8 · Geodesy

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INTRODUCTION

Essential to an understanding of the Arabs' contribution to mapmaking is their approach to geodesy—the measurement of distances on the curved surface of the earth.[1] Such distances can be measured either in linear units, such as the Arabic mile, or in angular units—longitude and latitude. To convert from one to the other one must know the number of miles per degree or, equivalently, the radius of the earth.

In the Greek classical period, before the general use of latitude as an angular coordinate, the inhabited area (the *oikoumene*) was divided into zones, or climates, according to the length of the longest day in the central part of the zone.[2] Thus Ptolemy, in his *Almagest*, takes seven boundaries in steps of one-half hour, running from thirteen to sixteen hours. The practice continues in Islamic mathematical geography.[3] For example, al-Bīrūnī (362/973 to after 442/1050), in his *al-Qānūn al-Masʿūdī* (The Masʿūdic canon), draws up a table showing the sizes of the successive climates in Arabic miles and *farsakhs*. He defines the boundaries again in terms of regular half-hour increments in the day length, but starting at 12¾ hours, the southern boundaries of his seven climates ranging from 12;39 to 47;11 degrees.[4] The actual measurement of latitude was a relatively straightforward matter and could be carried out by observing the altitudes of the stars or the sun. Such methods were well established in the classical world, and the tradition continued among Arab scientists.

Determining longitude raised far greater difficulties than determining latitude. Here as elsewhere, Arabic methods continued those of classical antiquity. In the classical world longer distances were taken from travelers' reports, while shorter distances were fixed by some device such as a waywiser.[5]

The first Arab astronomers and geographers began to estimate longitude from a knowledge of earlier work—Syriac, Greek, and Indian. Pre-Islamic astronomical tables were drawn up for a particular reference meridian, such as Alexandria or Ujjain.[6] In the earliest phase of Arabic astronomy, handbooks of tables were brought to Baghdad, and it was essential to recalculate them. In the subsequent development of astronomy in the Islamic world, many centers came to be used as meridians of reference.[7] This necessitated knowing the correct difference in longitude and so provided a strong motive for determining accurate geographical distances and coordinates.

1. The spherical nature of the earth had been recognized long before, in the classical world; and wherever Greek science went, in particular in the Islamic culture area, the concept of a spherical earth went also.

2. Ernst Honigmann, in his very important discussion of the history of Greek and Arabic geographical lists, sets the subject very much in the traditional context of the division of the earth into climates; see his *Die sieben Klimata und die πόλεις ἐπίσημοι* (Heidelberg: Winter, 1929).

3. The precise calculation of the corresponding latitudes depends on the obliquity of the ecliptic. This parameter was frequently revised in Arabic astronomy and was always less than Ptolemy's value of 23;51.

4. Abū al-Rayḥan Muḥammad ibn Aḥmad al-Bīrūnī, *Kitāb al-qānūn al-Masʿūdī fī al-hayʾah wa-al-nujūm*, bk. 5, chap. 9; see *al-Qānūnuʾl-Masʿūdī (Canon Masudicus)*, 3 vols. (Hyderabad: Osmania Oriental Publications Bureau, 1954–56), 2:542–45, and Ahmad Dallal, "Al-Bīrūnī on Climates," *Archives Internationales d'Histoire des Sciences* 34 (1984): 3–18. Here and elsewhere, where magnitudes are expressed sexagesimally, the division between the integer and the fraction is marked by a semicolon. This mode, rather than the decimal fraction, was a commonplace in Arabic texts and is a direct inheritance of Greek and Babylonian astronomy. When it is a question of an angle, the first and second numbers after the semicolon are the usual minutes and seconds.

5. Heron of Alexandria describes a type of waywiser, a device in which a gear train is driven by rolling a wheel over the ground, the accumulated distance being indicated by a slowly rotating pointer (*Opera quae supersunt omnia*, 5 vols. [Leipzig: Teubner, 1899–1914], vol. 3, *Rationes dimetiendi et commentatio dioptrica*, ed. Hermann Schöne, 313). A simpler form is described by Vitruvius and seems to have been used by the Romans when placing milestones; see Donald R. Hill, *A History of Engineering in Classical and Medieval Times* (London: Croom Helm, 1984), 122–23.

6. The prime meridian of Indian astronomy passed through Ujjain (longitude 75;46) in Madhya Pradesh. It came to be referred to as Arīn in Arabic texts. The *zīj*, or astronomical handbook, of al-Khwārazmī was based on this meridian, in line with its close dependence on the *Brāhmasphuṭasiddhānta*; see Raymond P. Mercier, "Astronomical Tables in the Twelfth Century," in *Adelard of Bath: An English Scientist and Arabist of the Early Twelfth Century*, ed. Charles Burnett (London: Warburg Institute, 1987), 87–118. On the demonstration that the Indian observations were actually referred to this meridian, see Raymond P. Mercier, "The Meridians of Reference of Indian Astronomical Canons," in *History of Oriental Astronomy*, Proceedings of an International Astronomical Union, Colloquium, no. 91, New Delhi, India, 13–16 November 1985, ed. G. Swarup, A. K. Bag, and K. S. Shukla (Cambridge: Cambridge University Press, 1987), 97–107.

7. Among these, Baghdad, Damascus, Raqqa, Cairo, Samarkand, Córdoba, Ghazna.

Lists of the geographical coordinates of places are commonly found in Arabic astronomical handbooks. These lists of coordinates are discussed thoroughly above (chap. 4), and recently data from as many as seventy-four such lists were collected and published.[8] Adjustments of latitude and longitude were introduced from time to time, but as a rule we are not told exactly how this was accomplished. The geodetic researches of al-Bīrūnī are a notable exception, as we will see below.

Triangulation, as it is known to the modern surveyor, appears to have played no part in determining longitudes. In classical antiquity some form of the properties of similar right triangles must have been used when it was desired, for example, to drive a tunnel through a hill, and for this purpose an instrument such as the dioptra of Heron of Alexandria would have been used to determine angles in the horizontal plane, as Heron himself explains.[9] There are no examples, however, where angles measured in that way were incorporated into a piecemeal accumulation of triangles resulting eventually in the determination of the distance between points well out of sight of each other. When we examine in detail al-Bīrūnī's determination of the longitude of Ghazna (modern Ghazni in eastern Afghanistan), it will be clear that he applies a trigonometric analysis to a succession of spherical triangles, but in each case the triangles initially known are found from latitude determinations and from distances provided by travelers.

Simultaneous observations of a lunar eclipse in two places provide in principle a means of determining the difference of longitude between places. If the two observers note the eclipse according to their local mean times, then the time difference between them is established, and hence the longitude difference is known. But such an approach proves to have no more than theoretical interest. It would have been necessary to use records of past eclipses, and in that case to reconstruct the features that were observed—for example, the time of onset or the moment of maximum obscuration. There were also difficulties in fixing the local time with sufficient precision, or indeed in agreeing on exactly what was meant by local mean time.[10] In determining longitude differences, the precision attained from the study of travelers' distances far exceeded what would have been available from the study of lunar eclipses. Al-Bīrūnī goes into the method and its problems in some detail,[11] but he makes no practical use of it, any more than others before him.

The conversion between linear and angular distances may be expressed either as a ratio—the number of units per degree of circumference—or as a proportion of the radius of the earth. The values of both these important ratios were known from various pre-Islamic sources, but the principal difficulty in employing this information arose from ignorance of the earlier units of measurement. Islamic astronomers knew, for example, of the ratio seventy-five miles per degree of latitude, which in fact is very accurate when the mile is the Roman mile of 1,480 meters. They also knew from Ptolemy's *Geography* of his measure of 500 stades per degree. They were evidently confused about such earlier results, principally because they lacked information about units such as the stade or the Roman mile. This confusion was clearly the principal reason early Abbasid astronomers, at the time of the caliph al-Ma'mūn (r. 198–218/813–33), undertook to repeat the basic measurements, measuring off the distance in terms of units familiar to them, as I shall explain below. This illustrates the interaction between translation and scientific observation that is strongly characteristic of early Islamic scholarship. Each activity assisted the other. It is important to understand that they were not simply carrying out an a priori measurement; the measurement was intended to clarify the received tradition.

Whatever geodetic investigations were carried out at the time of al-Ma'mūn were not universally used, or even understood, by later scientists and historians. Attempts to make sense of those investigations have not resulted, for either medieval or modern scholars, in a clear and convincing history. As we shall see, even the most authoritative accounts are schematic and lack convincing circumstantial details, and they contradict one another. They show confusion between traditions of pre-Islamic measurements of the length of a degree and whatever was determined at the time of al-Ma'mūn. Many Arab scholars continued to use, for example, Ptolemy's 66⅔ miles per degree,[12] plainly unconvinced by the earlier Arab effort.

We are fortunate to have one example of the use of the trigonometric conversion of travelers' distances to true coordinates, for this is well illustrated by the work

8. Edward S. Kennedy and Mary Helen Kennedy, *Geographical Coordinates of Localities from Islamic Sources* (Frankfurt: Institut für Geschichte der Arabisch-Islamischen Wissenschaften, 1987).

9. Heron; see Schöne's edition, *Rationes dimentiendi et commentatio dioptrica*, 215 (note 5). The dioptra is similar in concept to the modern theodolite.

10. Although the local time in nonuniform hours is easier to fix, one must convert this to mean time for purposes of the longitude difference. There was not then, as there is now, an agreed definition of local mean time; Raymond P. Mercier, "Meridians of Reference in Pre-Copernican Tables," *Vistas in Astronomy* 28 (1985): 23–27.

11. Al-Bīrūnī, *al-Qānūn al-Masʿūdī*, bk. 5, chap. 1; see the 1954–56 edition, 2:507 (note 4).

12. Abū al-Fidā' Ismāʿīl ibn ʿAlī, *Taqwīm al-buldān*; see *Géographie d'Aboulféda: Texte arabe*, ed. and trans. Joseph Toussaint Reinaud and William MacGuckin de Slane (Paris: Imprimerie Royale, 1840), and *Géographie d'Aboulféda: Traduite de l'arabe en français*, 2 vols. in 3 pts. (vol. 1, *Introduction générale à la géographie des Orientaux*, by Joseph Toussaint Reinaud; vol. 2, pt. 1, trans. Reinaud; vol. 2, pt. 2, trans. S. Stanislas Guyard) (Paris: Imprimerie Nationale, 1848–83), 1:CCLXVIIIff. and vol. 2, pt. 1, 17–18.

of al-Bīrūnī. We have a number of his works bearing on geodesy, and in the sections below summary accounts are given of his attempts to determine the earth's radius as well as the difference between the longitude of Ghazna and that of Baghdad. Ghazna served to define the meridian of reference of al-Bīrūnī's astronomical tables, *al-Qānūn al-Masʿūdī*. The strengths and weaknesses of his work are apparent in these efforts.

Arabic Metrology

In early Islamic work, the units in use were the *farsakh* (Persian *farsāng*), mile (Arabic *mīl*, following Syriac *mīl*), cubit (*dhirāʿ*), and digit (*iṣbaʿ*). The *farsakh* equaled three Arabic miles, while the mile was 4,000 cubits. As far as geodetic work was concerned, it has been demonstrated by Mahmoud Bey that the cubit, of 24 digits, was equal to the ancient Babylonian cubit of 49.3 centimeters, making the mile 1,972 meters, and the *farsakh* 5,916 meters.[13]

The cubit used for geodetic measurements, and other scientific work reported in Arabic, was the "black" (*sawdāʾ*) cubit, which we are told was adopted by al-Ma'mūn. At that time another cubit was known to Arabic scientists, the traditional Egyptian cubit.[14] This was used to calibrate the nilometer at Rawḍah (Roda), an island very near Cairo. Indeed, in the ninth century A.D. the caliph al-Mutawakkil (r. 232–47/847–61) ordered a renovation of that nilometer, and among those involved were the well-known scientists of the time al-Khwārazmī and al-Farghānī.[15]

In his investigation of the cubit in early Arabic geodesy, Mahmoud Bey initially considered the nilometer cubit, then turned to other cubit measures he found in use in Egypt, such as the canonical (*shāfʿī*) cubit. After some remarkable and ingenious efforts to determine their metric equivalents, he found that they had an average length of 49.3 centimeters. That, according to Arabic writers on geodesy, direct measurements of the length of one degree gave 56⅔ miles convinced Mahmoud Bey that the mile of 4,000 cubits must have been based on this cubit of 49.3 centimeters, which he had established by other means, and definitely not on the Egyptian cubit. Based on this value, the length of the degree is then 111,747 meters, a close approximation to the correct value of 110,959 meters for the latitude of Baghdad.

The cubit of this length had been used in Mesopotamia for a very long time. It is attested, for example, on each of the two statues of the Sumerian ruler Gudea, in which a remarkable measuring scale forms part of the plan of a building.[16] The mile of 4,000 cubits appears to be in use in the pre-Islamic period,[17] and indeed in a cuneiform text of Nebuchadnezzar II (604–561 B.C.) it is stated that "for 4000 cubits . . . to the westward of Babylon I constructed an enclosing wall,"[18] so we may believe that the Arabic mile is an ancient Mesopotamian unit, like the Arabic cubit itself.

In some classical texts the Roman mile (ca. 1,480 meters) is given as 3,000 cubits, so that evidently the same cubit was implied: 1,480/3,000 = 0.493 meter. Arabic writers make references to the Roman mile, but it is not clear how well they understood its relation to their own. Certainly some assumed that the ancient and Arabic miles were the same. For example, in the thirteenth century A.D., Abū al-Fidāʾ states that the ancient cubit consisted of 32 digits,[19] in contrast to the Arabic cubit of 24 digits, and so infers that the ancient mile, which he correctly

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13. Mahmoud Bey has given the principal arguments underlying these evaluations; see his "Le système métrique actuel d'Egypte: Les nilomètres anciens et modernes et les antiques coudées d'Egypte," *Journal Asiatique*, ser. 7, vol. 1 (1873): 67–110. His work was resumed by Carlo Alfonso Nallino, who gave supplementary arguments leading to the same results in "Il valore metrico del grado di meridiano secondo i geografi arabi," *Cosmos* 11 (1892–93): 20–27, 50–63, 105–21; republished in *Raccolta di scritti editi e inediti*, 6 vols., ed. Maria Nallino (Rome: Istituto per l'Oriente, 1939–48), 5:408–57. Both Mahmoud Bey and Nallino were very much concerned with the geodetic context. Henry Sauvaire collected a mass of data from Arabic sources, related to matters other than geodesy, which he presented without much effort at critical evaluation in his "Matériaux pour servir à l'histoire de la numismatique et de la métrologie Musulmanes, quatrième et dernière partie: Mesures de longueur et de superficie," *Journal Asiatique*, 8th ser., 8 (1886): 479–536. Many of Sauvaire's reports gave the ratio of one unit to another, or the difference between two similar units, as so many digits. The survey by Walther Hinz, *Islamische Masse und Gewichte: Umgerechnet ins metrische System*, Handbuch der Orientalistik, ed. B. Spuler, suppl. vol. 1, no. 1 (Leiden: E. J. Brill, 1955), depended on such relations quoted from Sauvaire, and the units were given an absolute value by the assumption that the "black" cubit was equal to that of the nilometer at Rawḍah, which is incorrect.

14. The Egyptian units are relatively well documented and are summarized by Wolfgang Helck, "Masse und Gewichte," in *Lexikon der Ägyptologie*, ed. Wolfgang Helck and Eberhard Otto (Wiesbaden: Otto Harassowitz, 1975–), 3:1199–1209. The cubit, known to be 52.5 centimeters in the New Kingdom, appears to have been somewhat longer in the Ptolemaic period, when the cubit on the Ptolemaic nilometer was 53 centimeters.

15. K. A. C. Creswell, *Early Muslim Architecture: Umayyads, Early ʿAbbāsids and Ṭūlūnids*, 1st ed., 2 pts., (Oxford: Clarendon Press, 1932–40), pt. 2, 296–302.

16. François Thureau-Dangin, "L'u, le qa et la mine: Leur mesure et leur rapport," *Journal Asiatique*, 10th ser., 13 (1909): 79–110; for an illustration of one of the statues, see A. R. Millard, "Cartography in the Ancient Near East," in *The History of Cartography*, ed. J. B. Harley and David Woodward (Chicago: University of Chicago Press, 1987–), 1:107–16, esp. figs. 6.2 and 6.3.

17. Theodor Mommsen, "Syrisches Provinzialmass und römischer Reichskataster," *Hermes* 3 (1869): 429–38, drew attention to a Syriac text of A.D. 501 in which routes were measured in terms of a mile of 4,000 cubits.

18. Stephen Herbert Langdon, *Building Inscriptions of the Neo-Babylonian Empire: Part 1, Nabopolassar and Nebuchadnezzar* (Paris: Ernest Leroux, 1905), 65, 133, and 167.

19. Abū al-Fidāʾ, *Taqwīm al-buldān*; see *Géographie d'Aboulféda*, Arabic text, 15; translation, vol. 2, pt. 1, 18 (note 12).

says has 3,000 cubits, is of the same length as the Arabic mile of 4,000 24-digit cubits, and this leads him to a wrong interpretation of the Ptolemaic length of the degree, 66⅔ miles.

Measurements of the Length of a Degree

In converting from linear distances on the earth's surface to angular measurement, the equivalence of one degree to 56⅔ miles was used by many Arab scientific writers. Other values, however, were commonly found, namely, 66⅔ miles and 75 miles per degree. The ratio 66⅔ miles presumably arises from Ptolemy's assumption of 500 stades per degree, assuming 7½ stades to the mile.

The ratio 75 miles per degree is attributed to al-Khwārazmī by Ibn al-Faqīh (fl. 290/903), and he is followed by Yāqūt (575–626/1179–1229) in his geographical dictionary[20] and quoted by many other Arabic writers.[21] This is indeed a most accurate value if the mile is taken as Roman (1,480 m), for then 75 miles = 111,000 meters, while the true value at a latitude of 36° is 110,959 meters. It presumably reflects estimates and measurements made in the late Roman Empire. Certainly the figure of 75 miles does not originate with al-Khwārazmī but was presumably taken from Syriac sources, as indeed was much else in al-Khwārazmī's geography.[22] When converted to Arabic miles, the ratio of 75 is replaced by 56¼.

One of the earliest texts reporting the ratio 56⅔ is that of al-Farghānī: "We find in this way that to one celestial degree corresponds on the earth's surface 56⅔ miles, of which each contains 4,000 cubits, called black [*al-sawdā*ʾ]. So it was determined in the time of al-Maʾmūn of glorious memory, by a number of scholars brought together for this measurement."[23]

The ratio 56⅔ miles was apparently based on direct geodetic surveys first carried out early in the ninth century by teams appointed by al-Maʾmūn, although none of the accounts of that activity gave precisely that figure. There are various extant reports of this activity, and we may quote directly from Ḥabash al-Ḥāsib (fl. 240/850), from al-Bīrūnī, who quotes him, and from Ibn Yūnus (d. 399/1009), who owes his account to those of both Sind ibn ʿAlī and Ḥabash al-Ḥāsib.

In his *Kitāb taḥdīd nihāyāt al-amākin li-taṣḥīḥ masāfāt al-masākin* (The determination of the coordinates of positions for the correction of distances between cities), al-Bīrūnī quotes at length from the account of Ḥabash al-Ḥāsib, according to which al-Maʾmūn directed certain astronomers to a place in the desert of Sinjār nineteen *farsakhs* from Mosul and forty-three from Samarra, from which point two parties set out to the north and south, respectively, each determining that fifty-six miles were equivalent to one degree.[24]

A portion of Ḥabash al-Ḥāsib's work *Kitāb al-ajrām wa-al-abʿād* (Book of bodies and distances) is extant, and a translation has recently been published.[25] It confirms al-Bīrūnī's quotation in all its essentials. The passage is as follows:

> The Commander of the Faithful al-Maʾmūn desired to know the size of the earth. He inquired into this and found that Ptolemy mentioned in one of his books that the circumference of the earth is so and so many thousands of stades. He asked the commentators about the meaning of "stade," and they differed about

20. Shihāb al-Dīn Abū ʿAbdallāh Yāqūt ibn ʿAbdallāh al-Ḥamawī al-Rūmī al-Baghdādī, *Muʿjam al-buldān*; see *Jacut's geographisches Wörterbuch*, 6 vols., ed. Ferdinand Wüstenfeld (Leipzig: F. A. Brockhaus, 1866–73), 1:16. As Jwaideh remarks (*The Introductory Chapters of Yāqūt's "Muʿjam al-buldān,"* ed. and trans. Wadie Jwaideh [Leiden: E. J. Brill, 1959; reprinted, 1987], 24 n. 2), Yāqūt derives this from Aḥmad ibn Muḥammad ibn al-Faqīh al-Hamadhānī: *Kitāb al-buldān*; see *Compendium libri kitâb al-boldân*, ed. Michael Jan de Goeje, Bibliotheca Geographicorum Arabicorum, vol. 5 (Leiden: E. J. Brill, 1885; reprinted 1967), 5.

21. Hans von Mžik, "Ptolemaeus und die Karten der arabischen Geographen," *Mitteilungen der Kaiserlich-Königlichen Geographischen Gesellschaft in Wien* 58 (1915): 152–76, esp. 171–72; further quotations by Nallino in "Il valore metrico," 50–53 (note 13).

22. Hans von Mžik has convincingly argued for this Syriac dependence in "Afrika nach der arabischen Bearbeitung der Γεωγραφικὴ ὑφήγησις des Claudius Ptolemaeus von Muḥammad ibn Mūsā al-Ḫwārizmī," *Denkschriften der Kaiserlichen Akademie der Wissenschaften in Wien: Philosophisch-Historische Klasse* 59 (1917), Abhandlung 4, i–xii, 1–67, although the complications have been further discussed by Hubert Daunicht, *Der Osten nach der Erdkarte al-Ḫuwārizmīs: Beiträge zur historischen Geographie und Geschichte Asiens*, 4 vols. in 5 (Bonn: Selbstverlag der Orientalischen Seminars der Universität, 1968–70), 1:203–14. Indeed, Jacob of Edessa (d. A.D. 708) adopts the same figure for the geographical discussions in his *Hexameron* (see *Etudes sur l'Hexameron de Jacques d'Edesse*, trans. Arthur Hjelt [Helsinki, 1892], 20).

23. Abū al-ʿAbbās Aḥmad ibn Muḥammad al-Farghānī, *Elementa astronomica, arabicè et latinè*, ed. and trans. Jacob Golius (Amsterdam, 1669), 30 (Arabic and Latin). The "black" cubit is again referred to by al-Bīrūnī in the *Kitāb al-tafhīm*, "Each mile is a third of a *farsakh*, or 4,000 cubits, called black in Iraq, each of which equals 24 digits" (author's translation); see also Robert Ramsey Wright, ed. and trans., *The Book of Instruction in the Elements of the Art of Astrology* (London: Luzac, 1934), 208.

24. Al-Bīrūnī, *Taḥdīd*; see *The Determination of the Coordinates of Positions for the Correction of Distances between Cities*, trans. Jamil Ali (Beirut: American University of Beirut, 1967), 178–80. The distances from Mosul and Samarra fix the town of Sinjār itself on the northern edge of the desert.

Al-Bīrūnī's *Taḥdīd* is a work in twenty-five chapters giving a specialized treatment of fundamental geodetic questions such as the determination of distances on the earth and derivation therefrom of geographical coordinates. Some of the topics are treated again, more summarily, in his later *al-Qānūn al-Masʿūdī* (mentioned above). This is a far larger treatise covering every sort of astronomical theme, including geodetic topics, which are given in bk. 5, providing a scientific foundation for the long table of geographical coordinates that follows.

25. Y. Tzvi Langermann, "The Book of Bodies and Distances of Ḥabash al Ḥāsib," *Centaurus* 28 (1985): 108–28.

the meaning of this. Since he was not told what he wanted, he directed Khālid ibn ʿAbd al-Malik al-Marwarrūdhī, ʿAlī bin ʿĪsā al-Asṭurlābī [from the cognomen evidently an instrument maker], and Aḥmad ibn al-Bukhturī al-Dhāriʿ [al-Dhāriʿ means surveyor] with a group of surveyors and some of the skilled artisans including carpenters and brassmakers, in order to maintain the instruments which they needed. He transported them to a place which he chose in the desert of Sinjār. Khālid and his party headed for the north pole of Banāt Naʿsh [Ursa Minor], and ʿAlī and Aḥmad and their party headed to the south pole. They proceeded until they found that the maximum altitude of the Sun at noon had increased, and differed from the noon altitude which they had found at the place from which they had separated, by the amount of one degree, after subtracting from it the sun's declination along the path of the outward journey, and there put arrows. Then they returned to the arrows, testing the measurement a second time, and so found that one degree of the earth was 56 miles, of which one mile is 4,000 black cubits. This is the cubit adopted by al-Ma'mūn for the measurement of cloths, surveying of fields, and the distribution of way-stations.

Ḥabash al-Ḥāsib concluded by saying he had heard this account directly from Khālid.

Another much less detailed account is given by al-Bīrūnī in his *al-Qānūn al-Masʿūdī*:

> Al-Ma'mūn son of al-Rashīd, wished to verify it [the amount given by the Greeks], and for this appointed a commission of scholars, who set out to determine the amount in the Plain of Sinjār, who found the degree to be 56⅔ miles. Multiplying this by 360 gives 20,400 miles, the length of the circumference.[26]

Al-Bīrūnī expressed his concern about the discrepancy between the figures 56 and 56⅔, wondering if it arose from the two early attempts in Sinjār or from some other reason.

In his *al-Zīj al-kabīr al-Ḥākimī* (Hakimite tables), chapter 2, Ibn Yūnus writes:

> Sind ibn ʿAlī reports that al-Ma'mūn ordered that he and Khālid ibn ʿAbd al-Malik al-Marwarrūdhī should measure a degree of a great circle of the earth's surface. We left together, he says, for this purpose. He gave the same order to ʿAlī ibn ʿĪsā al-Asṭurlābī and ʿAlī [sic] ibn al-Bukhturī who took themselves to another direction [or region]. Sind ibn ʿAlī said, I and Khālid ibn ʿAbd al-Malik traveled to the area between Wāsa [or Wāmia] and Tadmor, and there we determined a degree of the great circle of the terrestrial equator, which was 57 miles. ʿAlī ibn ʿĪsā and ʿAlī [sic] ibn al-Bukhturī found the same and these two reports containing the same measure arrived from the two regions [or directions] at the same time.
>
> Aḥmad ibn ʿAbdallāh, named Ḥabash, reported in his treatise on observations made at Damascus by the authors of the *Mumtaḥan* [Verified tables], that al-Ma'mūn ordered the measurement of a degree of the terrestrial great circle. He said that for this purpose they traveled in the desert of Sinjār, until the noon altitudes between the two measurements in one day changed by one degree. They then measured the distance between the two places, which was 56¼ miles, of each mile was 4,000 cubits, the black cubits adopted by al-Ma'mūn.[27]

From these two quotations then, we appear to have from Aḥmad ibn ʿAbdallāh, called Ḥabash:

1. A survey along a southward path in Sinjār by ʿAlī ibn ʿĪsā al-Asṭurlābī and Aḥmad ibn al-Bukhturī al-Dhāriʿ: number of miles not stated.
2. A survey along a northward path in Sinjār by Khālid ibn ʿAbd al-Malik al-Marwarrūdhī: 56, or 56¼ miles (Ibn Yūnus).

And from Sind ibn ʿAlī:

3. Sind ibn ʿAlī and Khālid ibn ʿAbd al-Malik al-Marwarrūdhī, in the region of Wāsa/Wāmia and Tadmor [ancient Palmyra]: 57 miles.
4. ʿAlī ibn ʿĪsā al-Asṭurlābī and Aḥmad ibn al-Bukhturī al-Dhāriʿ in another direction/region: 57 miles.

These two accounts are inconsistent because Khālid is said in one to have gone to Sinjār and in the other, to Tadmor and Wāsa/Wāmia (see fig. 8.1). It is also odd that Sind ibn ʿAlī is not mentioned by Ḥabash.

This is not the only difficulty. The terrain between Palmyra and Raqqa is unsuitable for this type of survey, and Wāsa/Wāmia is certainly not recognized as an Arabic place-name.[28] Moreover, al-Bīrūnī in the *Taḥdīd*

26. Al-Bīrūnī, *al-Qānūn al-Masʿūdī*, bk. 5, chap. 7; see the 1954–56 edition, 2:529 (note 4).

27. This passage is found in the manuscript in Leiden, MS. Or. 143, pp. 81–82, and in the manuscript in Paris, Bibliothèque Nationale, MS. Arabe 2495, fols. 44r–v; only the former has historical value, the latter being merely copied from it. Tranlations of this passage were given by J. J. A. Caussin de Perceval, *Le livre de la grande table Hakémite* (Paris: Imprimerie de la Républic, 1804), 94–95; and by Nallino, "Il valore metrico," 54–55 (note 13). The Leiden manuscript appears to have "Wāsa," which the copyist of the Paris manuscript read as "Wāmia."

28. Ptolemy in his *Geography* (*Claudii Ptolemaei Geographia*, 2 vols. and tabulae, ed. Karl Müller [Paris: Firmin-Didot, 1883–1901], 15.14.13) lists a place named "θεμα," with the coordinates longitude 71;30, latitude 35;30, which is on the same meridian as Palmyra and 1;30 due north of it. It may have been misread as "οεμα," hence "Wamia."

Alternatively, it has been conjectured that "Wāmia" is a corruption of "Fāmia," that is, Greek "Apamea," which was the name of a number of towns, including not only the one near Ḥims, to the west of Palmyra, but also that due north of Palmyra, near Zeugma. The former is wrongly placed to be the correct reference, however, and it seems, moreover, that the latter Apamea was known as Birejik (Bīreğik); Kurt Regling, "Zur historischen Geographie des mesopotamischen Parallelogramms," *Klio* 1 (1901): 443–76, esp. 446.

The extensive studies of this region by travelers and scholars have revealed nothing to clarify this difficulty. One should also note that a Roman road, the Via Diocletiana, ran north-east from Palmyra and then due north to meet the Euphrates at a point just west of Raqqa. This road is represented by a track, which has been explored in modern

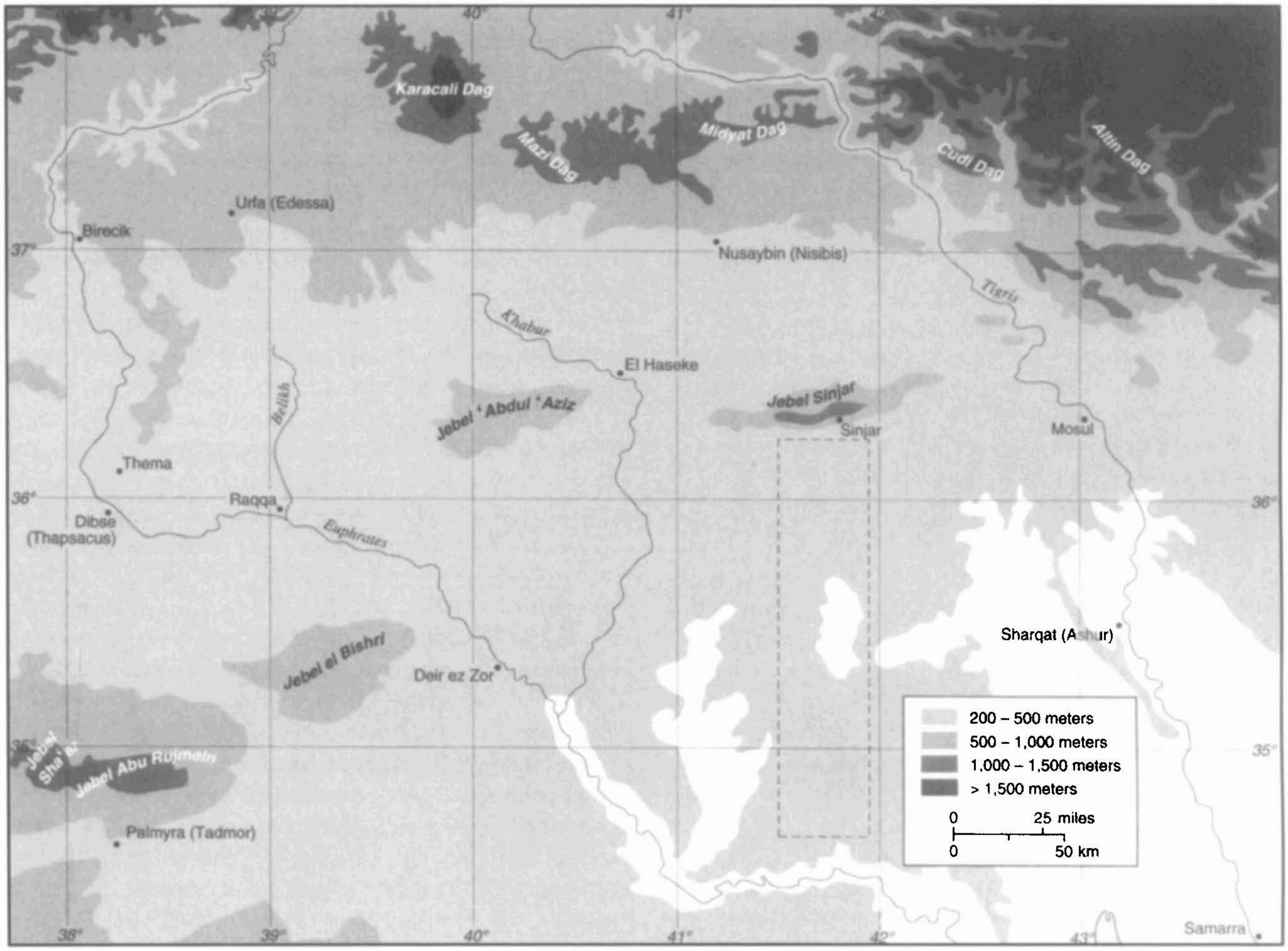

FIG. 8.1. REFERENCE MAP OF THE REGION OF PALMYRA AND SINJĀR. According to the reports, the geodetic survey at the time of al-Maʾmūn was carried out along a line running south from Sinjār. Here the terrain is very level and suitable for such a survey. The other reported survey was in the region including Raqqa and Palmyra (ancient Tadmor), which is generally less suitable.

remarks: "It has been transmitted in books that the ancients found that two towns, Raqqa and Tadmor (Palmyra), are on the same meridian, and that the distance between them is ninety miles."[29] He goes on to express his own doubts about this matter, suggesting that the manuscripts are corrupt. This remark may be linked usefully to two points. First, according to the coordinates of al-Khwārazmī's geographical list, Tadmor and Raqqa lie on the same meridian at the longitude of 66°, and at latitudes 35° and 36°, respectively, whereas in fact Raqqa lies 0;48 to the west of Tadmor, and 1;21 to the north.[30] Second, Jacob of Edessa reported that, according to some, one degree was equivalent to 90 miles.[31] From these considerations it begins to appear that we are dealing not with observations at the time of al-Maʾmūn, but with some pre-Islamic tradition of measurements near Raqqa and Tadmor, a tradition that is recast in these later Arabic accounts.

Without discrediting Sind ibn ʿAlī, who was, we understand, a creditable observer of the time,[32] but regarding only the report by Ibn Yūnus as corrupt, we may appre-

times. Abū al-Qāsim ʿUbayd Allāh ibn ʿAbdallāh ibn Khurradādhbih, *Kitāb al-masālik wa-al-mamālik*; see the edition by Michael Jan de Goeje, *Kitâb al-masâlik waʾl-mamâlik (Liber viarum et regnorum)*, Bibliotheca Geographorum Arabicorum, vol. 6 (Leiden: E. J. Brill, 1889; reprinted 1967), Arabic text, 73, translation, 53; Regling, "Des mesopotamischen Parallelogramms"; Alois Musil, *Palmyrena: A Topographical Itinerary* (New York, 1928); Antoine Poidebard, *La trace de Rome dans le désert de Syrie: Le limes de Trajan à la conquête arabe, recherches aériennes (1925–1932)* (Paris: P. Geuthner, 1934); and René Mouterde and Antoine Poidebard, *Le "limes" de Chalcis: Organisation de la steppe en haute Syrie romaine* (Paris: P. Geuthner, 1945).

29. Al-Bīrūnī, *Taḥdīd*; see Ali's translation, 176–77 (note 24).

30. Even al-Bīrūnī, in *al-Qānūn al-Masʿūdī* (1954–56 edition, 2:567 [note 4]), places them on the same meridian, although with the correct difference in latitude.

31. Hjelt's translation, *Etudes sur l'Hexameron*, 20 (note 22).

32. Ibn Yūnus, in his Hakimite tables, gives details of important solar measurements made by Sind ibn ʿAlī (*Le livre de la grand table Hakémite*, 56, 66, 146, 166 [note 27]). Aydın Sayılı discusses the observa-

ciate that pre-Islamic measurements involving Tadmor had somehow come to be included in the account of the Sinjār expedition.

Matters may be more secure regarding the survey in the desert of Sinjār. It would be natural to read this as describing two traverses, northward and southward from the same starting point, each giving (approximately) the same result, as Abū al-Fidā' believed.[33] Nevertheless it is the absence of clear circumstantial details that may give rise to serious doubts about even this account. For example, a competent astronomer would not advance along the meridian until the altitude had changed by precisely one degree. He would, of course, advance by any distance and calculate the ratio between the change in altitude of the celestial equator and the terrestrial distance. Moreover, the measurements along the two directions, north and south, are bound to have differed somewhat, but we are told nothing of this, nor why the generally accepted figure was 56⅔. In any case, the result 56¼ Arabic miles may be derived from the 75 Roman miles by a simple conversion of the units.

We have no information about the methods used to fix the latitude, and no details about the instruments or the observations. Ibn Yūnus only considers the matter in a general way and writes as follows, continuing directly the passage quoted above:

> These measurements are not without certain conditions, and it is necessary in fixing the difference of one degree in the meridian altitude, that the measurements be always in the plane of this meridian. To attain this, after having selected for the measurements two level and open places, it is necessary to lay down the meridian at the place from which the measurement starts, to take two very fine and faultless cords [*ḥabl*] of about fifty cubits each. Run one of them out to its end exactly along the meridian that was derived; then put the head of the second on the midpoint of the first and run it along the first. Continue always in this way, noting the direction and the altitude in the meridian. Then take the first cord and put its head at the midpoint of the second, etc. Continue always in this way, noting the direction, and the altitude in the meridian that changes between the first place where the meridian was derived and the second place, until the change of altitude of the celestial equator in a day is exactly one degree by two precise instruments in each of which the minutes are shown, so measuring what is between the two places. Then the [number of] cubits is the [number of] cubits of one degree of a great circle covering the sphere of the earth.
>
> It is possible to maintain the direction by means of three bodies instead of the two cords, one of them hiding the others [in line of sight], and extended along the direction of the meridian; one advances by fixing the nearest one by sight, then the second, the third, and so on.[34]

Ibn Yūnus here might seem at first to show how much care was taken with the work, but in fact this is very much an "armchair" description, quite lacking in the vital circumstantial details of an actual survey. The proposal about the cords of fifty cubits stretched from midpoint to midpoint might therefore not reflect the exact technique that was used.

Whether the result is 56, 56¼, 56⅔, or 57 miles, the fact remains that, given the cubit of 0.493 meter, this is an accurate result, indeed probably too accurate to have been determined by the methods claimed. For every minute of error in the measurement of the sun's elevation, the error would be approximately one mile, and even if the instruments were calibrated to show minutes of arc, as Ibn Yūnus said should be the case, the overall error would be much greater. The measurement of the angle of elevation of the sun involves many difficulties, not least because of the large diameter of the disk. Yet it is true that astronomers of the time had carried out new and accurate measurements of the obliquity[35] involving similar difficulties. Later generations would go to great lengths to cope with this problem, such as the installation of the aperture gnomon at Maragheh and Samarkand.[36] The balance of likelihood here is that these geodetic expeditions were intended to settle the choice among the various received values, such as 75 or 66⅔, rather than to confirm the value of 75 miles, converted to Arabic units.

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tional work of these astronomers in *The Observatory in Islam and Its Place in the General History of the Observatory* (Ankara: Türk Tarih Kurumu, 1960; reprinted New York: Arno Press, 1981), chap. 2 (50–87) passim.

33. He found that one of the two traverses gave 56⅔, the other 56: Abū al-Fidā', *Taqwīm al-buldān*; see *Géographie d'Aboulféda*, Arabic text, 14; translation, vol. 2, pt. 1, p. 17 (note 12).

34. Leiden, MS. Or. 143, p. 82; Paris, Bibliothèque Nationale, MS. Arabe 2495, fol. 44v; see also Caussin de Perceval, *Le livre de la grande table Hakémite*, 95 (note 27); and Nallino, "Il valore metrico," 55–56 (note 13).

35. The value 23;35 determined at Baghdad was more accurate than, and differed considerably from, Ptolemy's 23;51.

36. At the observatories of Maragheh (thirteenth century) and probably also at Samarkand (fifteenth century), following the pioneering work of al-Khujandī (below, note 58), use was certainly made of a technique in which the sun's light was admitted through a very narrow aperture in the roof of a darkened chamber, and then fell on a meridian scale, typically a sextant. This is a camera obscura in which the sun's image appears projected onto the scale as a well-defined disk, so permitting very precise measurements of its altitude and so forth; see, for example, Sayılı, *Observatory in Islam*, 194, 198, 283 (note 32). The instrument came to be called the Suds (sextant) al-Fakhrī. On the continuation of this technique at the seventeenth-century Indian observatories of Jai Singh in Delhi and Jaipur, see Raymond Mercier, "The Astronomical Tables of Rajah Jai Singh Sawā'i," *Indian Journal of History of Science* 19 (1984): 143–71, esp. 161–63, 167, 170–71.

AL-BĪRŪNĪ'S MEASUREMENT OF THE RADIUS OF THE EARTH

In the *Taḥdīd*, al-Bīrūnī told how he had devised another method for measuring the circumference of the earth. He explained that it did not "require walking in deserts"[37] but involved determining the radius of the earth based on the observation of the distant horizon from a mountain peak.[38] The angle of dip of that line of sight below the local horizontal determines the ratio between the height of the mountain and the radius of the earth. In figure 8.2 the line of sight from the peak H grazes the horizon at A. The peak is taken to be at a height h = HJ above the plain JA. The angle of dip d is equal to the angle subtended at the center of the earth by the arc JA, and the formula that gives the radius R in terms of h is

$$R = h \cos d/(1 - \cos d).$$

In al-Bīrūnī's observation, he stationed himself on a peak in the Salt Range, a short mountain range situated west of Jhelum in the Punjab. In figure 8.3, the peak is immediately to the southwest of Nandana, a fort situated at the southern end of a pass through the range.[39] In the *Taḥdīd*, al-Bīrūnī explains that he was detained there, at which time he came to appreciate that the site was suitable for such an observation. He had originally hoped to employ the usual geodetic method by measuring the length of the meridian line in the plains north of Dehistān, in the Jurjān region, near the southwestern shore of the Caspian Sea; there he was frustrated, apparently by lack of support. According to his accounts in *al-Qānūn al-Masʿūdī* and the *Taḥdīd*, he proceeded as follows. As he says:

> I changed to another way owing to having found in a region in India a mountain peak facing toward a wide flat plain whose flatness served as the smooth surface of the sea. Then on its peak I gauged the intersection of heaven and earth,—the horizon—in the

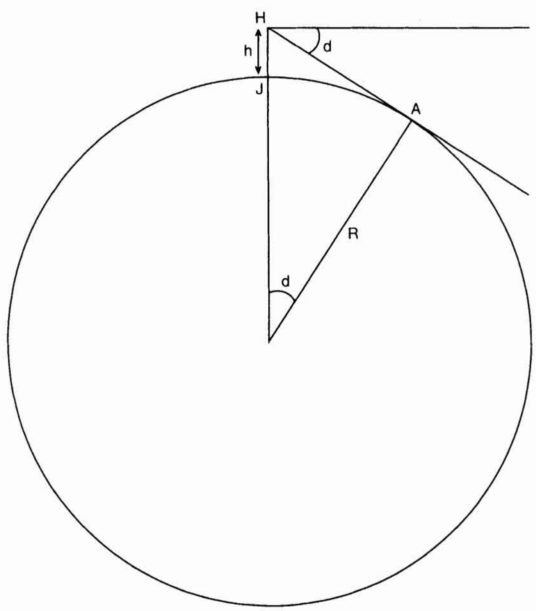

FIG. 8.2. DIP OF THE HORIZON MEASURED FROM THE MOUNTAIN. From the top of the mountain H, the visible horizon is in the direction HA, dipping below the local horizontal by the angle d. If refraction is neglected (following al-Bīrūnī) then the angle at the center of the earth between the mountain and the point A is also d. When the height of the mountain h is known, the radius of the earth R can be found from d and h. When refraction is included, the line of sight HA is curved, concave to the earth, the angle at the center is larger than d, and there is a different relation between d and h.

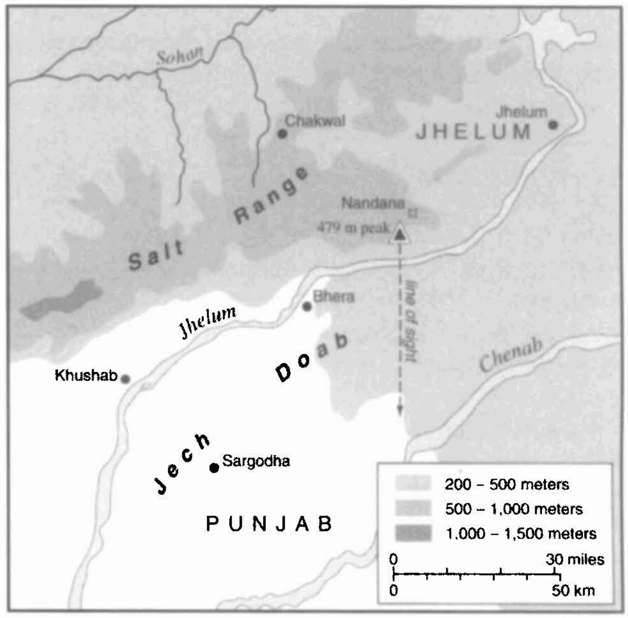

FIG. 8.3. AL-BĪRŪNĪ'S OBSERVATION AT NANDANA. Nandana, in the Jhelum district of Pakistan, is about 110 kilometers south of Islamabad. About 1.7 kilometers south-southwest of the fort at Nandana there is a peak from which al-Bīrūnī's observation was made. This peak is at an altitude of 1,570 feet (479 m) above sea level. When the atmosphere permits a view of the horizon to the south, the line of sight would graze the horizon at a point whose latitude is smaller by an amount approximately equal to the angle of dip, about 30 minutes, or about 55 kilometers to the south.

37. Al-Bīrūnī, *Taḥdīd*; see Ali's translation, 183 (note 24).

38. Al-Bīrūnī, *al-Qānūn al-Masʿūdī*, bk. 5, chap. 7; see the 1954–56 edition, 2:528 (note 4), and also in al-Bīrūnī's earlier work on geodesy, *Taḥdīd*; see Ali's translation, 188–89 (note 24). Syed Hasan Barani, "Muslim Researches in Geodesy," in *Al-Bīrūnī Commemoration Volume, A.H. 362-A.H. 1362* (Calcutta: Iran Society, 1951), 1–52, esp. 35–39, translated the passage in *al-Qānūn* from manuscripts accessible to him.

39. Locating this pass is assisted in that Sir Aurel Stein explored the region in his successful effort to determine where Alexander the Great entered the Indian Plain just before his famous battle with Poros; Mark Aurel Stein, "The Site of Alexander's Passage of the Hydaspes and the Battle with Poros," *Geographical Journal* 80 (1932): 31–46.

> prospect, and I found it by an instrument to incline from the East-West line a little less than 1/3 1/4 degree [0;35], and I took it as 0;34. I derived the height of the mountain by taking the summit in two places, and I found it to be 652 1/20 cubits [652;3,18].[40]

He carries out the construction shown in figure 8.2 and proceeds to derive the result above, leading eventually to 56;5,50 miles per degree.[41] This result, as he says, is near that found in the report from the teams sent out by al-Ma'mūn, 56 2/3, that is, 56;40 miles. Al-Bīrūnī accepts the earlier value because "their instrument was more refined, and they took greater pains in its accomplishment."[42]

The most likely peak is one situated about 1.4 kilometers south-southwest of Nandana, at an altitude of 478 meters above sea level, and about 265 meters, or 537 cubits, above the plain to the south. However, al-Bīrūnī gives the height as 652 1/20 cubits (321.5 m), in marked disagreement with this.[43] Rizvi, in his recent study, certainly seems to have this peak in mind (he worked from a map on a scale similar to that shown in fig. 8.3) and gave its height as 1,795 feet (547 m) above sea level, and 1,055 feet (321.5 m) above the plain, evidently not read from the map, but designed only to agree exactly with al-Bīrūnī's report of the height above the plain.[44]

As we would expect, al-Bīrūnī does not allow for refraction.[45] Indeed, astronomers of that time were not aware of refraction in astronomical observations,[46] and one would imagine that, if questioned, al-Bīrūnī would have assumed that the atmosphere is uniform up to its top, so that there is no refraction within it. On the assumption that the peak is situated 321 meters above the plain, the effect of refraction is to reduce the observed angle of dip to about 0;32 and to increase the angle subtended at the center of the earth to about 0;37. In fact the peak is 265 meters above the plain, which makes the dip, with refraction, about 0;29. Without refraction it would be 0;31,20. We have little information about his instrument and cannot judge whether he would have been able to observe the dip angle more precisely.

The difficulty of actually seeing the horizon and fixing the line of sight to it is considerable. Rizvi relates that he sought to view the horizon from the same peak and that after a number of unsuccessful attempts when it was obscured by dust, and so forth, he finally found one day that, after rainfall had cleared the air, he could see it clearly.[47] Unfortunately he does not report any measurement of the dip of the horizon. In his analysis of al-Bīrūnī's work he does not note the need to allow for refraction.

When one considers the errors in al-Bīrūnī's reported observations of both the height of the peak above the plain and the dip of the horizon, there can be no doubt that, having found the observations excessively difficult, he worked out the angle 0;34 from his assumed height of the peak and from the known length of the degree. It would not be the first time in the history of astronomy that fictitious results have been presented in lieu of true observations.[48]

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40. Al-Bīrūnī, *al-Qānūn al Mas'ūdī*, bk. 5, chap. 7; see the 1954–56 edition, 2:530 (note 4), and *Taḥdīd*, see Ali's translation, 188 (note 24).

41. The ratio between the radius of the earth and the height of the mountain is cos 0;34/(1 − cos 0;34). Al-Bīrunī calculates cos 0;34 as sin 89;26 = 0;59,59,49,2,28, giving the denominator 0;0,0,10,57,32. The height of the mountain is 652;3,18, making the radius of the earth 12,851,369;50,42 cubits. The value of π is taken as 22/7, making the circumference 80,780,039;1,33 cubits, and the length of one degree 224,388;59,50 cubits, or 56;5,50 miles. The chief error in the calculation arises from the smallness of the denominator, where the sine should be 0;59,59,49,26. This would lead to 58;11,37 miles per degree, or 58;10,13 if a better value of π is used. It is only by accident, therefore, that he gets a result so close to the received value of 56;40 miles.

42. Al-Bīrūnī, *al-Qānūn al-Mas'ūdī*, bk. 5, chap. 7 see the 1954–56 edition, 2:531 (note 4).

43. In his account in the *Taḥdīd*, al-Bīrūnī says that the cubit in question was used for measuring cloth, so it possibly is not the one used for geodetic purposes (see Ali's translation, 188 [note 24]). However, if his altitude was correct, this would mean a cubit of about 40 centimeters, very different from any value encountered elsewhere.

44. Saiyid Samad Husain Rizvi, "A Newly Discovered Book of al-Bīrūnī, 'Ghurrat-uz-Zījāt' and al-Bīrūnī's Measurements of Earth's Dimensions," in *Al-Bīrūnī Commemorative Volume*, Proceedings of the International Congress held in Pakistan on the occasion of the Millenary of Abū Raihān Muhammed ibn Ahmad al-Bīrūnī (973–ca. 1051 A.D.) November 26, 1973 through December 12, 1973, ed. Hakim Mohammed Said (Karachi: Times Press, 1979), 605–80.

45. None of the commentators on al-Bīrūnī's work have realized that refraction is in fact a substantial part of the observed angle of dip *d*. The refracted ray is curved concave to the earth, and the exact departure from a straight line can be calculated only if one knows the pressure and temperature, and the vertical temperature gradient, at all points along the ray. In practice such detailed information is not available, and surveyors tend to use a rule of thumb according to which the path followed by the ray is an arc whose radius is seven times the radius of the earth. This is the case when typical values of pressure and temperature are assumed. Equivalently, one may assume that the angle of dip is reduced by one-fourteenth of the arc subtended at the center of the earth. This approximate rule is less secure for grazing rays, which a modern surveyor would try to avoid.

46. Indeed, calculations of the height of the atmosphere by Mu'ayyad al-Dīn al-'Urḍī al-Dimishqī and others in the thirteenth century show clearly that refraction at the top of the atmosphere played no part in their argument; George Saliba, "The Height of the Atmosphere according to Mu'ayyad al-Dīn al-'Urḍī, Quṭb al-Dīn al-Shīrāzī, and Ibn Mu'ādh," in *From Deferent to Equant: A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honor of E. S. Kennedy*, ed. David A. King and George Saliba, Annals of the New York Academy of Sciences, vol. 500 (New York: New York Academy of Sciences, 1987), 445–65.

47. Rizvi, "Newly Discovered Book," 619 (note 44).

48. In the sixteenth and seventeenth centuries similar methods were proposed by Francesco Maurolico, Johannes Kepler, and Giovanni Baptista Riccioli, who either ignored or underestimated the role of refraction. The matter was settled finally by Jean Picard, the father of modern geodesy. Writing of Maurolico's suggestion that one should discover from what distance at sea Mount Etna would be visible, he said (according to a contemporary English translation of Picard's 1671 work): "But

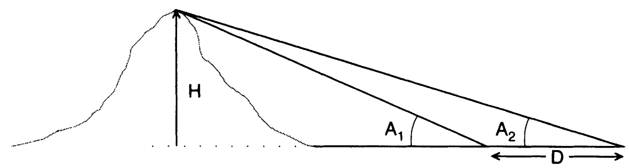

FIG. 8.4. MEASUREMENT OF THE HEIGHT OF A MOUNTAIN USING TWO ALTITUDES. Al-Bīrūnī's method made use of the altitude of the mountain taken from two different places, distance D apart. The height of the mountain, H, would then be equal to D/(cot A_1 − cot A_2).

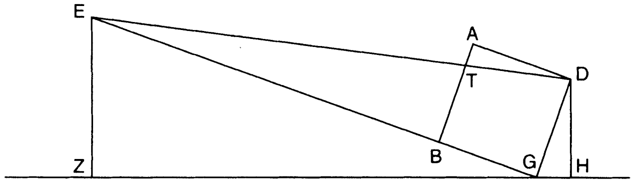

FIG. 8.5. USE OF A QUADRANT TO MEASURE THE HEIGHT OF THE MOUNTAIN. The quadrant ABGD is arranged so that the lower edge BG, and also the alidade DT, are in line of sight to the top E of the mountain ZE. The sides of the quadrant are each 1 cubit (49.3 cm). If the distance GZ is, for example, 500 meters, and the height 320 meters, then the interval AT is 0.041 centimeters. The angle ADT is arctan(0.041/49.3) = 0;2,51.

As to the measurement of the height of the mountain, in *al-Qānūn al-Masʿūdī* al-Bīrūnī says he derived it after taking its altitude in two places. That means that if the altitude is taken from two places spaced apart by a distance *D* in a line leading away from the peak, so that A_1 and A_2 are the altitudes, then the height of the mountain is $D/(\cot A_1 - \cot A_2)$ (fig. 8.4).[49] This method seems practical enough, but as we have seen, his result was inaccurate.

In the *Taḥdīd*, on the other hand, al-Bīrūnī explains how the altitude is determined with the use of a square plate equipped with an alidade, as shown in figure 8.5.[50] He gives the side of the quadrant as one cubit. If, for the sake of example, ZG is 0.5 kilometer, the mountain being 0.32 kilometer in height, then the angle ADT is 0;2,51, and AT is about 0.4 millimeter. Even when the scale is supplemented by transversals, as was the case in the sixteenth-century version of the instrument (shown in fig. 8.6), this is at the limit of precision. Besides, he advises that the interval at the base GH, which is also needed, is to be determined not by means of the instrument, but by dropping a stone from the corner D! This quite impractical proposal can only be intended as a jeu d'esprit.

Determining the Longitude of Ghazna

Al-Bīrūnī's substantial treatise on geodesy, the *Kitāb taḥdīd nihāyāt al-amākin li-tasḥīḥ masāfāt al-masākin*, was composed in the interval 409–16/1018–25,[51] some time before *al-Qānūn al-Masʿūdī*, which appears to date from about 420/1030. In both works he gives an account of his investigations into the longitude of Ghazna, the capital of his patron Maḥmūd,[52] and the meridian of reference of the tables in *al-Qānūn*. The chapter in *al-Qānūn* is a summary of the work described in the *Taḥdīd*. In the *Taḥdīd* he follows a much more complicated series of routes from Baghdad to Ghazna.[53]

The city of Ghazna was taken by al-Bīrūnī to define the meridian of reference of the mean longitudes in his *al-Qānūn*. He naturally wished to determine the difference in longitude between it and other cities such as Baghdad and Alexandria, which had served as meridians of reference for other tables, so that anyone using his

the refractions which are yet greater upon the sea than upon the land, render this practice fallacious, because they enable us to discover objects at a much greater distance than the convexity of the sea ought to permit, and by consequence make the earth appear much greater than in effect it is"; Jean Picard, *The Measure of the Earth*, trans. Richard Waller (London, 1688).

49. In figure 8.6 this type of measurement is illustrated in the left part of the lower border.

50. There are two pairs of similar triangles, DAT ≡ EGD and EZG ≡ DGH, giving respectively the ratios AT/AD = GD/GE and ZE/GE = GH/GD. Thus

$$GE = GD \times AD/AT \text{ and } EZ = GE \times GH/GD.$$

The altitude of the mountain, EZ, is given by the final step. In practice it is necessary to determine AT when the angle ADT is very small, this being essentially the parallax over the line DG. The quadrant is equipped to give AT but not, however, to give GH.

In figure 8.6 this use of the quadrant is illustrated in the center of the left border. In the arrangement shown, only the interval AT can be found, giving the distance EG, but not the height EZ.

51. Editions of the Arabic text of al-Bīrūnī's *Kitāb taḥdīd nihāyāt al-amākin li-taṣḥīḥ masāfāt al-masākin* include one edited with an introduction by Muḥammad Tāwīt al-Ṭanjī (Ankara, 1962), and one edited by P. G. Bulgakov, verified by Imām Ibrahīm Aḥmad, in *Majallat Maʿhad al-Makhṭūṭāt al-ʿArabīyah* (Journal of the Institute of Arabic Manuscripts of the Arab League), special no., vol. 8 (pts. 1 and 2) (Cairo, 1962). Translations include Ali's (note 24) and a Russian translation and commentary by P. G. Bulgakov, *Abu Reihan Biruni, 973–1048: Izbrannie Proizvedeniya* (Selected works), vol. 3, *Opredelenie Granitz Mest dlya Utochneniya Rasstoyanii Mejdu Naselennimi Punktami* (*Kitāb taḥdīd nihāyāt al-amākin li-taṣḥīḥ masāfāt al-masākin*) Geodeziya (Geodesy), investigation, translation, and commentary (Tashkent: Akademia Nauk Uzbekskoi SSR, 1966); and see also Edward S. Kennedy, *A Commentary upon Bīrūnī's "Kitāb taḥdīd al-amākin": An 11th Century Treatise on Mathematical Geography* (Beirut: American University of Beirut, 1973).

52. Al-Bīrūnī, *al-Qānūn al-Masʿūdī*, bk. 6, chap. 2; see the 1954–56 edition, 2:609–16 (note 4), and al-Bīrūnī, *Taḥdīd*, see Ali's translation, 192–240 (note 24).

53. A number of other topics are treated in the *Taḥdīd*, including an account of the determination of latitudes, and of the obliquity of the ecliptic, of which many determinations by Arabic astronomers are cited in circumstantial detail. The work ends with an account of the determination of the qibla (the direction of Mecca). Al-Bīrūnī gives here, as well as in *al-Qānūn*, an account of his determination of the radius of the earth, which has been discussed above.

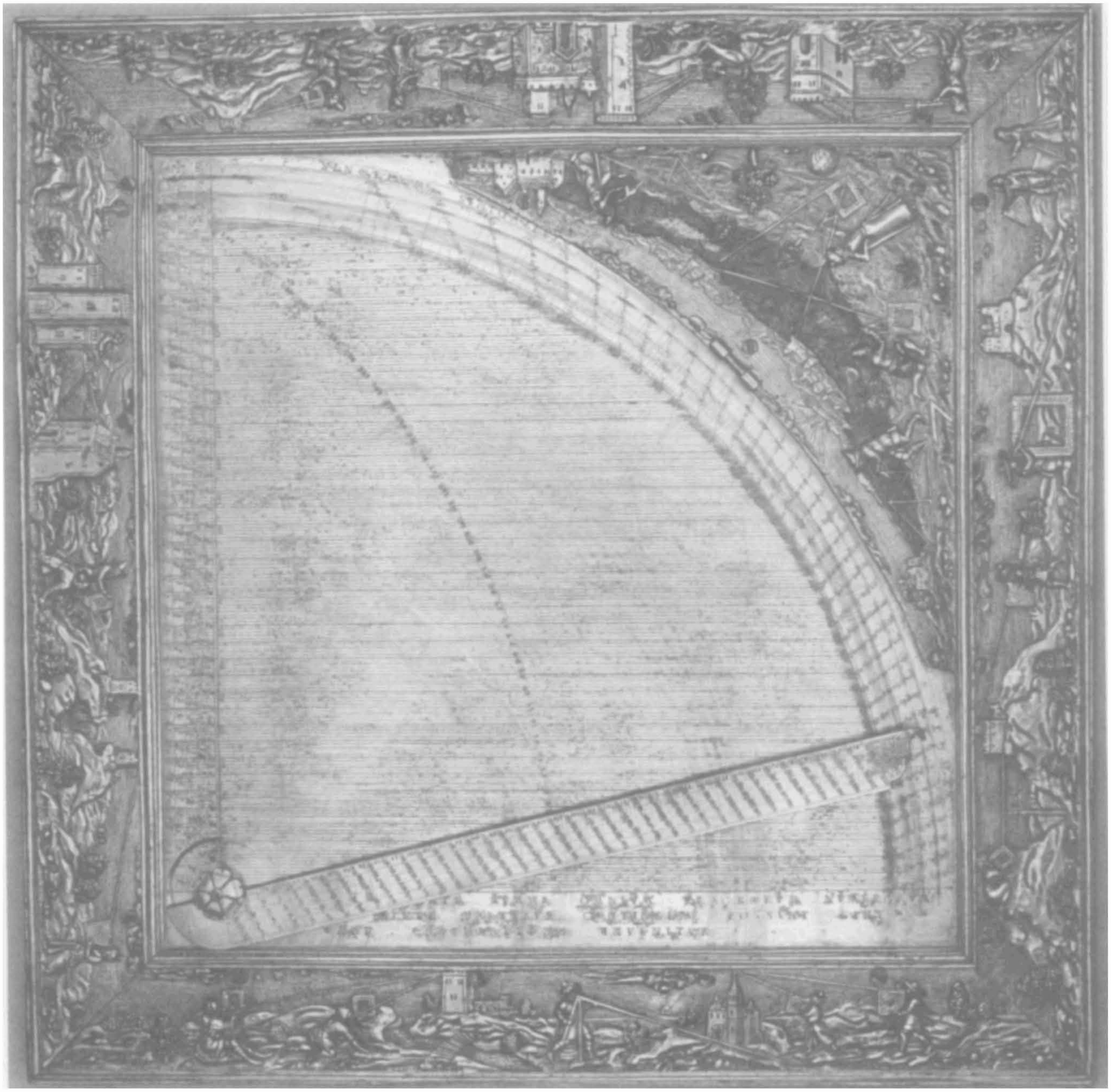

FIG. 8.6. SIXTEENTH-CENTURY QUADRANT WITH ALIDADE. The uses of the quadrant are illustrated in a series of fine vignettes on the reverse of an early European example, the sixteenth-century *Quadraticum geometricum* of Christoph Schissler. Echoes of its Oriental origin are evident in these fine pictures that decorate the four edges, showing the various turbaned observers. The history and use of the instrument, along with descriptions of the extant examples, are given by Herbert Wunderlich, *Das Dresdner "Quadratum geometricum" aus dem Jahre 1569 von Christoph Schissler d.A., Augsburg, mit einem Anhang: Schisslers Oxforder und Florentiner "Quadratum geometricum" von 1579/1599* (Berlin: Deutscher Verlag der Wissenschaften, 1960). The side of the inner, calibrated, square measures approximately 30 centimeters. In this square the scale along the edge is divided into two hundred parts, but with the aid of the transversal subdivision each of these is further divided into five parts, each therefore representing about 0.3 millimeter. There is, however, nothing to suggest that the square plate used by al-Bīrūnī was enhanced by such a transversal. That technique was used on other instruments of the sixteenth century, including those of Tycho Brahe.
Size of the original: 34.5 × 34.5 × 1.1 cm. Museum of the History of Science, Oxford (inv. no. 52–83). By permission of the Bettman Archive, New York.

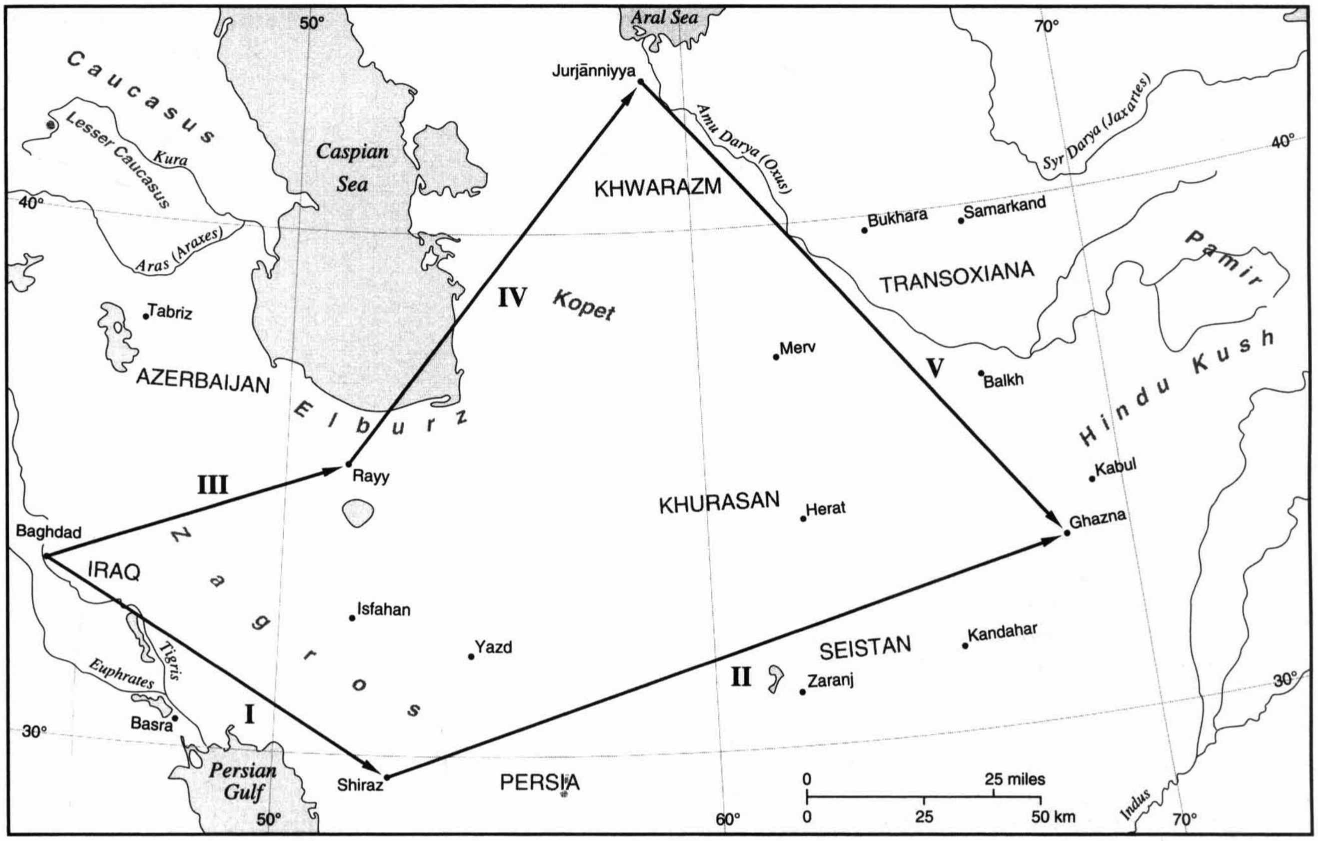

FIG. 8.7. "TRIANGULATION" BETWEEN BAGHDAD AND GHAZNA. Al-Bīrūnī arranged his "triangulation" of the region between Baghdad and Ghazna to follow two independent paths, through Rayy (near Tehran) and Jurjānīyah (near Urgench) to Ghazna, and also through Shiraz in southern Persia. In the *Taḥdīd* he subdivided the route between Shiraz and Ghazna into a number of shorter steps. The distances along these routes were taken from travelers' reports, while the latitudes were found accurately by astronomical means.

tables could calculate from them the mean longitudes at any other meridian. He proceeds, then, by a series of steps, as shown in figure 8.7, in which Ghazna is referred to Baghdad through Shiraz (steps I, II) or through Jurjānīyah[54] and Rayy[55] (steps III, IV, V). These various differences I to V are taken from travelers' accounts, and since such estimates are regarded as generally in excess, he subtracts a proportion, say one-tenth or one-sixth, according to his understanding of the particular terrain and the extent to which the traveler is likely to have to follow a crooked path. In a separate calculation he determines the difference in longitude between Baghdad and Alexandria, but in this brief summary of his work only the Baghdad-Ghazna calculations will be considered. In each of these steps he quotes the latitudes for the pair of places, and these, together with the direct distance between them, suffice to give him the longitude differences. Distances are converted to arcs in the proportion of 56⅔ miles per degree or, since the *farsakh* is 3 miles, 18;53,20 *farsakhs* per degree. He had given a somewhat different analysis of this problem in the *Taḥdīd*, but the following is based on the later *al-Qānūn al-Masʿūdī*.[56]

In figure 8.8 two places (A and B) are on the meridians TAJ, TBD, T being the North Pole and JD being an arc of the equator. We are given the latitudes of the two places, that is, the arcs JA, DB, and also the arc separating

54. Jurjānīyah (in Persian "Gurgānj") is situated in Khwārazm (Khorezm), al-Bīrūnī's native country. The site is now named Kunya Urgench (Old Urgench). A modern city named Urgench is located to the southeast of the old site. In his works, al-Bīrūnī notes a number of observations he has carried out in this region. The word "Bīrūnī" is derived from the Persian *bīrūn*, "outside." It is thus frequently suggested that his birthplace was in a suburb of Kāth, the erstwhile capital, but that is essentially speculation. Kāth is now a ruin, at a site known as Shah Abbas Wali; nearby is a modern city named Biruni in honor of the astronomer.

55. Rayy (Rai) is the ancient Rhagae, very near Tehran.

56. This chapter of *al-Qānūn* has been translated twice, by Carl Schoy, "Aus der astronomischen Geographie der Araber," *Isis* 5 (1923): 51–74, and later by J. H. Kramers, who corrected some errors of translation in "Al-Bīrūnī's Determination of Geographical Longitude by Measuring the Distances," in *Al-Bīrūnī Commemoration Volume, A.H. 362-A.H. 1362* (Calcutta: Iran Society, 1951), 177–93; reprinted in *Analecta Orientalia: Posthumous Writings and Selected Minor Works of J. H. Kramers*, 2 vols. (Leiden: E. J. Brill, 1954–56), 1:205–22. Neither reviewed the calculations.

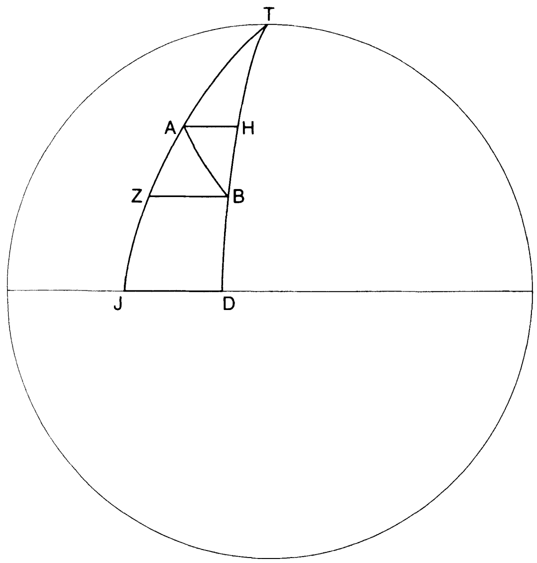

FIG. 8.8. TRIGONOMETRIC CONSTRUCTION TO DETERMINE THE DIFFERENCE OF LONGITUDE. Between the North Pole T of the earth and its equator JD, the great circle arcs TJ and TD are drawn. The arcs AH and ZB are drawn parallel to the equator (and so are not great circles), and AB is the great circle joining two places A, B, which lie at different latitudes. The latitudes are represented by the arcs JA (= DH) and BD (= JZ), with the difference represented by the arc JD. The formula derived in note 57 gives JD in terms of the known latitudes and known distance AB.

them along a great circle AB, and the problem is to find the difference of longitude JD.[57] Al-Bīrūnī proceeds to analyze the problems of the chords subtending the various arcs and derives the equation

$$\mathrm{ch}(AZ)^2 + \mathrm{ch}(AH)^2\cos(DB)/\cos(JA) = \mathrm{ch}(AB)^2,$$

in which ch(AZ) means the chord joining the points A and Z. In this equation all the quantities are known except ch(AH), for which one solves it, and then from AH, JD is derived.

He uses this equation repeatedly in his analysis, always referring back to the same diagram, with the points A and B representing in turn the successive pairs of places. He quotes either his own or earlier observations for the latitudes, the values being given in table 8.1 along with their modern values.[58]

In his account al-Bīrūnī gives all the numerical work, so that it is possible to observe the level of accuracy he achieves and also his occasional mistakes. An error to which he is prone is the accidental interchange of the two cosine terms in the ratio cos(DB)/cos(JA) in the equation above.[59]

Table 8.2 summarizes the angular distances between pairs of sites and the longitude differences, both according to al-Bīrūnī and also according to calculations based on the modern coordinates.

He averages the values obtained by the two routes to get (24;54,26 + 23;44,2)/2 = 24;19,14. Without any errors of calculation he would have had (25;36,14 + 24;57,1)/2 = 25;16,37. In the geographical table in *al-*

TABLE 8.1 Al-Bīrūnī's Latitude Values

Place	al-Bīrūnī	Modern
Baghdad	33;25	33;20
Shiraz	29;36	29;38
Ghazna	33;35	33;33
Rayy	35;34,39	35;35
Jurjānīyah	42;17	42;18

57. Draw the arcs AH and ZB parallel to the equator and lying in planes parallel to the equatorial plane. In the following we distinguish arcs and chords by writing the arc simply as AH and the chord as ch(AH), which equals 2cos(AJ) sin(JD/2) = cos(AJ)ch(JD). It may be shown that the four points A, H, B, Z lie on a circle and that as a consequence,

$$\mathrm{ch}(AZ)\mathrm{ch}(BH) + \mathrm{ch}(ZB)\mathrm{ch}(AH) = \mathrm{ch}(ZH)\mathrm{ch}(AB).$$

In the present application, AB = ZH, AZ = BH,

$$\mathrm{ch}(AZ)^2 + \mathrm{ch}(ZB)\mathrm{ch}(AH) = \mathrm{ch}(AB)^2.$$

We know AZ from the difference of latitudes and AB from the direct distance, and we wish to find JD. Substituting ch(AH) = cos(JA)ch(JD), ch(ZB) = cos(DB)ch(JD),

$$\mathrm{ch}(AZ)^2 + \mathrm{ch}(JD)^2\cos(DB)\cos(JA) = \mathrm{ch}(AB)^2.$$

Although this would provide ch(JD) directly, al-Bīrūnī usually writes the second term of the last equation in terms of ch(AH),

$$\mathrm{ch}(AZ)^2 + \mathrm{ch}(AH)^2\cos(DB)/\cos(JA) = \mathrm{ch}(AB)^2,$$

which he solves for ch(AH), getting ch(JD) from ch(JD) = ch(AH)/cos(JA).

58. Al-Bīrūnī gives the sources for the latitudes as follows:
1. Al-Bīrūnī using the Yamīnīyah ring in the period 409–10/1018–20.
2. Abū al-Ḥusayn ʿAbd al-Raḥmān ibn ʿUmar al-Ṣūfī (d. 372/983), using the ʿAḍūdī ring.
3. Al-Bīrūnī using a quadrant in 410/1019–20.
4. Abū Maḥmūd al-Khujandī, in 384/994, using his large mural sextant, equipped with a device for sharpening the sun's shadow.
5. Al-Bīrūnī using the Shāhīyah ring.

The rings in these applications are the most elementary of astronomical instruments, designed to measure altitude directly. They are likely to be brass circles, calibrated in degrees, equipped with a pointer carrying sights, and mounted securely in some way in the meridian plane.

59. In the case of the first stage, Baghdad to Shiraz, he finds that the direct distance is 170 *farsakhs*, which he reduces by one-tenth to 153. The corresponding arc is 8;6 and the chord 0;8,28,31, which he gives as 0;8,28,32. The latitude difference is 33;25–29;36 = 3;49, the chord of which is 0;3,59,46, which he gives as 0;3,59,40. The cosine ratio becomes cos(29;36)/cos(33;25) = 0;52,10,11/0;50,4,52, which he gives as 0;52,10,10/0;50,4,52. He obtains ch(AH) = 0;7,28,27, although from his figures he should have 0;7,19,27, and from a precise calculation 0;7,19,26. For the longitude difference ch(JD) = ch(AH)/cos(33;25), from which he obtains 0;8,57,16 correctly from his ch(AH); this gives the arc JD = 8;33,32. From the correct ch(AH) he would have ch(JD) = 0;8,46,28, arc 8;23,11.

The longitude of Baghdad is 70 (measured in the scale of al-Khwārazmī's geographical tables), so that of Shiraz is 78;33,32, according to his calculation. He remarks that this agrees with the received value 79.

TABLE 8.2 Summary of al-Bīrūnī's Angular and Longitudinal Differences between Pairs of Places

	Distance		Longitude Difference	
Places	al-Bīrūnī	Modern	al-Bīrūnī	Modern
Baghdad–Rayy	8;6	7;44	8;33,32	8;8
Rayy–Ghazna	15;2,7	14;5	16;20,54	15;54
Total			24;54,26	24;2
Baghdad–Rayy	7;0,21	6;12	8;5,20	7;1
Rayy–Jurjānīyah	8;10,14	9;0	6;1,26	7;43
Jurjānīyah–Ghazna	12;10,37	11;24	9;37,16	9;18
Total			23;44,2	24;2

Qānūn al-Masʿūdī he gives the longitude of Ghazna as 94;20,[60] evidently based on these calculations, whereas a better value would have been 95;17.

Except in one case, al-Bīrūnī has overestimated the angular distances, by some 8 percent on the average. This is due to an even greater overestimate of the distances in miles, because the conversion ratio 56⅔ miles per degree is itself some 0.7 percent too large. This, as one would expect, is a major source of error, since the latitudes are all well observed. The true difference in longitude between Baghdad and Ghazna is 24;2, whereas al-Bīrūnī gets 24;19,14 and would have arrived at 25;16,37 had he made no error of calculation.

Seen against the historical background of geographical coordinates, however, the results are impressive. An error of one degree in such a large distance marks a decided improvement over Ptolemy's geographical coordinates. The order of the errors in longitude would not be reduced by a further order of magnitude until the end of the seventeenth century, when observations of Jupiter's satellites were exploited.

60. Al-Bīrūnī, *al-Qānūn al-Masʿūdī*, bk. 5, chap. 10; see the 1954–56 edition, 2:561 (note 4).