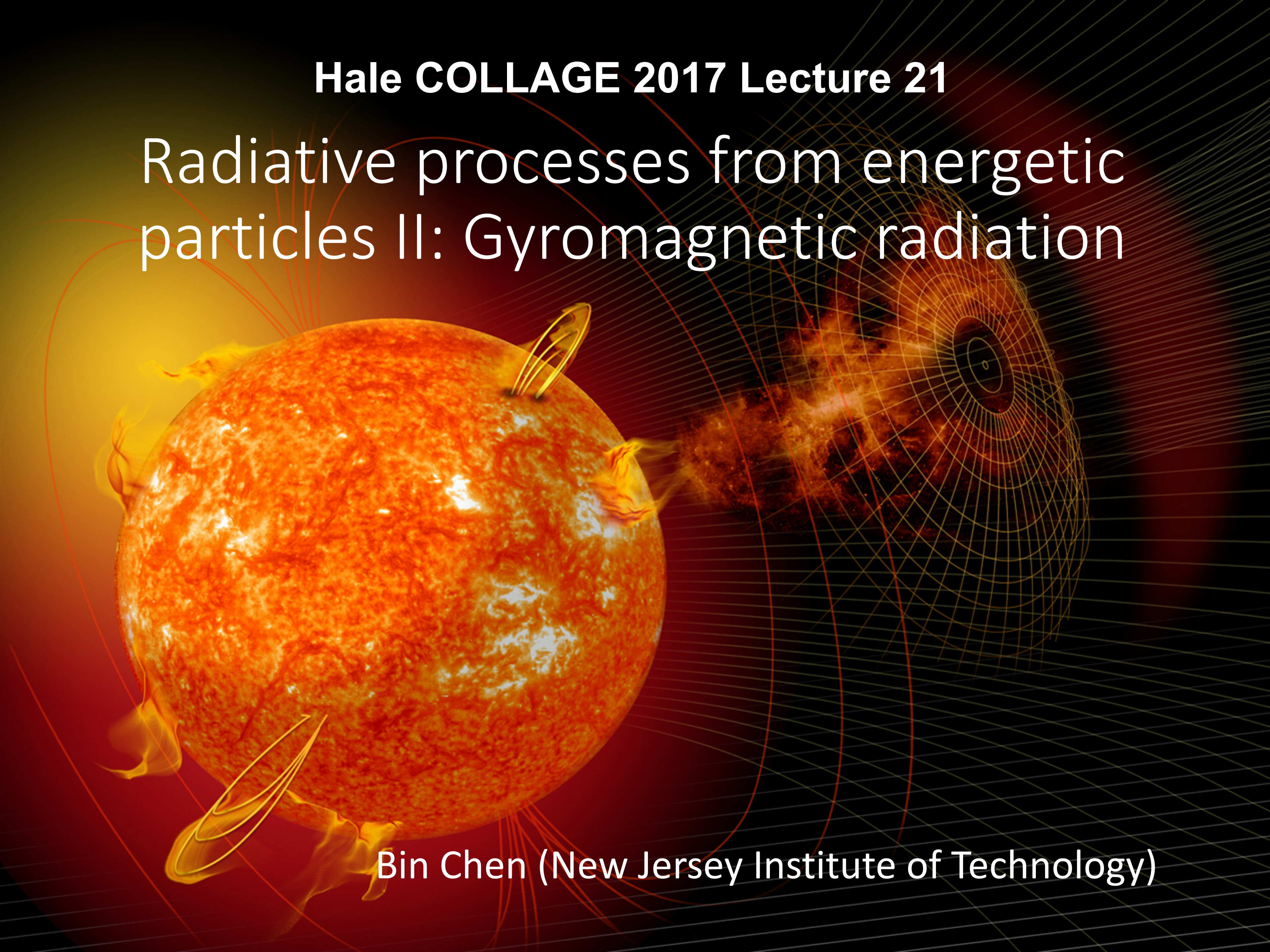
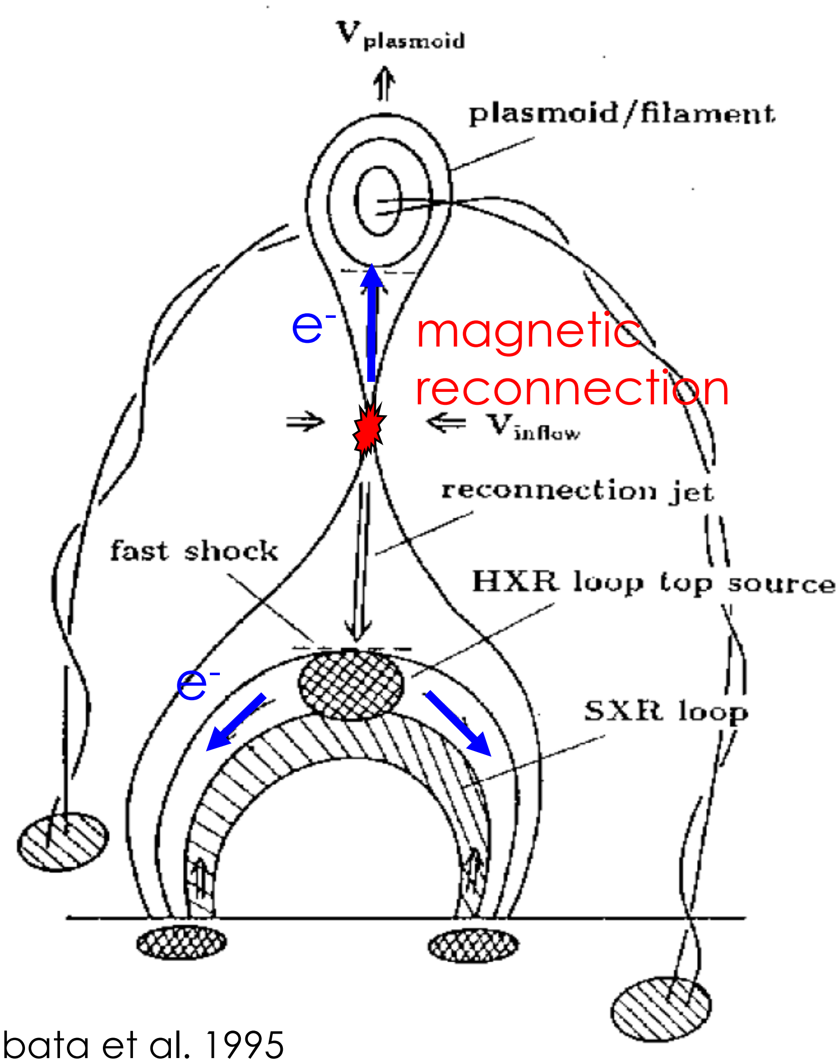
## Hale COLLAGE 2017 Lecture 21

# Radiative processes from energetic particles II: Gyromagnetic radiation



Bin Chen (New Jersey Institute of Technology)



Shibata et al. 1995

## Previous lectures

1) Magnetic reconnection and energy release
2) Particle acceleration and heating
3) Chromospheric evaporation, loop heating and cooling

Following lectures:

How to diagnose the accelerated particles and the environment?

- What?
- Where? → How?
- When?

# Outline

- Radiation from energetic particles
  - Bremsstrahlung → Previous lecture
  - Gyromagnetic radiation (“magnetobremsstrahlung”)

    → This lecture
  - Other radiative processes → Briefly in the next lecture
    - Coherent radiation, inverse Compton, nuclear processes
- Suggested reading:
  - Synchrotron radiation: Chapter 5 of “Essential Radio Astronomy” by Condon & Ransom 2016
  - Gyroresonance radiation: Chapter 5 of Gary & Keller 2004
  - Gyrosynchrotron radiation: Dulk & Marsh 1982
- Next two lectures: Diagnosing flare energetic particles using radio and hard X-ray imaging spectroscopy

# Radiation from an accelerated charge

**Larmor formula**:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\mathbf{a}^2 \sin^2\theta \qquad P = \frac{2q^2}{3c^3}\mathbf{a}^2$$

**Relativistic Larmor formula**:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\frac{(a_\perp^2 + \gamma^2 a_\parallel^2)}{(1-\beta\cos\theta)^4}\sin^2\theta$$

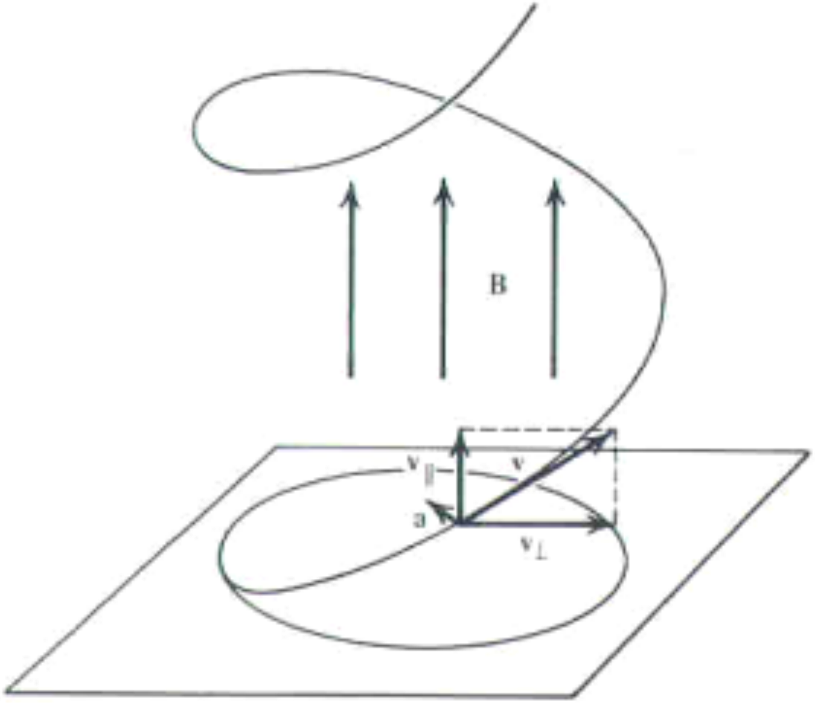
$$P = \frac{2q^2}{3c^3}\gamma^4({a_\perp}^2 + \gamma^2 a_\parallel^2)$$

Radio and HXR/gammy-ray emission in flares:

- Acceleration experienced in the Coulomb field: **bremsstrahlung**
- Acceleration experienced in a magnetic field: **gyromagnetic radiation**

# Gyromagnetic radiation

- Gyromagnetic radiation (sometimes called "gyroemission") is due to the acceleration experienced by an electron as it gyrates in a B field due to the Lorentz force.
- Acceleration is perpendicular to $v_e$



# Gyroemission from a single electron

- Let’s start from Larmor’s formula:

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\mathbf{a}^2 \sin^2\theta \qquad P = \frac{2q^2}{3c^3}\mathbf{a}^2$$

- Perpendicular acceleration: $a_\perp = \omega_{ce} v_\perp$, where $\omega_{ce}$ is the (angular) electron gyrofrequency

$$\omega_{ce} = 2\pi\nu_{ce} = \frac{eB}{m_e c} \approx 2\pi \cdot 2.8B\ \text{MHz}$$

- (Direction integrated) Larmor’s equation becomes:

$$P = \frac{2e^2}{3c^3}\omega_{ce}^2 v_\perp^2$$

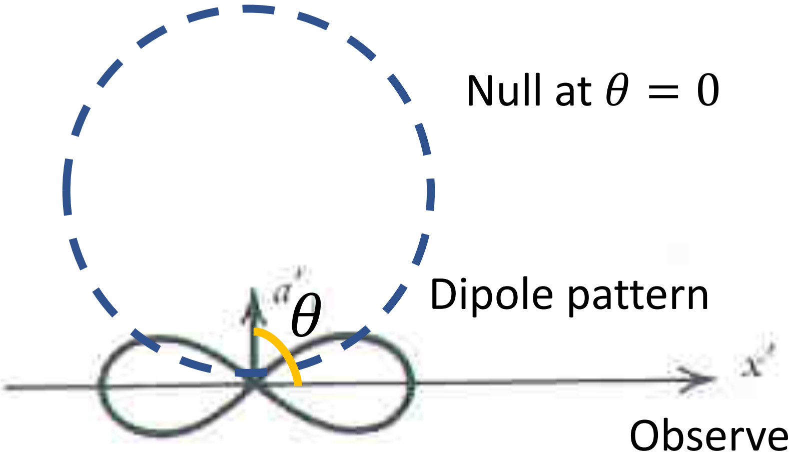
- Relativistic case:

$$P = \frac{2e^2}{3c^3}\gamma^4 \omega_B^2 v_\perp^2, \text{ with } \omega_B = \frac{eB}{\gamma m_e c} = \frac{\omega_{ce}}{\gamma}$$

# Radiation pattern: non-relativistic

- Larmor's Equation

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c^3}\mathbf{a}^2 \boxed{\sin^2\theta}$$



Null at $\theta = 0$

Dipole pattern

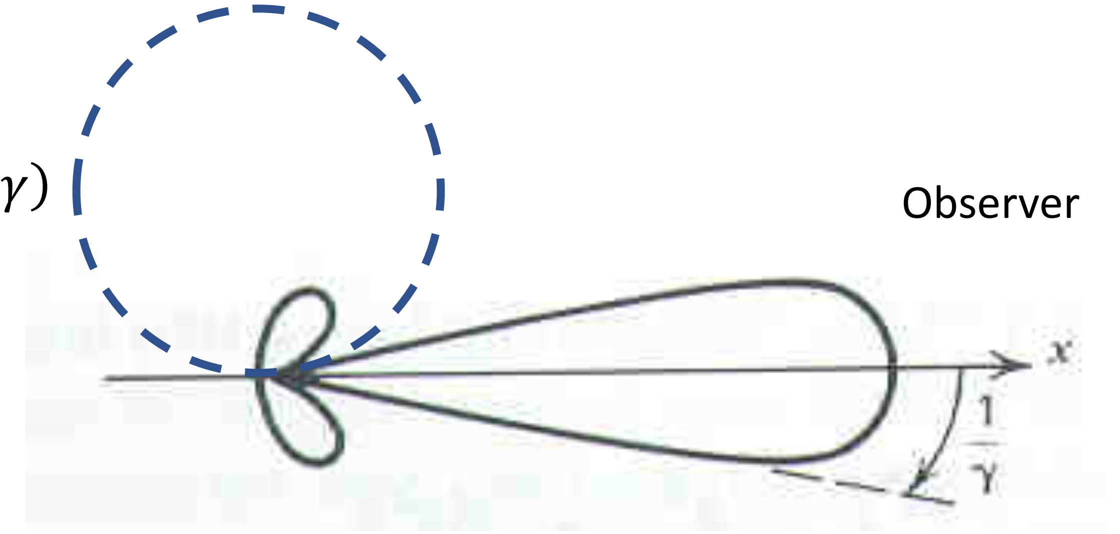
Observer

# Radiation pattern: relativistic

- Relativistic case ($\gamma \gg 1$)
  - In the rest frame of the electron

$$\frac{dP'}{d\Omega'} = \frac{q^2}{4\pi c^3}\boldsymbol{a}^2 \sin^2\theta'$$

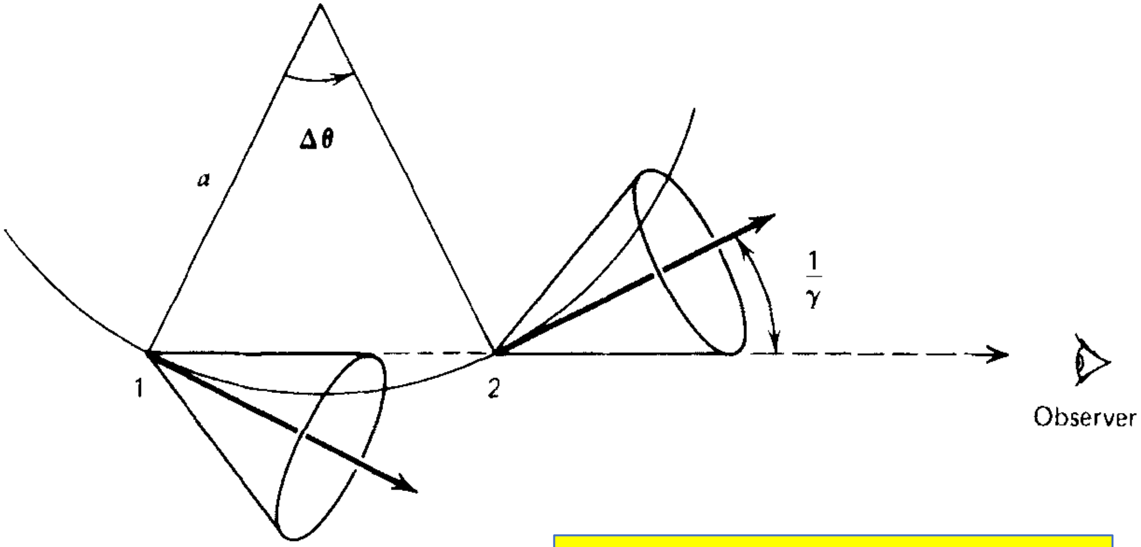
  - In the observer's frame, radiation pattern found from Lorentz transform from the electron rest frame

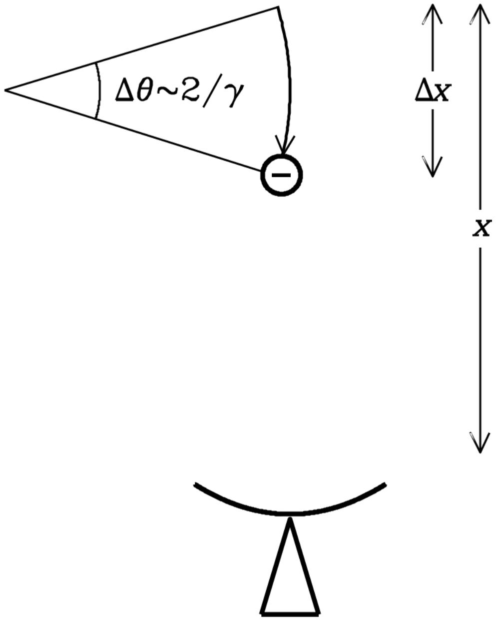


Null occurs at $\theta = \pm \arccos(1/\gamma)$

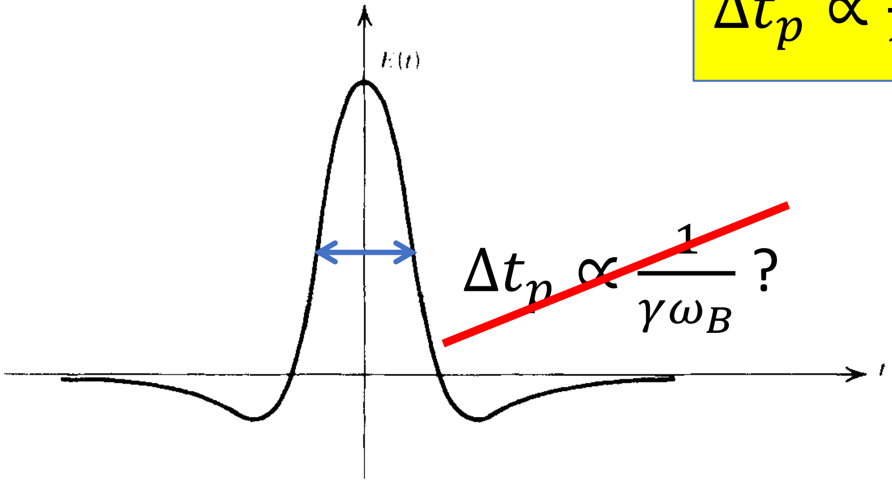
Strongly beamed forward along the direction of the electron!

# Relativistic gyroemission: sharply pulsed radiation





$$\boxed{\Delta t_p \propto \frac{1}{\gamma^3 \omega_B} = \frac{1}{\gamma^2 \omega_{ce}}}$$

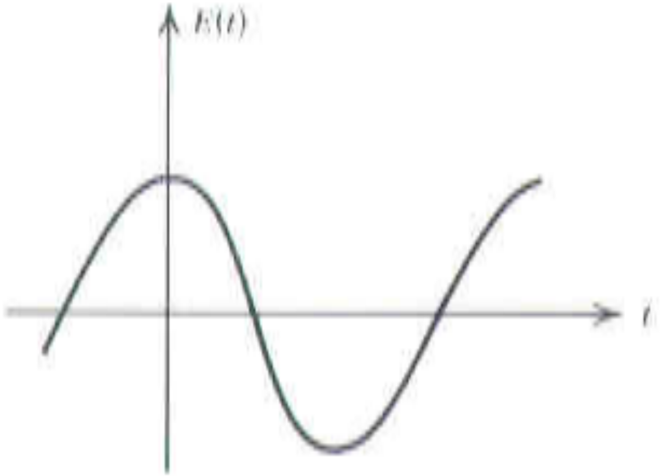


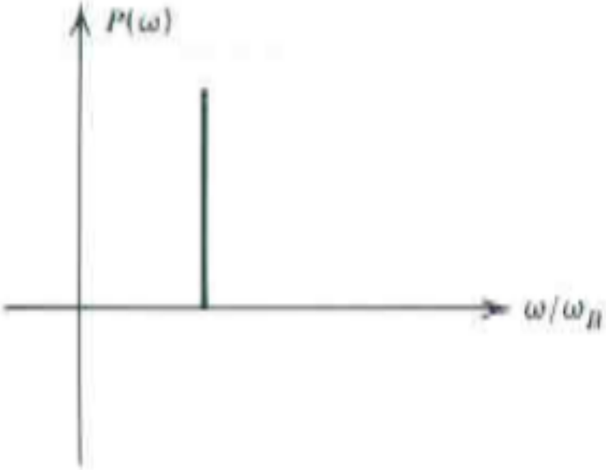
$$\cancel{\Delta t_p \propto \frac{1}{\gamma \omega_B}}\ ?$$

$$\Delta t_p = t\ \text{(end of pulse)} - t\ \text{(start of pulse)}$$
$$= \frac{\Delta x}{v} + \frac{x - \Delta x}{c} - \frac{x}{c} = \frac{\Delta x}{v}\left(1 - \frac{v}{c}\right) \ll \frac{\Delta x}{v} = \Delta t$$

# Power spectrum $P(\nu)$

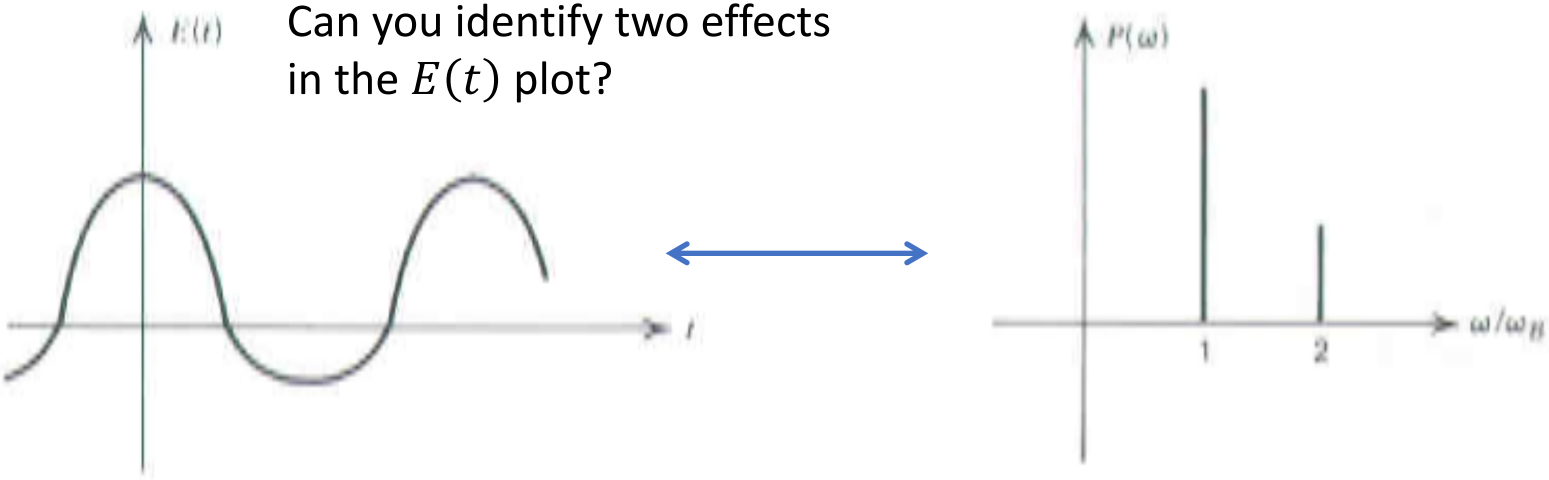
- For a nonrelativistic electron, radiation field $E(t)$ is a sinusoid with frequency $\omega_{ce}$
- Power spectrum is a single tone at the electron gyrofrequency





# Power spectrum $P(\nu)$

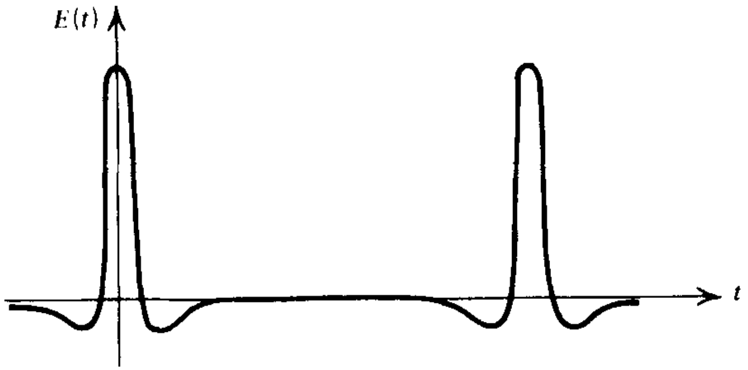
- As the electron speed picks up, mild beaming effect takes place, $E(t)$ is non-sinusoidal
- Low harmonics of electron gyrofrequency show up in the power spectrum

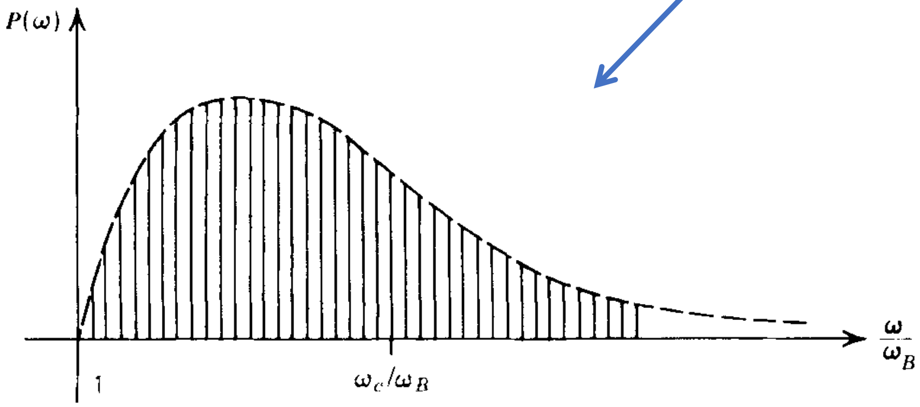


Can you identify two effects in the $E(t)$ plot?

# Power spectrum $P(\nu)$

- When the electron is relativistic $E(t)$ is highly pulsed





- The power spectrum shows contribution from many harmonics

# Types of gyromagnetic radiation

- Gyromagnetic radiation behaves very differently with different electron distributions
- A precise general expression valid for all electron energies is *not* available. Instead, we use approximate expressions for various electron energy regimes

❖ Non-relativistic or thermal $(\gamma - 1 \ll 1)$:

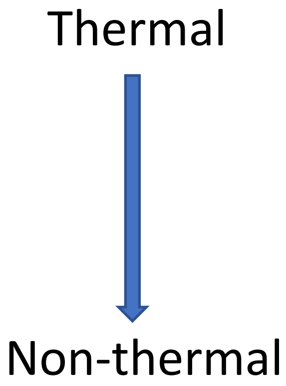
Gyroresonance or cyclotron radiation

❖ Mildly relativistic $(\gamma - 1 \sim 1 - 5)$:

Gyrosynchrotron radiation

❖ Ultra-relativistic $(\gamma - 1 \gg 1)$:

Synchrotron radiation

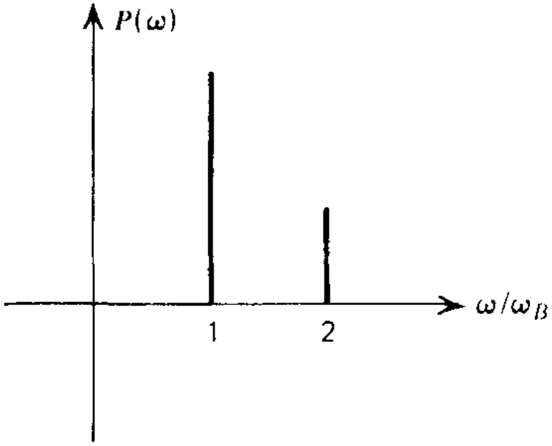


# Thermal gyroresonance radiation

- At a given B, thermal gyroresonance radiation is essentially a "spectral line" centered at $s\nu_{ce}$, where $s = 1, 2, 3 \ldots$ is the harmonic number
- Particularly relevant above active regions at microwave frequencies – Why?
- Spectral width of a given resonance line

$$\Delta\nu / s\nu_{ce} \approx \sqrt{\frac{k_B T}{m_e c^2}}$$

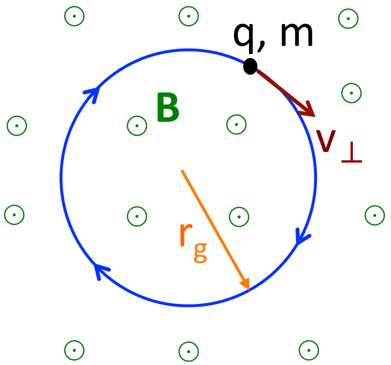
Very narrow in the corona (~1/3000)

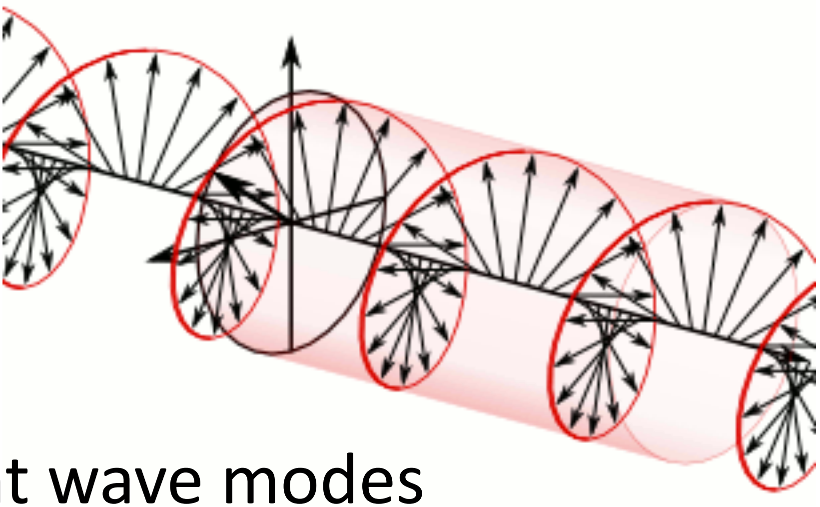


- High opacity only at these "resonance layers"

# Thermal gyroresonance opacity

- Two different wave modes: ordinary (o mode) and extraordinary (x mode, gyrates with the same sense of rotation as an electron)





- Opacity for two different wave modes

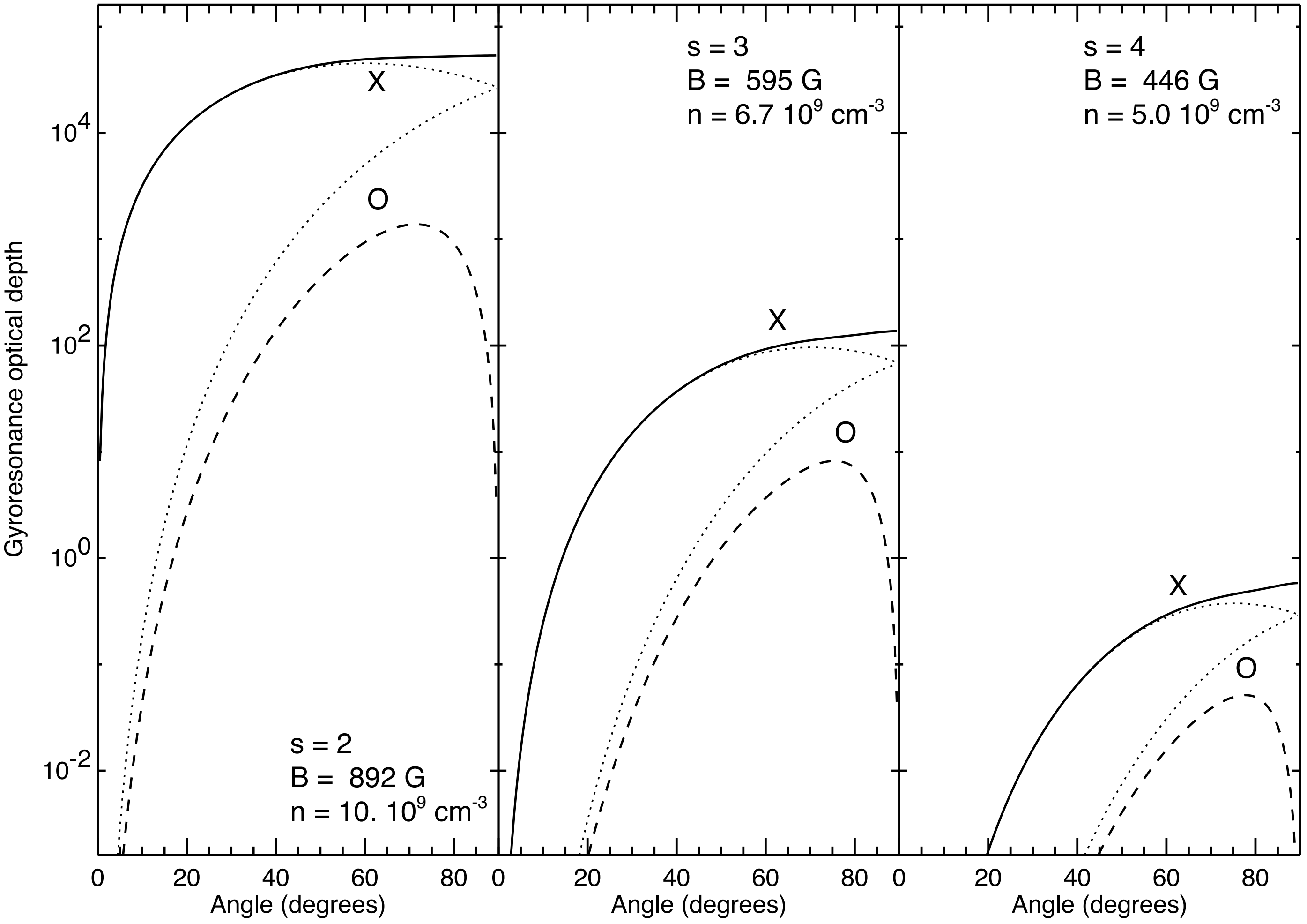
$$\tau_{x,o}(s,\nu,\theta) \;=\; .0133\,\frac{n_e\,L_B(\theta)}{\nu}\;\frac{s^2}{s!}\left(\frac{s^2\,\sin^2\theta}{2\,\mu}\right)^{s-1} F_{x,o}(\theta)$$

Where $F_{x,o}(\theta) \approx (1 - \sigma\cos\theta)^2$ and $\mu = m_e c^2/k_B T$

$\sigma = -1$ for x mode and 1 for o mode, $L_B$ is the scale length of B

**Which mode has a larger opacity? Why?**

# Thermal gyroresonance opacity

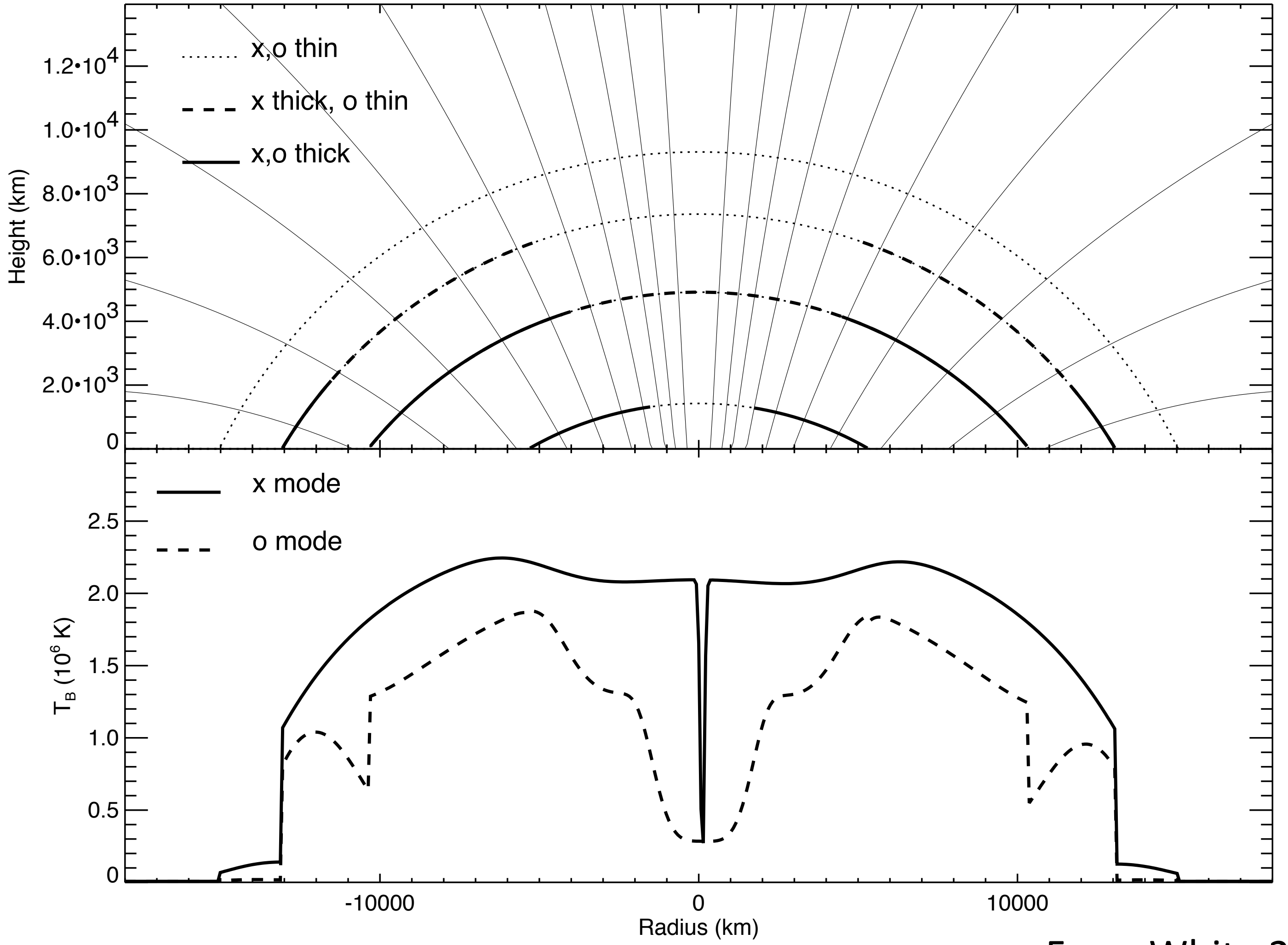


$\nu = 5$ GHz
$T = 3$ MK
$L_B = 10$ Mm

Typically, optically thick at S=2 (o-mode) and 3 (x-mode)

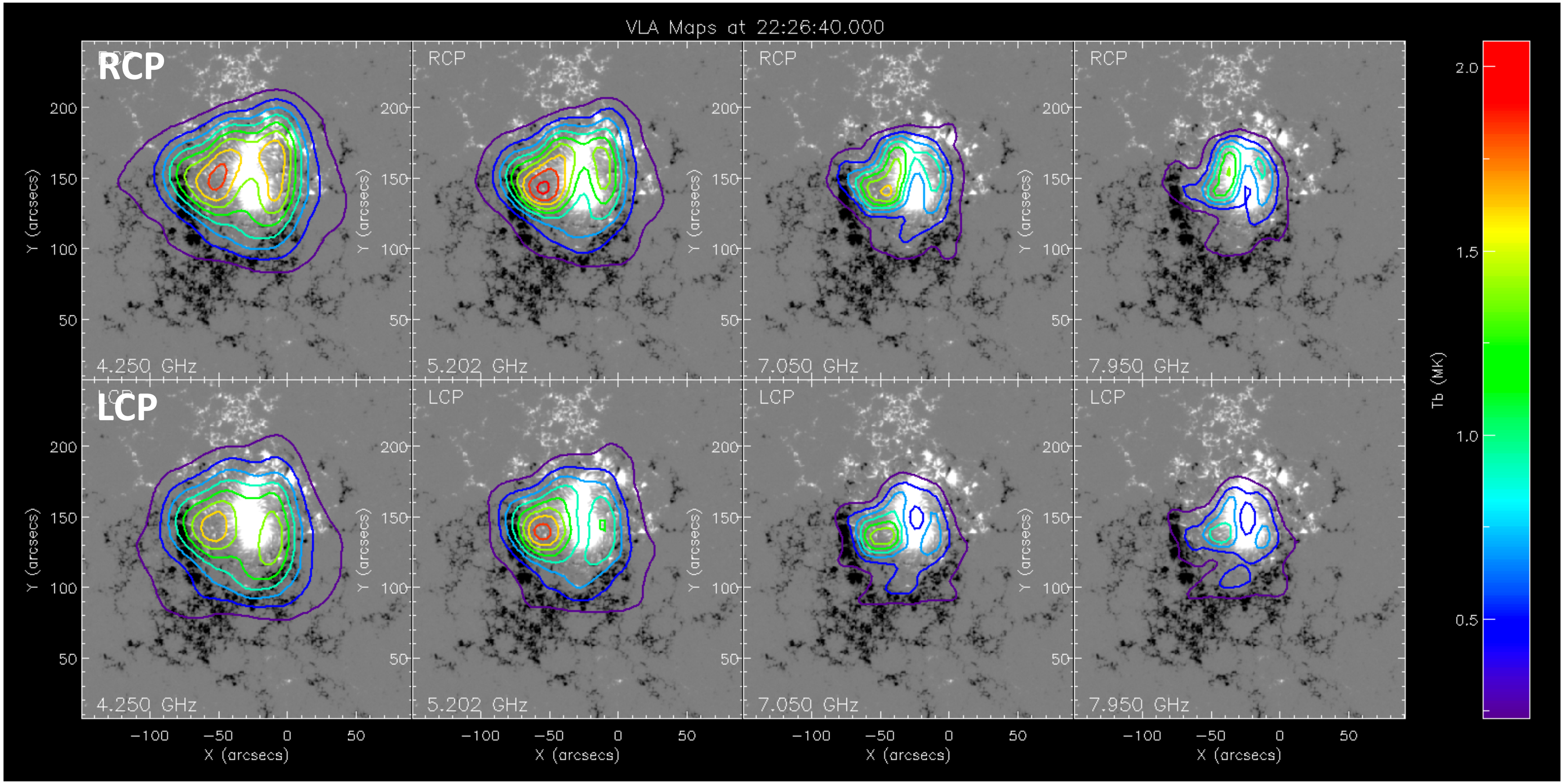
From White 2004

# Gyroresonance emission of a sunspot



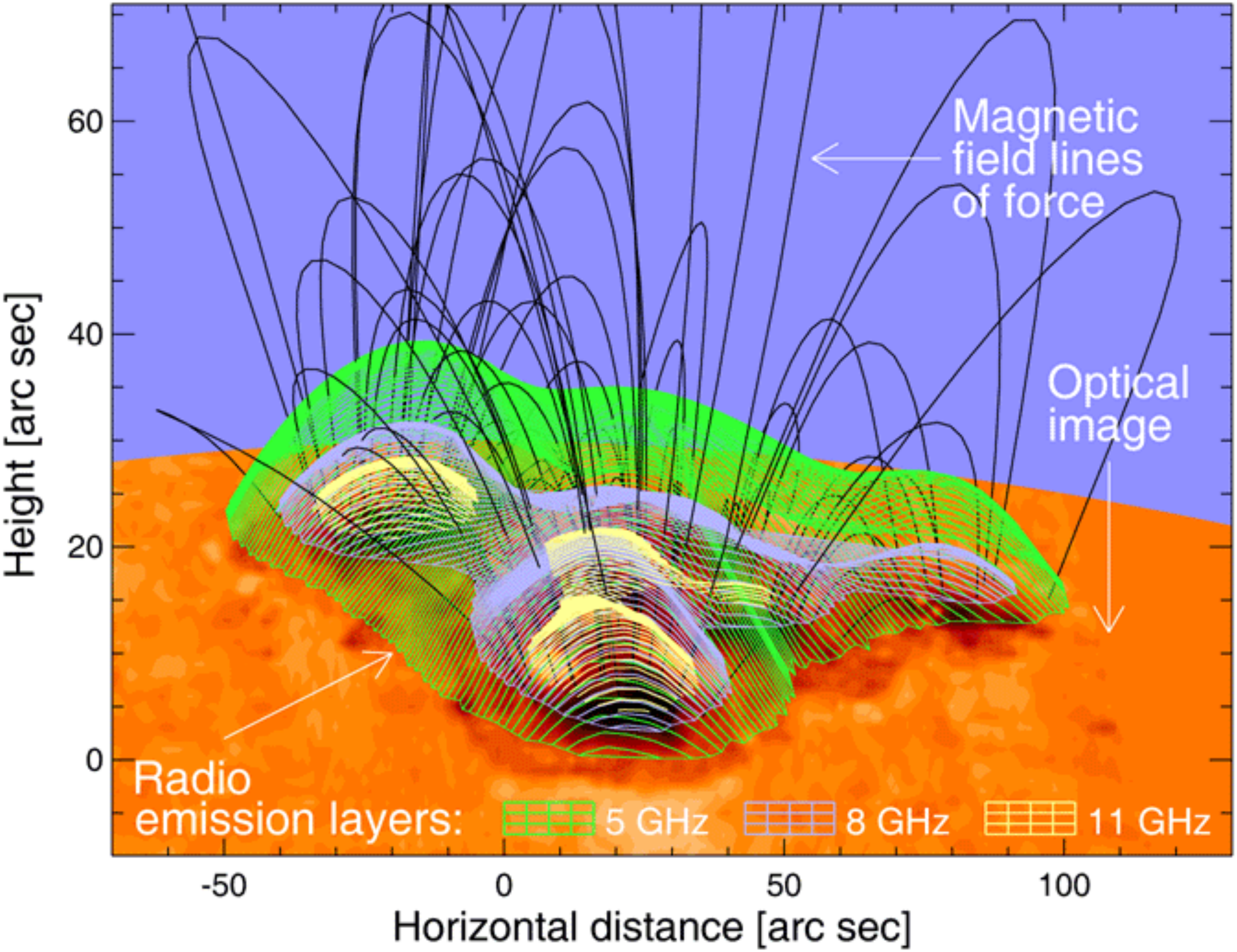
From White 2004

# Actual observation from the VLA



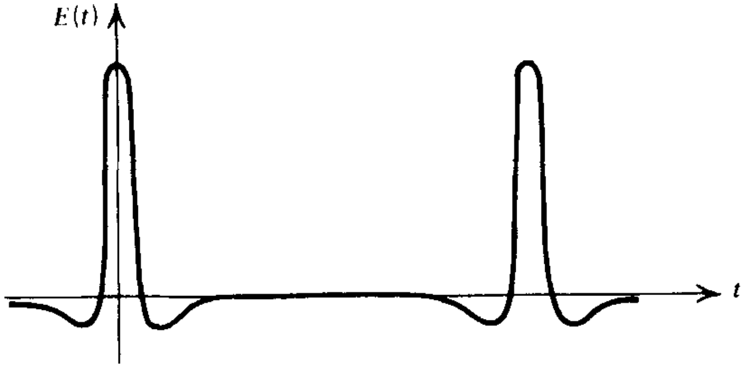
Q: Which polarization is the x-mode?

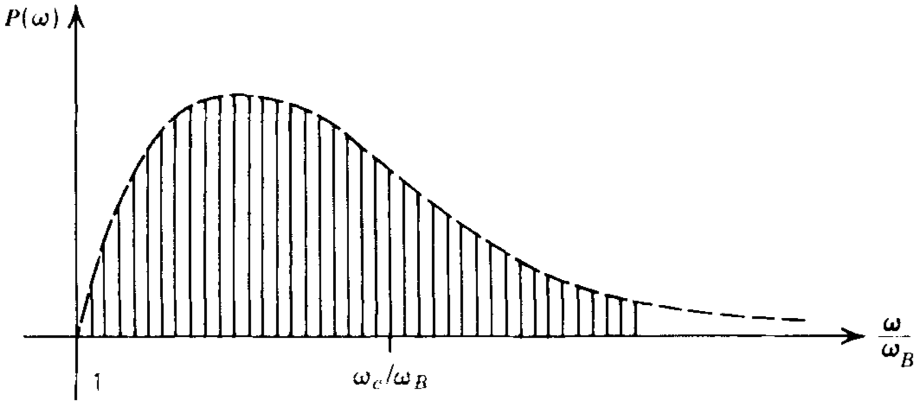
Made by B. Chen for AR 12158
(unpublished)


Magnetic field lines of force
Optical image
Radio emission layers:
5 GHz
8 GHz
11 GHz
Height [arc sec]
Horizontal distance [arc sec]
60
40
20
0
-50
0
50
100

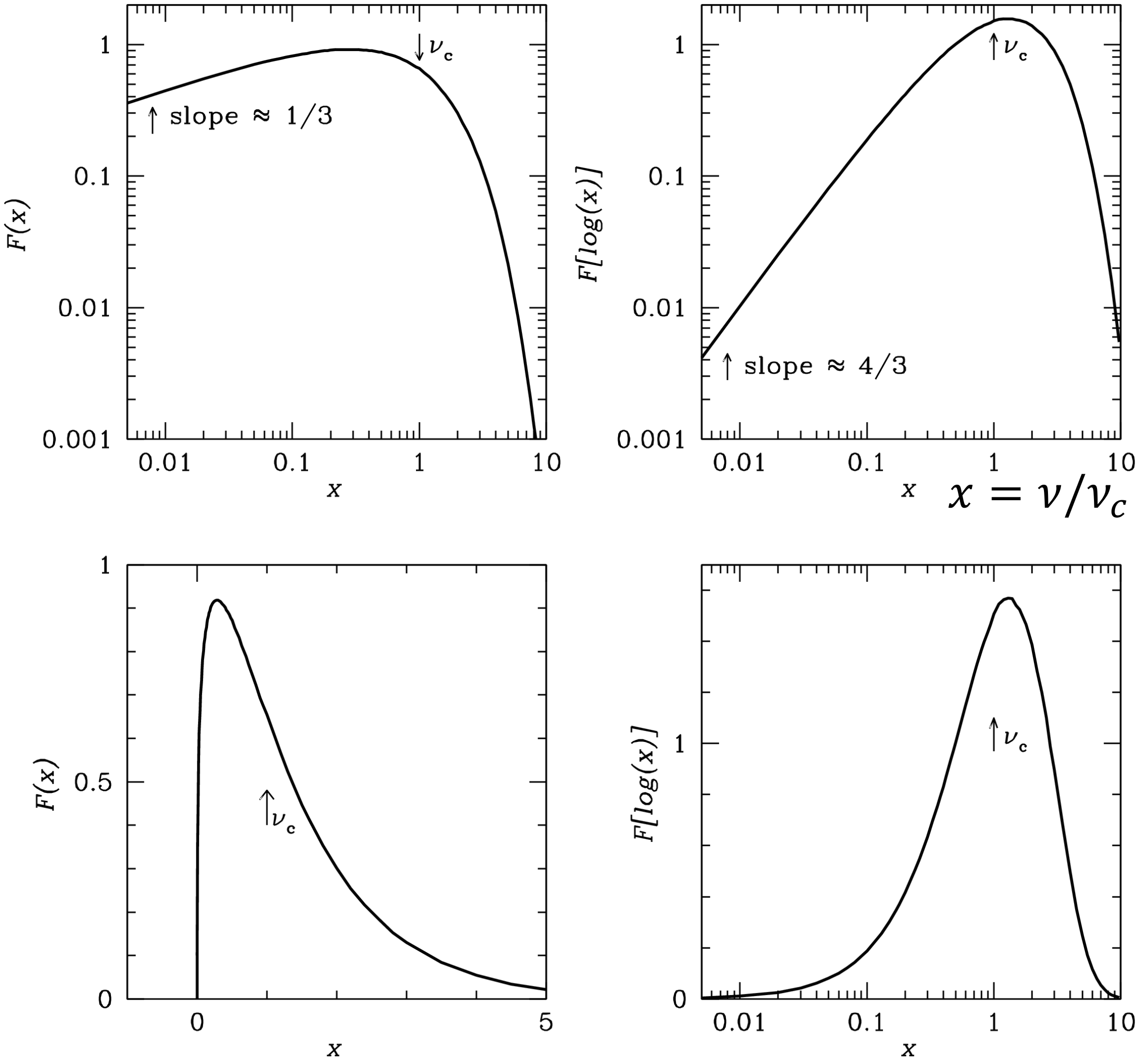
# Nonthermal synchrotron radiation

- Ultra-relativistic ($\gamma - 1 \gg 1$)
- From a single electron, adjacent ”spikes” are separated in frequency by only $\Delta\nu = \frac{\nu_{ce}}{\gamma}$
- Fluctuations in electron energy, B strength, or pitch angle cause “broadening” of the spikes
- Spectrum is virtually continuous





# Synchrotron spectrum $P(\nu)$ from a single electron



$x = \nu/\nu_c$

Most of the energy is emitted at $\nu \approx \nu_c$, where

$$\boxed{\nu_c = \frac{3}{2}\gamma^2 \nu_{ce} \sin\alpha}$$

is the **critical frequency**

($\alpha$ is the pitch angle)

# Synchrotron spectrum of an optically thin source

- One electron of electron $E$ nearly emits all energy at a single frequency $\nu \approx \gamma^2 \nu_{ce}$
- Optically thin source → to get emissivity $j_\nu$ in $(\nu, \nu + d\nu)$, just add $P(\nu) = -dE/dt$ up from all electrons within $(E, E + dE)$:

$$j_\nu d\nu = -\frac{dE}{dt} f(E) dE$$

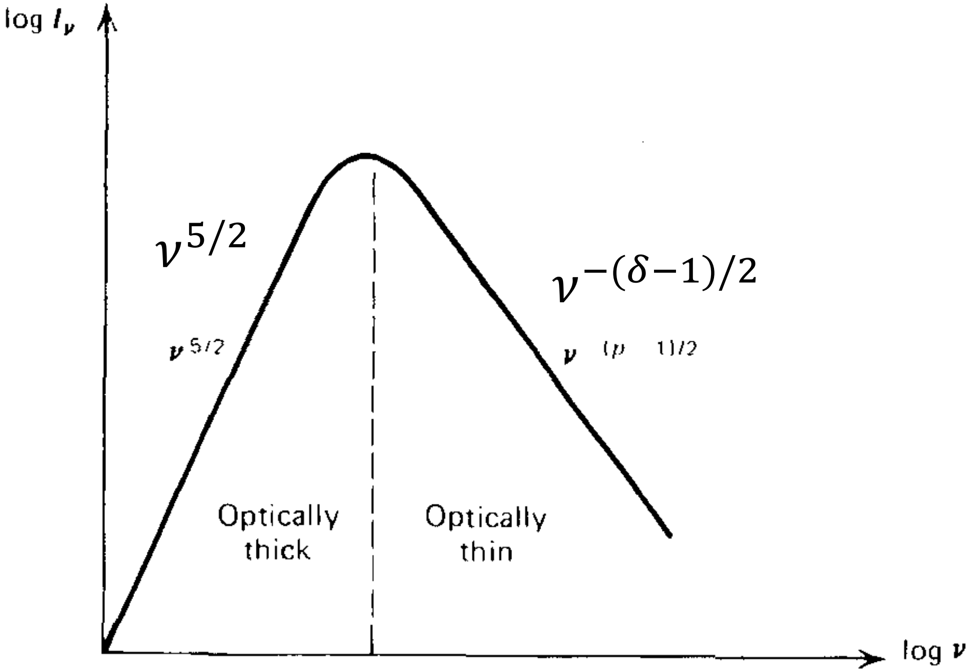
- Assume a power law electron energy distribution:

$$f(E) = C n_e E^{-\delta}$$

- The emissivity $j_\nu \propto \nu^{-(\delta-1)/2}$

# Synchrotron spectrum: optically thick regime

- Synchrotron brightness cannot be arbitrarily high → self-absorption becomes important at low frequencies
- The spectrum has a power law of slope 5/2 for optically thick source

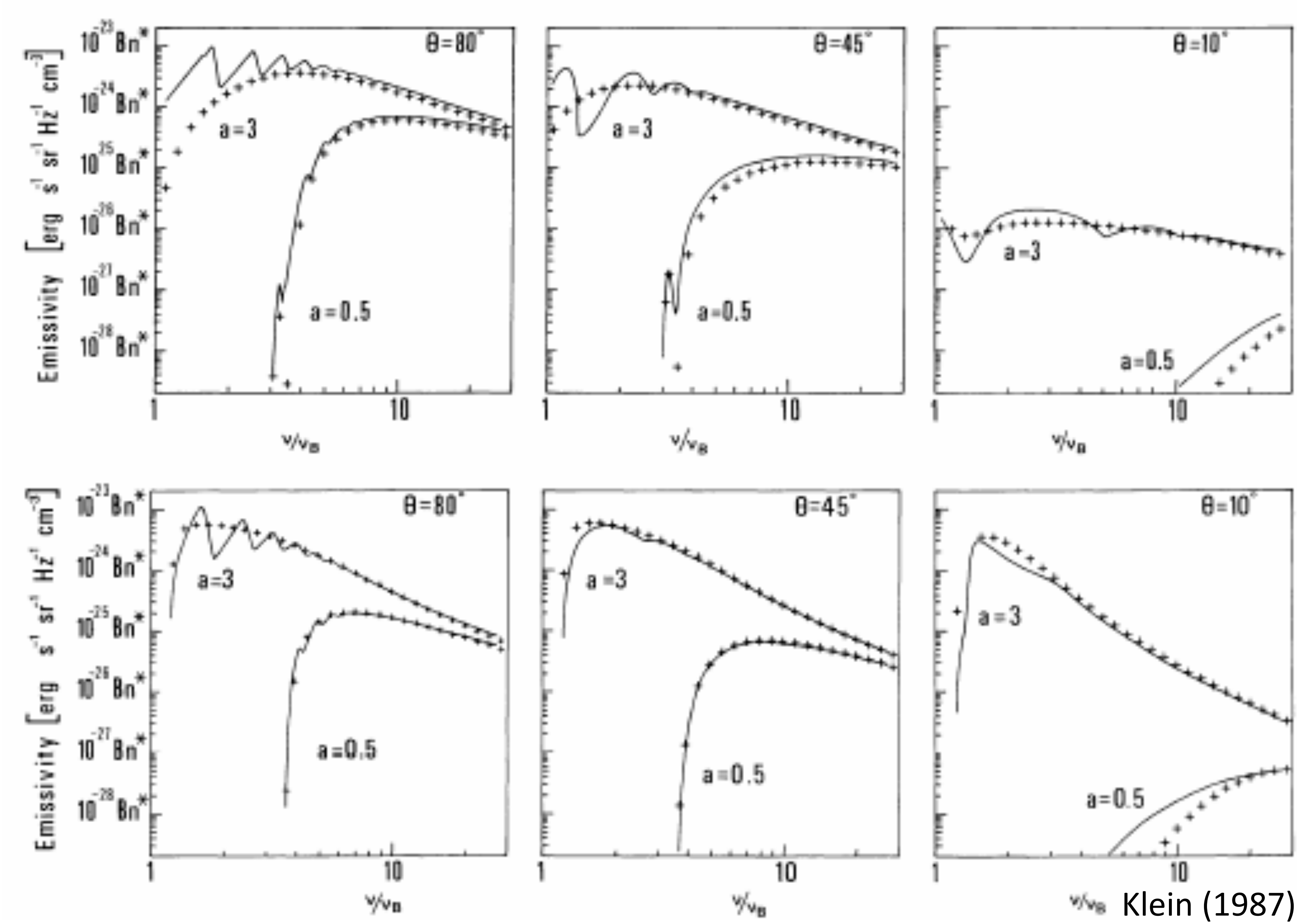


# Gyrosynchrotron radiation

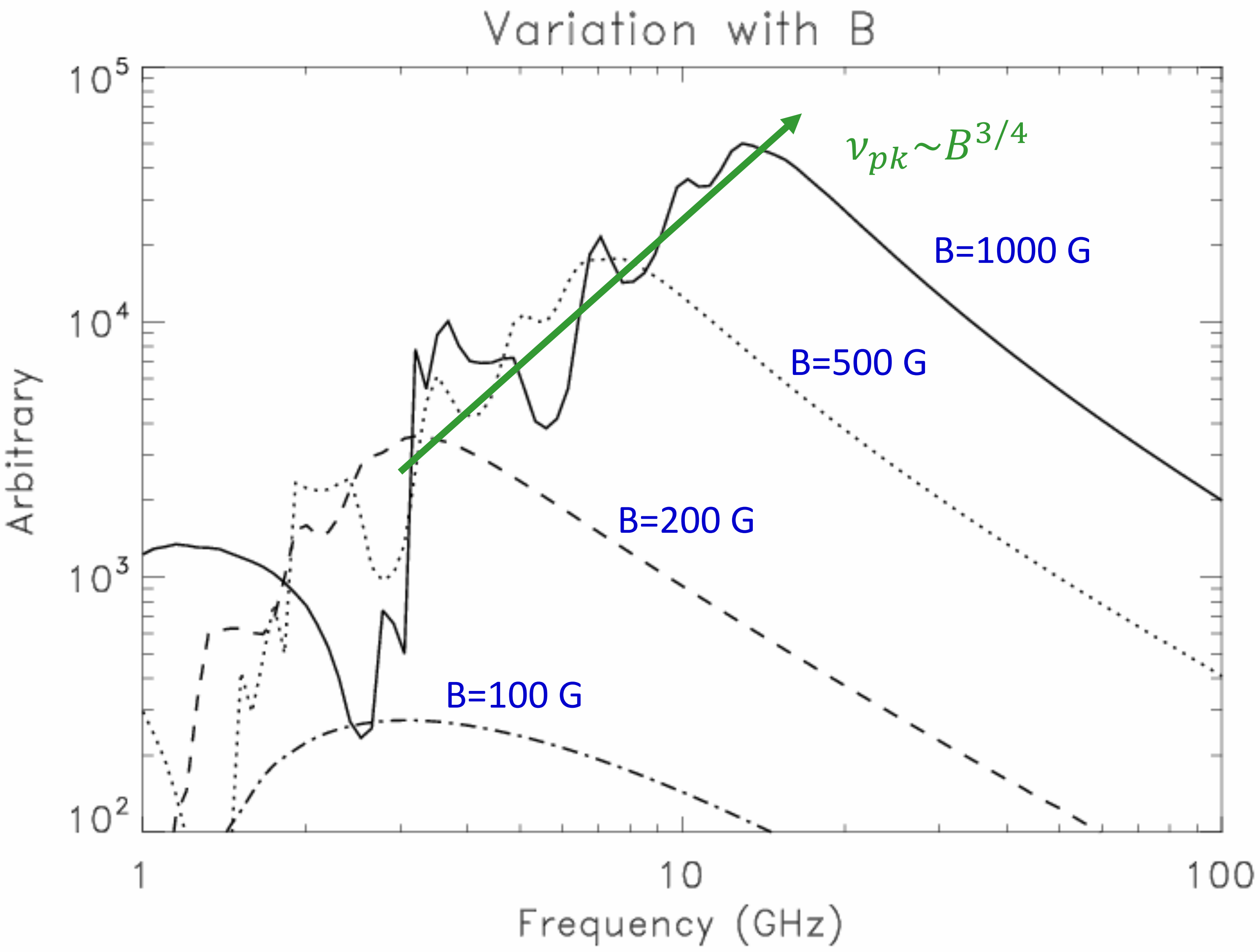
- From mildly relativistic electrons (~1 to several MeV)
- Expressions for the emission and absorption coefficient are much more complicated than the nonrelativistic (thermal gyroresonance) and ultra-relativistic (synchrotron) case

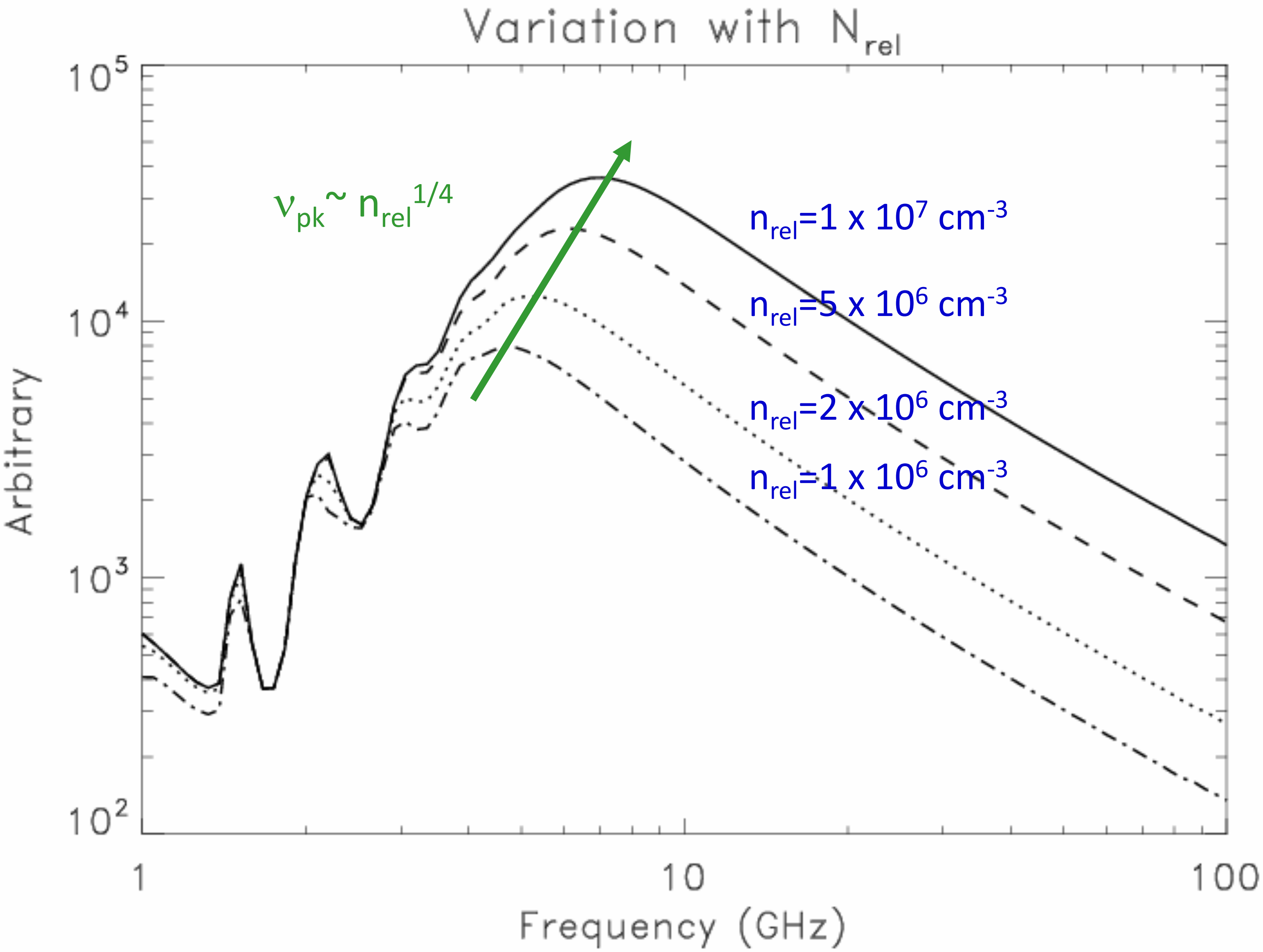
| “exact” | approximate |
|---|---|
| Ramaty 1969 | Petrosian 1981 |
| Benka & Holman 1992 | Dulk & Marsh 1982, 1985 |
| | Klein 1987 |

# Spectrum is also more complicated



Klein (1987)


Variation with B
$\nu_{pk} \sim B^{3/4}$
B=1000 G
B=500 G
B=200 G
B=100 G
Arbitrary
$10^5$
$10^4$
$10^3$
$10^2$
1
10
100
Frequency (GHz)



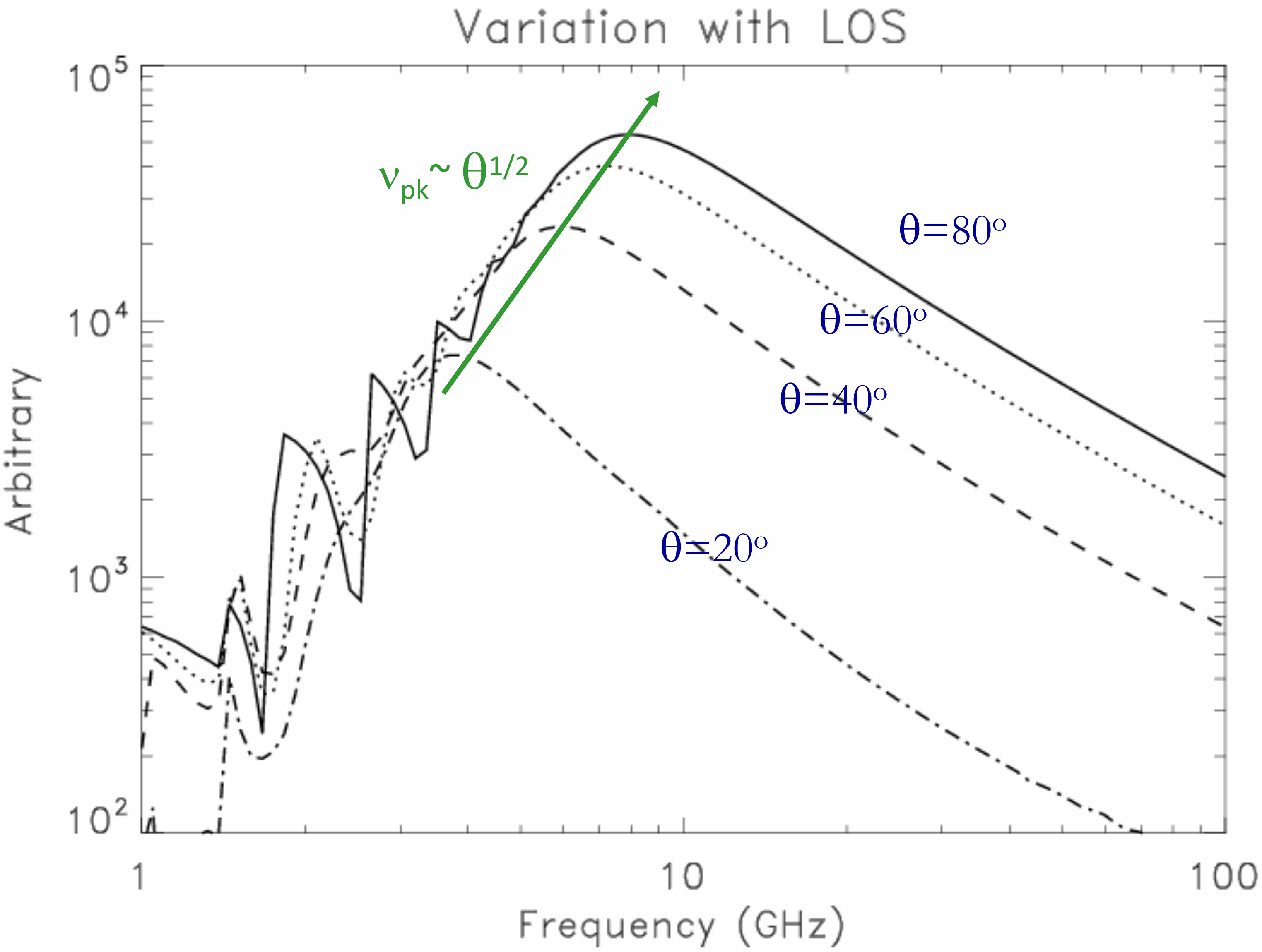
$\nu_{pk} \sim n_{rel}^{1/4}$

$n_{rel} = 1 \times 10^7$ cm$^{-3}$

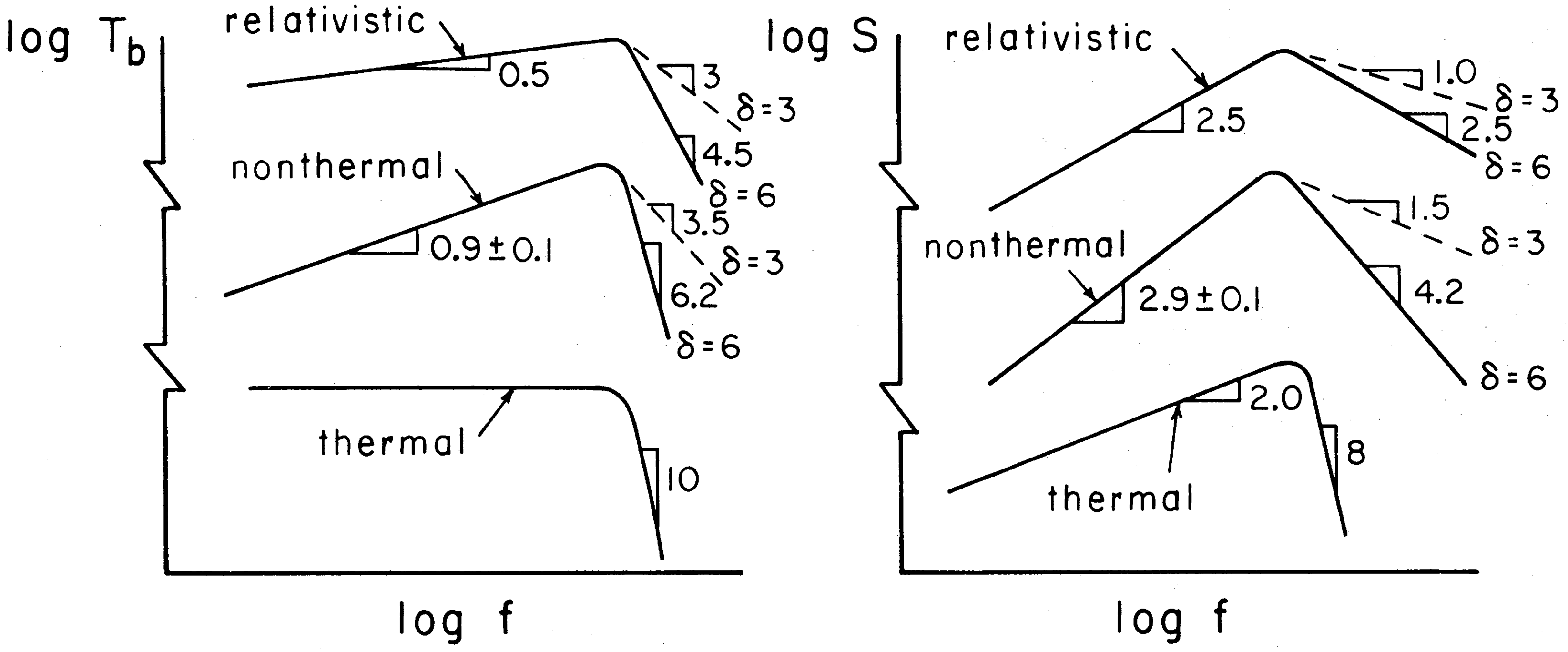
$n_{rel} = 5 \times 10^6$ cm$^{-3}$

$n_{rel} = 2 \times 10^6$ cm$^{-3}$

$n_{rel} = 1 \times 10^6$ cm$^{-3}$

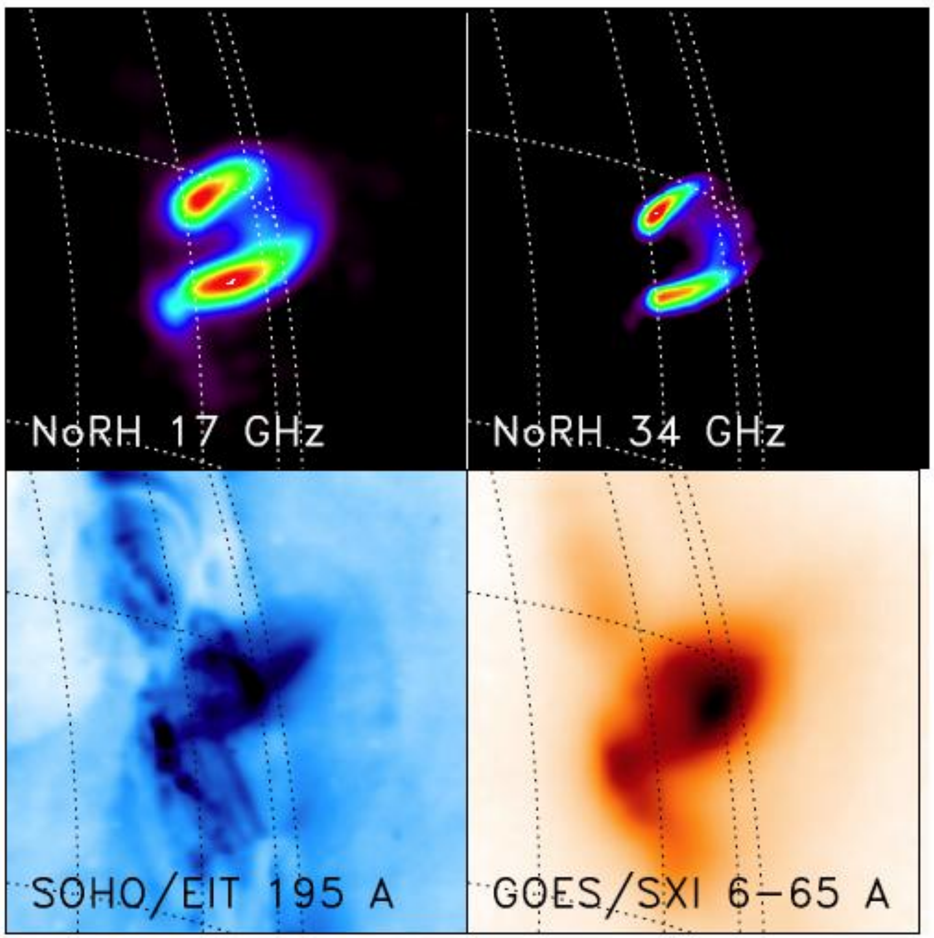

Variation with LOS
$10^5$
$10^4$
$10^3$
$10^2$
Arbitrary
1
10
100
Frequency (GHz)
$\nu_{pk} \sim \theta^{1/2}$
$\theta = 80^o$
$\theta = 60^o$
$\theta = 40^o$
$\theta = 20^o$

# (Gyro)synchrotron spectrum



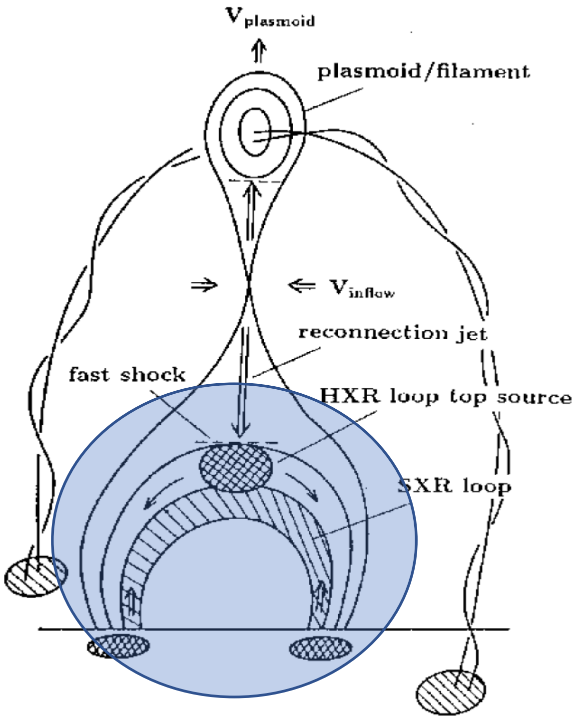
Schematic diagram from Dulk & Marsh 1982

# Gyrosynchrotron in flares



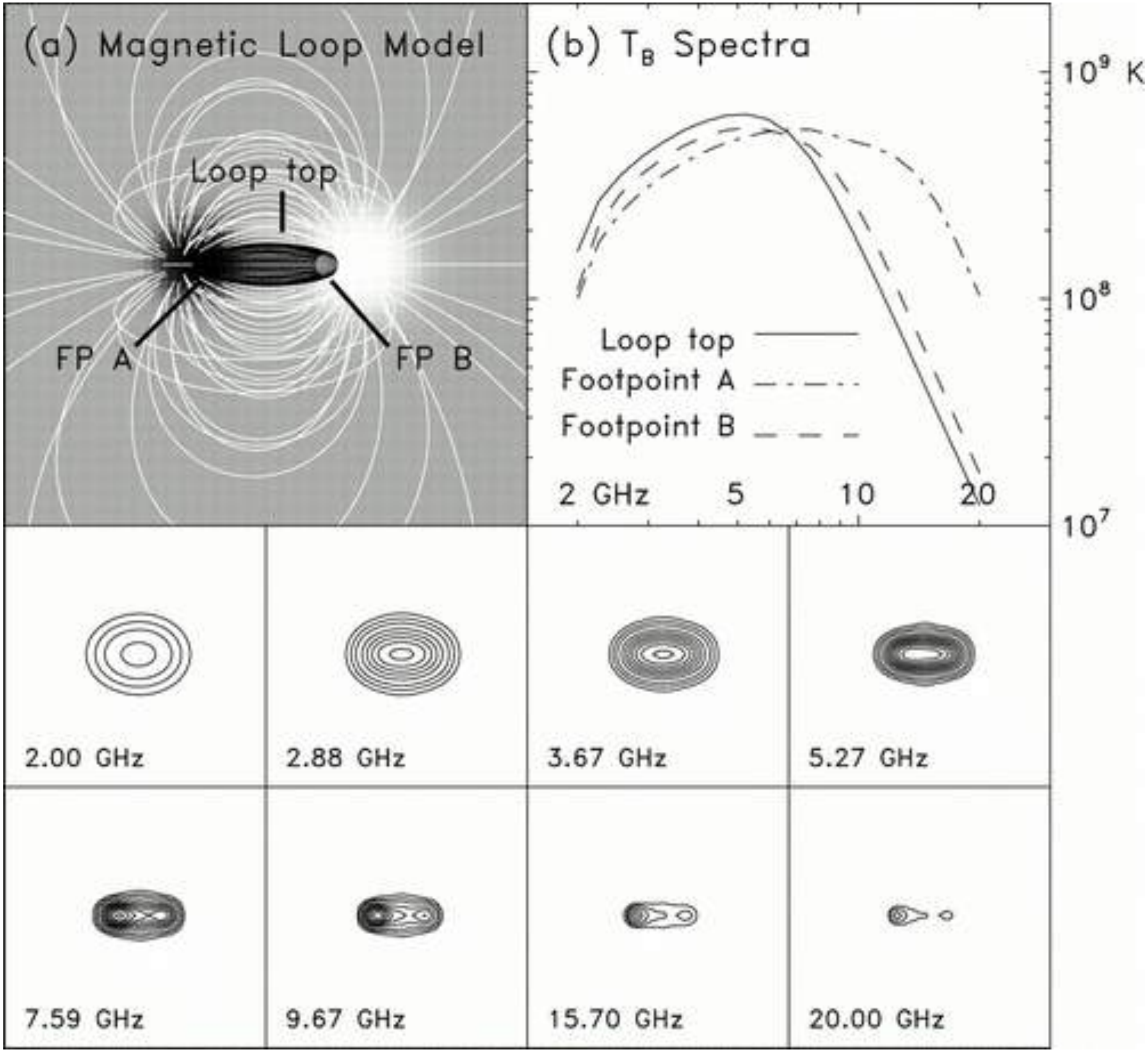
Flare observed by SOHO, GOES, and Nobeyama Radioheliograph at 17 and 34 GHz

- Microwave: gyrosynchotron
- EUV/SXR: hot thermal plasma



From T. Bastian

# A schematic model of a flare loop



Bastian et al 1998

# Summary

- Gyromagnetic radiation results from electrons accelerated in the magnetic field
- Three different regimes based on energy of the source electrons: gyroresonance, gyrosynchrotron, and synchrotron
- Gyroresonance can be used to diagnose B fields in active regions
- Gyrosynchrotron can be used to probe flare-accelerated electrons and diagnose B field in flare loops
- Synchrotron is more relevant to cosmic sources, but still possible on the Sun (e.g., the mysterious sub-THz flare component)