The book by Hu, *Integer programming and network flows*, was the first in this area and remains a useful reference. The areas covered well are network flows, cutting planes, and Gomory's group problem. In particular, Gomory's original, ground-breaking papers on the group problem are reproduced here. Of the other books, only Salkin gives an adequate survey of this work and some of its present directions.

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Rings with involution, by I. N. Herstein, Univ. Chicago Press, Chicago and London, 1976, x + 247 pp., $5.50.

For me, a "week-end" associative ring theorist, reading this book is like reading a letter from a not-too-distant relative who writes periodically to inform us (with a certain amount of pride and joy) of what his branch of the family has been doing. The recent work of Professor Herstein's immediate family (among them Baxter, Lanski, Martindale, and Montgomery) as well as the work of older family members (Jacobsen, Kaplansky, and Herstein himself) play a central role in this book. (This list of names is not meant to be a complete family tree.) The author states "I have tried to give in this book a rather intense sampler of the work that has been done recently in the area of rings endowed with an involution. There has been a lot of work done on such rings lately, in a variety of directions. I have not attempted to give the last minute results, but, instead I have attempted to present those whose statements and proofs typify." Such a "letter" must perforce have its main interest with those already familiar with the "family" and no effort is made to interest outsiders (aside from a careful and lucid presentation which highlights the intrinsic interest of the material). Applications and motivation from outside associative ring theory (from Jordan and quadratic Jordan algebras, and from operator and Banach algebras) are purposefully omitted in order to achieve the author's goal efficiently. Indeed, one really should be familiar with the letter of several years ago, *Topics in ring theory* [2] (= TRT in the remainder of the review) in order to read the present one. The general theme of the current letter is: Given a ring R with involution $*$: $R \to R$, define the subsets $S = \{x \in R | x = x^*\}$, $K = \{x \in R | x^* = -x\}$, $T = \{x + x^* | x \in R\}$, $K_0 = \{x - x^* | x \in R\}$ and then try to (1) determine what effect on R the imposition of certain hypotheses (e.g. regularity, periodicity, or the satisfaction of a polynomial identity) on the elements of S, K, T, or K_0 will have, and (2) characterize (or extend) mappings on R (or on S, K, T, or K_0) which preserve properties of, or operations on S, K, T, or K_0. (Beware "linear" reader! The sets S and T are not necessarily the same since $\frac{1}{2}$ may not be present. Similarly R is not necessarily the span of its selfadjoint elements. In fact, one of the lessons that a nonspecialist, such as myself, can learn from this book is how nice it is to have linearity instead of just additivity.) In the absence of further restrictions, these questions usually cannot be answered so that R is almost always assumed to be simple (no two-sided ideals), prime (the product of two nonzero two-sided ideals is nonzero), semiprime (no nonzero nilpotent two-sided ideals), primitive (possessing an irreducible module), or to contain a few idempotents.

The book is divided into six chapters and a brief description of, a typical result from, or a comment about each one will help to give the flavour and direction of the material. Chapter 1 is devoted to developing preliminary definitions and theorems. The material ranges from easy definitions such as that of a prime ring, on to characterizations of primitive rings with a minimal right ideal, and then to more difficult material on polynomial identities including Formanek's result that for $n \geqslant 1$ there exists a nonconstant central polynomial (in several noncommuting variables) on F_n, the ring of $n \times n$ matrices. (For $n = 2$, the polynomial is $(xy - yx)^2$.) The proofs of these latter results are representative of those in the book in that no ponderous machinery is necessary to achieve quite general and frequently very pretty results. Rather, the proofs rely on clever ideas and computation. Chapter 2 deals, in the main, with rings in which each element of S, K, T, or K_0, is invertible (or "regular or nilpotent", or some similar condition). Under this type of condition a semiprime ring turns out (roughly) to be a domain, a sum of a domain and its opposite, or a subring of F_2. An unsolved problem (the McCrimmon Conjecture) related to this circle of ideas is whether the nilpotency of every element of S will force R to be nil. This chapter introduces and relies upon results from TRT which analyze the nonassociative Lie structure of an associative semiprime (and later, simple) ring, R, induced by the bracket multiplication $[x, y] = xy - yx$, $x, y \in R$. Interest in the Lie structure arises (1) because so many naturally interesting subsets of a ring with involution R are Lie ideals (or Lie ideals and associative subrings which is even better), e.g., E, the additive subgroup of R generated by its idempotents, $\bar{S}$ and $\bar{T}$, the subrings generated by S and T, S^2, T^2, K^2, and K_0^2 are all Lie ideals, and (2) because of the potential to analyze the associative structure of R via the analysis of the induced Lie structure. For example, a basic theorem in this vein, whose statement contains no mention of Lie structure, is that if R is a simple noncommutative ring with characteristic $\neq 2$ and contains a nontrivial idempotent then $\bar{E} = R$. This theorem rests on the fact that if R is simple and U a Lie ideal of R, then either $U \subseteq Z$, the centre of R, or $[R, R] \subseteq U$, except if R is of characteristic 2 and $\dim_Z R = 4$. (Here $[R, R]$ is the additive subgroup generated by all $[x, y]$ for $x, y \in R$.) It follows from this fact that if R is simple and contains a nontrivial idempotent then $[R, R] \subseteq E$. If, in addition, characteristic $R \neq 2$, $\bar{E} = R$, since it can be shown in this case that $\overline{[R, R]} = R$. Chapter 3 is concerned with proving theorems which have a periodicity hypothesis on the elements of S, K, T, or K_0 and which parallel the theorem of Jacobsen which states that if $x^{n(x)} = x$, $n(x) > 1$ for all $x \in R$, then R is commutative. An easy example shows that such a condition on S (even for R simple) will not force the elements of S to commute. However, when R is a division ring the periodicity condition on either S or K implies R is commutative, and for any ring R with involution the periodicity condition on K implies that members of K commute. In Chapter 4 mappings between rings with involution which preserve certain *-operations or certain algebraic operations on S, K, T, or K_0 are studied. A good portion of the chapter is allotted to proving a theorem of Martindale that under the rather general conditions a Jordan homomorphism $\phi: S \to R$ can be extended to a unique associative homomorphism. A related result not mentioned in this book but which shows the power of the methods used is that of J. M. Cusack [1] which states that if R is any ring (with or without *) such that $2x = 0$ implies $x = 0$ and which is semiprime or has a commutator which is not a zero divisor, then any Jordan derivation of R is an associative derivation. Chapter 5 is devoted to proving Amitsur's theorem that algebras which satisfy an identity of the form $p(x_1, \ldots, x_n; x_1^*, \ldots, x_n^*)$, p a polynomial, must satisfy a standard identity. Chapter 6 is titled "Potpourri", but most of the results here state that if a certain subset $A \subseteq R$ is invariant with respect to some "natural" operation then A is contained in the centre of R or contains a two-sided ideal of R.

The author certainly achieves his purpose with this book, but some exposition of the origins of the problems discussed would be most welcome by a nonspecialist reader. Though some of the hypotheses of theorems appear strange to an operator algebraist, even one with strong ring theoretic prejudices such as myself, there is a distillation from other, more familiar (to me), mathematical specialities of "essence of ring theory" which makes the book particularly appealing. One attraction of this type of ring theory is that it offers the clear view of an algebraic skeleton of certain problems without the flesh of topology or even linearity. The tantalizing thought is that the solution to such problems may rest upon the "bare bones" of algebra. ("The rest of the flesh is transient, strung like laundry upon a lattice. To dwell upon bone is to contemplate the fate of man. Bone is the keepsake of the earth, all that remains of a man when the rest has long since melted and seeped and crumbled away. It endures for a million years and, if then dug up from the ground, suggests still to anthropologists the humps of meat that once it wore, and to poets the much that was from the little that remains" [3].) Poetic considerations aside, an example of the type of problem I have in mind will illustrate the principle. A factor von Neumann algebra of type II_1 is an algebra of operators on Hilbert space, is algebraically simple so that by the above discussion it is generated as a ring by its projections, and it is the weak operator closure of the linear span of its projections (projections being selfadjoint idempotents). This type of algebra has been the object of intense study and yet it is unknown whether it is the (unclosed) linear span of its projections. Could it be that this result is a particular instance of a more general (and as yet unproved) theorem to the effect that any simple algebra containing a nontrivial idempotent is the linear span of its idempotents? This theorem is posed as a question in Professor Herstein's book, but without a hint of its possible difficulty.

This book will no doubt be especially welcomed by thesis advisors with students in the field and by anyone who enjoys this sort of ring theory. The material is accessible, the presentation lucid, and the results elegant and general. The nonspecialist will, however, occasionally be overwhelmed by the number of variations on a theorem theme. In short, the family seems to be doing quite well, and I look forward to hearing about them again.

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Courant in Göttingen and New York–The story of an improbable mathematician, by Constance Reid, Springer-Verlag, New York, Heidelberg, Berlin, 1976, 314 pp. + 16 pp. photographs, $12.80.

Richard Courant's life was split into two very different parts on August 21, 1934, the day when he arrived with his family to take up residence in the United States. Ms. Reid tells the story of his life in this absorbing book, in some respects a sequel to her well-known biography of Hilbert. Urged by some of Courant's associates and admirers, she agreed to assist him in preparing his reminiscences at a time when he was already in his eighties. However, as she reports on her third page, "it very soon became clear that I had come too late for the project which Friedrichs had had in mind. Courant had neither the vigor not the desire to go back over his life meaningfully . . . much as he admired what Klein had done [i.e. in compiling his collected works (Reviewer)], Courant could not bring himself to do something similar. He took comparatively little satisfaction in his past achievements. He was concerned about the future of mathematics and of the institute he had created, and he was frustrated and unhappy because he could no longer help. Not only did he lack the physical and mental energy, but mathematics had passed him by." Nevertheless, from her numerous conversations with Courant and his associates, from documents available to her in Courant's files, and from her extensive work on Hilbert's life and times, she found that she had enough material for a book about Courant. As it now appears in print she calls it a "life-story" rather than a "biography". The distinction is a valid one. In writing a biography she would have needed to go farther afield for her material, consulting additional sources less intimately involved with Courant and his circle. The second part of Courant's life can hardly be put in final perspective without such a broader background. The first half of his career, as student, professor, and administrator at Göttingen, where he was guided by his loyalty and admiration for Klein and–above all–Hilbert, does not seem to require quite the same amount of biographical probing. It is the details of his subsequent attempt to reestablish in America the lost paradise of Göttingen with its high traditions and congenial intellectual atmosphere that merit closer and more extensive examination if the latter half of his life is to be properly understood.

Ms. Reid is to be thanked for the very useful service she has performed in putting down a coherent account of the information she has gathered about Richard Courant. At the same time she is sure to entertain many a curious