Seismic Experiments on the Fracturing and Overturning of Columns. *2nd Paper.*

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With Plates I-VI.

1. *Introduction.* The results of the Shaking Table experiments on the overturning and fracturing of columns by horizontally applied motion, carried on in 1898 and 1899, have been given in the "Publications of the Earthquake Investigation Committee in foreign languages," No. 4. The papers subsequently published by the present author which relate to Practical Seismology are as follows :—

Notes on Applied Seismology.	*Verhandlungen d. 1. internat. seismolog. Konferenz,*	1901.
On the Deflection and Vibration of Railway Bridges	The *Publications,*	No. 9.
On the Overturning and Sliding of Columns	„	, No. 12.
Note on the Vibration of Chimneys	„	, „
Note on the Vibration of Railway Bridge Piers....................	„	, „
Motion of a Brick Wall produced by Earthquakes. 2nd Paper	„	, „
Application of Seismographs to the Measurement of the Vibration of Railway Carriages. 1st Paper.................................	„	, No. 15.
Application of Seismographs to the Measurement of the Vibration of Railway Carriages. 2nd Paper	„	, No. 20.
Earthquake Measurement in a Brick Building. 3rd Paper.	„	, „
Vibrations of a Railway Bridge Pier.............................	The *Bulletin,*	Vol. I.
Deflection and Vibration of Railway Bridges. 2nd Paper..........	„	, „
Note on the Seismic Stability of the Piers of the Naisha-*gawa* Railway Bridge, Formosa.	„	, Vol. II.
Example of a simple Brick Structure damaged by Earthquake	„	, „
Experiment on the Vibration of Brick Columns....................	„	, „

The present paper gives an account of the seismic experiments on the sliding and overturning of columns carried on, in 1900, by means of the Shaking Table,* which was driven by a steam engine and was made to execute simple harmonic motion, confined, as in the former cases, to the horizontal component.

The method of experiment consisted in placing a column or plate on the Shaking Table and causing it, by giving proper movements to the latter, to be overturned, to be put into rocking motion, or to undergo sliding displacement. The movement of the Shaking Table was mechanically registered in the form of a diagram, while the exact moment of the rocking, overturning, or sliding of the column was carefully watched and electrically recorded. The shaking intensity deduced from the diagram of motion of the table was then compared with the theoretical value of the acceleration necessary for the overturning or sliding. In the different experiments, which lasted 1 to 3 minutes, the motion of the Shaking Table was at first slow, but was gradually lessened until the desired result was produced; the amplitude of motion being, in each case, fixed and unalterable, except some slight increase consequent to the quickening of period. The double amplitude, the complete period, and the corresponding maximum acceleration of the motion of the table are denoted by the symbols $2a$, T, and A, respectively.

SLIDING.

2. *Sliding and coefficient of friction.* Let $ABCD$ be a column resting on the ground; the intensity of the earthquake motion, supposed to be horizontal, being represented by its maximum acceleration ($=\alpha$). If m be the mass of the column, y the height of the centre of gravity (G), x half-width of the base AB,

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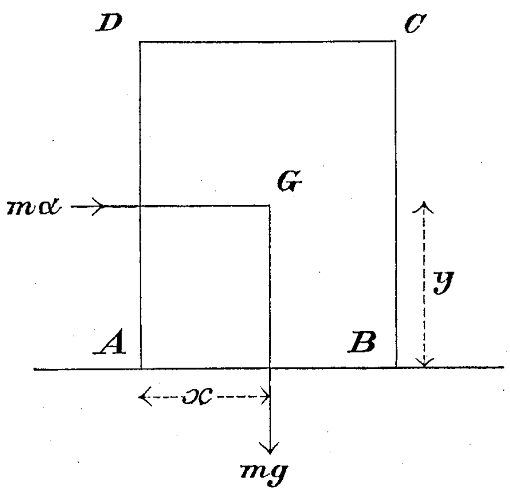
* Described in the "Publications," No. 3 and No. 4.

Fig. 1.



we have, for the overturning of the body, the following relation :—

$$a = g \times \frac{x}{y} \quad \ldots\ldots\ldots\ldots(1)$$

g being the acceleration due to the gravity. Again, if S be the frictional force between the surfaces in contact at AB, which acts in the direction BA, we have :

$$S = ma$$

or

$$\mu\, mg = mg \times \frac{x}{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

μ being the coefficient of friction of plane surfaces. As the frictional force S can not be greater than μmg, the relations (1) and (2) do not exist unless the ratio x/y be smaller than μ. In other words, the column would suffer a sliding, but not be overturned, by an earthquake, if

$$\frac{x}{y} > \mu \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The values of μ for different surfaces of contact are as follows* :—

MEAN COEFFICIENT OF FRICTION OF REPOSE.

Surfaces.	Dry	Damp with water.
Wood on wood.	0.50	0.68
Metal on metal.	0.18	—

* Taken from Molesworth's Pocket-Book of Engineering Formulæ.

Surfaces.	Dry	Damp with water.
Wood on metal.	0.60	0.65
Stones on stones or bricks.	0.71	—
Stones on wrought iron.	0.45	—
Wood on stones.	0.60	—

By means of the relation (3), we can determine the least value of the ratio x/y, which indicates, for a given material, the height, relatively to the base, of a column capable of resisting the overturning action by the earthquake motion. Thus, for the base of unit length, the approximate height in question would be :—

For wood on wood ....Height = 2.0
,, wood on stones .... ,, = 1.7
,, stones on stones or bricks.......... ,, = 1.4

Again, the relation

$$a = \mu g \qquad (4)$$

determines the approximate value of the intensity (a) of earthquake motion, at which a given column would commence to slide. Thus we have :—

For wood on wood (dry) .... $a = 5000$ mm/sec. ..(5)
,, ,, ,, ,, (damp with water) .. $a = 6700$,, ..(6)
,, wood on stones .... $a = 6000$,, ..(7)
,, stones on stones or bricks .... $a = 7000$,, ..(8)

3. *Experiments on the sliding of bodies.* The following experiments relate to the sliding of wooden and stone plates subjected to strong rectilinear horizontal movements; illustrative diagrams being given in Pls. III and IV.

EXP. 1. Stone plate No. 1 ($24 \times 39 \times 76$ cm), of whose two faces one was smooth and the other rough, put on the Table longitudinally, i.e., parallel to the direction of motion. (See Fig. 2.)

Sliding began at $2a=82$ mm, $T=$?. Max. $2a=91$ mm, $T=0.36$ sec., $A=13840$ mm/sec^2. During the shaking, the sliding amounted to about 80 mm. After the experiment, the stone was found displaced 32 mm.

EXP. 2. Stone plate No. 1, put broadside on, namely, with its longer side perpendicular to the direction of motion.

Sliding began at $2a=96$ mm, $T=0.43$ sec., $A=10180$ mm/sec^2., this latter being also the maximum intensity of motion during the experiment.

Amount of sliding$=85$ mm.

EXP. 3. Same experiment repeated.

Sliding began at $2a=86$ mm, $T=0.44$ sec., $A=8770$ mm/sec^2.

Max. $2a=99$ mm, $T=0.36$ sec., $A=15100$ mm/sec^2.

Amount of sliding$=75$ mm.

EXP. 4. Two exactly similar stone plates, No. 1 and No. 2, put broadside on, one upon the other ; with their smooth surfaces in contact, so that the lower plate, rested with its rough surface on the Shaking Table. (See Fig. 3.)

Sliding began at $2a=84$ mm, $T=1.0$ sec., $A=1660$ mm/sec^2. The upper plate tumbled over the lower at $2a=91$ mm, $T=0.44$ sec., $A=9300$ mm/sec^2.

Max. $2a=101$ mm, $T=0.35$ sec., $A=16240$ mm/sec^2.

EXP. 5. Two stone plates, No. 1 and No. 2, put broadside on, the upper one resting with its rough surface on the smooth surface of the lower. (See Fig. 4.)

Sliding began at $2a=84$ mm, $T=0.62$ sec., $A=4320$ mm/sec^2., the two plates moving together as a single body.

Max. $2a=94$ mm, $T=0.29$ sec., $A=22050$ mm/sec^2.
The amount of sliding was about 160 mm., accompanied by some rotation.

Exp. 6. Two stone plates, No. 1 and No. 2, put one upon the other with their rough surfaces in contact. The lower plate was fixed in position by means of wooden beams and nails.

Sliding began at $2a=91$ mm, $T=0.50$ sec., $A=7180$ mm/sec^2.

Max. $2a=99$ mm, $T=0.43$ sec., $A=10560$ mm/sec^2.

On account of the violence of motion, the wooden beams gave way, and the lower plate rotated slightly, while the upper plate slid on the lower through a distance of 160 mm.

Exp. 7. Same experiment repeated; the lower plate being fixed tightly by means of two strong wooden beams bolted to the Shaking Table. (See Figs. 5 and 6.)

During the strongest part of the shaking the upper plate moved through a distance of about 60 mm. When, however, the motion was decreasing, the sliding of the upper plate became much larger, the final displacement amounting to 185 mm. The lower plate slid transversally 32 mm within the timber bracings. Sliding (of the upper plate) began at $2a=86$ mm, $T=0.52$ sec., $A=6260$ mm/sec^2.

Max. $2a=99$ mm, $T=0.40$ sec., $A=12220$ mm/sec^2.

Exp. 8. Two square hollow wooden columns or boxes (i) and (ii) each put side by side, longitudinally, and on one of its sides. (See Fig. 7.)

Sliding began at $2a=88$ mm, T$=0.47$ sec., $A=7860$ mm/sec^2.

Max. $2a=106$ mm, $T=0.48$ sec., $A=9050$ mm/sec.^{2}

The amount of sliding of the column (i) was 370 mm, and that of the column (ii) was 550 mm; the sliding being accompanied in each case by some lateral movement and slight rotation.

EXP. 9. Same experiment repeated.

Sliding began at $2a=86$ mm, $T=0.50$ sec., $A=6800$ mm/sec^2.

Max. $2a=98$ mm, $T=0.35$ sec., $A=15800$ mm/sec.^{2}

The amount of sliding of the column (i) was 500 mm, and that of the column (ii) was 1000 mm.

EXP. 10. The same two boxes, within each of which was placed a block of masonry composed of 12 bricks.

Sliding began at $2a=88$ mm, $T=0.48$ sec., $A=7510$ mm/sec^2.

Max. $2a=95$ mm, $T=0.32$ sec., $A=18300$ mm/sec^2.

The amount of sliding of the column (i) was 50 mm, and that of the column (ii) was 900 mm.

EXP. 11. Same experiment repeated.

Sliding began at $2a=92$ mm, $T=0.45$ sec., $A=8960$ mm/sec^2.

Max. $2a=98$ mm, $T=0.37$ sec., $A=14140$ mm/sec^2.

The column (i) was only slightly displaced, while the column (ii) slid through a distance of 940 mm.

EXP. 12. A wooden box ($65\times65\times32$ cm) filled with concrete.

Sliding began at $2a=87$ mm, $T=0.55$ sec., $A=5700$ mm/sec^2.

Max. $2a=96$ mm, $T=0.45$ sec., $A=9340$ mm/sec^2.

EXP. 13. A wooden box ($76\times54\times32$ cm) filled with concrete.

Sliding began at $2a=93\frac{1}{2}$ mm, $T=0.47$ sec., $A=8340$ mm/sec^2.

Max. $2a=98$ mm, $T=0.43$ sec., $A=10430$ mm/sec^2.

The box moved 125 mm, accompanied by some rotation.

SUMMARY. The results obtained from the preceding experiments are summarized in the following table.

TABLE I. EXPERIMENTS ON SLIDING.

No. of Expt.	Friction Surfaces.	Amount of Sliding.	Intensity of motion at which Sliding began.	Maximum intensity of motion during each Experiment.
		mm	mm/sec².	mm/sec².
1	Stone (rough) on wood	32	—	13840
2	„ „	85	10180	10180
3	„ „	75	8770	15100
	Mean..........................		9480	
4	Stone on stone (smooth surfaces)	200	1660	16240
5	Stone on stone (rough and smooth surfaces)	160	4320	22050
6	Stone on stone (rough surfaces)	160	7180	10560
7	„ („)	185	6260	12220
	Mean..........................		6720	
8	Wood on wood.	550	7860	9050
9	„ „	500	6800	15800
10	„ „ (with load)	900	7510	18300
11	„ „ („)	940	8960	14140
12	„ „ („)	—	5700	9340
13	„ „ („)	125	8340	10430
	Mean..........................		7530	
Experiments Nos. 14–23. next §	Wood on wood.	6–26½ inches	5820	

The maximum range of motion, or double amplitude, of the Shaking Table varied between 91 and 106 mm. The actual amount of the sliding was in most cases far greater than these limits, and reached, when the shaking was continued in fovourable conditions and sufficiently long, from 500 mm to nearly 1 metre. Again, a markedly large amount of sliding was produced, not necessarily in the epoch of the maximum intensity of the movement of the Shaking Table, but often when the shaking was being rapidly lessened.

From these experimental results it may be inferred that in the case of a violent prolonged earthquake the amount of the sliding of bodies may be several times larger than the actual range of motion of the ground, being the accumulation of the effects due to a series of different vibrations composing the principal portion of the seismic disturbance.

According to the preceding table, the mean intensity of motion at which the sliding began was, for the wooden friction surfaces, 5820 to 7530 mm/sec^2., which are practically equal to the calculated values given in (5) and (6). For the rough stone friction surfaces, the intensity of motion in question was 6720 mm/sec^2., which is again nearly identical with the calculated value given in (8). With the contact of stone surfaces, one of which was smooth and the other rough, the sliding took place at an intensity of motion of 4320 mm/sec^2.; while with the contact of two smooth surfaces, the sliding began already at an intensity of 1660 mm/sec^2. The experimental value of the intensity of motion found for the stone and wooden friction surfaces was appreciably higher than the calculated one given in (7), the discrepancy arising probably from the fact that the platform of the Shaking Table, which was of wood, was not very hard, giving in consequence a high value of the coefficient of friction between it and the heavy stone plates placed on it. On the whole the formula (4) seems to give the approximate intensity of the earthquake motion required to produce the sliding of different friction surfaces. Japanese houses, temples, etc., which are wooden framed structures resting on the foundations of stones, may be regarded as cases, which involve the contact of wooden and stones friction surfaces.

4. ***Effect of overloading.*** To test the effect of overloading on the sliding of a body, I have taken two similar flat square wooden boxes or rather vessels (I) and (II) of the form and dimensions indicated in Fig. 8. These were placed side by side on the Shaking Table, and subjected to strong horizontal movements, one being empty and the other loaded with a cubical block of brickwork (W_1), about 25 kg in weight, or with a concrete block (W_2) measuring $65 \times 65 \times 32$ cm. It was found that the two boxes always began, and stopped, to slide practically at the same instant. The details of the experiments are given in the following table.

Fig. 8.

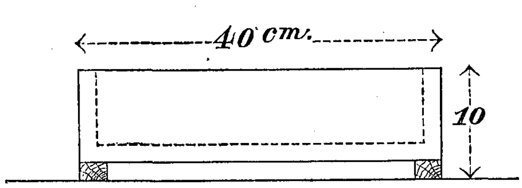


TABLE II.

No. of Expt.	Sliding began at			Sliding stopped at			Remarks.
	$2a$	T	A	$2a$	T	A	
	mm	sec.	mm/sec^2.	mm	sec.	mm/sec^2.	
14	87	0.67	3830	84.5	0.45	8240	(I) with the load W_1.
15	87.5	0.45	8520	—	—	—	Two boxes interchanged in position; (II) with the load W_1.
16	87	0.50	6860	86	0.65	4020	Same experiment repeated.
17	86.5	0.51	6550	86	0.53	6030	(I) with the load W_1.
18	86.5	0.54	5850	84.5	0.53	5920	(I) and (II) interchanged in position; (I) with the load W_1.
19	85.5	0.48	7300	84.5	0.52	6170	Same experiment repeated.

No. of Expt.	Sliding began at			Sliding stopped at			Remarks.
	$2a$ (mm)	T (sec.)	A (mm/sec^2.)	$2a$ (mm)	T (sec.)	A (mm/sec^2.)	
20	84.5	0.64	4080	84.5	0.50	6680	(I) and (II) interchanged in position; (II) with the load W_1.
21	85	0.65	3970	88	0.54	5950	(II) loaded with the weight W_2, rigidly fixed.
22	85	0.70	3430	85	0.50	6720	(I) and (II) interchanged in position.
23	84.5	0.54	5710	83.6	0.60	4580	Same as before, only the weight W_2 not rigidly fixed.
Mean.	85.6	0.57	5610	84.7	0.53	6030	

According to the above table, the sliding began and stopped on the average respectively at $A=5610$ mm/sec^2. and $A=6030$ mm/sec^2., the mean of these two values being 5820 mm/sec^2.

In most cases, the movement of the boxes during the shaking was greater than the final displacement or the amount of sliding found after the experiment. The motion of the Shaking Table, corresponding to the commencement of the markedly active sliding of the boxes, was as follows :—

Exp. 18.... $2a=87.0$ mm, $T=0.42$ sec., $A=9740$ mm/sec^2.
Exp. 19.... $2a=92.4$,, $T=0.35$,, $A=14900$,,
Exp. 20.... $2a=88.5$,, $T=0.39$,, $A=11500$,,

The amount of the final displacements of the two boxes in the different experiments is shown in the following table :—

TABLE III. EFFECT OF OVERLOADING ON THE SLIDING.

No. of Experiment.	Box *without* Load. Displacement $=d_1$.		Box *with* Load. Displacement $=d_2$.		Ratio, d_2/d_1
		inches		inches	
14	(II)	—	(I)	—	—
15	(I)	4	(II)	7	1.8
16	(I)	2	(II)	6	3.0
17	(II)	3.5	(I)	14.5	4.1
18	(II)	11.5	(I)	18	1.6
19	(II)	13	(I)	22	1.7
20	(I)	10	(II)	18	1.8
21	(I)	8.5	(II)	26.5	3.1
22	(I)	3.5	(II)	12	3.4
23	(I)	4	(II)	10.5	2.6
Mean	....	6.7	....	14.9	2.6

Thus it will be seen that the sliding of the boxes when loaded was on the average 2.6 times larger than when empty, due probably to the raising of the height of centre of gravity in the former case. It is likely, as may be inferred from the relations (1), (2) and (3), that the phenomena of the displacement depend essentially on the coefficient of friction between the surfaces in contact, and is not influenced, within certain limits at least, by the absolute weight of the sliding body.

OVERTURNING OF COLUMNS.

5. ***Cases of large amplitude.*** In the *Publications*, No. 4, are given the results of a series of the experiments on the overturning by horizontally applied motion of 42 uniform section columns, whose height $2y$ varied between 242 and 1150 mm, and whose base dimension $2x$ varied between 90 and 300 mm; the least and greatest values of the ratio y/x being respectively 3.0 and 8.0. The columns, with the exception of two iron tubes, were of wood or of brick, the section being square or hollow square, as follows:—

Columns	k_1, k_2	Iron tube.
„	l_1, l_2, l_3,	Brick, solid square.
„	m_1, m_2, n, o_1, o_2	Brick, hollow square.
„	$A_1, A_2, B, C_1, C_2, C_3, D_1, D_2, D_3,$ $E_1, E_2, E_3, E_4, F_1, F_2, F_3, F_4, F_5,$	Wood, hollow square.
„	$G_1, G_2, G_3, G_4, G_5, G_6, G_7,$ $H_1, H_2, H_3, H_4, H_5, H_6, H_7,$	Wood, solid square.

The overturning experiments carried on in 1898–9 were 170 in number, and may, with regard to the double amplitude $2a$, and the complete vibration period, T, of the Shaking Table, be divided into the following three groups:—

Group	(i)	: $2a = 100^{\text{mm}} — 117^{\text{mm}}$	; $T = 0.63^{\text{sec.}} — 1.31^{\text{sec.}}$	....(87 Expts)
„	(ii)	: $2a = 55 — 57\frac{1}{2}$	; $T = 0.54 — 0.81$	....(4 „)
„	(iii)	: $2a = 29\frac{1}{2} — 35$	; $T = 0.41 — 0.91$	....(79 „)

An examination of the results of the experiments shows that, for the group (ii), in which the $2a$ was large and varied between 100 mm and 117 mm, the seismic stability of the columns against overturning, namely, $a = \frac{x}{y} \times g$, agreed very well with the maximum acceleration of the motion of the Shaking Table at the moment of the overturning, namely, $A = 4\pi^2 a/T^2$ as will be seen from the following table.

TABLE IV. OVERTURNING OF COLUMNS.

($2a = 100$–117 mm.)

Material.	Column.	Calculation.			Experiment.			Ratio
		$2x$	$2y$	α	$2a$	T	A	A/α
Square brick column (solid)	l_1	mm 230	mm 1150	mm/sec². 1960	mm 100	sec. 0.89	mm/sec². 2480 (1)	1.27
,,	l_2	230	700	3220	104	0.69	4280 (1)	1.33
,,	l_3	230	660	3400	114	0.77	3780 (1)	1.11
Square brick column (hollow)	m_1	230	1100	2050	110	1.11	1760 (1)	0.86
,,	m_2	230	790	2860	111–110	0.92	2620 (2)	0.92
,,	n	233	900	2540	111–110	0.95	2430 (2)	0.96
,,	o_1	185	950	1910	108	1.27	1310 (1)	0.69
,,	o_2	183	600	3040	111	0.83	3180 (1)	1.05
Iron (tube)	k_1	152	940	1580	113–110	1.31	1360 (2)	0.86
,,	k_2	150	480	3060	114–113	0.92	2630 (3)	0.86
Square wooden column (hollow)	A_1	300	900	3260	114	0.79	3600 (1)	1.11
,,	B	274	818	3290	116–114	0.73	4420 (2)	1.34
,,	C_1	242	970	2450	113–112	0.94	2540 (2)	1.04
,,	C_2	242	727	3260	116–111	0.76	4040 (4)	1.24
,,	D_1	210	850	2420	113–110	0.92	2640 (3)	1.09
,,	D_2	210	634	3250	114–111	0.82	3330 (4)	1.03
,,	E_1	180	908	1940	112	1.05	2010 (2)	1.04
,,	E_2	180	727	2430	113–110	0.94	2420 (7)	1.00

Material.	Column.	Calculation.			Experiment.			Ratio
		$2x$	$2y$	a	$2a$	T	A	A/a
		mm	mm	mm/sec^2.	mm	sec.	mm/sec^2.	
Square wooden column (hollow)	E_3	180	544	3240	114	0.81	3440 (3)	1.06
,,	E_4	180	362	4870	116	0.63	5850 (2)	1.21
,,	F_1	152	910	1640	113–110	1.24	1480 (2)	0.90
,,	F_2	152	754	1960	112	1.09	1860 (1)	0.95
,,	F_3	152	602	2470	112	1.04	2040 (1)	0.83
,,	F_4	152	450	3310	114–113	0.87	3000 (2)	0.91
,,	F_5	152	304	4900	117–116	0.66	5270 (2)	1.08
Square wooden column (solid)	G_1	120	970	1210	110	1.30	1280 (1)	1.06
,,	G_2	120	844	1390	110	1.30	1280 (1)	0.92
,,	G_3	120	728	1620	111	1.28	1330 (1)	0.82
,,	G_4	120	596	1970	112–108	1.20	1520 (2)	0.77
,,	G_5	120	480	2450	112–110	1.02	2120 (3)	0.87
,,	G_6	120	362	3260	114–111	0.84	3040 (6)	0.90
,,	G_7	120	242	4860	116	0.66	5200 (3)	1.07
,,	H_1	90	726	1220	111	1.24	1430 (2)	1.17
,,	H_2	90	666	1330	111	1.24	1430 (2)	1.08
,,	H_3	90	638	1390	11 –111	1.20	1540 (2)	1.11
,,	H_4	90	544	1620	112	1.15	1710 (2)	1.06
,,	H_5	90	454	1940	112	1.13	1740 (2)	0.90
,,	H_6	90	362	2440	113	1.02	2160 (2)	0.89
,,	H_7	90	270	3260	116–110	0.87	3040 (3)	0.93

In the above table, the values of the $2a$, T, and A for each column are the means deduced from a number of experiments made on the overturning of the latter, as indicated by a figure enclosed in brackets put after the A. The general average value of the ratio A/α, which varied in the majority of cases between 0.8 and 1.2, was 1.007; thus indicating the almost perfect identity of α and A when the amplitude of motion is sufficiently large. The base dimension $2x$ of the different columns was from 90 to 300 mm, namely, 0.8 to 2.6 times the range of motion $2a$. For the experiments of the groups (ii) and (iii), in which the range of motion of the Shaking Table was smaller and varied between $29\frac{1}{2}$ and $57\frac{1}{2}$ mm, the agreement of the two quantities was not so uniform.

6. ***Cases of comparatively small amplitude.*** The second series of experiments carried on in 1900, related to the overturning of 17 uniform section columns, also by the horizontally applied motion. Eight of the columns were of wood and hollow square in section, and the dimension ($2x$) of whose external side varied between 186 and 455 mm; while the remaining nine were solid or hollow brick columns, square or rectangular in section, whose external dimension ($2x$) in direction of the motion of the Shaking Table varied between 113 and 342 mm. The height $2y$ varied between 358 and 1821 mm, the least and greatest values of the ratio y/x being respectively 3.0 and 11.0. The sectional forms of the different columns are shown in Figs. 9–17, Pl. II.

TABLE V. OVERTURNING EXPERIMENTS (1900.) LIST OF COLUMNS.

No. of Column.	Material.	Height.	Section.
1	Wood.	1821 mm	Hollow Square: $\overline{445}^2-\overline{420}^2$ mm
2	”	1217	” $\overline{304}^2-\overline{268}^2$
3	”	911	” ”
4	Brick* ($\frac{1}{5}$)	1161	” $\overline{186}^2-\overline{143}^2$
5	” (”)	1125	” ”
6	” (”)	996	” ”
7	” (”)	988	” ”
8	” ($\frac{1}{2}$)	1098	” $\overline{224}^2-\overline{118}^2$
9	” ($\frac{1}{2}$)	1243	Rectangle: 113×228
10	” (”)	461	” ”
11	” (”)	358	” ”
12	” ($\frac{1}{2}$)	895	Hollow Rectangle: $\left\{\begin{matrix}226 \times 348 \\ -117 \times 240\end{matrix}\right.$
13	” (”)	891	” ”
14	” (1)	820	Square: $\overline{224}^2$
15	” (”)	1015	” $\overline{219}^2$
16	” (1)	1672	Hollow Square: $\overline{342}^2-\overline{123}^2$
17	” (”)	1464	” ”

* The figures $\frac{1}{5}$, $\frac{1}{2}$, and 1, enclosed in brackets, indicate that the linear dimensions of the bricks were respectively $\frac{1}{5}$ of, $\frac{1}{2}$ of, and equal to, those of ordinary bricks.

The $2a$ of the Shaking Table in the different experiments was from 18.6 to 99 mm, the period varying between 0.29 and 1.08 sec. The amplitude was in many cases small in comparison to the base dimension of the columns, so that some of the latter could not be overturned even with the corresponding utmost intensity of motion of the Shaking Table. I give next the results of the experiments relating to those columns which were overturned.

TABLF VI. COLUMNS EASILY OVERTURNED.
($2a=62.5-87.7$ mm)

No. of Column.	Calculation.			Experiment.			Ratio A/a
	$2x$	$2y$	$a=\frac{x}{y}\times g$	$2a$	T	A	
	mm	mm	mm/sec^2.	mm	sec.	mm/sec^2.	
6	186	996	1830	87.7	0.92	2040	1.12
4	186	1161	1570	87.5	0.88	2230	1.42
5	186	1125	1620	86.6	0.92	2020	1.25
7	186	988	1845	86.5	0.87	2260	1.23
15	221	1015	2134	85	0.80	2620	1.23
8	224	1098	1999	87	0.88	2220	1.11
14	225	1012	2179	62.5	0.77	2080	0.96
10	113	461	2402	87	0.81	2610	1.09
11	113	358	3093	87	0.73	3210	1.04
					Mean..	....	**1.16**

The base dimension $2x$ of the nine columns here considered was 113 to 225 mm, or 1.3 to 3.6 times the range of motion $2a$, which varied between 62.5 to 87.7 mm. The average value of the ratio A/a was 1.16, indicating that the intensity of motion, A, required for overturning the columns was slightly greater than the calculated quantity a. The difference $(A-a)$ became markedly large with the increase of the base dimension $2x$ relative to the extent of motion $2a$. Thus, for the column No. 9 (Table IX), for which the ratio $2x/2a$ was 6.1, the value of the overturning acceleration A was nearly three times greater than the quantity a. Finally, the two columns Nos. 16 and 17 and the two columns Nos. 10 and 11 could not be overturned respectively with the ranges of motion of 60 mm and of 20.5 mm, as shown in the following table.

TABLE VII. COLUMNS NOT OVERTURNED.

(Max. $2a \doteq 60$ mm or 20.5 mm.)

No. of Column.	$2x$	$2y$	$a=\frac{x}{y}\times g$	Maximum Motion.		
				$2a$	T	A
	mm	mm	mm/sec^2.	mm	mm	mm/sec^2.
16	342	1672	2005	58.5	0.43	6245
				57	0.52	4161
				60	0.42	6714
17	342	1464	2289	55	0.60	3016
				60	0.39	7787
10	113	461	2402	20.5	0.29	4810
11	113	358	3093	”	”	”
C_3 *	242	484	4900	119	0.51	9000
				120	0.47	10700

* This is taken from the first series of the overturning experiments, the "Publications," No. 4.

The ratio $2x/2a$ was about 5.7 for the two columns Nos. 16 and 17, and 5.5 for the two columns Nos. 10 and 11. The column C_3 for which the ratio y/x was only 2, could not be overturned on account of the smallness of height $2y$.

ROCKING OF COLUMNS.

7. The rocking phenomena were markedly shown by the short and large columns, which could not be overturned even when subjected to strong movements of the Shaking Table. In the case of the columns of small base dimension $2x$, the rocking took place immediately before, or almost simultaneously with, the overturning. Again, when the motion of the Shaking Table was very quick, the period of the rocking of the columns was the same as that of the latter. When, however, the motion of the Shaking Table was comparatively long, the columns made rockings with their proper periods. I give next the period τ corresponding to the movements of different amplitudes of the three brick columns, Nos. 18, 19, and 20, deduced directly from the diagrams obtained by fixing a pointer to the top of each of the columns, which were caused to make rockings by being pushed with hand; the motion being written on a record-receiver put on an independent support.

TABLE VIII. PERIOD OF ROCKING.

Brick Column.	$2a$ at the top of Column (Individual Vibration.)	Period of Rocking $=\tau$.
No. 18 (230* × 226 × 2078 mm)	170.0 mm — 140.0 mm	1.58 sec.
	131.5—123.0	1.30

Brick Column.	$2a$ at the top of Column (Individual Vibration.)	Period of Rocking $=\tau$.
No. 18 (230* x 226 x 2078 mm)	117.5 mm — 111.0 mm	1.22 sec.
	106.4—101.0	1.20
	95.5— 90.0	—
	84.5— 80.0	1.13
	77.2— 73.2	0.92
	70.0— 66.5	0.86
	63.8— 61.0	0.90
No. 19 (111* x 228 x 1738 mm)	45.0— 34.0	0.99
	55.0— 37.0	1.01

Brick Column.	$2a$.	Average T.
No. 20 (225 x 225 x 1827 mm)	60.0 mm — 37.0 mm	0.71 sec.
	71.0— 37.0	0.75

Thus, for the column No. 18, whose base dimension $2x$ was 230 mm, or equal to 1/9 of the height $2y$, the period of rocking varied from 0.90 to 1.58 sec., within the limits of $2a$ between 61 and 170 mm. For the column No. 20, whose $2x$ was 225 mm, or 1/8 of $2y$, the period was 0.71 to 0.75 sec., within the limits of $2a$ of 37 to 71 mm. Finally, for the column No. 19, whose $2x$ was only 111 mm, or 1/16 of $2y$, the period of rocking was about 1.0 sec. at the $2a$ of 34 to 55 mm. From these examples it may be inferred that the rocking lengthens its period rapidly with the increase of the amplitude. The period of rocking of the

* Sectional dimension in direction of the motion (rocking).

taller and larger ones among the columns used in the 2nd series of the experiments (Table V), when the latter were about to be overturned, would probably be some 2 sec., being thus considerably longer than the period of the motion of the Shaking Table at the maximum of its intensity. The following table gives the elements of motion relating to the rocking of some of the columns. (See Pls. V and VI.)

TABLE IX. OVERTURNING AND ROCKING OF COLUMNS.

No. of Column.	$2x$	$2y$	$v=\frac{y}{x}\times g$	Rocking began at			Max. Motion.			Overturned at		
				$2a$	T	A	$2a$	T	A	$2a$	T	A
	mm	mm	mm/s²	mm	sec.	mm/s²	mm	sec.	mm/s²	mm	sec.	mm/sec².
				86	0.85	2350	—	—	—	87	0.46	8100
				87	0.81	2610	88.5	0.46	8250	(Not overturned)		
1	455	1821	2449	87	0.82	2490	88.5	0.53	6210	(	,,	)
				—	—	—	73	0.45	7100	(	,,	)
				—	—	—	99	0.34	16900	(	,,	)
				Mean	....	**2480**						
				—	—	—	73	0.45	7100	(Not overturned)		
3	303	911	3260	—	—	—	99	0.34	16900	(	,,	)
				—	—	—	53	0.35	8540	(	,,	)
				—	—	—	—	—	—	86	0.66	3880
				—	—	—	—	—	—	81	0.96	1740
				—	—	—	—	—	—	81	0.60	4450
				—	—	—	—	—	—	86	0.75	3020
2	305	1217	2456	—	—	—	—	—	—	85	0.69	3520
				—	—	—	—	—	—	86	0.71	3370
				87	0.88	2220				Mean	......	**3220**

No. of Column.	$2x$	$2y$	$a=\frac{x}{y}\times g$	Rocking began at			Max. Motion.			Overturned at		
				$2a$	T	A	$2a$	T	A	$2a$	T	A
	mm	mm	mm/s²	mm	sec.	mm/s²	mm	sec.	mm/s²	mm	sec.	mm/s²
14	225	1012	2179	62	0.85	1700	—	—	—	62.5	0.77	2080
13	226	891	2486	87	0.81	2620	—	—	—	90.0	0.62	4620
12	226	895	2475	87	0.89	2170	—	—	—	90.0	0.66	4080
9	113	1243	891	—	—	—	—	—	—	18.6	0.38	2540

In one of the five experiments with the $2a$ of 73 to 99 mm, the column No. 1, of $2x=455$ mm, was overturned accidentally at an acceleration of 8100 mm/sec²., but could not in the other four cases be overturned with the acceleration of 6210 to 16900 mm/sec². Again, in one of the four experiments with the $2a$ of 53 to 99 mm, the column No. 3, of $2x=303$ mm, was overturned at an acceleration of 3880 mm/sec²., but could not be overturned in the other three cases even with the acceleration of 7100 to 16900 mm/sec². The columns Nos. 2, 12, and 13 were overturned with the $2a$ of 62.5 to 90 mm at accelerations much higher than their a'. The rocking of the different columns, however, occurred when the intensity, A, of the motion of the Shaking Table became nearly equal to $a=\frac{x}{y}\times g$, as follows :—

Col. No. 1 : $a = 2449$ mm/sec². ...... Rocking began at $A = 2480$ mm/sec².
„ No. 2 : $a = 2456$ ...... „ „ $A = 2220$
„ No. 14 : $a = 2179$ ...... „ „ $A = 1700$
„ No. 13 : $a = 2486$ ...... „ „ $A = 2620$
„ No. 12 : $a = 2475$ ...... „ „ $A = 2170$

Mean.... : $a = 2409$ *Mean*.......... $A = 2224$

At the epoch of the maximum motion of the Shaking Table the period of the latter varied from 0.34 to 0.53 sec., being much shorter than the natural rocking period of the columns. Under these circumstances, the columns made forced rockings, having the same period as the motion of the Shaking Table.

This sort of effect is entirely different from that caused by the synchronizing of the rocking motion of a body with, for instance, regular vibrations of the ground. Thus, the infinitesimally small and insensible tremors of the ground due to the working of a dynamo machine or a steam engine may cause considerable rattling of windows or rocking of bottles put on a table even at a distance from the source of disturbance. In such cases, the maximum acceleration of the motion of the ground may be less than 1 mm/sec².*

Effect of Vertical Motion on Sliding and Overturning.

8. If there exist, in addition to the horizontal earthquake motion, also more or less considerable amount of the vertical component, the relation (4) is, for the case of the sliding of bodies, to be modified into

$$a' = \mu (g - a'') \qquad (9)$$

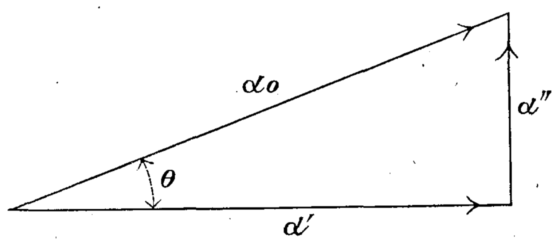
in which a' and a'' are respectively the horizontal and vertical seismic accelerations. The a in the equation (4) is, in the case under consideration, to be regarded as the *equivalent* horizontal intensity of motion which alone produces the sliding effect equal to that due to the coexistence of a' and a''. From the equations (4) and (9) we have:—

$$\frac{g}{a} = \frac{g}{a'} - \frac{a''}{a'} \qquad (10)$$

* See the "Publications", No. 18.

If α_0 be the actual, or resultant, intensity of the earthquake motion, we have :—

Fig. 18.



$$\alpha = \alpha_0 \cos\theta\,; \quad \alpha'' = \alpha_0 \sin\theta\,;$$

in which θ is the angle of emergence. Equation (10) can be changed into the following two forms :—

$$\frac{g}{\alpha'} = \frac{g}{\alpha} + \tan\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$\alpha_0 \left(\cos\theta + \frac{\alpha}{g}\sin\theta\right) = \alpha \ldots\ldots\ldots\ldots (12)$$

For the case of the overturning of a column, the equation (1) is to be changed into

$$\alpha' = \frac{x}{y}(g - \alpha'')\,, \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

which leads to the relation

$$\alpha_0 \left(\cos\theta + \frac{x}{y}\sin\theta\right) = \alpha \quad \ldots\ldots\ldots\ldots (14)$$

This equation is, in virtue of (1), can also be put into the form of the equation (12).

Equation (12) or equation (14) enables us to calculate the value of the real earthquake intensity α_0 from the angle of emergence θ and the equivalent horizontal intensity α. It will be observed that the difference in the values of the real and equivalent intensities for the case of the sliding is the same as that for the case of the overturning.

From the equations (12) and (14), we see that, for a given equivalent intensity α, the value of the real intensity

(i) α_0 is minimum, when $\theta = \theta_1$, which angle is determined by the equation

$$\tan \theta_1 = n \,; \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

where $n = \frac{\alpha}{g}$, or $n = \frac{x}{y}$;

(ii) α_0 is equal to α, when $\theta = 0^\circ$, and also when $\theta = \theta_2$, which latter angle is determined by the equation

$$\tan \theta_2 = \frac{2n}{1-n^2}, \ldots\ldots\ldots\ldots (16)$$

n having the same meaning as in (15). From the equations (15) and (16), it will at once be observed that

$$\theta_2 = 2\theta_1 \quad \ldots\ldots\ldots\ldots\ldots (17)$$

Thus, for instance, if $\alpha = 4000$ mm/sec^2., then n is equal to 40/98 , and the real intensity α_0 attains its minimum value of 3702 mm/sec^2. at an angle of emergence of 22° 12′, while it becomes equal to 4000 mm/sec^2. at an angle of emergence of 44° 24′. Again, if $\alpha = 4900$ mm/sec^2., then n is equal to 0.5 and α_0 has the minimum value of 4383 mm/sec^2. at an angle of emergence of 26° 34′, and becomes equal to 4900 mm/sec^2. at an angle of emergence of 53° 8′.

To see the amount of difference between the actual and the equivalent horizontal intensities in the case of a very violent earthquake motion, I give below, for $\alpha = 4000$ mm/sec^2., and $\alpha = \frac{g}{2} = 4900$ mm/sec^2., the values of the accelerations α_0, α', and α'', corresponding to different angles of emergence θ, from 0° up to 70°.

TABLE X. RELATIONS BETWEEN THE ACTUAL AND THE EQUIVALENT HORIZONTAL INTENSITIES.

$a=4000$ mm/sec².

Angle of Emergence $=\theta$.	Actual Intensity $=a_0$	$a_1=$ Horizontal component of a_0	$a''=$ Vertical component of a_0
0°	4000 mm/sec².	4000 mm/sec².	0 mm/sec².
5°	3877	3862	338
10°	3789	3732	658
15°	3733	3605	966
20°	3706	3483	1268
22° 12′	**3703** (min.)	3429	1399
25°	3708	3360	1567
30°	3738	3237	1869
35°	3798	3111	2178
40°	3890	2981	2500
44° 24′	**4000** $(=a)$	2858	2799
45°	4016	2840	2840
50°	4186	2691	3207
55°	4406	2527	3609
60°	4687	2343	4059
65°	5047	2133	4574
70°	5513	1886	5180

TABLE XI. RELATIONS BETWEEN THE ACTUAL AND THE EQUIVALENT HORIZONTAL INTENSITIES.

$$a = \frac{g}{2} = 4900 \text{ mm/sec}^2.$$

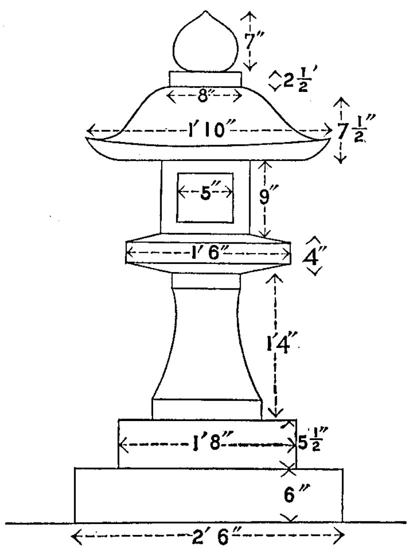
Angle of Emergence $= \theta$	Actual Intensity $= a_0$	$a' =$ Horizontal component of a_0	$a'' =$ Vertical component of a_0
0°	4900 mm/sec².	4900 mm/sec².	0 mm/sec².
5°	4713	4695	411
10°	4573	4503	794
15°	4474	4321	1158
20°	4412	4146	1509
25°	4384	3974	1853
26° 34′	**4383** (min.)	3920	1960
30°	4391	3802	2195
35°	4431	3630	2541
40°	4506	3452	2896
45°	4620	3267	3267
50°	4777	3070	3659
53° 8′	**4900** $(=a)$	2940	3921
55°	4984	2859	4083
60°	5252	2626	4548
65°	5595	2365	5071
70°	6036	2064	5671

According to the first of the above tables, the value of the actual intensity a_0, corresponding to the given equivalent horizontal intensity of $\alpha = 4000$ mm/sec^2., varied, for the angle of emergence θ of 0° up to 55°, between 3703 mm/sec^2. (minimum) and 4406 mm/sec^2.; the maximum amount of the difference between the two quantities, namely, $\alpha_0 - \alpha$, being within the limits of θ here considered -297 and $+406$ mm/sec^2. Especially, for a place at an epicentral distance equal to the focal depth, for which θ is 45°, and where the horizontal intensity of motion is maximum, the values of α_0 and α are practically identical. Again, for $\alpha = g/2 = 4900$ mm/sec^2., the value of the actual intensity α_0 varied, for the angle of emergence θ of 0° up to 55° between 4383 mm/sec^2. (minimum) and 5252 mm/sec^2. ; the difference $\alpha^0 - \alpha$ varying between -517 and $+352$ mm/sec^2., and being equal to 280 mm/sec^2. for $\theta = 45^\circ$. Thus, for the cases of $\alpha = 4000$ and $\alpha = 4900$ mm/sec^2., the real earthquake intensity α_0 does not differ by much above 10 % of its amount from the equivalent horizontal intensity, or the acceleration estimated on the supposition that the motion was entirely horizontal.

In the *Publications*, No. 4, I have given the values of the equivalent horizontal intensity α determined from the observations of overturned bodies at the different places within the meizoseismal area of the great Mino-Owari earthquake of Oct. 28, 1891. Amongst others, the values of α for the two cities of Nagoya and Gifu were found respectively to be 2600 mm/sec^2. and 3000 mm/sec^2. Judging from the ordinary seismograph record of the earthquake motion obtained at the Meteorological Observatory of Nagoya, the amplitude of the vertical component there seemed to be about one-third of the horizontal, corresponding to an angle of emergence of $18^\circ 26'$. For this value of θ, the actual intensities α_0 for the two places in question would be respectively 2518 and 2869 mm/sec^2. ; the difference $\alpha - \alpha_0$ being 82 and 131 mm/sec². Again, if we suppose the θ to be 45°, the values of α_0 for Nagoya and Gifu would be respectively 2906 and 3248 mm/sec².; the difference $\alpha_0 - \alpha$ being 306 and 248 mm/sec²., or 1/8 and 1/12 of the α in the two cases. From these examples it may be assumed that the values of the α given in the *Publications* No. 4, do not materially differ from those of the α_0 even where the angle of emergence was considerable,

9. *Sliding of bodies in the epifocal region of the Mino-Owari earthquake.* In the "Publications", No. 12, I have described some of the seismic effects on *ishidoro* (or stone lanterns for Japanese gardens and temple grounds, one of which is illustrated in Fig. 19) in the Neo-Valley, the epifocal zone of the great Mino-Owari earthquake of 1891. On account of the violence of motion, these were not only overthrown, but had their pedestals displaced to the maximum amount of 2′ 6″; the sliding in these cases being that of stone surfaces, one of which is rough and the other smooth. According to Exp. 5 (§ 3) which relates to the contact of the sort under consideration, the sliding first took place when the acceleration of the motion of the Shaking Table reached 4320 mm/sec². Thus the equivalent horizontal intensity α of motion in the Neo-Valley, where the sliding phenomena were remarkable, is to be assumed to have been over $g/2$ or 5000 mm/sec².

Fig. 19.



With regard to the range of motion in the destructive area of the Mino-Owari earthquake, the $2a$ in the city of Nagoya, where the α was equal to 2600 mm/sec²., was probably about 233 mm/sec²., the period of vibration being assumed to be 1.3 sec. (See the *Publications*, No. 4.). With the assumption of the same period for Gifu, where α was equal to 3000 mm/sec²., the range of motion would have been about 257 mm. In both of these cities, the ground is hard. Again, at differents towns and villages along the Kiso-*gawa* the ground is soft, and the α attained, at some of the places, values over 4300 mm/sec². According to the seismographical observations at Hitotsubashi, in Tokyo, the period of large earthquake movement in soft soil seems to be about 1.0 sec. This would give for $\alpha =$ 4300 mm/sec²., a range of motion of 218 mm. These examples seem to indicate that a range of motion of 2 ft, or about 600 mm, may be considered as nearly limiting, if not beyond, the most violent earthquake motion. Such a motion would, for a period of 1.3 sec., give an $\alpha = 7120$ mm/sec²., corresponding to the $2a$ of 2 ft, and the amount of the sliding produced may be from a few inches to a few feet, namely, of the same order as those observed in the Neo-Valley. From considerations like these, the intensity (α) of motion in the last named locality may be assumed as having been probably between 5000 and 7000 mm/sec².

Experiment on Sliding.

Showing how the plates and boxes were put on the Shaking Table.

Fig. 2.

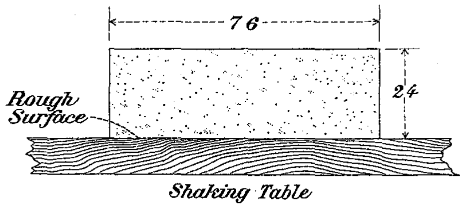


Fig. 3.

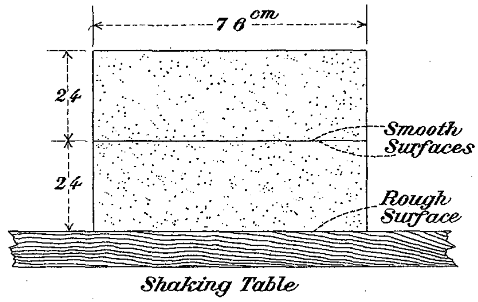


Fig. 5.

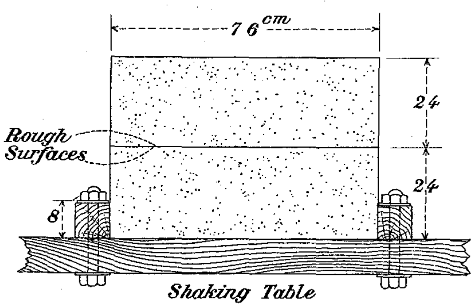


Fig. 4.

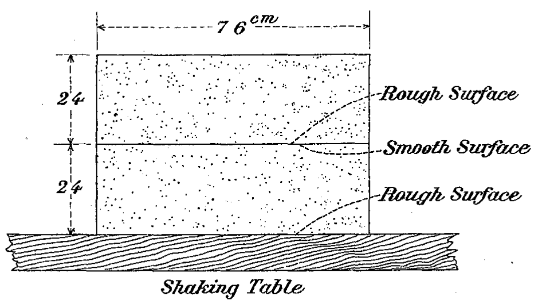


Fig. 6.

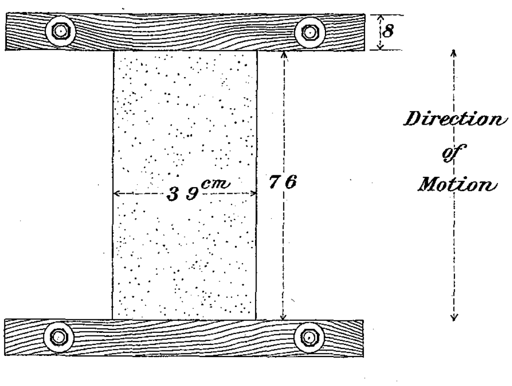
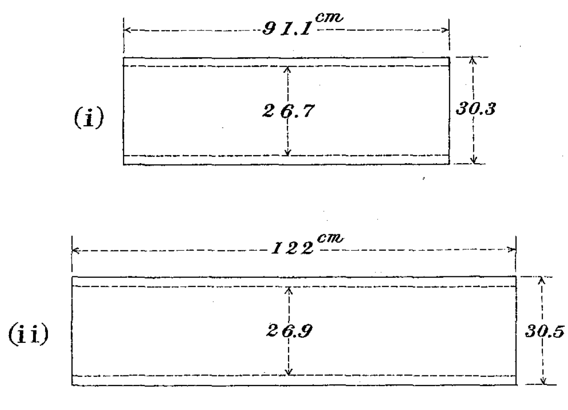


Fig. 7.



Overturning Experiment.

Showing the Sections of the different Columns.

Fig. 9.

Col. No. 1.

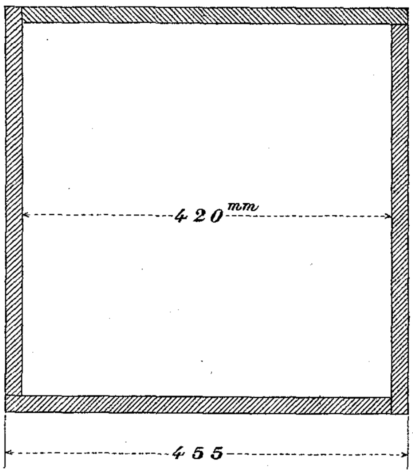


Fig. 10.

Col. Nos. 2, 3.

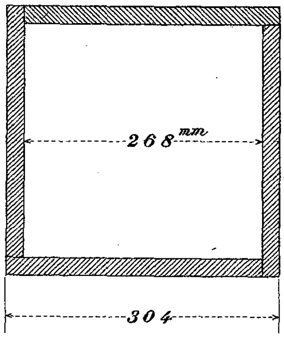


Fig. 11.

Columns Nos. 4, 5, 6, 7.

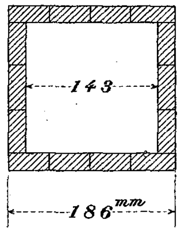


Fig. 12.

Col. No. 8.

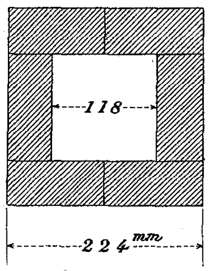


Fig. 13.

Col. Nos. 16, 17.

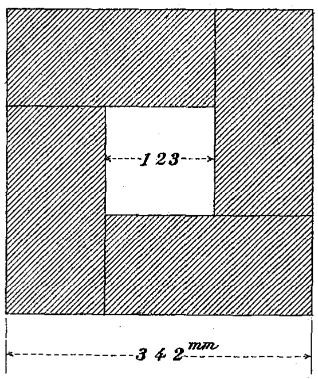


Fig. 14.

Columns Nos. 12, 13.

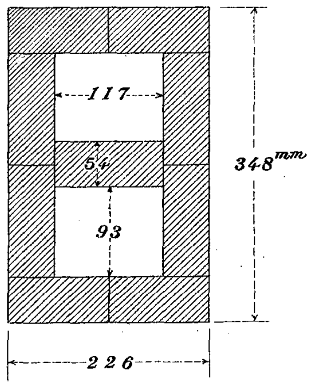


Fig. 15.

Col. No. 14.

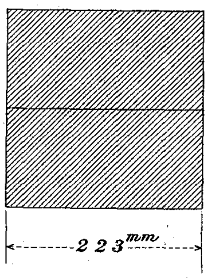


Fig. 16.

Col. No. 15.

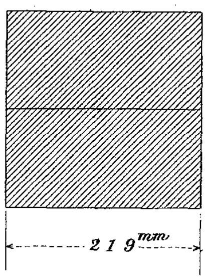


Fig. 17.

Columns Nos. 9, 10, 11.

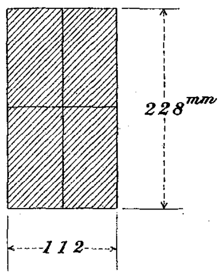
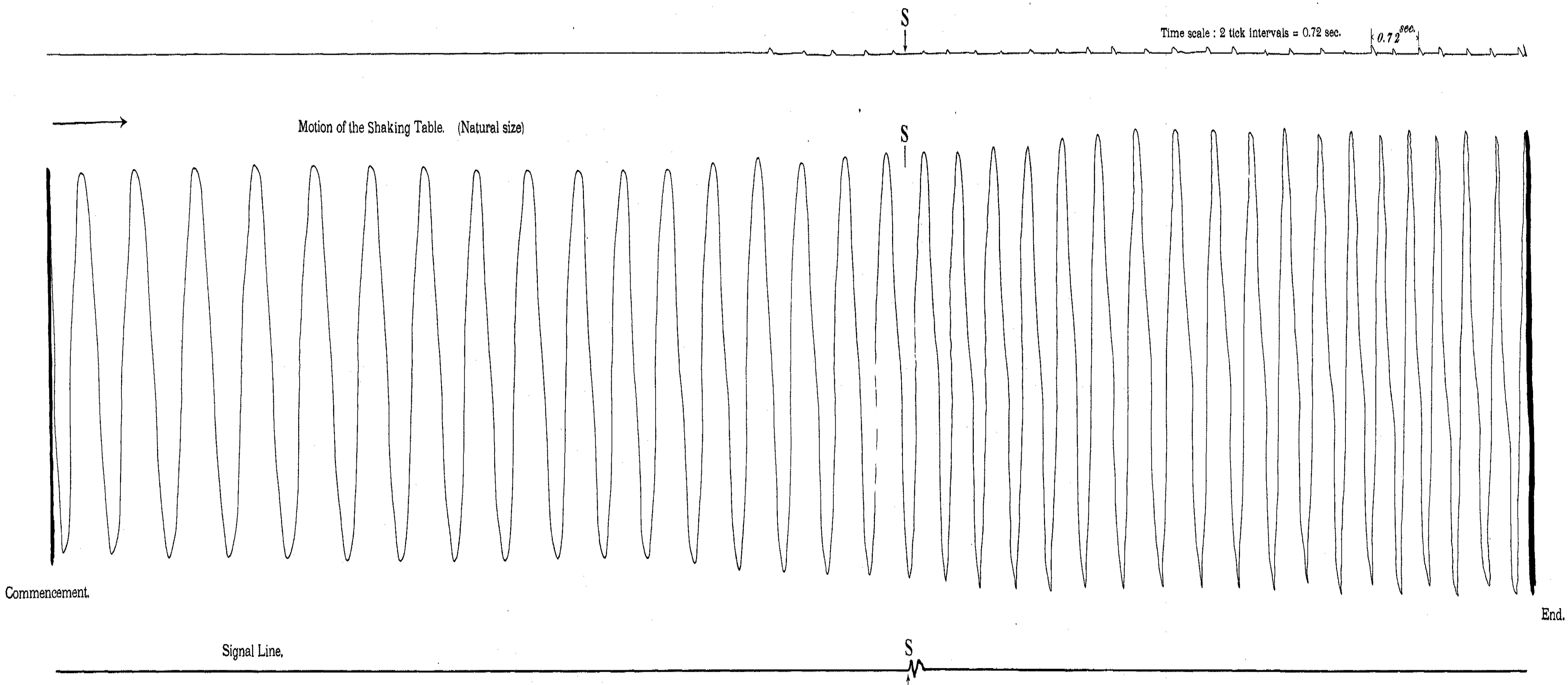
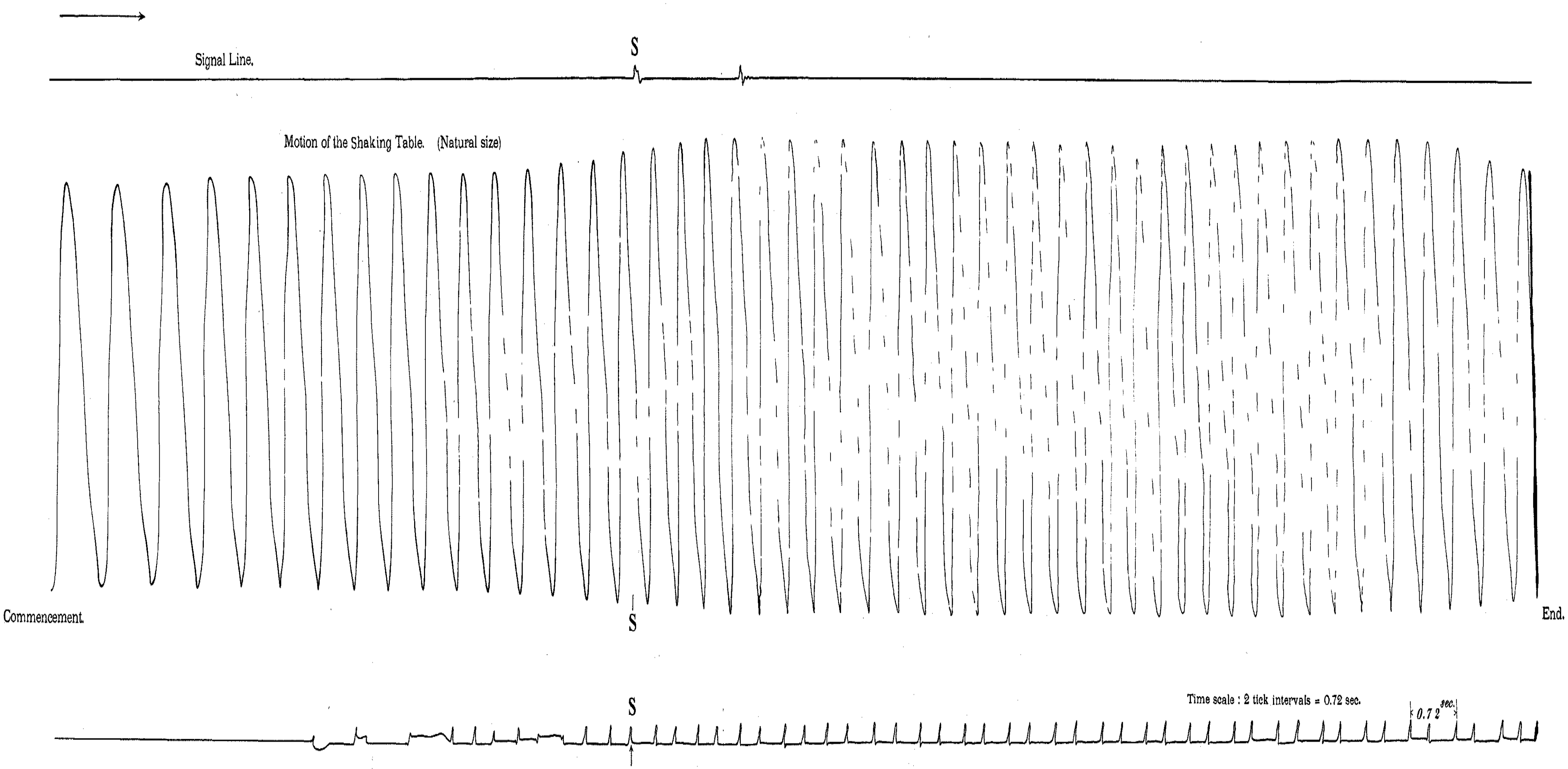


Fig. 19. Experiment on the Sliding of Stone Plates No. 1 and No. 2. (Expt. 6, Page 6.)



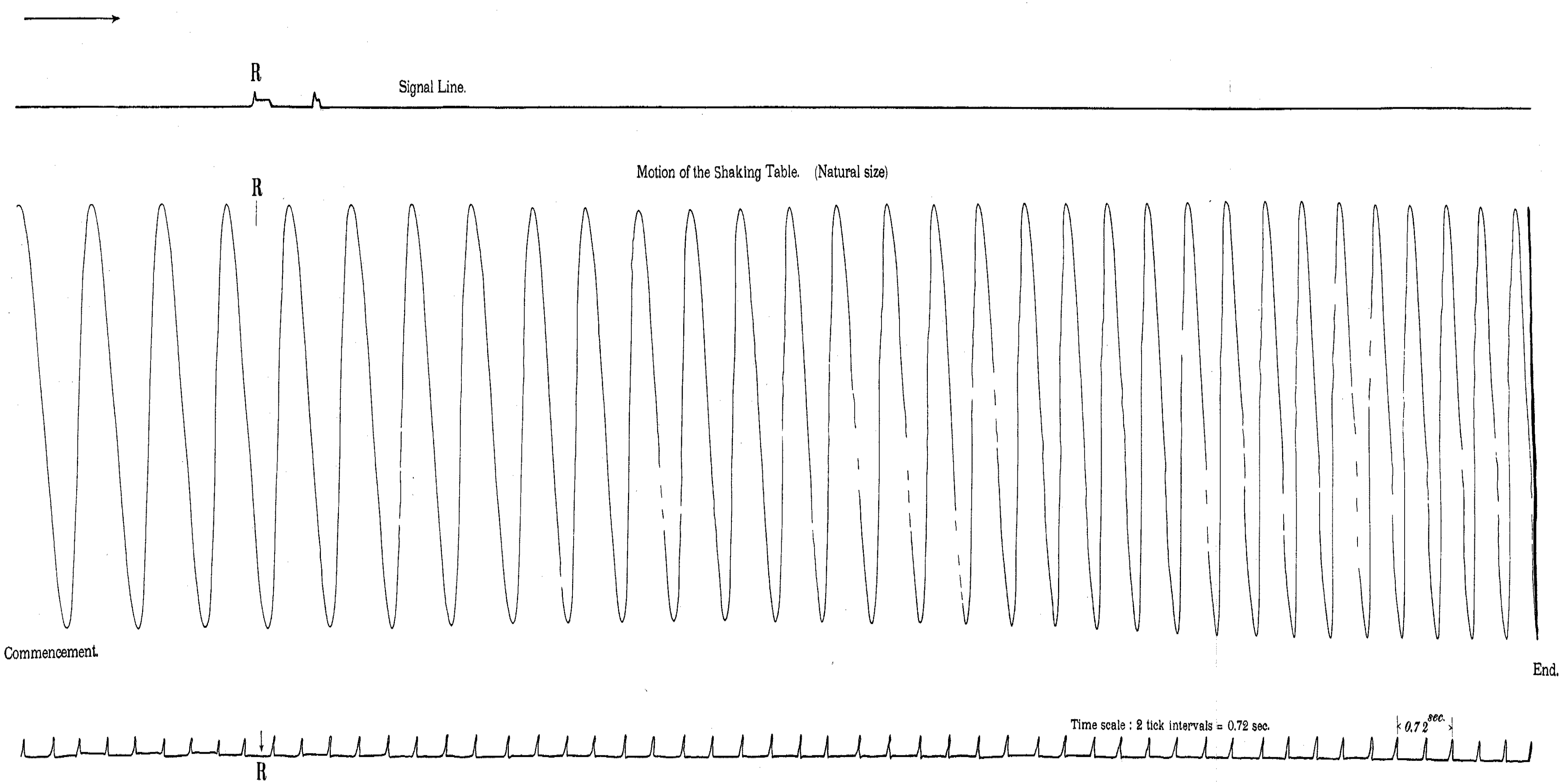
S ...... Commencement of the Sliding.

Fig. 20. Experiment on the Sliding of a heavy wooden Box. (Expt. 13, Page 7.)



S ...... Commencement of the Sliding.

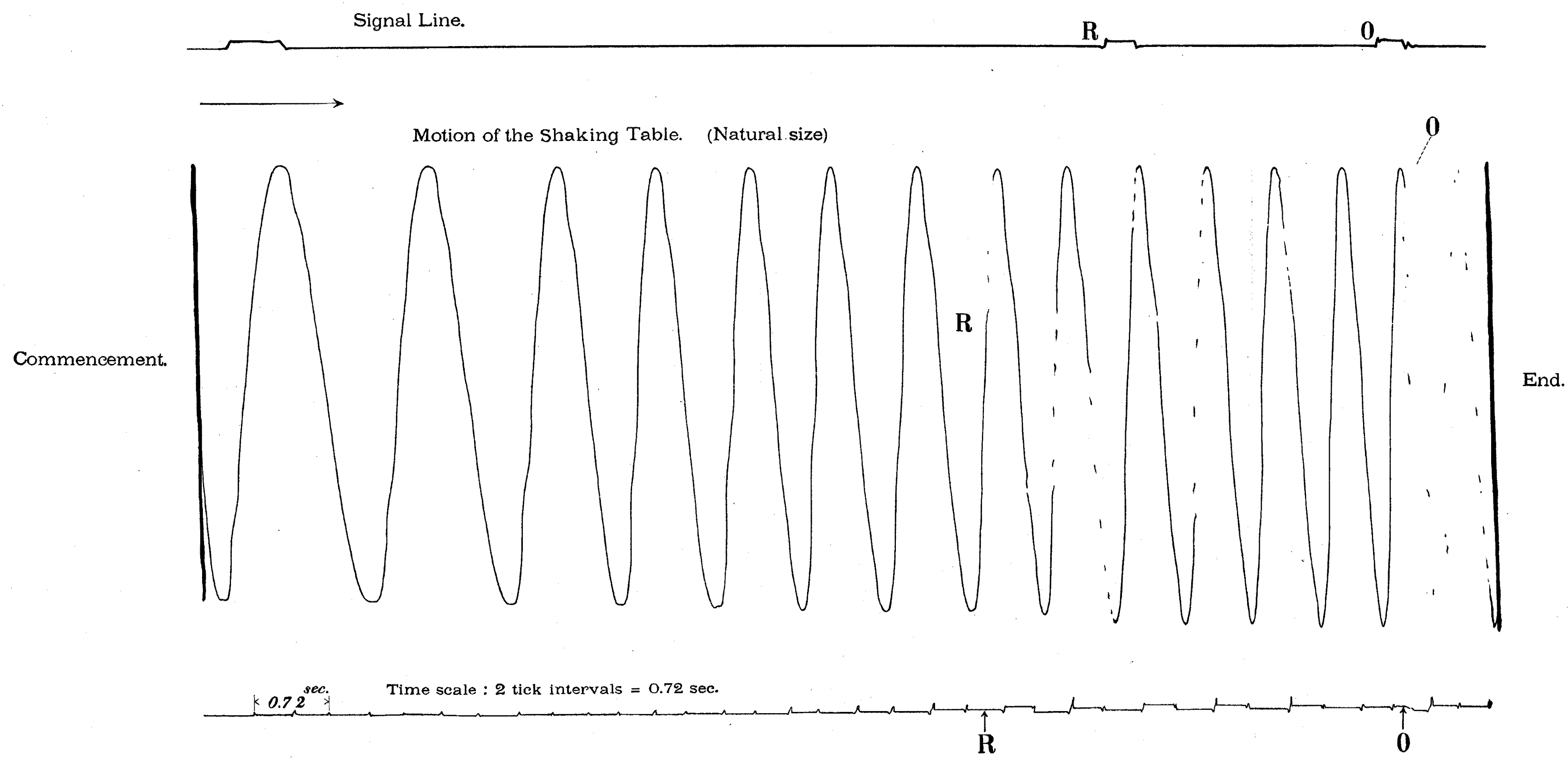
Fig. 21. Experiment on the Rocking of wooden Column, No. 1. (Page 17.)



R...... Commencement of the Rocking.

Fig. 22. Experiment on the Rocking and Overturning of wooden Column, No. 13.

(See Page 17.)



R ...... Commencement of the Rocking.

O ...... Overturning of the Column.