seminar in Algebraic Aspects of Darboux Transformations, Quantum Integrable Systems and Supersymmetric Quantum Mechanics, which was held at the Universidad Sergio Arboleda in Santa Marta, Colombia.

The papers cover the fields of Supersymmetric Quantum Mechanics and Quantum Integrable Systems from an algebraic point of view. Some results presented in this volume correspond to the analysis of Darboux Transformations in higher order as well as some exceptional orthogonal polynomials.

The reader will find an interesting Galois approach to study finite gap potentials.

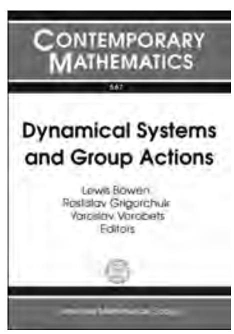
This item will also be of interest to those working in mathematical physics.

Contents: Y. V. Brezhnev, Spectral/quadrature duality: Picard-Vessiot theory and finite-gap potentials; **D. Dutta** and **P. Roy**, Darboux transformation, exceptional orthogonal polynomials and information theoretic measures of uncertainty; **D. Gómez-Ullate**, **N. Kamran**, and **R. Milson**, On orthogonal polynomials spanning a non-standard flag; **M. A. Gonzalez Leon**, **M. T. Mayado**, **J. M. Guilarte**, and **M. J. Senosiain**, On the supersymmetric spectra of two planar integrable quantum systems; **Y. Grandati** and **A. Bérard**, Solvable rational extension of translationally shape invariant potentials; **V. Ovsienko**, The pentagram map: Geometry, algebra, integrability; **E. G. Reyes**, Jet bundles, symmetries, Darboux transforms; **A. Schulze-Halberg**, Explicit higher-dimensional Darboux transformations for the time-dependent Schrödinger equation; **V. P. Spiridonov**, Elliptic beta integrals and solvable models of statistical mechanics.

Contemporary Mathematics, Volume 563

March 2012, 211 pages, Softcover, ISBN: 978-0-8218-7584-1, LC 2011050423, 2010 *Mathematics Subject Classification:* 12H05, 33E30, 81Q60, 81Q80, 82B23, 33E99, **AMS members US$63.20**, List US$79, Order code CONM/563

Geometry and Topology



Dynamical Systems and Group Actions

Lewis Bowen, **Rostislav Grigorchuk**, and **Yaroslav Vorobets**, *Texas A & M University, College Station, TX*, Editors

This volume contains cutting-edge research from leading experts in ergodic theory, dynamical systems and group actions. A large part of the volume addresses various aspects of ergodic theory of general group actions including local entropy theory, universal minimal spaces, minimal models and rank one transformations. Other papers deal with interval exchange transformations, hyperbolic dynamics, transfer operators, amenable actions and group actions on graphs.

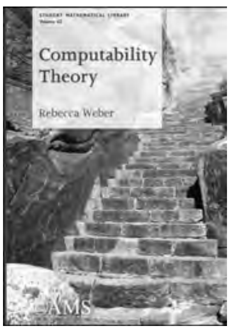
This item will also be of interest to those working in algebra and algebraic geometry.

Contents: M. Abért and **G. Elek**, Hyperfinite actions on countable sets and probability measure spaces; **A. B. Antonevich**, **V. I. Bakhtin**, and **A. V. Lebedev**, A road to the spectral radius of transfer operators; **M. Boshernitzan**, A condition for weak mixing of induced IETs; **L. Bowen**, Every countably infinite group is almost Ornstein; **L. A. Bunimovich**, Fair dice-like hyperbolic systems; **G. Chinta**, **J. Jorgenson**, and **A. Karlsson**, Complexity and heights of tori; **A. I. Danilenko**, Flows with uncountable but meager group of self-similarities; **E. Glasner** and **Y. Gutman**, The universal minimal space of the homeomorphism group of a H-homogeneous space; **W. Huang** and **X. Ye**, Generic eigenvalues, generic factors and weak disjointness; **E. Janvresse**, **T. de la Rue**, and **V. Ryzhikov**, Around King's rank-one theorems: Flows and $\mathbb{Z}^n$-actions; **V. A. Kaimanovich** and **F. Sobieczky**, Random walks on horospheric products; **Y. G. Sinai**, Statistics of gaps in the sequence $\{\sqrt{n}\}$; **W. A. Veech**, Invariant distributions for interval exchange transformations; **Y. Vorobets**, Notes on the Schreir graphs of the Grigorchuk group; **B. Weiss**, Minimal models for free actions.

Contemporary Mathematics, Volume 567

May 2012, 264 pages, Softcover, ISBN: 978-0-8218-6922-2, LC 2011051433, 2010 *Mathematics Subject Classification:* 37Axx, 37Bxx, 37Dxx, 20Exx, 20Pxx, 20Nxx, 22Fxx, **AMS members US$79.20**, List US$99, Order code CONM/567

Logic and Foundations



Computability Theory

Rebecca Weber, *Dartmouth College, Hanover, NH*

What can we compute—even with unlimited resources? Is everything within reach? Or are computations necessarily drastically limited, not just in practice, but theoretically? These questions are at the heart of computability theory.

The goal of this book is to give the reader a firm grounding in the fundamentals of computability theory and an overview of currently active areas of research, such as reverse mathematics and algorithmic randomness. Turing machines and partial recursive functions are explored in detail, and vital tools and concepts including coding, uniformity, and diagonalization are described explicitly. From there the material continues with universal machines, the halting problem, parametrization and the recursion theorem, and thence to computability for sets, enumerability, and Turing reduction and degrees. A few more advanced topics round out the book before the chapter on areas of research. The text is designed to be self-contained, with an entire chapter of preliminary material including relations, recursion, induction, and logical and set notation and operators. That background, along with ample explanation, examples, exercises, and suggestions for further reading, make this book ideal for independent study or courses with few prerequisites.

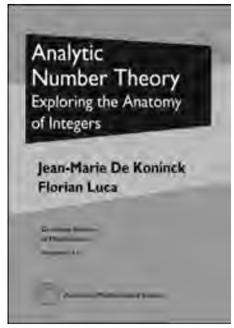
This item will also be of interest to those working in applications.

Contents: Introduction; Background; Defining computability; Working with computable functions; Computing and enumerating sets; Turing reduction and Post's problem; Two hierarchies of sets; Further tools and results; Areas of research; Mathematical asides; Bibliography; Index.

Student Mathematical Library, Volume 62

May 2012, approximately 206 pages, Softcover, ISBN: 978-0-8218-7392-2, LC 2011050912, 2010 *Mathematics Subject Classification:* 03Dxx; 68Qxx, **AMS members US$29.60**, List US$37, Order code STML/62

Number Theory



Analytic Number Theory

Exploring the Anatomy of Integers

Jean-Marie De Koninck, *Université Laval, Quebec, QC, Canada*, and **Florian Luca**, *Universidad Nacional Autonoma de México, Morelia, Michoacan, México*

The authors assemble a fascinating collection of topics from analytic number theory that provides an introduction to the subject with a very clear and unique focus on the anatomy of integers, that is, on the study of the multiplicative structure of the integers. Some of the most important topics presented are the global and local behavior of arithmetic functions, an extensive study of smooth numbers, the Hardy-Ramanujan and Landau theorems, characters and the Dirichlet theorem, the *abc* conjecture along with some of its applications, and sieve methods. The book concludes with a whole chapter on the index of composition of an integer.

One of this book's best features is the collection of problems at the end of each chapter that have been chosen carefully to reinforce the material. The authors include solutions to the even-numbered problems, making this volume very appropriate for readers who want to test their understanding of the theory presented in the book.

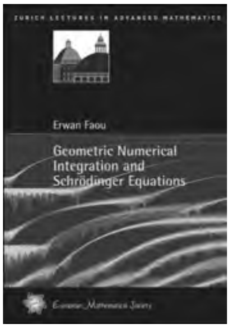
Contents: Preliminary notions; Prime numbers and their properties; The Riemann zeta function; Setting the stage for the proof of the prime number theorem; The proof of the prime number theorem; The global behavior of arithmetic functions; The local behavior of arithmetic functions; The fascinating Euler function; Smooth numbers; The Hardy-Ramanujan and Landau theorems; The *abc* conjecture and some of its applications; Sieve methods; Prime numbers in arithmetic progression; Characters and the Dirichlet theorem; Selected applications of primes in arithmetic progression; The index of composition of an integer; Appendix. Basic complex analysis theory; Solutions to even-numbered problems; Bibliography; Index.

Graduate Studies in Mathematics, Volume 134

June 2012, approximately 420 pages, Hardcover, ISBN: 978-0-8218-7577-3, LC 2011051431, 2010 *Mathematics Subject Classification:* 11A05, 11A41, 11B05, 11K65, 11N05, 11N13, 11N35, 11N37, 11N60, 11B39, **AMS members US$60**, List US$75, Order code GSM/134

New AMS-Distributed Publications

Applications



Geometric Numerical Integration and Schrödinger Equations

Erwan Faou, *ENS Cachan Bretagne, Bruz, France*

The goal of geometric numerical integration is the simulation of evolution equations possessing geometric properties over long periods of time. Of particular importance are Hamiltonian partial differential equations typically arising in application fields such as quantum mechanics or wave propagation phenomena. They exhibit many important dynamical features such as energy preservation and conservation of adiabatic invariants over long periods of time. In this setting, a natural question is how and to which extent the reproduction of such long-time qualitative behavior can be ensured by numerical schemes.

Starting from numerical examples, these notes provide a detailed analysis of the Schrödinger equation in a simple setting (periodic boundary conditions, polynomial nonlinearities) approximated by symplectic splitting methods. Analysis of stability and instability phenomena induced by space and time discretization are given, and rigorous mathematical explanations are provided for them.

The book grew out of a graduate-level course and is of interest to researchers and students seeking an introduction to the subject matter.

This item will also be of interest to those working in analysis.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Introduction; Finite-dimensional backward error analysis; Infinite-dimensional and semi-discrete Hamiltonian flow; Convergence results; Modified energy in the linear case; Modified energy in the semi-linear case; Introduction to long-time analysis; Bibliography; Index.

Zurich Lectures in Advanced Mathematics, Volume 15

January 2012, 146 pages, Softcover, ISBN: 978-3-03719-100-2, 2010 *Mathematics Subject Classification:* 65P10, 37M15, 35Q41, **AMS members US$30.40**, List US$38, Order code EMSZLEC/15

Differential Equations

Œuvres Scientifiques I, II, III with DVD

Laurent Schwartz

This first volume of selected mathematical papers of Laurent Schwartz covers the first half of his work in Analysis and PDE. After a foreword by Claude Viterbo, followed by some pictures, the reader will find a notice about the scientific works, by Laurent Schwartz himself, a few documents (letters and preparatory lecture notes), Bernard Malgrange's presentation of the theory of distributions (the theory for which Laurent Schwartz received the Fields Medal in 1950), and selected papers from 1944 to 1954.

This second volume of selected mathematical papers of Laurent Schwartz covers the second half of his work in Analysis and PDE. This volume includes Alain Guichardet's discussion of Laurent Schwartz and the seminars, followed by a selection of Schwartz's papers from 1954 to 1966.

This third volume of selected mathematical papers of Laurent Schwartz covers his work on Banach spaces (1970–1996) and Probability Theory (1968–1987) and contains some papers of a historical nature (1951–1994). This volume also includes presentations by Gilles Godefroy on Laurent Schwartz's influence on the theory of Banach spaces and by Michel Émery on Laurent Schwartz the probabilist.

The set includes all three volumes with a DVD.

This item will also be of interest to those working in analysis and probability and statistics.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: *Contents for Volume I:* Remerciements; Présentation des Œuvres de Laurent Schwartz; Quelques photos; Notice sur les travaux scientifiques de Laurent Schwartz; *Partie I. Travaux en Analyse et EDP:* La théorie des distributions (by B. Malgrange); Quelques documents; Articles retenus (1944–1954); Liste des publications de Laurent Schwartz; *Contents for Volume II:* Remerciements; Laurent Schwartz et les séminaires (by A. Guichardet); *Partie I (suite). Travaux en Analyse et EDP:* Articles retenus (1954–1966); Liste des publications de Laurent Schwartz; *Contents for Volume III:* Remerciements; *Partie II. Travaux sur les espaces de Banach:* L'influence de Laurent Schwartz en théorie des espaces de Banach (by G. Godefroy); Articles retenus (1970–1996); *Partie III. Travaux en Calcul des probabilités:* Laurent Schwartz probabiliste (by M. Émery); Articles retenus (1968–1987); *Partie IV. Travaux historiques et divers:* Articles retenus (1951–1994); Liste des publications de Laurent Schwartz.

Documents Mathématiques

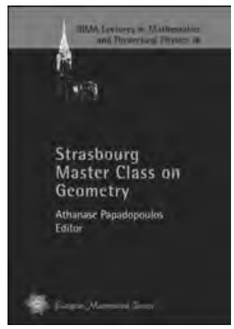
Volume I: December 2011, 523 pages, Hardcover, ISBN: 978-2-85629-317-1, 2010 *Mathematics Subject Classification:* 35-XX, 35Jxx, 46Axx, 46Fxx, 46Bxx, 60Gxx, **Individual member US$94.50**, List US$105, Order code SMFDM/9

Volume II: December 2011, 507 pages, Hardcover, ISBN: 978-2-85629-318-8, 2010 *Mathematics Subject Classification:* 35-XX, 35Jxx, 46Axx, 46Fxx, 46Bxx, 60Gxx, **Individual member US$94.50**, List US$105, Order code SMFDM/10

Volume III: December 2011, 619 pages, Hardcover, ISBN: 978-2-85629-319-5, 2010 *Mathematics Subject Classification:* 35-XX, 35Jxx, 46Axx, 46Fxx, 46Bxx, 60Gxx, **Individual member US$94.50**, List US$105, Order code SMFDM/11

Set: December 2011, 1649 pages, Hardcover, ISBN: 978-2-85629-337-9, 2010 *Mathematics Subject Classification:* 35-XX, 35Jxx, 46Axx, 46Fxx, 46Bxx, 60Gxx, **Individual member US$256.50**, List US$285, Order code SMFDMSET

Geometry and Topology



Strasbourg Master Class on Geometry

Athanase Papadopoulos, *Université de Strasbourg, France*, Editor

This book contains carefully revised and expanded versions of eight courses that were presented at the University of Strasbourg during two geometry master classes in 2008 and 2009.

The aim of the master classes was to give fifth-year students and Ph.D. students in mathematics the opportunity to learn new topics that lead directly to the current research in geometry and topology. The courses were taught by leading experts. The subjects treated include hyperbolic geometry, three-manifold topology, representation theory of fundamental groups of surfaces and of three-manifolds, dynamics on the hyperbolic plane with applications to number theory, Riemann surfaces, Teichmüller theory, Lie groups, and asymptotic geometry.

The text is aimed at graduate students and research mathematicians. It can also be used as a reference book and as a textbook for short courses on geometry.

A publication of the European Mathematical Society. Distributed within the Americas by the American Mathematical Society.

Contents: **N. A'Campo** and **A. Papadopoulos**, Notes on non-Euclidean geometry; **F. Dal'Bo**, Crossroads between hyperbolic geometry and number theory; **F. Herrlich**, Introduction to origamis in Teichmüller space; **P. Korablev** and **S. Matveev**, Five lectures on 3-manifold topology; **G. Link**, An introduction to globally symmetric spaces; **J. Marché**, Geometry of the representation spaces in SU(2); **C. Petronio**, Algorithmic construction and recognition of hyperbolic 3-manifolds, links, and graphs; **V. Schroeder**, An introduction to asymptotic geometry.

IRMA Lectures in Mathematics and Theoretical Physics, Volume 18

January 2012, 461 pages, Softcover, ISBN: 978-3-03719-105-7, 2010 *Mathematics Subject Classification:* 51-01, 51-02, 57-01, 57-02, **AMS members US$54.40**, List US$68, Order code EMSILMTP/18

Classified Advertisements

Positions available, items for sale, services available, and more

MASSACHUSETTS

NORTHEASTERN UNIVERSITY
Department of Mathematics
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The Department of Mathematics at Northeastern University invites applicants for a tenure-track position at the Assistant/Associate Professor level to start as early as September of 2012. Appointments are based on exceptional research contributions in mathematics combined with strong commitment and demonstrated success in teaching. Applications from those with an interest and ability to connect across units in the university to the advantage of research at the interface of mathematics and other disciplines are a top priority. Outstanding candidates with research in any area of mathematics are encouraged to apply.

Minimum Qualifications. Candidates must have a Ph.D., research experience, and demonstrated evidence of excellent teaching ability. Review of applications will begin immediately. Complete applications received by October 1, 2011, will be guaranteed full consideration. Additional applications will be considered until the position is filled.

To apply, visit "Careers at Northeastern" at: `https://psoft.neu.edu/psc/neuhrprdpub/EMPLOYEE/HRMS/c/NEU_HR.NEU_JOBS.GBL`. Click on "Faculty Positions" and search for the current position under the College of Science. You can also apply by visiting the College of Science website at: `http://www.northeastern.edu/cos/` and clicking on the Faculty Positions button. Research statements, reference letters, and teaching statements should be submitted to `www.mathjobs.org` along with the other materials requested there.

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000106

NEW YORK

THE UNIVERSITY OF ROCHESTER
Department of Mathematics

The Department of Mathematics at the University of Rochester invites applications for an opening in pure mathematics, starting on July 1, 2013, or later, at the tenure-track Assistant Professor level; more senior candidates with outstanding research achievements may also be considered. The teaching load for this position is three one-semester courses per year. Applications are encouraged in the general areas currently represented in the department's research profile: algebraic topology; algebra and number theory; analysis and PDE; differential geometry and global analysis; and probability and mathematical physics. Qualifications include a Ph.D. in mathematics, exceptional promise and/or accomplishments in research, and excellence in teaching. Application materials consist of a current C.V.; a statement of current and future research plans; a statement of teaching philosophy and experience; and at least four letters of recommendation, one of which should specifically address teaching. Applications should be submitted electronically through the website: `http://www.rochester.edu/fort/mth`. Consideration of applications will begin on February 15, 2012, and continue on an ongoing basis. The University of Rochester, an Equal Opportunity Employer, has a strong commitment to diversity and actively encourages applications of candidates from groups underrepresented in higher education.

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HOME RENTALS AND SWAPS

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Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: May 2011 issue–February 28 2011; June/July 2011 issue–April 28, 2011; August 2011 issue–May 27, 2010; September 2011 issue–June 28, 2011; October 2011 issue–July 28, 2011; November 2012 issue–August 30, 2012.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

About the Cover

Data MINEing

April is Math Awareness Month, and this year the theme is data mining. The cover illustrates a simplified variant of the impressive tool MINE described in a recent *Science* article by David and Yakir Reshef and several others, "Detecting novel associations in large data sets", in volume 334 from last December).

"Data mining" is the term coined to describe what one does in order to extract valuable information from the huge amounts of (likely noisy) data that modern computers make available. Among the information one might want to extract are the relations that hold among various specific variables recorded in statistics, for example car accident rate and age. But one might also be given data on a large number of variables, and wonder without much prior information which of them are in fact related. This is what MINE helps with—it scans data for whatever pairs, possibly all, that you might think of interest, and produces for each pair a number—in fact a whole matrix of numbers—that measures the strength of relationship. The basic technique is to fit grids of various sizes to a set of pairs (x,y) in the plane in order to maximize the *mutual information coefficient* of data in the grid. Roughly speaking, the mutual information coefficient (*mic*), based on Shannon's information theory and first defined by the astronomer E. H. Linfoot in 1957, tells how much information about one variable is implied by the other. The cover shows a collection of sample small grids on artificial data.

The problem that Reshef, Reshef, et al. attack looks at first almost impossible. After all, the number of ways to partition a large planar set is (so to speak) astronomical. But because of the additive properties of entropy, a sub-grid of an optimal grid must also be optimal, and this suggests an approach by dynamic programming that turns out to make the task feasible.

One of the authors of the article has described some features of MINE, as well as the process of publishing in *Science*, on his website:

`http://mybiasedcoin.blogspot.com/2011/12/mic-and-mine-short-description.html`

—Bill Casselman
Graphics Editor

(`notices-covers@ams.org`*)*

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Lawrence, Kansas

University of Kansas

March 30 – April 1, 2012

Friday – Sunday

Meeting #1081

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: February 2012
Program first available on AMS website: March 8, 2012
Program issue of electronic *Notices*: March 2012
Issue of *Abstracts*: Volume 33, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Frank Calegari, Northwestern University, *Studying algebraic varieties through their cohomology—Recent progress in the Langlands program.*

Christopher Leininger, University of Illinois at Urbana-Champaign, *On complexity of surface homeomorphisms.*

Alina Marian, Northeastern University, *Strange duality for K3 and Abelian surfaces.*

Catherine Yan, Texas A&M University, *Enumerative combinatorics with fillings of polyominoes.*

Special Sessions

Algebraic Geometry and its Applications, **Yasuyuki Kachi**, **B. P. Purnaprajna**, and **Sarang Sane**, University of Kansas.

Combinatorial Commutative Algebra, **Christopher Francisco** and **Jeffrey Mermin**, Oklahoma State University, and **Jay Schweig**, University of Kansas.

Complex Analysis, Geometry and Probability, **Pietro Poggi-Corrandini** and **Hrant Hakobyan**, Kansas State University.

Dynamics and Stability of Nonlinear Waves, **Mat Johnson** and **Myunghyun Oh**, University of Kansas.

Enumerative and Geometric Combinatorics, **Margaret Bayer**, University of Kansas, **Joseph P. King**, University of North Texas, **Svetlana Poznanovik**, Georgia Institute of Technology, and **Catherine Yan**, Texas A&M University.

Geometric Representation Theory, **Zongzhu Lin**, Kansas State University, and **Zhiwei Yun**, Massachusetts Institute of Technology.

Geometric Topology and Group Theory, **Richard P. Kent IV**, University of Wisconsin-Madison, **Christopher J. Leininger**, University of Illinois Urbana-Champaign, and **Kasra Rafi**, University of Oklahoma.

Geometry of Moduli Spaces of Sheaves, **Alina Marian**, Northeastern University, and **Dragos Oprea**, University of California San Diego.

Harmonic Analysis and Applications, **Arpad Benyi**, Western Washington University, **David Cruz-Uribe**, Trinity College, and **Rodolfo Torres**, University of Kansas.

Interplay between Geometry and Partial Differential Equations in Several Complex Variables, **Jennifer Halfpap**, University of Montana, and **Phil Harrington**, University of Arkansas.

Invariants of Knots, **Heather A. Dye**, McKendree University, and **Aaron Kaestner** and **Louis H. Kauffman**, University of Illinois at Chicago.

Mathematical Statistics, **Zsolt Talata**, University of Kansas.

Mathematics of Ion Channels: Life's Transistors, **Bob Eisenberg**, Rush Medical Center at Chicago, **Chun Liu**, Penn State University, and **Weishi Liu**, University of Kansas.

Mirror Symmetry, **Ricardo Castano-Bernard**, Kansas State University, **Paul Horja**, Oklahoma State University, and **Zheng Hua** and **Yan Soibelman**, Kansas State University.

Nonlinear Dynamical Systems and Applications, **Weishi Liu** and **Erik Van Vleck**, University of Kansas.

Numerical Analysis and Scientific Computing, **Weizhang Huang**, **Xuemin Tu**, **Erik Van Vleck**, and **Honggou Xu**, University of Kansas.

Partial Differential Equations, **Milena Stanislavova** and **Atanas Stefanov**, University of Kansas.

Singularities in Commutative Algebra and Algebraic Geometry, **Hailong Dao**, University of Kansas, **Lance E. Miller**, University of Utah, and **Karl Schwede**, Pennsylvania State University.

Stochastic Analysis, **Jin Feng**, **Yaozhong Hu**, and **David Hualart**, University of Kansas.

Topics in Commutative Algebra, **Hailong Dao**, **Craig Huneke**, and **Daniel Katz**, University of Kansas.

Undergraduate Research, **Marianne Korten** and **David Yetter**, Kansas State University.

University Mathematics Education in an Online World, **Andrew G. Bennett** and **Carlos Castillo-Garsow**, Kansas State University.

Rochester, New York

Rochester Institute of Technology

September 22–23, 2012

Saturday – Sunday

Meeting #1082

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: May 2012
Program first available on AMS website: July 19, 2012
Program issue of electronic *Notices*: September 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: May 15, 2012
For abstracts: July 10, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

James Keener, University of Utah, *Title to be announced*.

Dusa McDuff, Barnard College, *Title to be announced*.

Steve Gonek, University of Rochester, *Title to be announced*.

Peter Winkler, Dartmouth College, *Title to be announced*.

Special Sessions

Analytic Number Theory (Code: SS 5A), **Steve Gonek**, University of Rochester, and **Angel Kumchev**, Towson University.

Applied and Computational Mathematics (Code: SS 11A), **Ludwig Kohaupt**, Beuth University of Technology, and **Yan Wu**, Georgia Southern University.

Continuum Theory (Code: SS 3A), **Likin C. Simon Romero**, Rochester Institute of Technology.

Difference Equations and Applications (Code: SS 10A), **Michael Radin**, Rochester Polytechnic Institute.

Financial Mathematics (Code: SS 1A), **Tim Siu-Tang Leung**, Columbia University.

Geometric, Categorical and Combinatorial Methods in Representation Theory (Code: SS 12A), **David Hemmer** and **Yiqiang Li**, State University of New York at Buffalo.

Inverse Problems and Nonsmooth Optimization: Celebrating Zuhair Nashed's 75th. Birthday (Code: SS 7A), **Patricia Clark**, **Baasansuren Jadama**, and **Akhtar A. Khan**, Rochester Institute of Technology, and **Hulin Wu**, University of Rochester.

Microlocal Analysis and Nonlinear Evolution Equations (Code: SS 2A), **Raluca Felea**, Rochester Institute of Technology, and **Dan-Andrei Geba**, University of Rochester.

Modern Relativity (Code: SS 6A), **Manuela Campanelli** and **Yosef Zlochower**, Rochester Institute of Technology.

New Advances in Graph Theory (Code: SS 9A), **Jobby Jacob**, Rochester Institute of Technology, and **Paul Wenger**, University of Colorado Denver.

Operator Theory and Function Spaces (Code: SS 4A), **Gabriel T. Prajitura** and **Ruhan Zhao**, State University of New York at Brockport.

Research in Mathematics by Undergraduates and Students in Post-Baccalaureate Programs (Code: SS 8A), **Bernard Brooks**, **Darren Narayan**, and **Tamas Wiandt**, Rochester Institute of Technology.

New Orleans, Louisiana

Tulane University

October 13–14, 2012

Saturday – Sunday

Meeting #1083

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: June 2012
Program first available on AMS website: September 6, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 3, 2012
For abstracts: August 28, 2012

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Anita Layton, Duke University, *Title to be announced*.

Lenhard Ng, Duke University, *Title to be announced*.

Henry K. Schenck, University of Illinois at Urbana-Champaign, *From approximation theory to algebraic geometry: The ubiquity of splines*.

Milen Yakimov, Louisiana State University, *Title to be announced*.

Akron, Ohio

University of Akron

October 20–21, 2012

Saturday – Sunday

Meeting #1084

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: August 2012
Program first available on AMS website: September 27, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 22, 2012
For consideration of contributed papers in Special Sessions: July 10, 2012
For abstracts: September 4, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Tanya Christiansen, University of Missouri, *Title to be announced*.

Tim Cochran, Rice University, *Title to be announced*.

Ronald Solomon, Ohio State University, *Title to be announced*.

Ben Weinkove, University of California San Diego, *Title to be announced*.

Special Sessions

Complex Analysis and its Broader Impacts (Code: SS 5A), **Mehmet Celik**, University of North Texas at Dallas, **Alexander Izzo**, Bowling Green State University, and **Sonmez Sahutoglu**, University of Toledo.

Complex Geometry and Partial Differential Equations (Code: SS 4A), **Gabor Szekelyhidi**, University of Notre Dame, **Valentino Tosatti**, Columbia University, and **Ben Weinkove**, University of California San Diego.

Extremal Graph Theory (Code: SS 2A), **Arthur Busch**, University of Dayton, and **Michael Ferrara**, University of Colorado Denver.

Groups, Representations, and Characters (Code: SS 1A), **Mark Lewis**, Kent State University, **Adriana Nenciu**, Otterbein University, and **Ronald Solomon**, Ohio State University.

Noncommutative Ring Theory (Code: SS 6A), **S. K. Jain**, Ohio University, and **Greg Marks** and **Ashish Srivastava**, St. Louis University.

Spectral, Scattering, and Inverse Scattering Theory (Code: SS 3A), **Tanya Christiansen**, University of Missouri, and **Peter Hislop** and **Peter Perry**, University of Kentucky.

Tucson, Arizona

University of Arizona, Tucson

October 27–28, 2012

Saturday – Sunday

Meeting #1085

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2012
Program first available on AMS website: October 4, 2012
Program issue of electronic *Notices*: October 2012
Issue of *Abstracts*: Volume 33, Issue 4

Deadlines

For organizers: March 27, 2012
For consideration of contributed papers in Special Sessions: July 17, 2012
For abstracts: September 11, 2012

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Michael Hutchings, University of California Berkeley, *Title to be announced*.

Kenneth McLaughlin, University of Arizona, Tucson, *Title to be announced*.

Ken Ono, Emory University, *Title to be announced* (Erdős Memorial Lecture).

Jacob Sterbenz, University of California San Diego, *Title to be announced*.

Goufang Wei, University of California, Santa Barbara, *Title to be announced*.

Special Sessions

Dispersion in Heterogeneous and/or Random Environments (Code: SS 2A), **Rabi Bhattacharya**, Oregon State University, Corvallis, and **Edward Waymire**, University of Arizona, Tucson.

Geometric Analysis and Riemannian Geometry (Code: SS 4A), **David Glickenstein**, University of Arizona, **Guofang Wei**, University of California Santa Barbara, and **Andrea Young**, Ripon College.

Geometrical Methods in Mechanical and Dynamical Systems (Code: SS 3A), **Akif Ibragimov**, Texas Tech University, **Vakhtang Putkaradze**, Colorado State University, and **Magdalena Toda**, Texas Tech University.

Harmonic Maass Forms and q-Series (Code: SS 1A), **Ken Ono**, Emory University, **Amanda Folsom**, Yale University, and **Zachary Kent**, Emory University.

Mathematical Physics: Spectral and Dynamical Properties of Quantum Systems (Code: SS 6A), **Bruno Nachtergaele**, University of California, Davis, **Robert Sims**, University of Arizona, and **Günter Stolz**, University of Alabama, Birmingham.

Representations of Groups and Algebras (Code: SS 5A), **Klaus Lux** and **Pham Huu Tiep**, University of Arizona.

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday

Meeting #1086

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Oxford, Mississippi

University of Mississippi

March 1–3, 2013

Friday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Chestnut Hill, Massachusetts

Boston College

April 6–7, 2013

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 6, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boulder, Colorado

University of Colorado Boulder

April 13–14, 2013

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 12, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Ames, Iowa

Iowa State University

April 27–28, 2013

Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: April 2013
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 27, 2012
For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Operator Algebras and Topological Dynamics (Code: SS 1A), **Leslie Hogben**, Iowa State University and American Institute of Mathematics, and **Bryan Shader**, University of Wyoming.

Zero Forcing, Maximum Nullity/Minimum Rank, and Colin de Verdiere Graph Parameters (Code: SS 2A), **Leslie Hogben**, Iowa State University and American Institute of Mathematics, and **Bryan Shader**, University of Wyoming.

Alba Iulia, Romania

June 27–30, 2013

Thursday – Sunday
First Joint International Meeting of the AMS and the Romanian Mathematical Society, in partnership with the "Simion Stoilow" Institute of Mathematics of the Romanian Academy.
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Louisville, Kentucky

University of Louisville

October 5–6, 2013

Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 5, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Philadelphia, Pennsylvania

Temple university

October 12–13, 2013

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 12, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

St. Louis, Missouri

Washington University

October 18–20, 2013

Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 20, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Riverside, California

University of California Riverside

November 2–3, 2013

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2013
For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday

Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lubbock, Texas

Texas Tech University

April 11–13, 2014

Friday – Sunday

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 18, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Tel Aviv, Israel

Bar-Ilan University, Ramat-Gan and Tel-Aviv University, Ramat-Aviv

June 16–19, 2014

Monday – Thursday

The 2nd Joint International Meeting between the AMS and the Israel Mathematical Union.

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday

Porto, Portugal

University of Porto

June 11–14, 2015

Thursday – Sunday

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 6–9, 2016

Wednesday – Saturday

Atlanta, Georgia

Hyatt Regency Atlanta and Marriott Atlanta Marquis

January 4–7, 2017

Wednesday – Saturday

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 10–13, 2018

Wednesday – Saturday

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Matthew Miller, Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 111 in the the January 2012 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

June 10–June 30, 2012: MRC Research Communities, Snowbird, Utah. (Please see http://www.ams.org/amsmtgs/mrc.html for more information.)

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