Vehicle Load Transfer

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Vehicle Load Transfer

Part I

General Load Transfer

Factors in Vehicle Dynamics

- Within any modern vehicle suspension there are ***many*** factors to consider during design and development.

- Factors in vehicle dynamics:
  - Vehicle Configuration
    - Vehicle Type (i.e. 2 dr Coupe, 4dr Sedan, Minivan, Truck, etc.)
    - Vehicle Architecture (i.e. FWD vs. RWD, 2WD vs.4WD, etc.)
    - Chassis Architecture (i.e. type: tubular, monocoque, etc. ; material: steel, aluminum, carbon fiber, etc. ; fabrication: welding, stamping, forming, etc.)
    - Front Suspension System Type (i.e. MacPherson strut, SLA Double Wishbone, etc.)
    - Type of Steering Actuator (i.e. Rack and Pinion vs. Recirculating Ball)
    - Type of Braking System (i.e. Disc (front & rear) vs. Disc (front) & Drum (rear))
    - Rear Suspension System Type (i.e. Beam Axle, Multi-link, Solid Axle, etc.)
    - Suspension/Braking Control Systems (i.e. ABS, Electronic Stability Control, Electronic Damping Control, etc.)

Factors in Vehicle Dynamics

- Factors in vehicle dynamics (continued):
  - Vehicle Suspension Geometry
    - Vehicle Wheelbase
    - Vehicle Track Width Front and Rear
    - Wheels and Tires
    - Vehicle Weight and Distribution
      - Vehicle Center of Gravity
      - Sprung and Unsprung Weight
      - Springs Motion Ratio
    - Chassis Ride Height and Static Deflection
    - Turning Circle or Turning Radius (Ackermann Steering Geometry)
    - Suspension Jounce and Rebound
    - Vehicle Suspension Hard Points:
      - Front Suspension
        - Scrub (Pivot) Radius
        - Steering (Kingpin) Inclination Angle (SAI)
        - Caster Angle
        - Mechanical (or caster) trail
        - Toe Angle
        - Camber Angle
        - Ball Joint Pivot Points
        - Control Arm Chassis Attachment Points
        - Knuckle/Brakes/Steering
        - Springs/Shock Absorbers/Struts
        - ARB (anti-roll bar)

Factors in Vehicle Dynamics

- Factors in vehicle dynamics (continued):
  - Vehicle Suspension Geometry (continued)
    - Vehicle Suspension Hard Points (continued):
      - Rear Suspension
        - Scrub (Pivot) Radius
        - Steering (Kingpin) Inclination Angle (SAI)
        - Caster Angle (if applicable)
        - Mechanical (or caster) trail (if applicable)
        - Toe Angle
        - Camber Angle
        - Knuckle and Chassis Attachment Points
          - Various links and arms depend upon the Rear Suspension configuration. (i.e. Dependent vs. Semi-Independent vs. Independent Suspension)
        - Knuckle/Brakes
        - Springs/Shock Absorbers
        - ARB (anti-roll bar)
  - Vehicle Dynamic Considerations
    - Suspension Dynamic Targets
      - Wheel Frequency
      - Bushing Compliance
      - Lateral Load Transfer with and w/o ARB
      - Roll moment
      - Roll Stiffness (degrees per g of lateral acceleration)
      - Maximum Steady State lateral acceleration (in understeer mode)
      - Rollover Threshold (lateral g load)
      - Linear Range Understeer (typically between 0g and 0.4g)

Factors in Vehicle Dynamics

- Factors in vehicle dynamics (continued):
  - Vehicle Dynamic Considerations (continued)
    - Suspension Dynamic Analysis
      - Bundorf Analysis
        - Slip angles (degrees per lateral force)
        - Tire Cornering Coefficient (lateral force as a percent of rated vertical load per degree slip angle)
        - Tire Cornering Forces (lateral cornering force as a function of slip angle)
        - Linear Range Understeer
      - Steering Analysis
        - Bump Steer Analysis
        - Roll Steer Analysis
        - Tractive Force Steer Analysis
        - Brake Force Steer Analysis
        - Ackerman change with steering angle
      - Roll Analysis
        - Camber gain in roll (front & rear)
        - Caster gain in roll (front & rear – if applicable)
        - Roll Axis Analysis
        - Roll Center Height Analysis
        - Instantaneous Center Analysis
        - Track Analysis
      - Load Transfer Analysis
        - Unsprung and Sprung weight transfer
        - Jacking Forces
      - Roll Couple Percentage Analysis
        - Total Lateral Load Transfer Distribution (TLLTD)

Vehicle Load Transfer

- While the total amount of factors may seem a bit overwhelming, it may be easier to digest if we break it down into certain aspects of the total.

- The intent of this document is to give the reader a better understanding of vehicle dynamic longitudinal and lateral load transfer as a vehicle accelerates/decelerates in a particular direction.

- The discussion will include:

  Part I – General Load Transfer Information
  - Load vs. Weight Transfer
  - Rotational Moments of Inertia
  - Sprung and Unsprung Weight

  Part II – Longitudinal Load Transfer
  - Vehicle Center of Gravity
  - Longitudinal Load Transfer
  - Suspension Geometry
    - Instant Centers
    - Side View Swing Arm
  - Anti-squat, Anti-dive, and Anti-lift

  Part III – Lateral Load Transfer
  - Cornering Forces
  - Suspension Geometry
    - Front View Swing Arm
    - Roll Center Heights
    - Roll Axis
  - Roll Stiffness
    - Anti-roll bars
    - Tire Rates
  - Roll Gradient
  - Lateral Load Transfer

Load vs. Weight Transfer

Load vs. Weight Transfer

- In automobiles, **load transfer** is the imaginary "shifting" of weight around a motor vehicle during acceleration (both longitudinal and lateral). This includes braking, or deceleration (which can be viewed as acceleration at a negative rate). Load transfer is a crucial concept in understanding vehicle dynamics.

- Often load transfer is misguidedly referred to as weight transfer due to their close relationship. The difference being load transfer is an imaginary shift in weight due to an imbalance of forces, while weight transfer involves the actual movement of the vehicles center of gravity (Cg). Both result in a redistribution of the total vehicle load between the individual tires.

Load vs. Weight Transfer

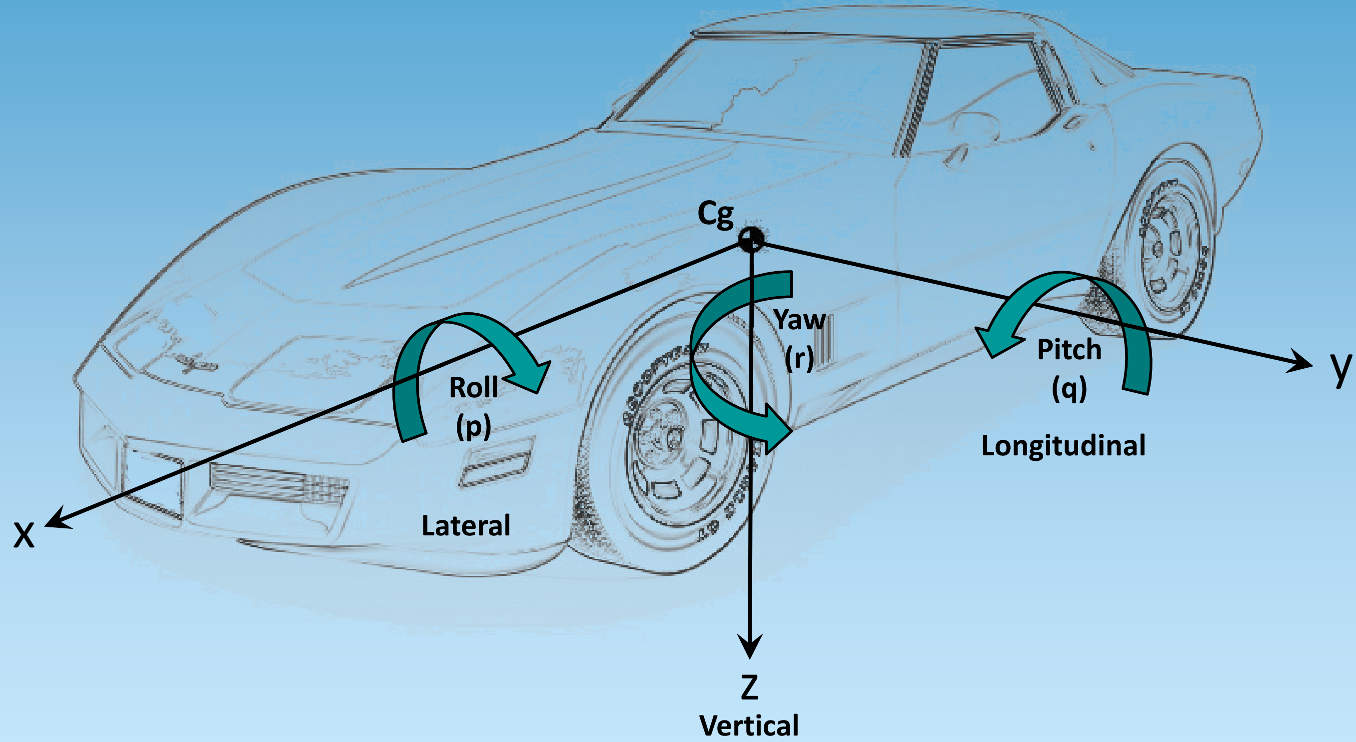
- **Weight transfer** involves the *actual* (small) movement of the vehicle Cg relative to the wheel axes due to displacement of liquids within the vehicle, which results in a redistribution of the total vehicle load between the individual tires.
- Liquids, such as fuel, readily flow within their containers, causing changes in the vehicle's Cg. As fuel is consumed, not only does the position of the Cg change, but the total weight of the vehicle is also reduced.
- Another factor that changes the vehicle’s Cg is the expansion of the tires during rotation. This is called “dynamic rolling radius” and is effected by wheel-speed, temperature, inflation pressure, tire compound, and tire construction. It raises the vehicle’s Cg slightly as the wheel-speed increases.

Load vs. Weight Transfer

- The major forces that accelerate a vehicle occur at the tires contact patch. Since these forces are not directed through the vehicle's Cg, one or more moments are generated. It is these moments that cause variation in the load distributed between the tires.

- Lowering the Cg towards the ground is one method of reducing load transfer. As a result load transfer is reduced in both the longitudinal and lateral directions. Another method of reducing load transfer is by increasing the wheel spacings. Increasing the vehicles wheel base (length) reduces longitudinal load transfer. While increasing the vehicles track (width) reduces lateral load transfer.

Rotational Moments of Inertia

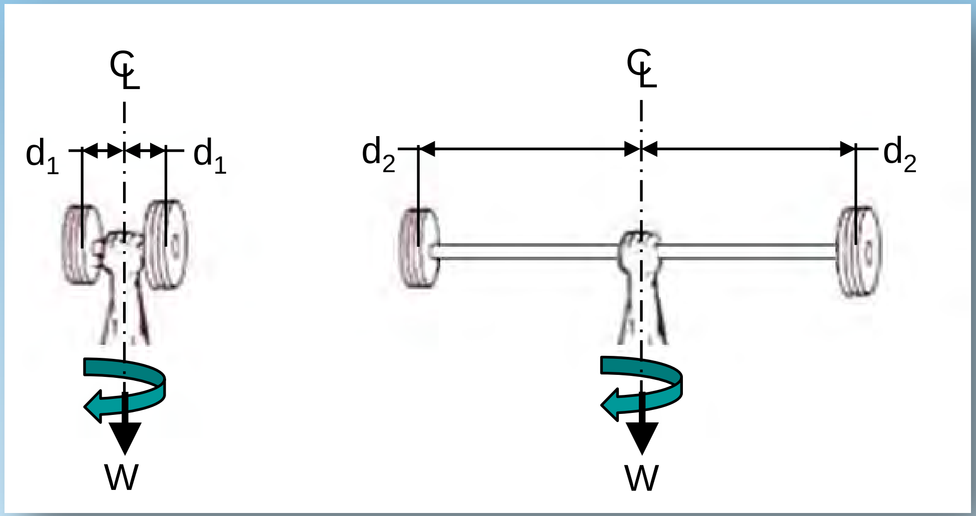


Moment of Inertia

- **Polar moment of inertia**
  - A simple demonstration of polar moment of inertia is to compare a dumbbell vs. a barbell both at the same weight. Hold each in the middle and twist to feel the force reacting at the center. Notice the dumbbell (which has a lower polar moment) reacts quickly and the barbell (which has a higher polar moment) reacts slowly.

$$I_o = Wd^2$$

Example:
W = 50 lb (25 lb at each end)
d_1 = 8 in
d_2 = 30 in



$$I_1 = 2 * 25 * (8)^2 = 3200 lb \cdot in^2 \qquad I_2 = 2 * 25 * (30)^2 = 45{,}000 lb \cdot in^2$$

Moment of Inertia

- **Sum the polar moments of inertia**
  - The total polar moment of inertia for a vehicle can be determined by multiplying the weight of each component by the distance from the component Cg to the Cg of the vehicle. The sum of the component polar moments of inertia would establish the total vehicle polar moment of inertia.
  - A vehicle with most of its weight near the vehicle Cg has a lower total polar moment of inertia is quicker to respond to steering inputs.
  - A vehicle with a high polar moment is slower to react to steering inputs and is therefore more stable at high speed straight line driving.

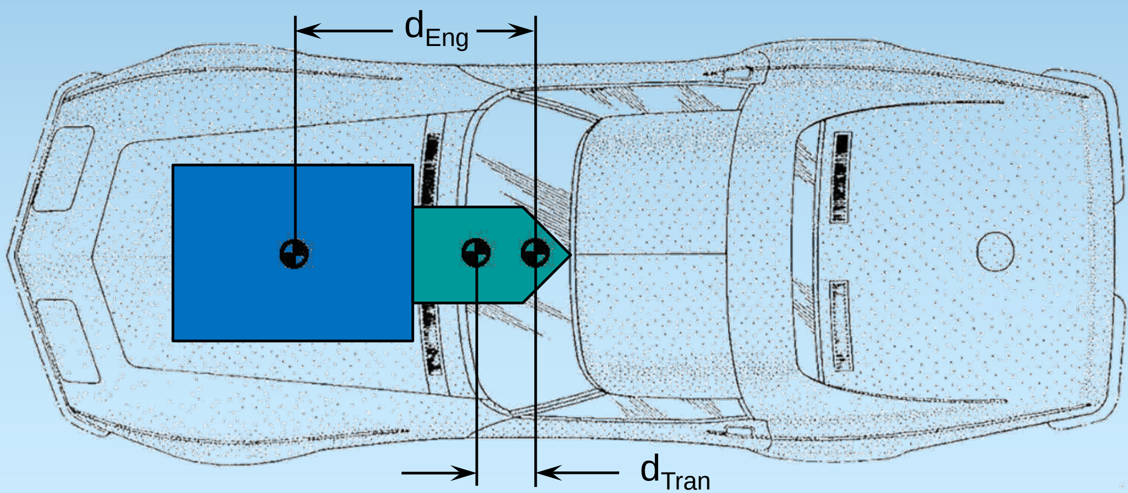
Moment of Inertia

- **Effects of polar moments of inertia**
  - Here is an example of a V8 engine with a typical transmission packaged into a sports car.

$$\sum M(\mathrm{I}_o) = W_{Eng}(d_{Eng})^2 + W_{Tran}(d_{Tran})^2$$

$$\sum M(\mathrm{I}_o) = 600lb(40in)^2 + 240lb(10in)^2 = 984{,}000 lb \cdot in^2$$

Example:
W_{Eng} = 600 lb
W_{Tran} = 240 lb
d_{Eng} = 40 in
d_{Tran} = 10 in



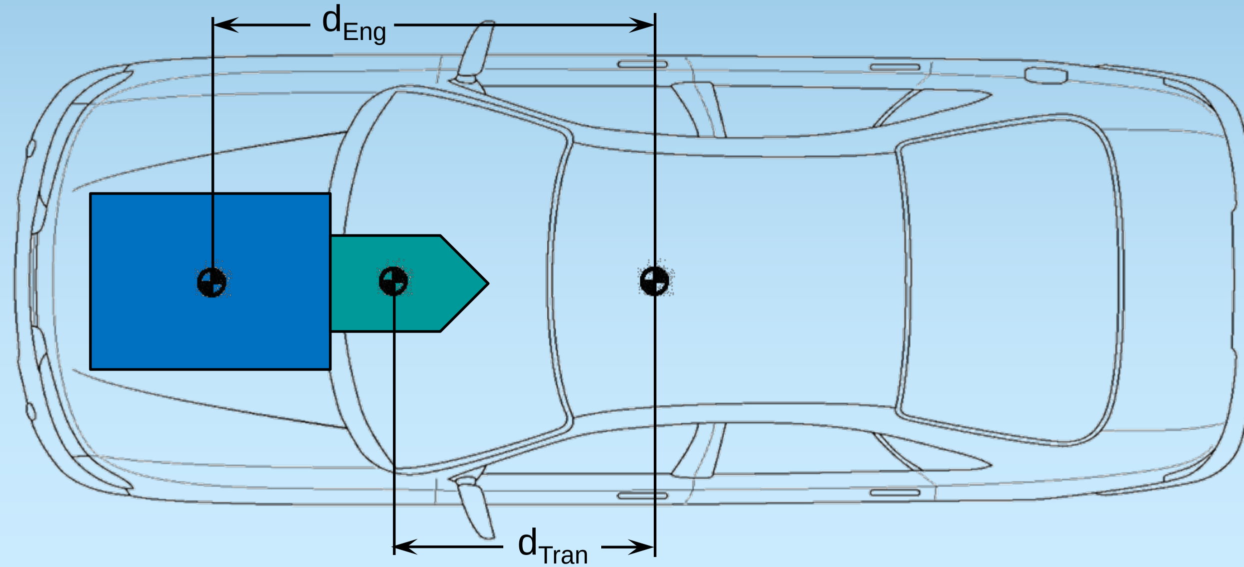
Moment of Inertia

- **Effects of polar moments of inertia**
  - Here is an example of a V8 engine with a typical transmission packaged into a sedan.

$$\sum M(\mathrm{I}_o) = W_{Eng}(d_{Eng})^2 + W_{Tran}(d_{Tran})^2$$

$$\sum M(\mathrm{I}_o) = 600lb(70in)^2 + 240lb(40in)^2 = 3{,}324{,}000 lb \cdot in^2$$

Example:
W_{Eng} = 600 lb
W_{Tran} = 240 lb
d_{Eng} = 70 in
d_{Tran} = 40 in



Load Transfer

Load Transfer

- **Load Transfer**

  - The forces that enable a road vehicle to accelerate and stop all act at the road surface.

  - The center of gravity, which is located considerably above the road surface, and which is acted upon by the accelerations resulting from the longitudinal forces at the tire patches, generates a moment which transfers load.

  - As asymmetric load results in differing traction limits, a vehicles handling is affected by the “dynamic load distribution”.

Load Transfer

Load Transfer equations & terms

$$Newton's\ Second\ Law:\ F = m\,a$$

$$Inertial\ Force = \frac{Vehicle\ Weight * Vehicle\ Acceleration}{g}$$

$$Load\ Transfer = \frac{Inertial\ Force * CG_{height}}{Wheelbase}$$

F = force
m = mass
a = acceleration
g = a_g = acceleration due to gravity
= 32.2ft/sec^2 = 9.8m/sec^2

a_x = acceleration in the x direction
a_y = acceleration in the y direction
a_z = acceleration in the z direction
Weight = mass * a_g

Load Transfer

- Load transfers between the Center of Gravity and the road surface through a variety of paths.

  - Suspension Geometry
    - Front: Location of instant centers
      (Side View Swing Arm)
    - Rear: Instant centers, Lift Bars
      (Side View Swing Arm)

  - Suspension Springs
    - Front: Coils, Air Springs, leafs or Torsion bars and Anti-roll bars
    - Rear: Coils, Air Springs, leafs or Torsion bars and Anti-roll bars

Load Transfer

Load transfer (continued)

- Dampers (Shock Absorbers)
    - During transient conditions

- Tires
    - During all conditions (where the rubber meets the road)

Where and how you balance the load transfer between the ***Springs, Geometry, Dampers and Tires*** *are key determinates as to how well the car will accelerate and brake and the stability associated with each condition.*

Load Transfer Control Devices

- **Dampers (Shock Absorbers)**

  - Along with the springs, dampers transfer the load of the rolling (pitching) component of the vehicle. They determine how the load is transferred to and from the individual wheels while the chassis is rolling and/or pitching.

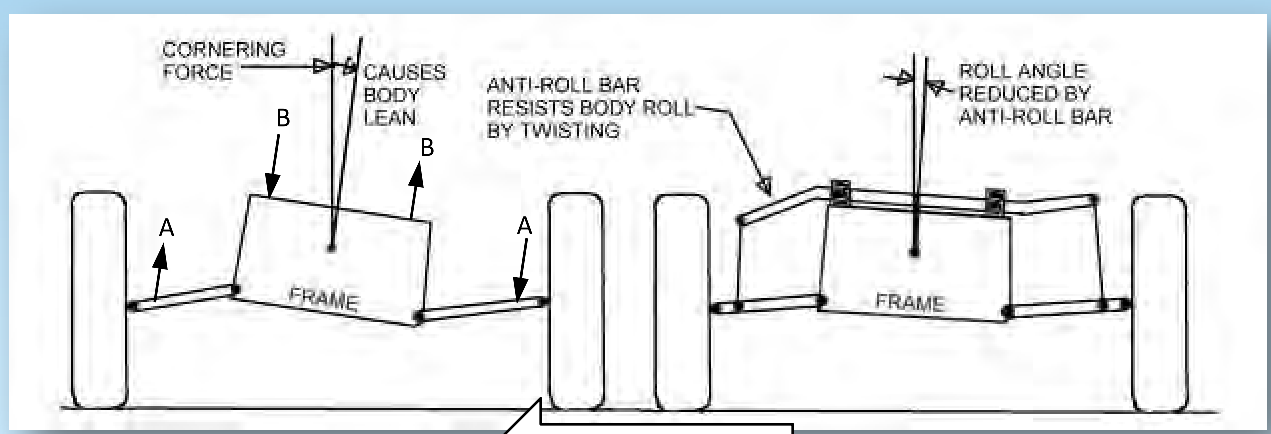
  - Within 65-70% critically damped is said to be the ideal damper setting for both handling and comfort simultaneously. Most modern dampers show some digression to them as well, meaning they may be 70% critically damped at low piston speeds but move lower to allow the absorption of large bumps. Damping is most important below 4 in/second as this is where car control tuning takes place.

Load Transfer Control Devices

- **Springs**
  - Along with the dampers (shock absorbers), springs transfer the load of the sprung mass of the car to the road surface. During maneuvers, depending on instant center locations, the springs and dampers transfer some portion of the (m x a), mass x acceleration, forces to the ground.
  - Spring Rate is force per unit displacement for a suspension spring alone .
  - For coil springs this is measured axially along the centerline.
  - For torsion bar springs it is measured at the attachment arm.
  - For leaf springs it is measured at the axle seat.
  - The spring rate may be linear (force increases proportionally with displacement) or nonlinear (increasing or decreasing rate with increasing displacement).
  - Units are typically lb/in.

Load Transfer Control Devices

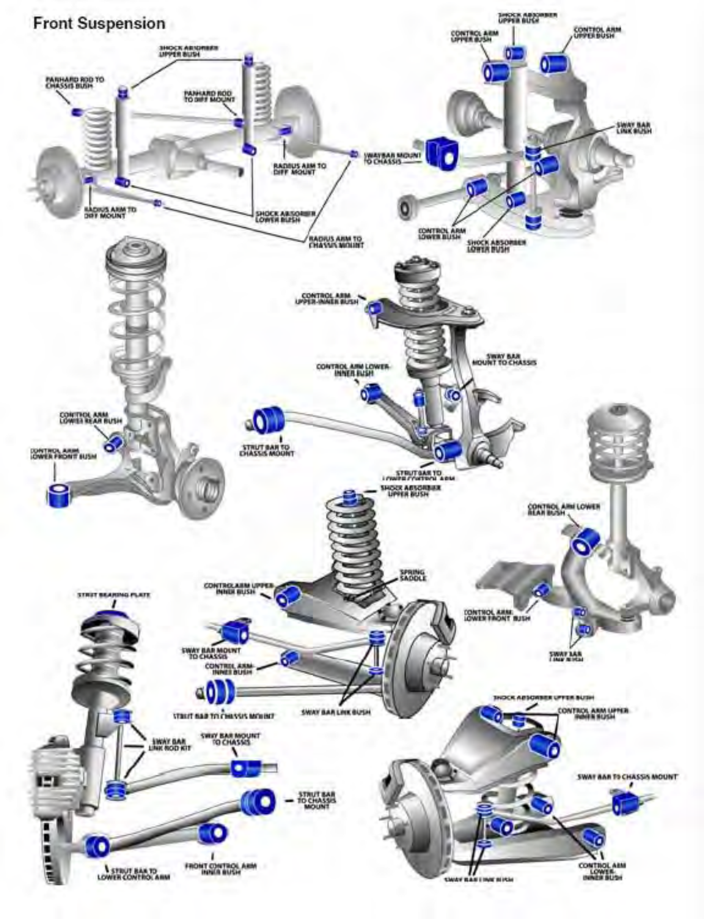
- **Anti-roll bars**
  - [Drawing 1] shows how an anti-roll bar (ARB) is twisted when the body rolls in a turn. This creates forces at the four points where the bar is attached to the vehicle. The forces are shown in [Drawing 2]. Forces A on the suspension increase [load] transfer to the outside tire. Forces B on the frame resist body roll. The effect is a reduction of body roll and an increase in [load] transfer at the end of the chassis which has the anti-roll bar. Because the total [load] transfer due to centripetal force is not changed, the opposite end of the chassis has reduced [load] transfer. [6]

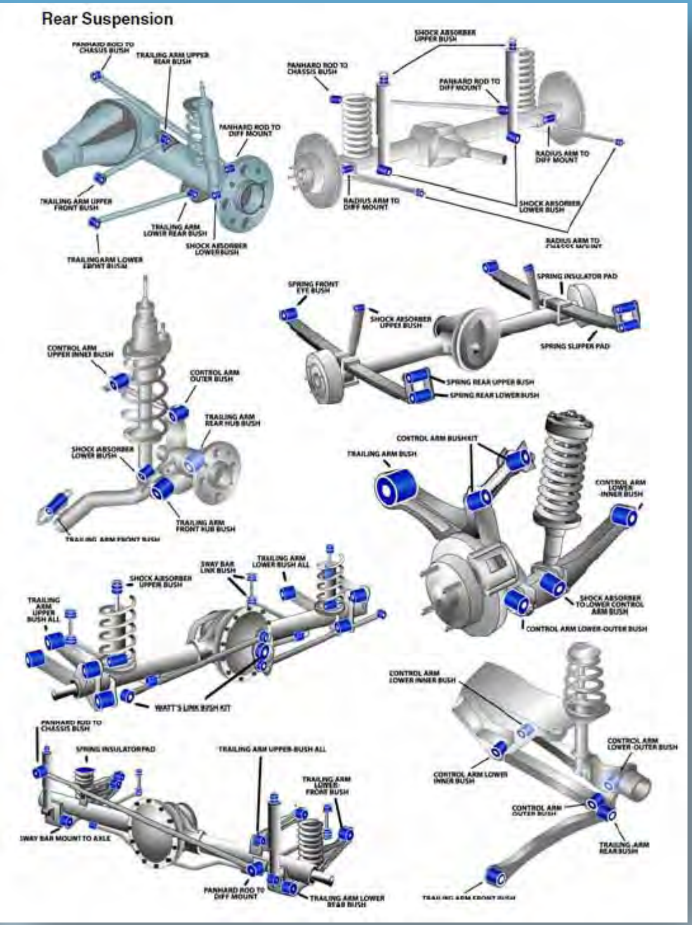


Drawing 2 Direction of Turn Drawing 1

Load Transfer Control Devices

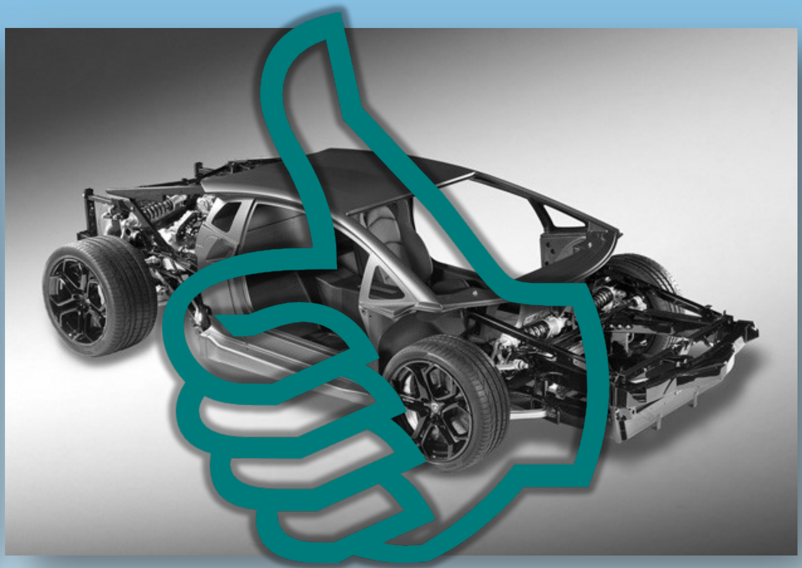
- **Bushing Deflection (suspension compliance)**
  - All of the calculations shown in this presentation **do not** include bushing deflection. There are many rubber bushings within a vehicle suspension to consider when analyzing suspension compliance.





Load Transfer Control Devices

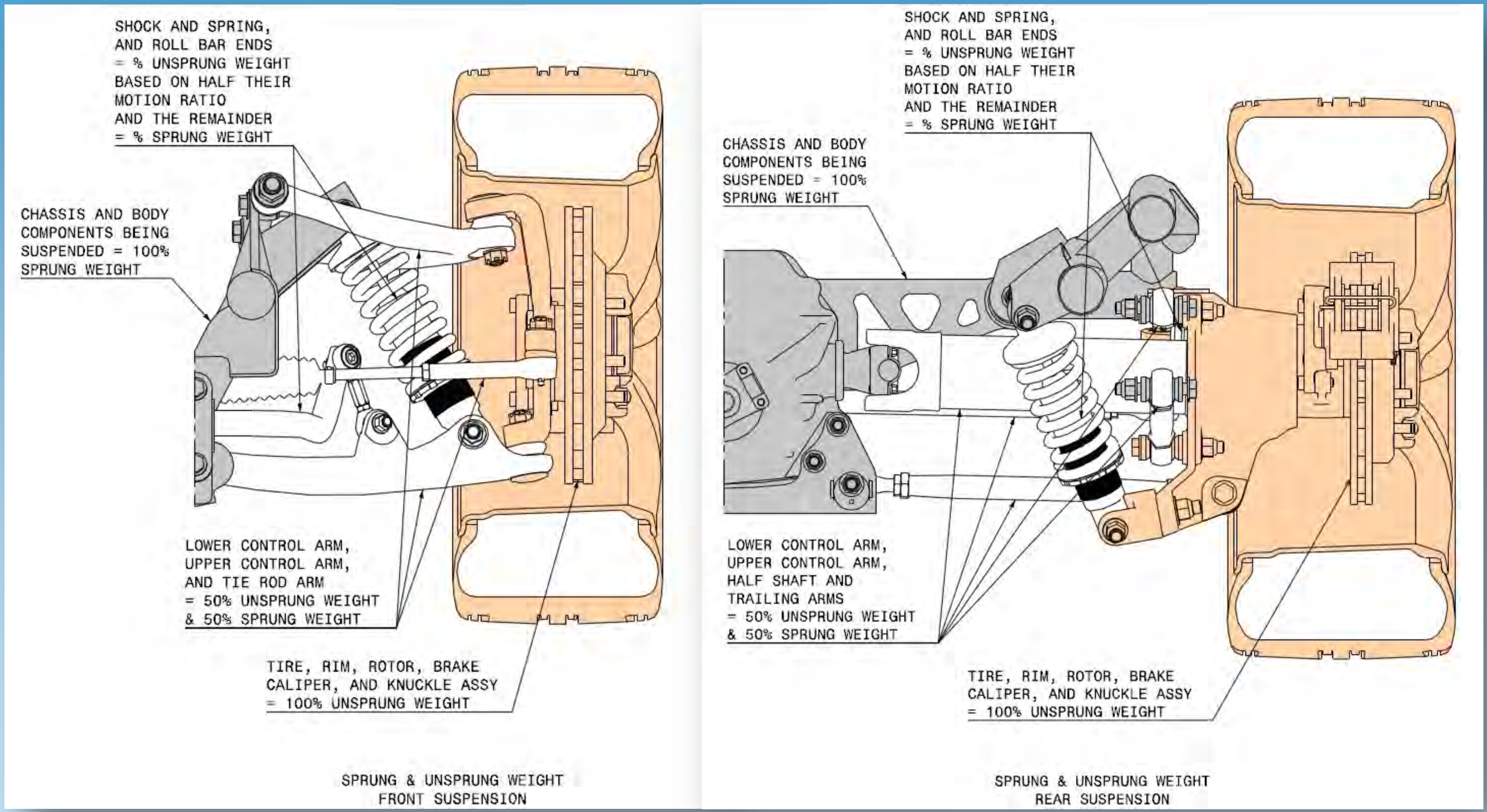
- **Frame/Chassis Deflection**
  - All of the calculations shown in this presentation are made under the assumption that the frame or chassis is completely rigid (both in torsion and bending). Of course any flexing within the frame/chassis will adversely effect the performance of the suspension which is attached to it.



Sprung and Unsprung Weight

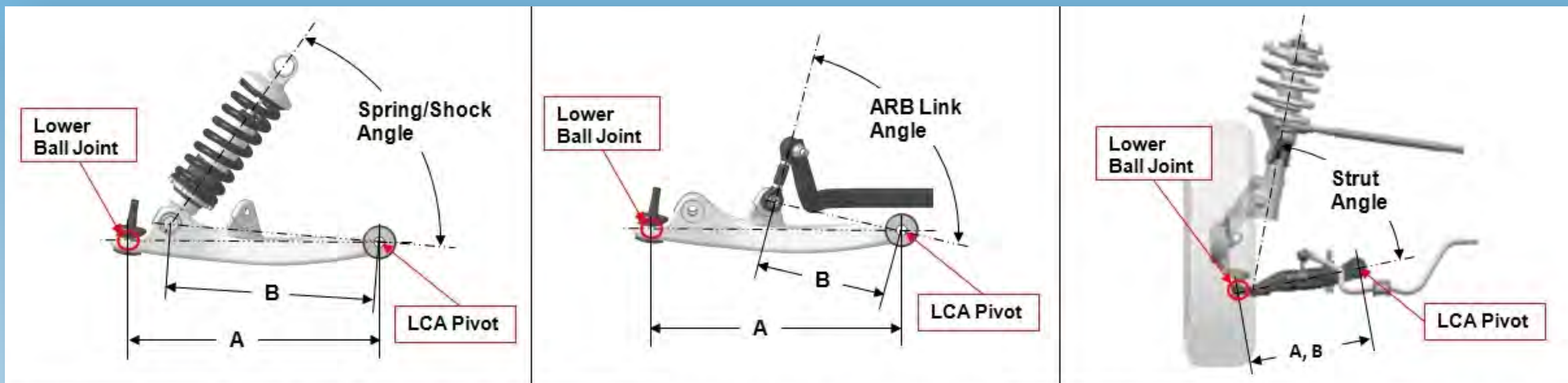
- 100% Unsprung weight includes the mass of the tires, rims, brake rotors, brake calipers, knuckle assemblies, and ball joints which move in unison with the wheels.
- 50% unsprung and 50% sprung weight would be comprised of the linkages of the wheel assembly to the chassis.
- The % unsprung weight of the shocks, springs and anti-roll bar ends would be a function of their motion ratio/2 with the remainder as % sprung weight.
- The rest of the mass is on the vehicle side of the springs is suspended and is 100% sprung weight.

Sprung and Unsprung Weight



Springs Motion Ratio

- The shocks, springs, struts and anti-roll bars are normally mounted at some angle from the suspension to the chassis.
- Motion Ratio: If you were to move the wheel 1 inch and the spring were to deflect 0.75 inches then the motion ratio would be 0.75 in/in.



For double-wishbone suspensions, B and A are usually different and line A is normally parallel with the ground at ride height.

Though the angle changes throughout jounce to rebound, for calculations done within this spreadsheet the Spring Angle will be taken at Ride Height.

For ARB (anti-roll bar) motion ratio, the link angle is used as the spring angle.

Bushing Deflection plays a major roll in the effectiveness of the ARB spring rate.

For strut based suspension, B and A are always the same due to the load path through the knuckle to the ball joint.

The Strut Angle is the angle between the lower ball joint/LCA pivot and upper strut mount.

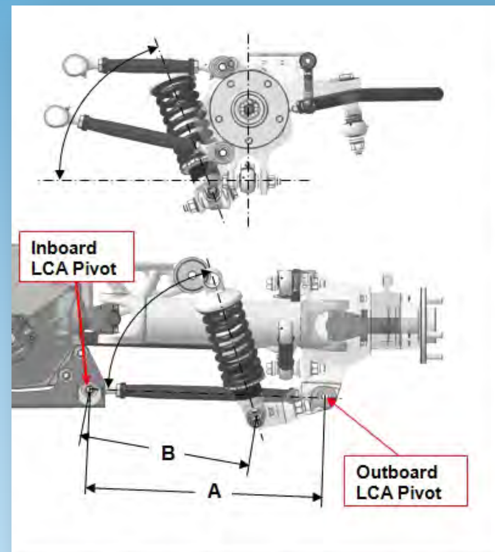
Distance to mounting point (B):	12.497	in	Distance to mounting point (B):	9.61	in	Distance to mounting point (B):	11	in
Total length (A):	14.94	in	Total length (A):	14.94	in	Total length (A):	11	in
Angle of spring/shock:	65.1	degrees	Angle of ARB link:	90	degrees	Angle of strut:	72	degrees
Spring Motion Ratio:	**0.759**	**(in/in)**	**ARB Motion Ratio:**	**0.643**	**(in/in)**	**Strut Motion Ratio:**	**0.951**	**(in/in)**

Motion Ratio = (B/A) * sin(spring angle)

Springs Motion Ratio

- The shocks, springs, struts and anti-roll bars are normally mounted at some angle from the suspension to the chassis.
- Motion Ratio: If you were to move the wheel 1 inch and the spring were to deflect 0.75 inches then the motion ratio would be 0.75 in/in.

Motion Ratio =
(B/A) * sin(spring angle)

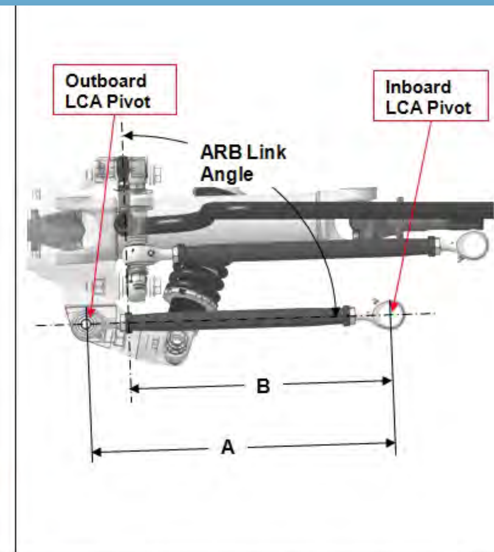


For multilink suspensions with coil-over shocks or struts, the spring and shock/strut are mounted along the same axis.

If the spring and shock/strut are mounted at compound angles (i.e. cross-car tilt and fore-aft tilt), the motion ratio would be a resultant of the compound angle.

The compound angle is measured from the plane between lines A & B and the centerline of the spring/shock.

Distance to mounting point (B):	13	in
Total length (A):	17.5	in
Compound Angle of spring/shock:	77	degrees
Spring Motion Ratio:	**0.724**	**(in/in)**



For ARB (anti-roll bar) motion ratio, the link angle is used as the spring angle.

Bushing Deflection plays a major roll in the effectiveness of the ARB spring rate.

Distance to mounting point (B):	13.13	in
Total length (A):	17.5	in
Angle of ARB link:	90	degrees
ARB Motion Ratio:	**0.750**	**(in/in)**

Wheel Rates

- Wheel Rates are calculated by taking the square of the motion ratio times the spring rate. Squaring the ratio is because the ratio has two effects on the wheel rate. The ratio applies to both the force and distance traveled.

- Because it's a force, and the lever arm is multiplied twice.
  - The motion ratio is factored once to account for the distance-traveled differential of the two points (A and B in the example below).
  - Then the motion ratio is factored again to account for the lever-arm force differential.

Example:

```
      K
      |
A----B------------P
```

Wheel Rate (lb/in) =
(Motion Ratio)^2* (Spring Rate)

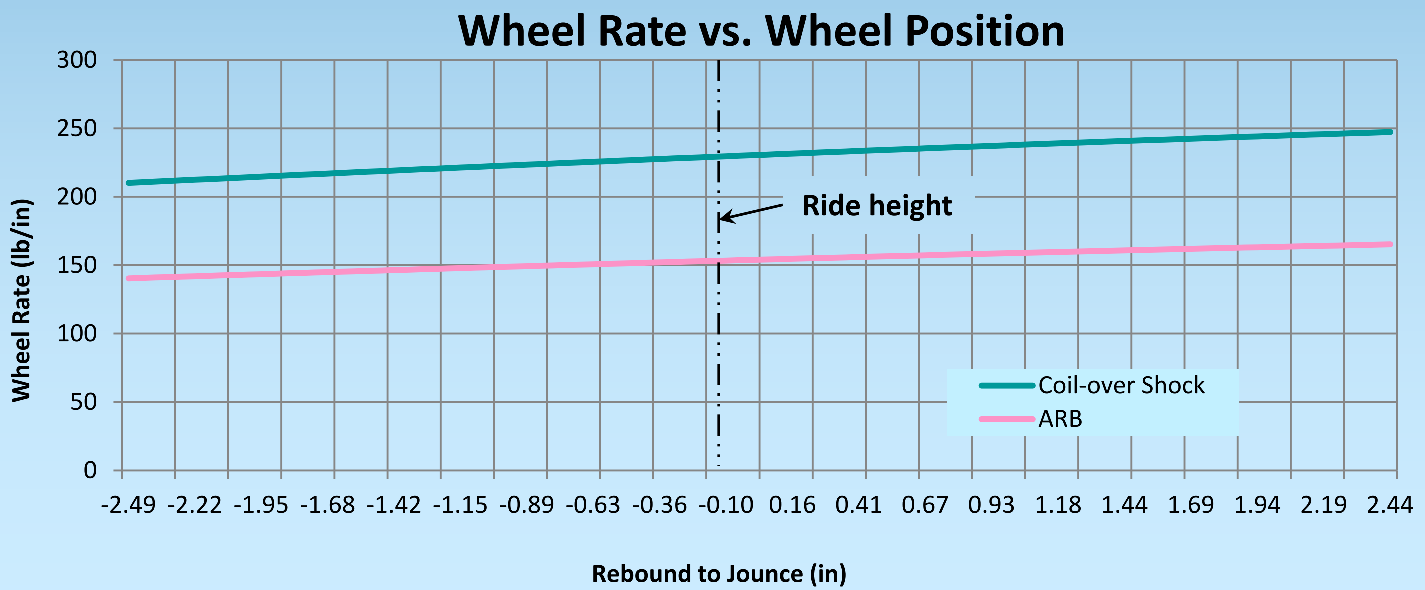
- P is the pivot point, B is the spring mount, and A is the wheel. Here the motion ration (MR) is 0.75... imagine a spring K that is rated at 100 lb/in placed at B perpendicular to the line AP. If you want to move A 1 in vertically upward, B would only move (1in)(MR) = 0.75 in. Since K is 100 lb/in, and B has only moved 0.75 in, there's a force at B of 75 lb. If you balance the moments about P, you get 75(B)=X(A), and we know B = 0.75A, so you get 75(0.75A) = X(A). A's cancel and you get X=75(0.75)=56.25. Which is [100(MR)](MR) or 100(MR)^2.

Wheel Rates

- Since the linkages pivot, the spring angles change as the components swing along an arc path. This causes the motion ratio to be calculated through a range. The graph below shows an example of these results for both coil-over shock and anti-roll bar for an independent front suspension from rebound to jounce positions.

Example:
Coil-over K_S = 400 lb/in (linear)
Coil-over MR = 0.72-.079 in/in
ARB K_S = 451.8 lb/in (body roll)
ARB MR = 0.56-0.61 in/in

K_W = Wheel Rate (lb/in) =
(Motion Ratio - range)^2* (Spring Rate - linear)
$K_W = MR^2 * K_S$

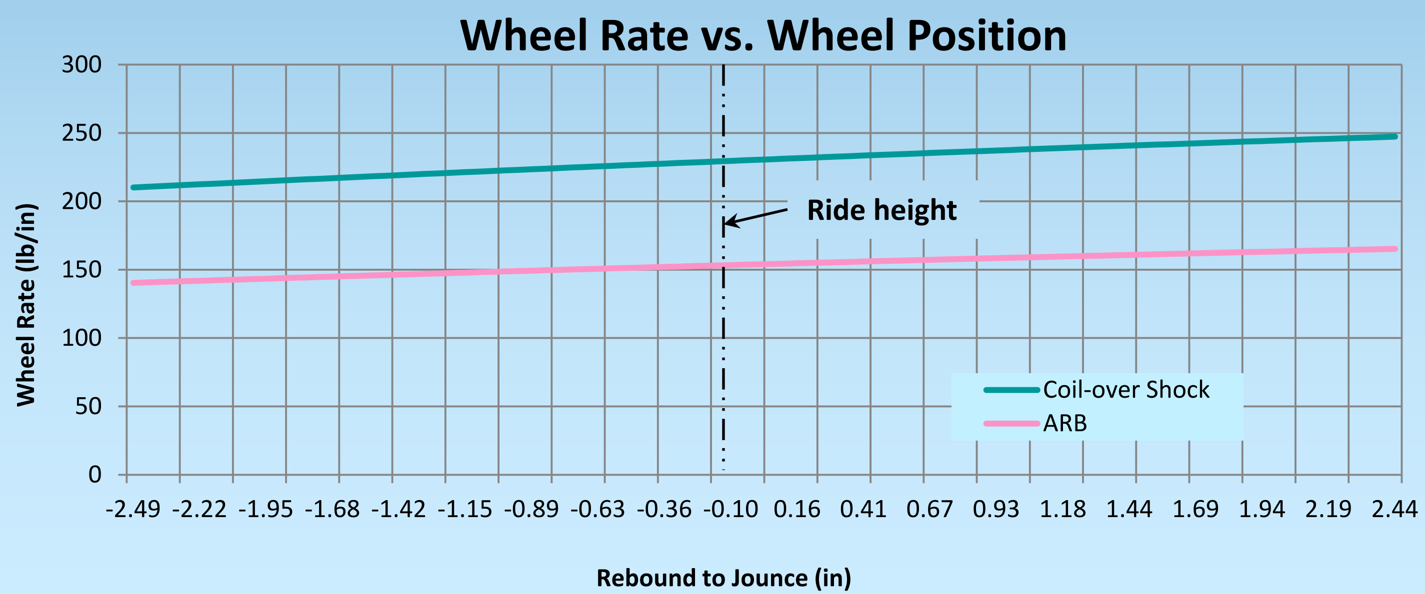


Wheel Rates

- In longitudinal pitch, the anti-roll bar (ARB) rotates evenly as the chassis moves relative to the suspension. Therefore, the ARB only comes into play during lateral pitch (body roll) of the vehicle (it also comes into play during one wheel bump, but that rate is not shown here).

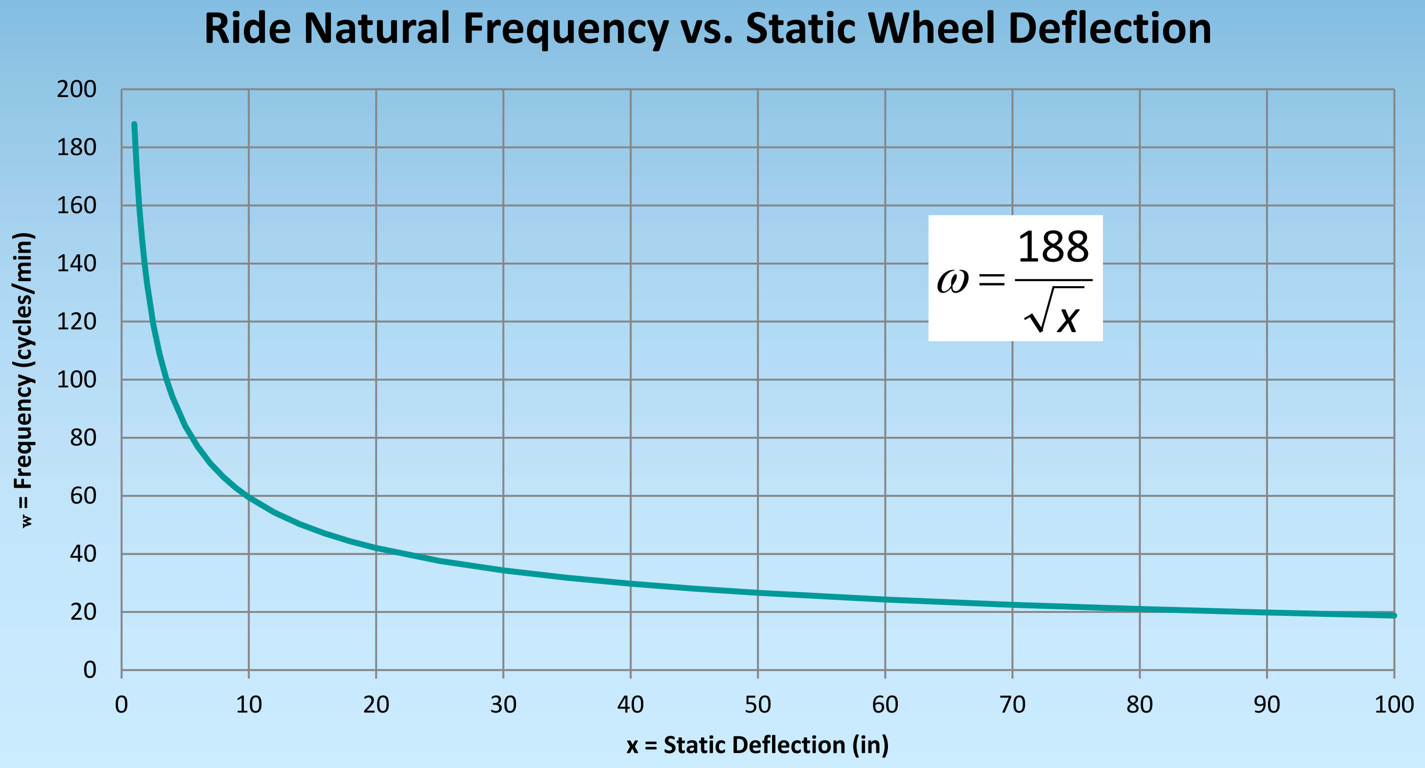
Example:
Coil-over K_S = 400 lb/in (linear)
Coil-over MR = 0.72-.079 in/in
ARB K_S = 451.8 lb/in (body roll)
ARB MR = 0.56-0.61 in/in

K_W = Wheel Rate (lb/in) =
(Motion Ratio - range)^2* (Spring Rate - linear)
$K_W = MR^2 * K_S$



Spring Rates/Ride Frequency

- The static deflection of the suspension determines its natural frequency.
- Static deflection is the rate at which the suspension compresses in response to weight.



Spring Rates/Ride Frequency

- Ride frequency is the undamped natural frequency of the body in ride. The higher the frequency, the stiffer the ride.

- Based on the application, there are ballpark numbers to consider.
  - 30 - 70 CPM for passenger cars
  - 70 - 120 CPM for high-performance sports cars
  - 120 - 300+ CPM for high downforce race cars

- It is common to run a spring frequency higher in the rear than the front. The idea is to have the oscillation of the front suspension finish at the same time as the rear.

- Since the delay between when the front suspension hits a bump and the rear suspension hits that bump varies according to vehicle speed, the spring frequency increase in the rear also varies according to the particular speed one wants to optimize for.

Spring Rates/Ride Frequency

- Once the motion ratios has been established, the front and rear spring rates can be optimized for a "flat" ride at a particular speed.

Spring Rate/Frequency Calculator

Back to Susp Project Tasks

Vehicle:

Vehicle Curb Weight:	3542 lb
Wheelbase	98 in
Weight Distribution Left	**50.0%**
Weight Distribution Right	50.0%

L/R weight distribution should be kept at 50/50 for most applications. Otherwise there will be different springs left to right.

Vehicle Sprung Weight:

Front axle weight:	1781.63 lb
Rear axle weight:	1760.37 lb
Sprung weight - front left spring:	785.81 lb
Sprung weight - front right spring:	785.81 lb
Sprung weight - rear left spring:	761.69 lb
Sprung weight - rear right spring:	761.69 lb

Front:

Weight Distribution:	50.3%
Motion Ratio:	**0.759**
Unsprung Weight:	105 lb/side
Spring rate:	**400** lb/in
Front Tire Stiffness Rate:	1567.49 lb/in

Front:

Spring Effect Wheel Rate:	230.26 lb/in
Front Ride Rate:	200.77 lb/in
Static Deflection:	3.91 in
Front natural freq:	95.03 CPM
Front natural freq:	**1.58** Hz

Rear:

Weight Distribution:	49.7%
Motion Ratio:	**0.724**
Unsprung Weight:	118.5 lb/side
Spring rate:	**600** lb/in
Rear Tire Stiffness Rate:	1764.16 lb/in

Rear:

Spring Effect Wheel Rate:	314.35 lb/in
Rear Ride Rate:	266.81 lb/in
Static Deflection:	2.85 in
Rear natural freq:	111.27 CPM
Rear natural freq:	**1.85** Hz

Rear spring rate for "flat" ride (less pitching):

Optimize for this speed:	60 mph

Recommended Rear Ride Rate:	268.27 lb/in
Rec. rear natural freq:	**1.86** Hz
Rec. rear spring rate:	**603.89** lb/in

Spring Rates/Ride Frequency

- Here are the equations from the previous spreadsheet:

$$K_W = K_S * (MR)^2 = Spring\ Effect\ Wheel\ Rate$$

$$K_R = \frac{K_T * K_W}{K_T + K_W} = Ride\ Rate$$

$$D_S = \frac{W_S}{K_R} = Static\ Deflection$$

$$\omega = \frac{188}{\sqrt{D_S}} = Natural\ Frequency\ (CPM)$$

$$\omega_{Rrec} = \frac{1}{(1/\omega_f) - (1/17.6) * (\ell / v)} = Rec.\ Rear\ Natural\ Frequency\ (Hz)$$

$$K_{RR\,rec} = \frac{4\pi^2 * \omega_{Rrec}{}^2 * W_S}{(12 * 32.2)} = Rec.\ Rear\ Ride\ Rate$$

$$K_{SR\,rec} = \frac{(K_{Trear} * K_{RRrec})/(K_{Trear} - K_{RRrec})}{(MR_{rear})^2} = Rec.\ Rear\ Spring\ Rate$$

Where:
K_S = Spring Rate (lb/in)
MR = Motion Ratio (in/in)
K_T = Tire Stiffness Rate (lb/in)
W_S = Sprung Weight (lb)
ω/60 = Hz
ω_f = Front Frequency (Hz)
ℓ= Wheelbase (in)
v = Vehicle Speed (mph)
1 mph = 17.6 in/sec

Ride Rate

- **Ride Rate (independent suspension)**
  - The **overall ride rate** for a suspension can be thought of as a series combination of two springs.
    1. The **wheel center rate** acting between the chassis and the wheel center.
    2. The **vertical tire rate** acting between the wheel center and the ground.

$$K_R = \frac{K_T * K_W}{K_T + K_W}$$

Where:

K_R = overall ride rate (lb-in)

K_T = vertical tire rate (lb/in)

K_W = spring wheel rate (lb/in)

(independent suspension)

Ride Rate

- **Ride Rate (solid axle)**
  - The **overall ride rate** for a suspension can be thought of as a series combination of two springs.
    1. The **wheel center rate** acting between the chassis and the wheel center.
    2. The **vertical tire rate** acting between the wheel center and the ground.

$$K_R = \frac{K_T * K_W}{K_T + K_W}$$

Where:

K_R= overall ride rate (lb-in)

K_T = vertical tire rate (lb/in)

K_W = vertical axle rate (lb/in)

(solid axle suspension)

Vehicle Load Transfer

Part III

Lateral Load Transfer

Lateral Load Transfer

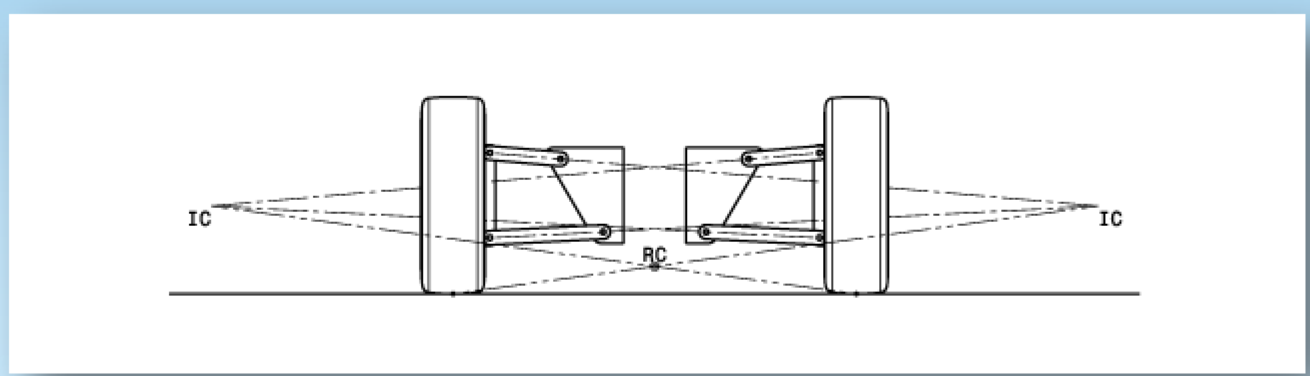
- **Lateral Load Transfer**
  - Now that the ride rate analysis is complete, we can move on to the roll analysis. We will want to calculate the anti-roll bars. To do this we will need the following information on the vehicle suspension:
    - Roll center heights front and rear
    - Roll Axis
    - Tire Static Load Radius
    - Tire Stiffness Rate
    - Spring motion ratio
    - ARB motion ratio
    - Track width (independent susp)
    - Leaf spring spacing (solid axle)
    - Sprung mass CG height
    - Sprung mass weight distribution
    - Roll Moment lever arm
    - Roll Moment per lateral g acceleration
    - Roll Stiffness Rate per Roll Gradient
    - Total Lateral Load Transfer Distribution (TLLTD)

Roll Centers and Roll Axis

- **Roll Centers and Roll Axis**
  - As the vehicle changes direction, the sprung mass (body) of a vehicle pivots about its roll axis in the opposite direction. This lateral load transfer is a result of the centripetal force acting on the moment (distance) between the roll axis and the CG of the vehicle.
  - How the roll centers react to the suspension dynamics is called the roll center characteristics. The roll center characteristics affects the roll center height as well as camber changes caused by movement of the roll center throughout the suspension travel.
  - There are many types of vehicle suspension designs; each has unique roll center characteristics.

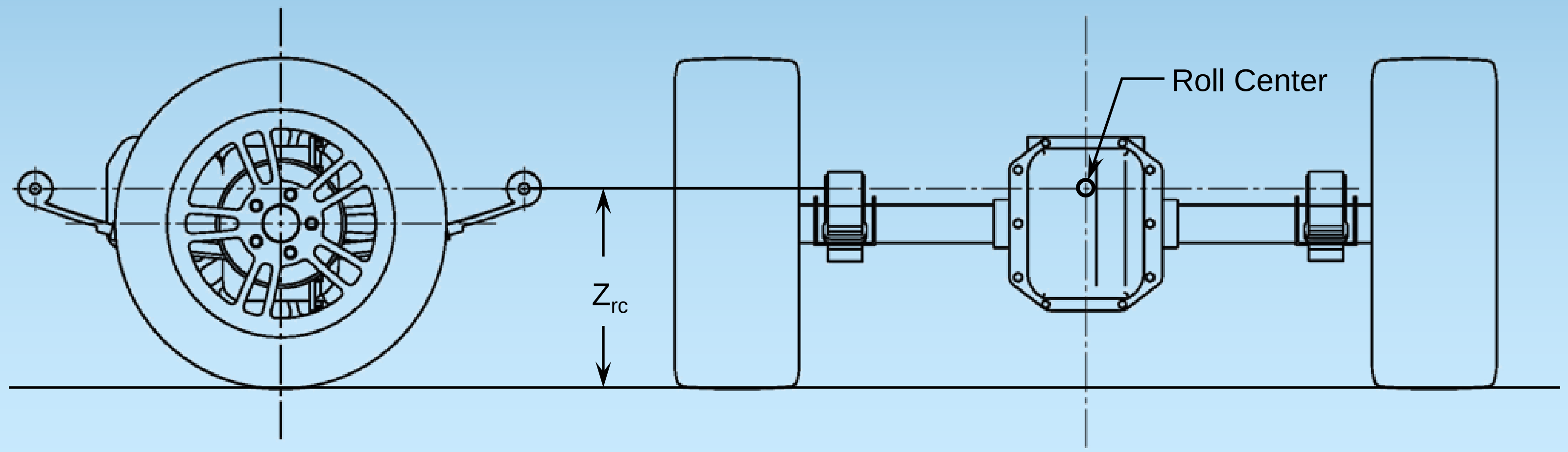
Roll Centers

- **Roll Centers**
  - Every vehicle has two roll centers, one at the front and one at the rear. Each roll center is located at the intersection of a line drawn from the center of the tire contact patch through the IC of that tire's suspension geometry.
  - As the IC moves during suspension travel, so too will the roll center.



Roll Centers

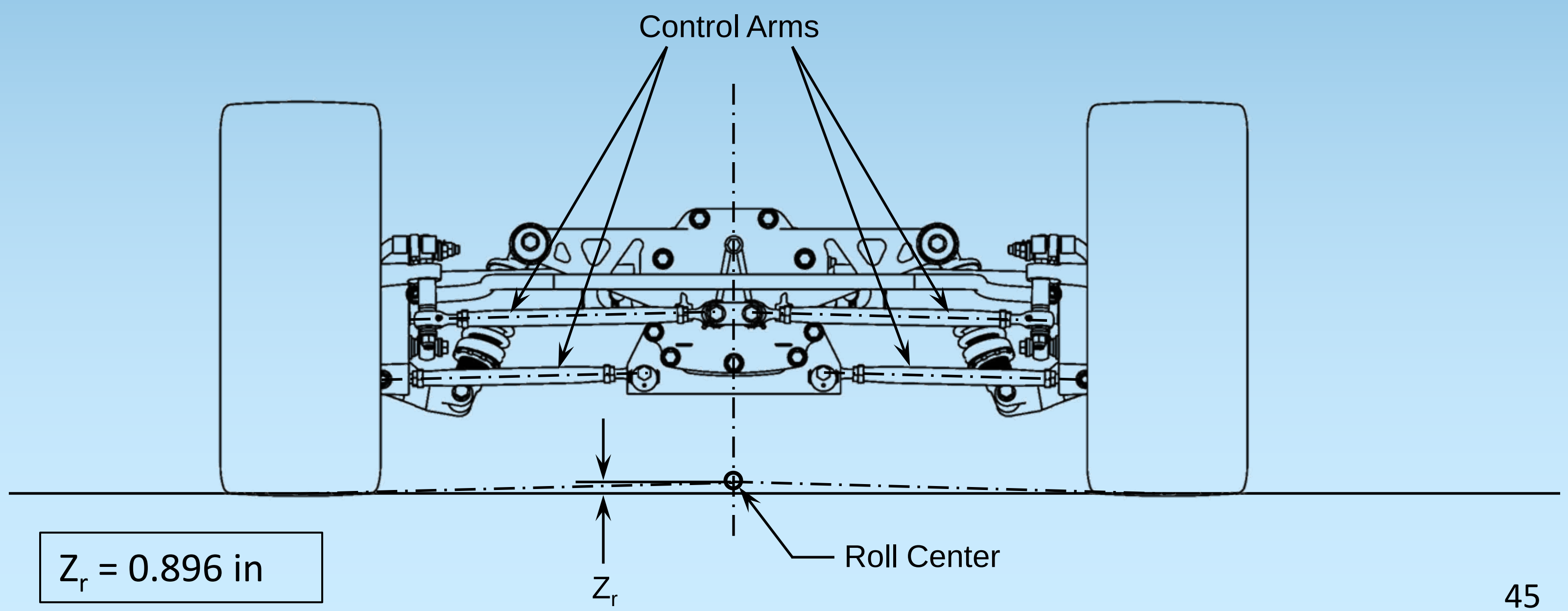
- **Roll Centers**
  - This example shows a solid rear axle with leaf springs. The axle/differential is suspended and moves with the wheel assemblies. The roll center height (Z_{rc}) is derived by intersecting a plane running through the spring pivots with a vertical plane running through the centerline of the axle. The roll center point is equal distance between the springs.



Z_{rc} = 16 in

Roll Centers

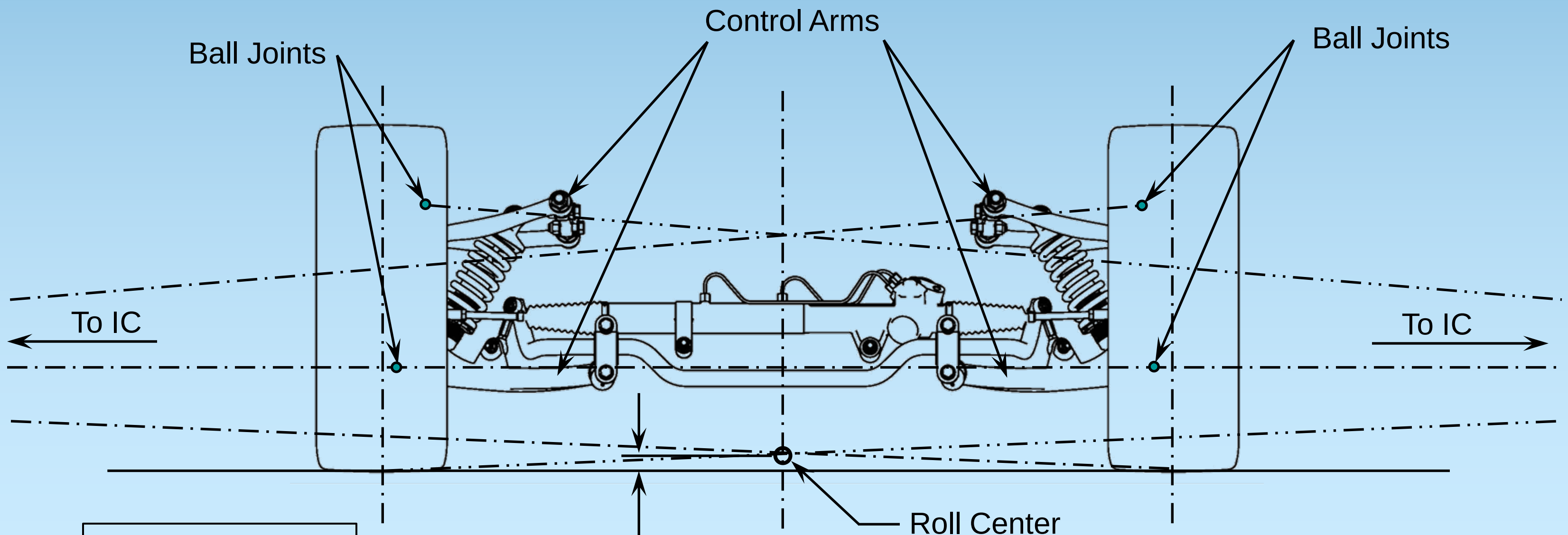
- **Roll Centers**
  - This example shows an independent rear suspension with the differential attached to the chassis and control arms suspending the wheel assemblies from the chassis. The control arms are parallel, therefore the IC is infinite. In this case the lines running from the center of the tire contact path to the roll center are parallel with the control arms.



Z_r = 0.896 in

Roll Centers

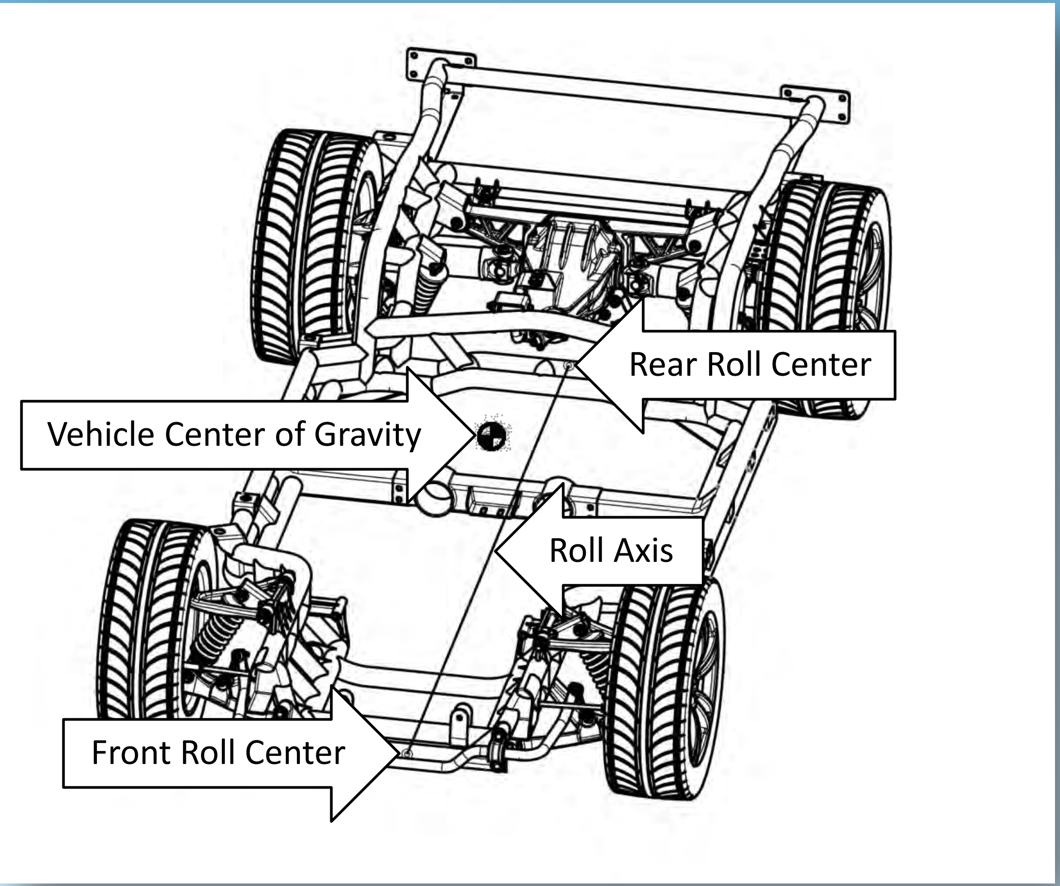
- **Roll Centers**
  - This example shows an independent front suspension with the control arms suspending the wheel assemblies from the chassis. Lines running through the control arm pivots and ball joints intersect at the IC. The lines running from the center of the tire contact path to the IC intersect at the roll center.



$Z_f = 0.497$ in

Roll Centers and Roll Axis

- **Roll Axis**
  - A line connecting the two roll centers is called the roll axis.



Tire Stiffness Rate

- **Tire Rate**
  - There are many ways to calculate tire rate.
    - Load-deflection (LD)
    - Non-rolling vertical free vibration (NR-FV)
    - Non-rolling equilibrium load-deflection (NR-LD)
    - Rolling vertical free vibration (R-FV)
    - Rolling equilibrium load-deflection (R-LD)
  - The simplest would be load deflection.
    - All tire manufacturers list a static load radius in their catalog for a specific tire. They will also list that tire's unloaded diameter. There will also be a chart showing the tire's maximum load rating. From these numbers the static deflection can easily be calculated and the static rate is load/deflection.

Tire Stiffness Rate

- **Tire Rate**
  - These examples are for the static rate of the tires shown.
  - The tire stiffness rate will change due to changes in air pressure (D1 bar can affect rate by 40%) and slightly (less than 1%) when the vehicle speed changes.

$$k_t = \frac{2039.3\,lb}{(28.95in/2) - 13.03in} = 1411.3\,lb/in$$

Example 1:
Tire = P 235/70 R 16 105 T
Static (max) load radius = 330.9mm (13.03in)
Unloaded diameter = 735.4mm (28.95in)
Max Load = 925kg (2039.3lb) at 2.5 bar

$$k_t = \frac{1521.2\,lb}{(25.69in/2) - 11.87in} = 1560.2\,lb/in$$

Example 2:
Tire = P 245/45 R 17 95 W
Static (max) load radius = 301.5mm (11.87in)
Unloaded diameter = 652.4mm (25.69in)
Max Load = 690kg (1521.2lb) at 2.5 bar

$$k_t = \frac{1708.6\,lb}{(26.66in/2) - 12.36in} = 1761.4\,lb/in$$

Example 3:
Tire = P 275/40 R 18 99 W
Static (max) load radius = 314mm (12.36in)
Unloaded diameter = 677.2mm (26.66in)
Max Load = 775kg (1708.6lb) at 2.5 bar

Source: BND TechSource Tire Data Calculator

Tire Static Load Radius

- **Tire Static Load Radius**
  - The term Static Load Radius used to determine the tire stiffness rate is given by the tire manufacturer at the maximum load value for that particular tire.
  - The Tire Loaded Radius (RL_F;RL_R) used in the following equations is calculated at the vehicle corner load values (deflection=load/rate).

$$RL = (28.95/2) - (992/1411.3) = 13.77in$$

Example 1:
Tire = P 235/70 R 16 105 T
Unloaded diameter = 735.4mm (28.95in)
Veh Corner Load = 450kg (992lb) at 2.5 bar
K_T = 1382.6lb/in

$$RL_F = (25.69/2) - (893/1560.2) = 12.27in$$

Example 2:
Tire = P 245/45 R 17 95 W
Unloaded diameter = 652.4mm (25.69in)
Veh Corner Load = 405kg (893lb) at 2.5 bar
K_T = 1560.2lb/in

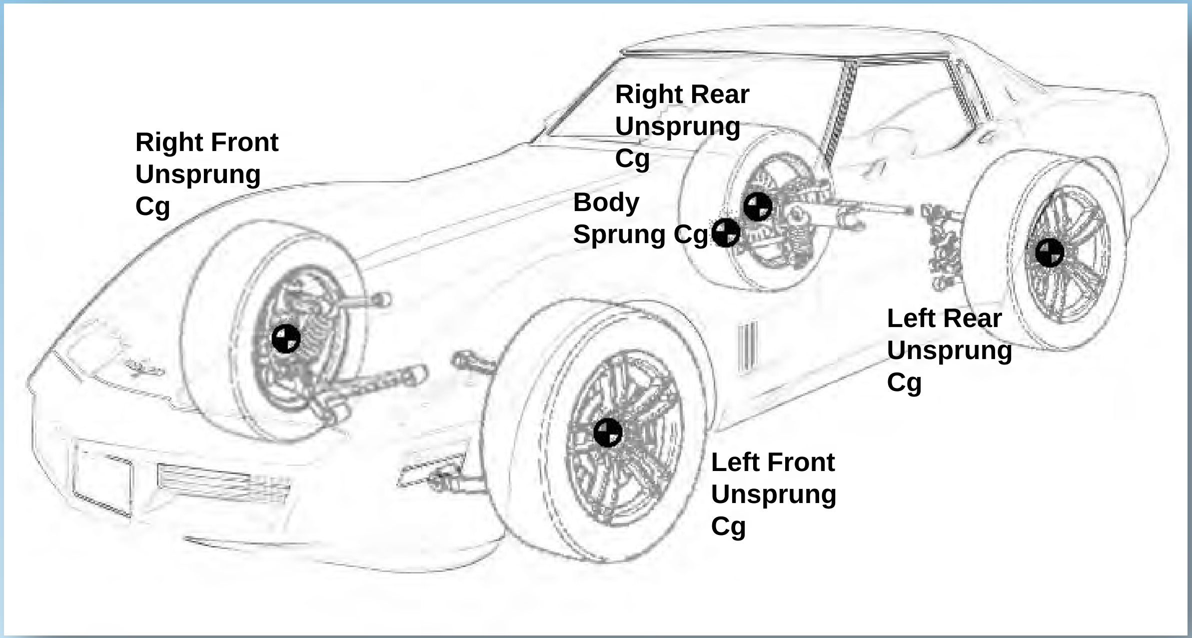
$$RL_R = (26.66/2) - (882/1761.4) = 12.83in$$

Example 3:
Tire = P 275/40 R 18 99 W
Unloaded diameter = 677.2mm (26.66in)
Veh Corner Load = 400kg (882lb) at 2.5 bar
K_T = 1761.4lb/in

Source: BND TechSource Tire Data Calculator

Sprung and Unsprung Weight

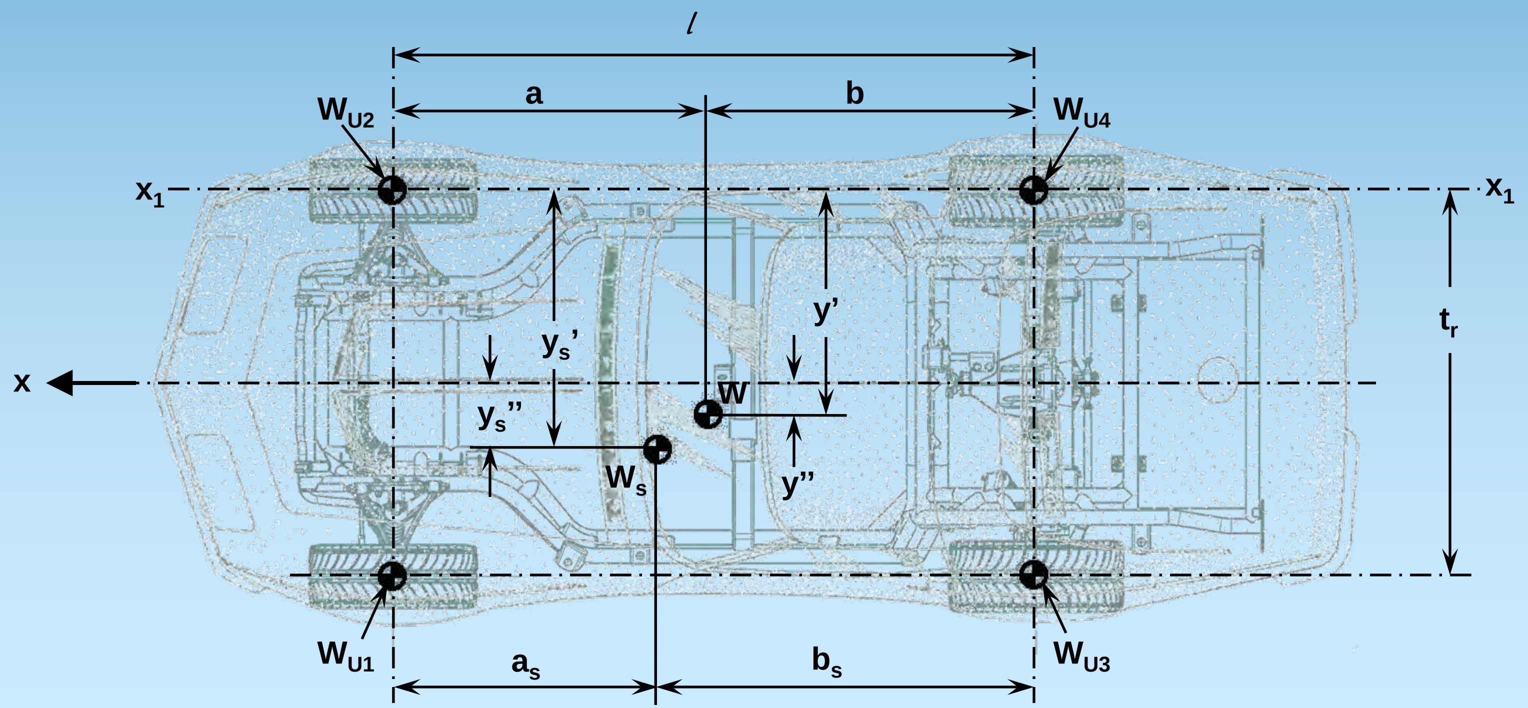
- **Sprung and Unsprung Weight**
  - An example of this would be the front unsprung weight is 11.5% (split equally left to right) of the vehicle weight. The rear unsprung weight is 13.5% (split equally left to right) and then the body would make up the remainder as sprung weight at 75%.



Sprung and Unsprung Weight

- **Sprung and Unsprung Weight**
  - The sprung weight of the vehicle is simply the total weight minus the unsprung weight.

$$W_s = W - W_{U1} - W_{U2} - W_{U3} - W_{U4} = Sprung\,Weight$$



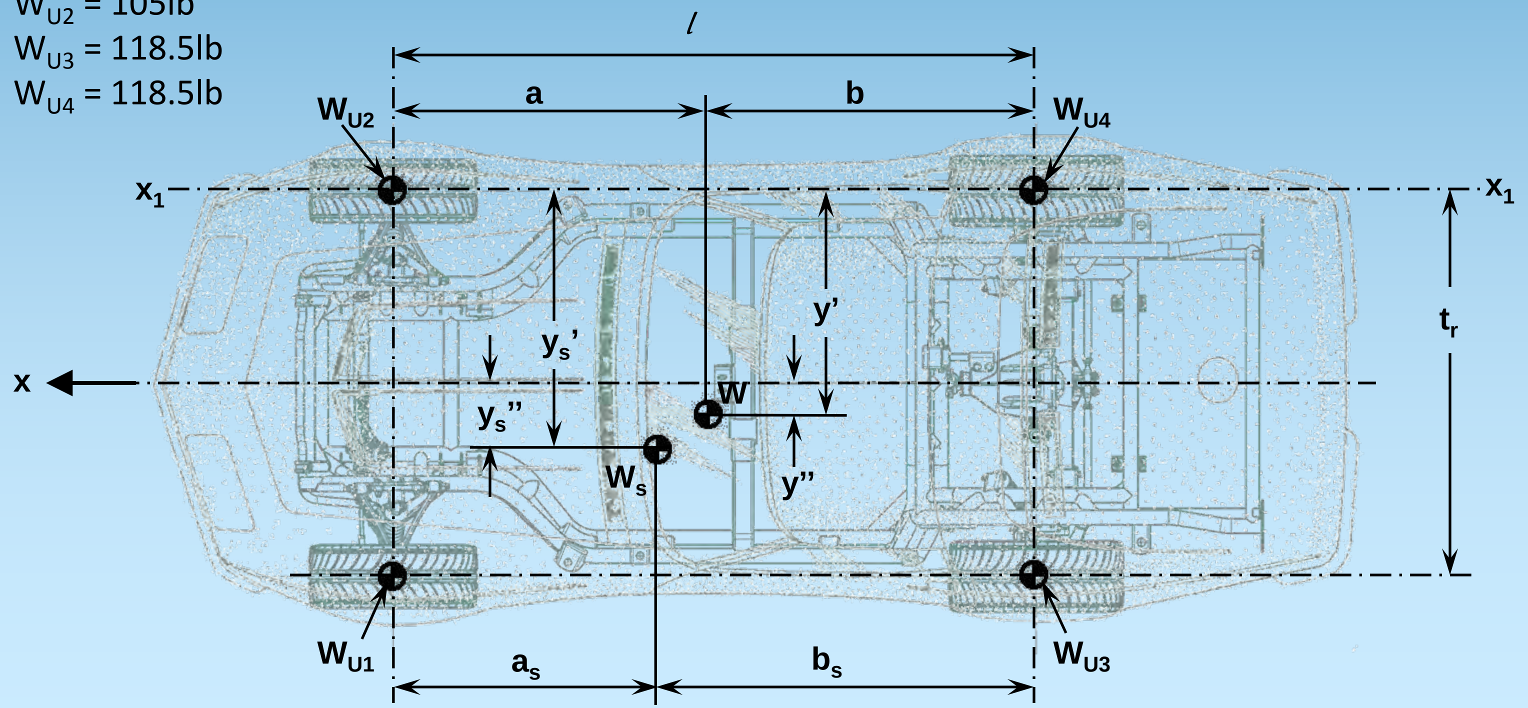
Sprung and Unsprung Weight

- **Sprung and Unsprung Weight**
  - The sprung weight of the vehicle is simply the total weight minus the unsprung weight.

Example C3 Corvette Upgrade:
W_S = Sprung weight (lb)
W_T = 3542lb
W_{U1} = 105lb
W_{U2} = 105lb
W_{U3} = 118.5lb
W_{U4} = 118.5lb

$$W_s = 3542 - 105 - 105 - 118.5 - 118.5$$

$$W_S = 3095 lb$$

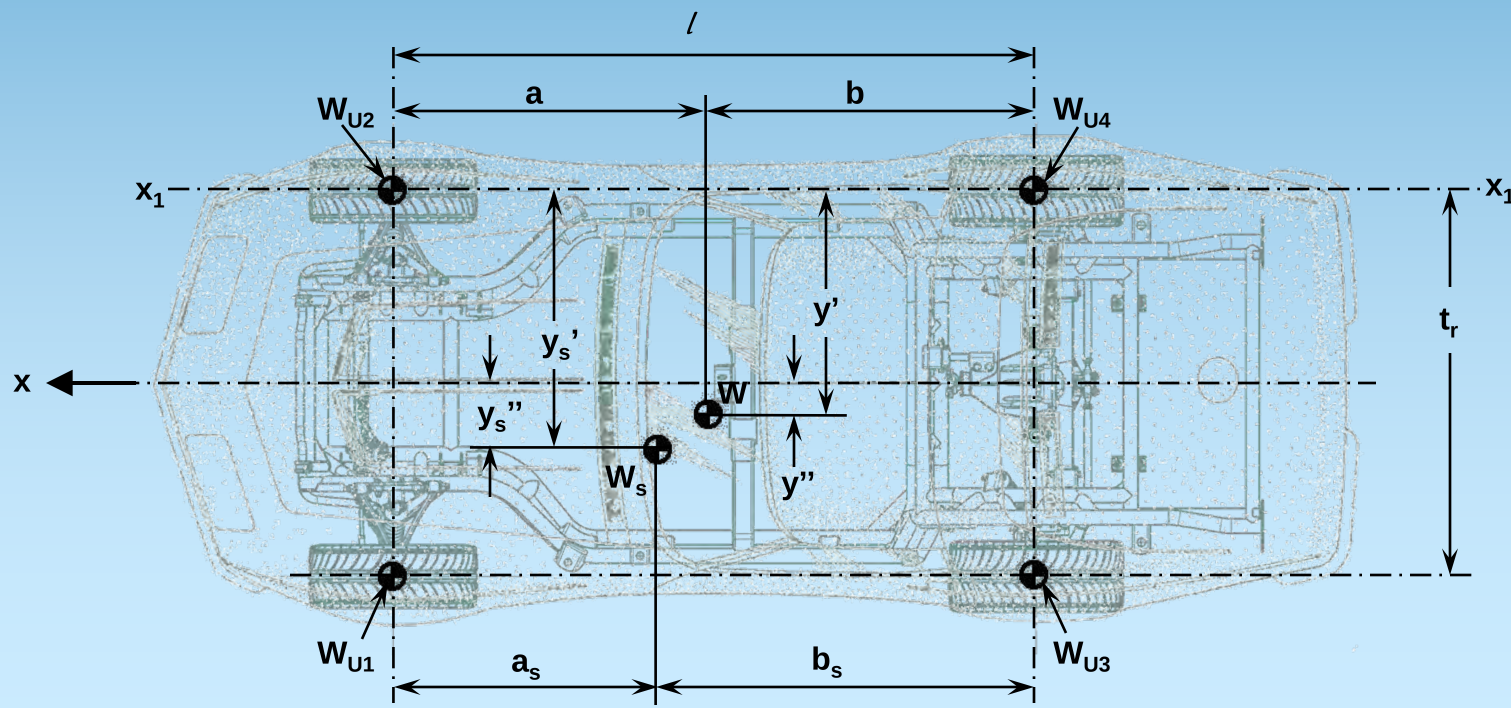


Sprung Weight

- **Sprung Weight Distribution**
  - The unsprung weight front and rear:

$$W_{UF} = W_{U1} + W_{U2} = Unsprung\ Weight\ Front$$

$$W_{UR} = W_{U3} + W_{U4} = Unsprung\ Weight\ Rear$$



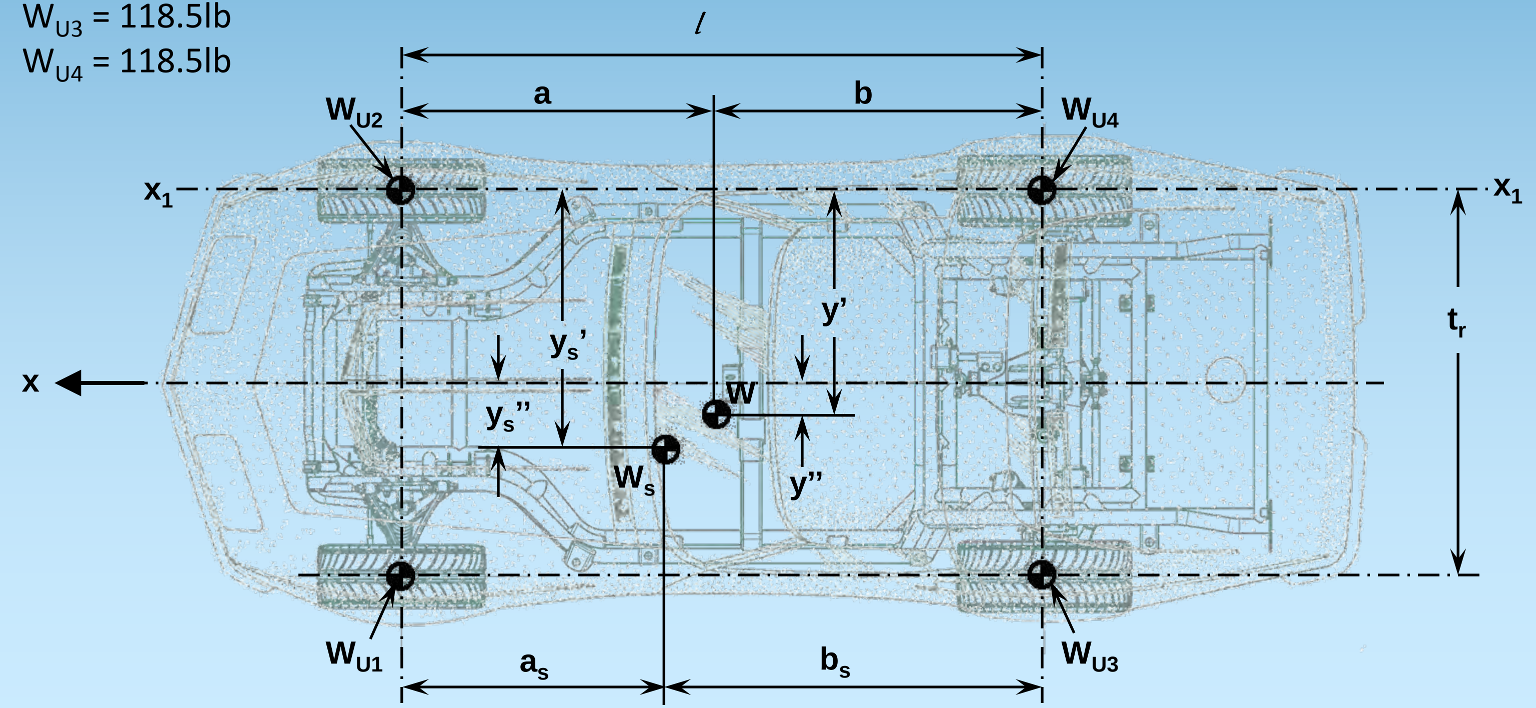
Sprung Weight

- **Sprung Weight Distribution**
  - The unsprung weight front and rear:

Example C3 Corvette Upgrade:
W_{UF} = Sprung weight front (lb)
W_{UR} = Sprung weight rear (lb)
W_{U1} = 105lb
W_{U2} = 105lb
W_{U3} = 118.5lb
W_{U4} = 118.5lb

$$W_{UF} = 105 + 105 = 210 lb$$

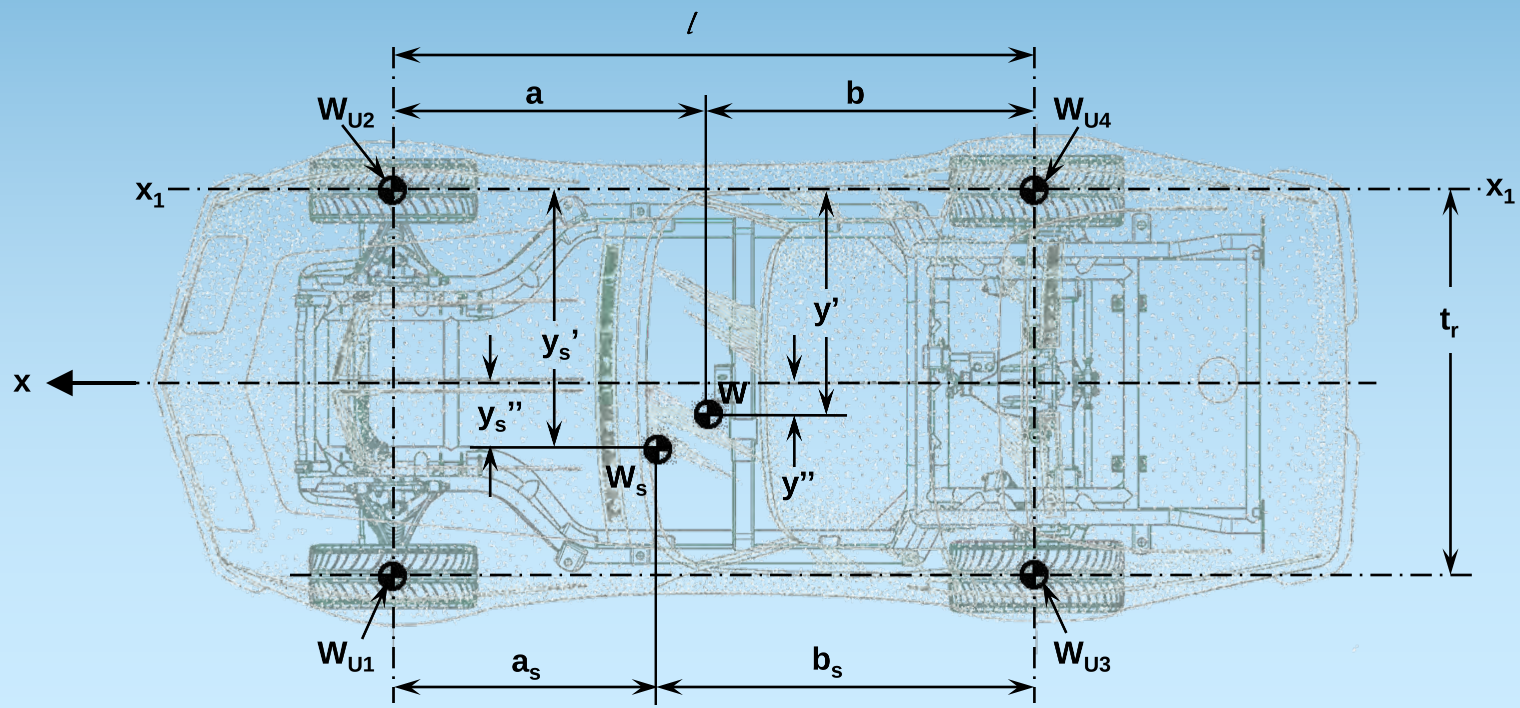
$$W_{UR} = 118.5 + 118.5 = 237 lb$$



Sprung Weight

- **Sprung Weight Distribution**
  - Taking the moments about the rear axle gives the longitudinal location of the sprung mass CG.

$$b_s = \frac{(W_T * b) - (W_{UF} * \ell)}{W_s} \quad \text{and} \quad a_s = \ell - b_s$$



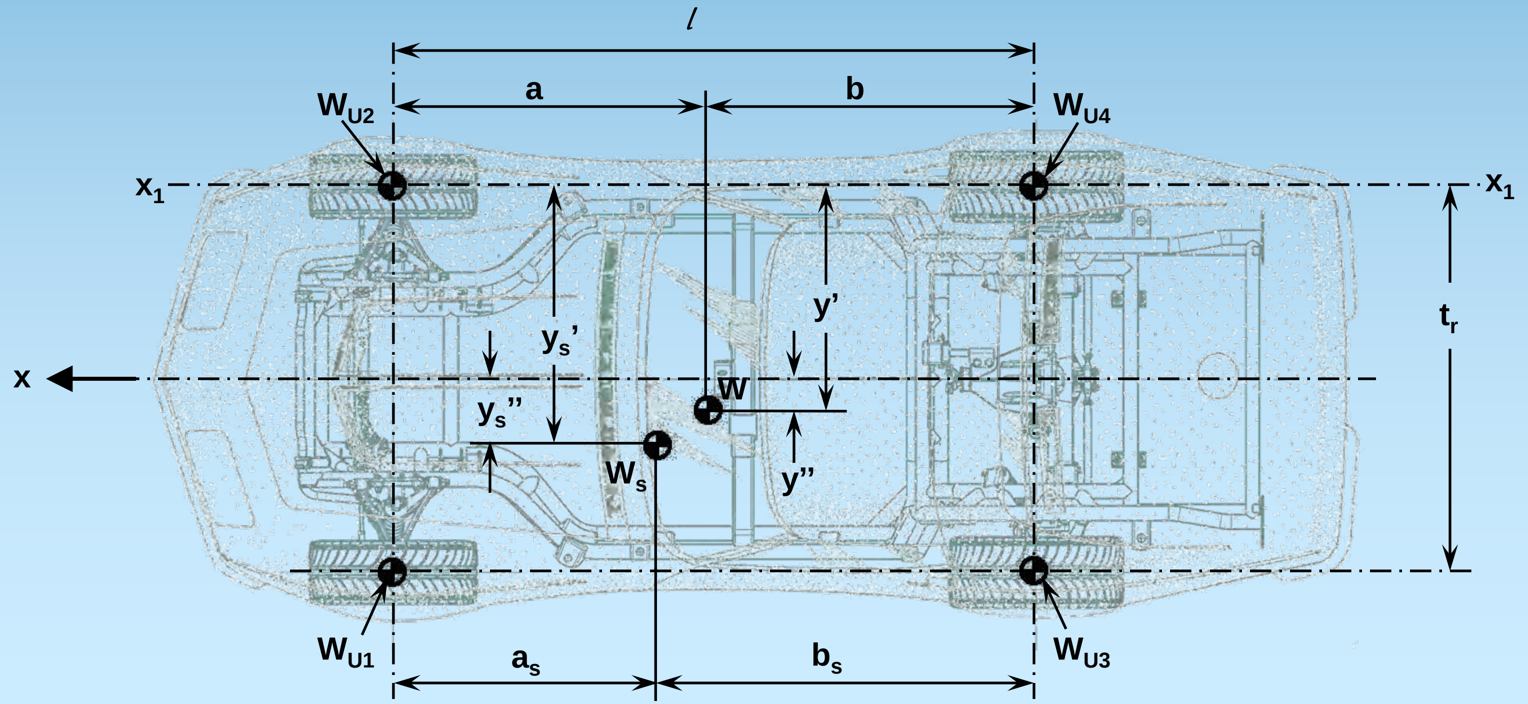
Sprung Weight

- **Sprung Weight Distribution**
  - Taking the moments about the rear axle gives the longitudinal location of the sprung mass CG.

Example C3 Corvette Upgrade:
W_T = 3542lb
W_S = 3095lb
W_{UF} = 210lb
b = 48.7in
l = 98in

$$b_s = \frac{(3542 * 48.7) - (210 * 98)}{3095} = 49.08 in$$

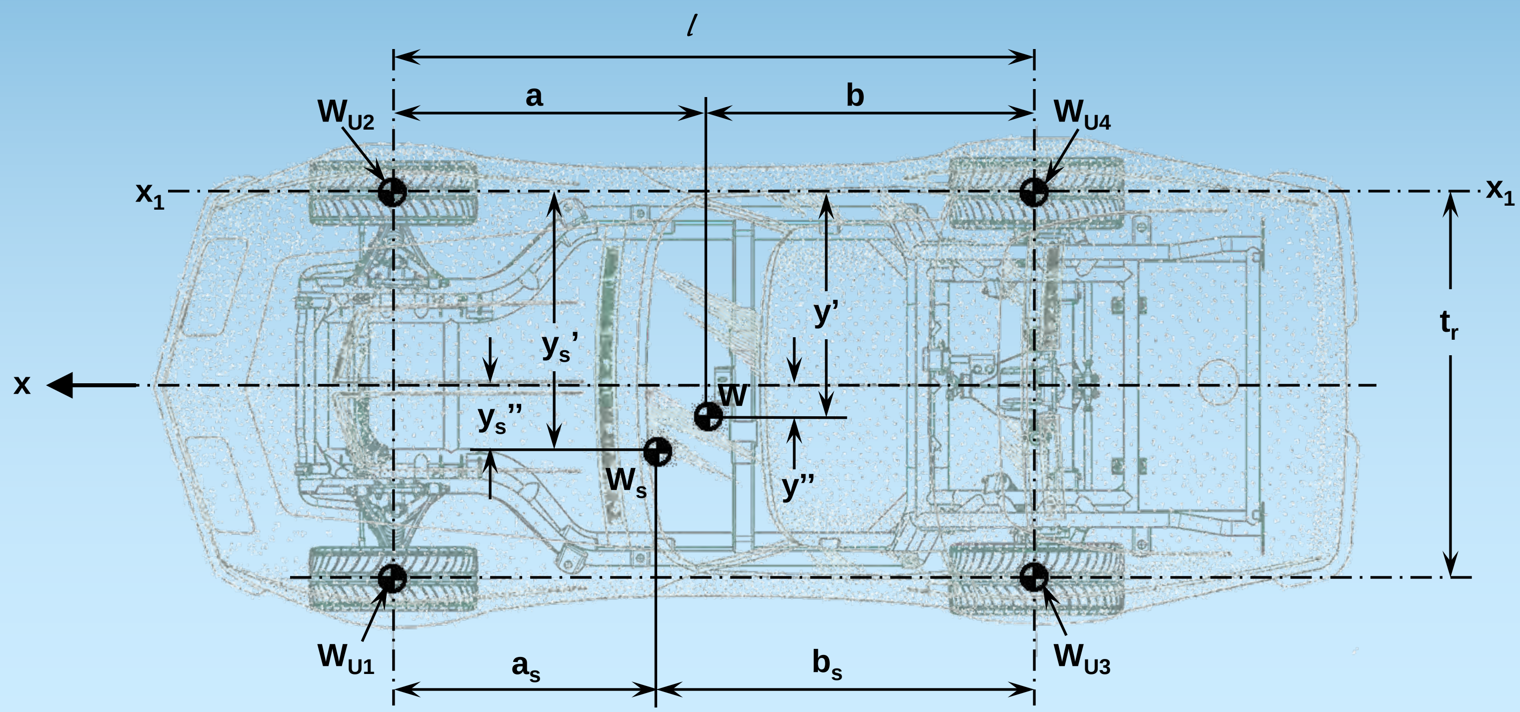
$$a_s = 98 - 49.08 = 48.92 in$$



Sprung Weight

- **Sprung Weight Distribution**
  - If the font and rear unsprung weight are equal side to side, and the front/rear tracks are the same, then the lateral location of the sprung mass CG is found by taking the moments about the x_1 axis as:

$$y'_s = \left(\frac{W_T}{W_s} * y'\right) - \left(\frac{W_{UR}}{2 * W_s} * t\right) - \left(\frac{W_{UF}}{2 * W_s} * t\right)$$



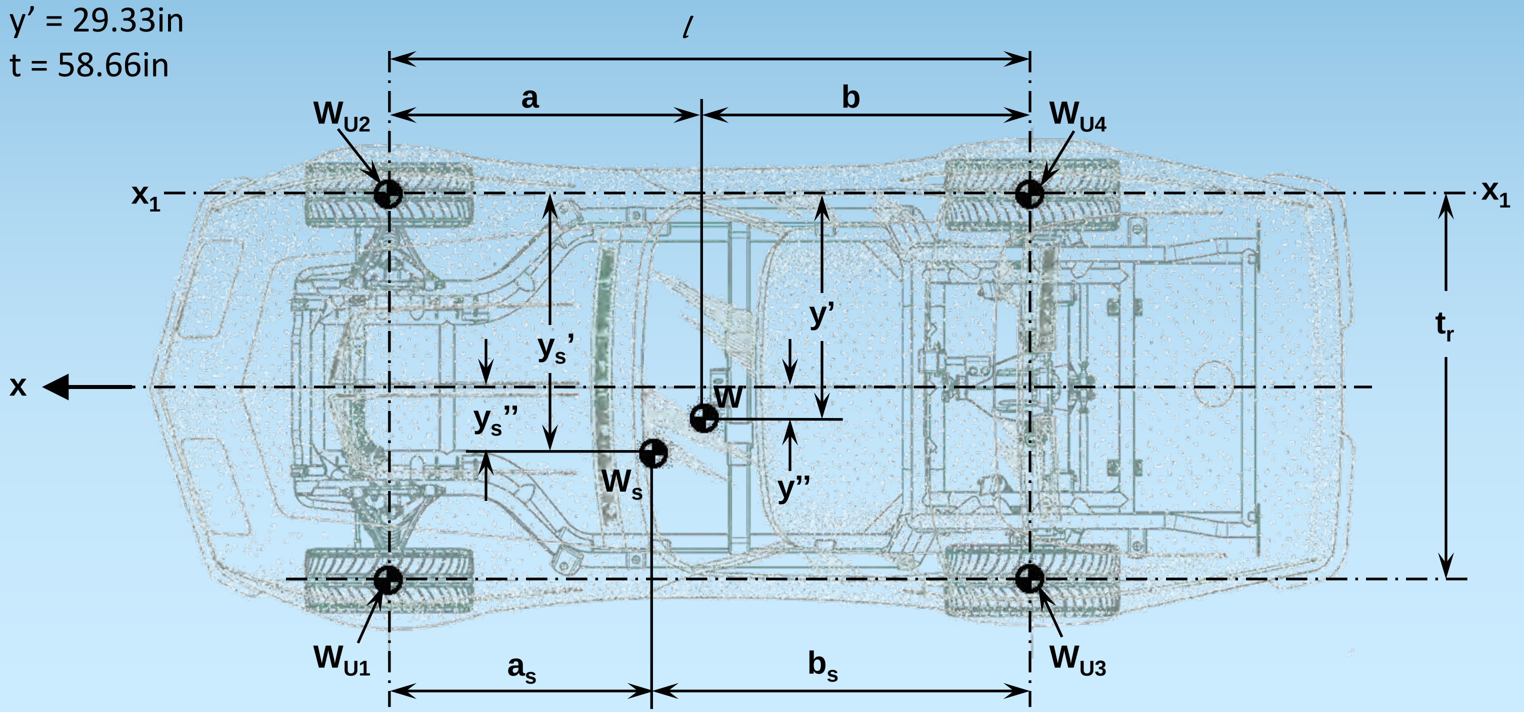
Sprung Weight

- **Sprung Weight Distribution**
  - If the font and rear unsprung weight are equal side to side, and the front/rear tracks are the same, then the lateral location of the sprung mass CG is found by taking the moments about the x_1 axis as:

Example C3 Corvette Upgrade:
W_T = 3542lb
W_S = 3095lb
W_{UF} = 210lb
W_{UR} = 237lb
y' = 29.33in
t = 58.66in

$$y'_s = \left(\frac{3542}{3095} * 29.33\right) - \left(\frac{237}{2 * 3095} * 58.66\right) - \left(\frac{210}{2 * 3095} * 58.66\right)$$

$$y'_s = 29.33 in$$

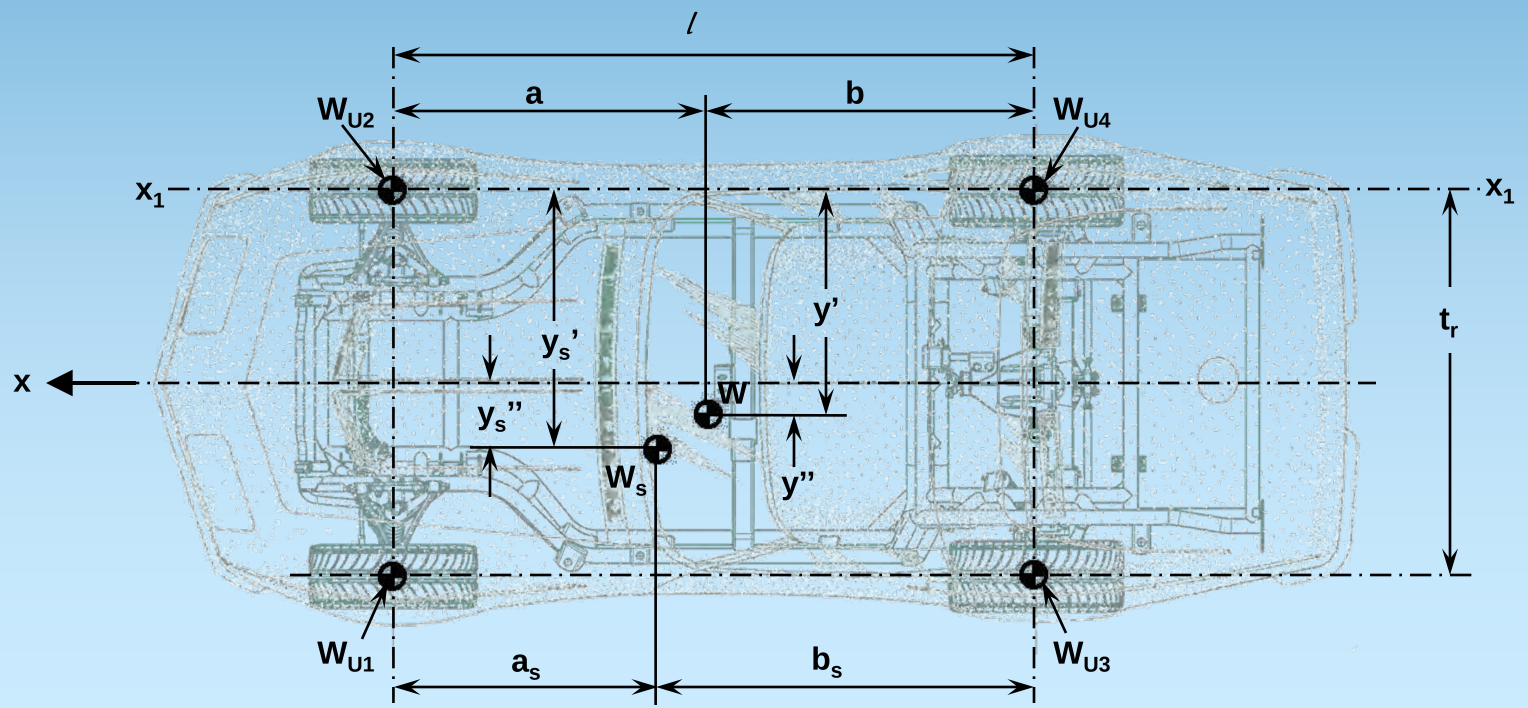


Sprung Weight

- **Sprung Weight Distribution**
  - The sprung weight front and rear:

$$W_{SF} = W_S * (a_s / \ell) = Sprung\ Weight\ Front$$

$$W_{SR} = W_S + (b_s / \ell) = Sprung\ Weight\ Rear$$



Sprung Weight

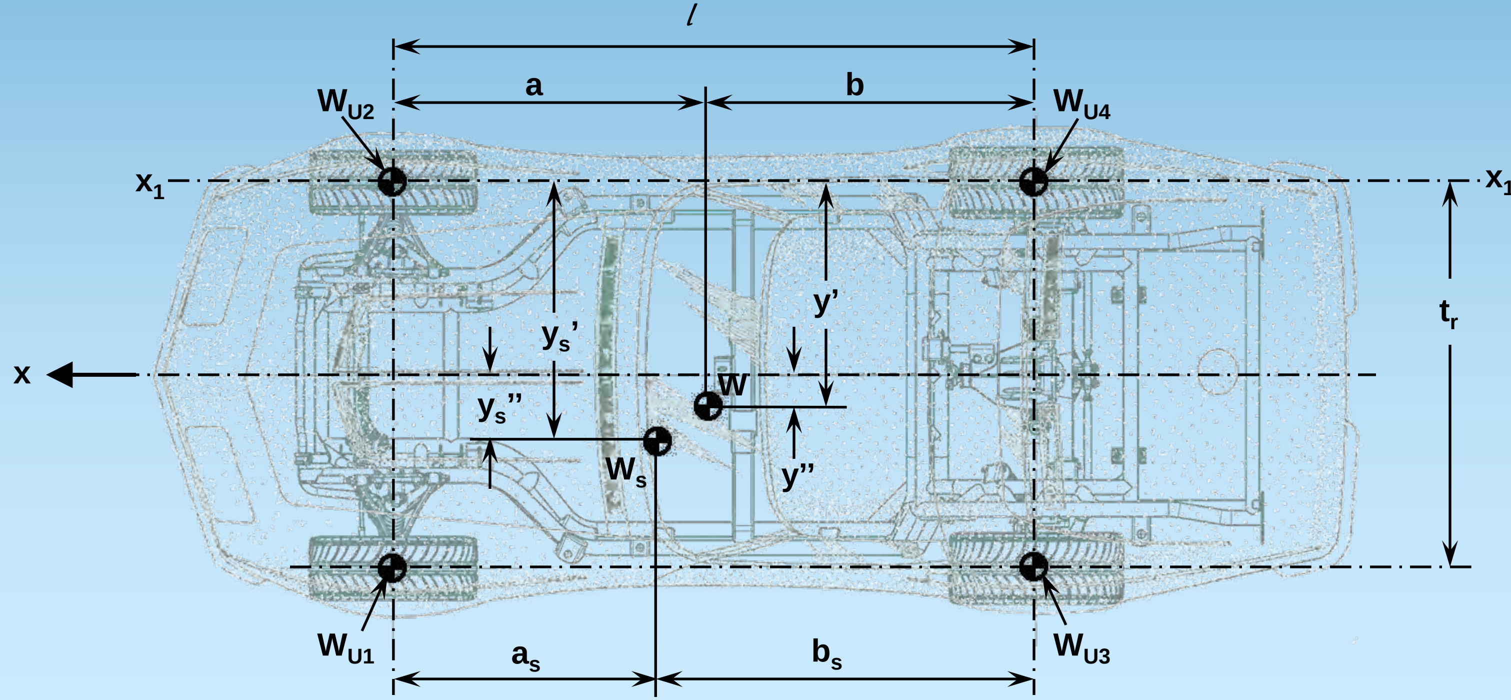
Sprung Weight Distribution

- The sprung weight front and rear:

Example C3 Corvette Upgrade:
W_S = 3095lb
a_S = 48.92in
b_S = 49.08in
l= 98in

$$W_{SF} = 3095 * (48.92/98) = 1544.4lb$$

$$W_{SR} = 3095 * (49.08/98) = 1550.6lb$$

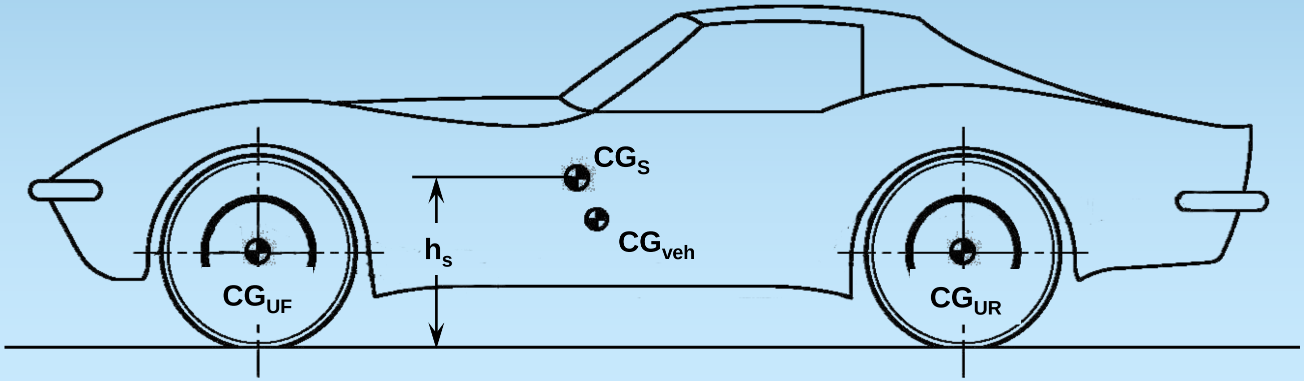


Sprung and Unsprung Weight

- **Sprung Weight CG height**

$$h_S = \frac{(W_T * h) - (W_{UF} * RL_F) - (W_{UR} * RL_R)}{W_S}$$

Where:
h_S = Sprung weight CG height (in)
W_T = Total vehicle weight (lb)
h = Vehicle CG height (in)
W_{UF} = Unsprung weight front (lb)
RL_F = Tire Loaded Radius front (in)
W_{UR} = Unsprung weight rear (lb)
RL_R = Tire Loaded Radius rear (in)
W_S = Sprung weight (lb)



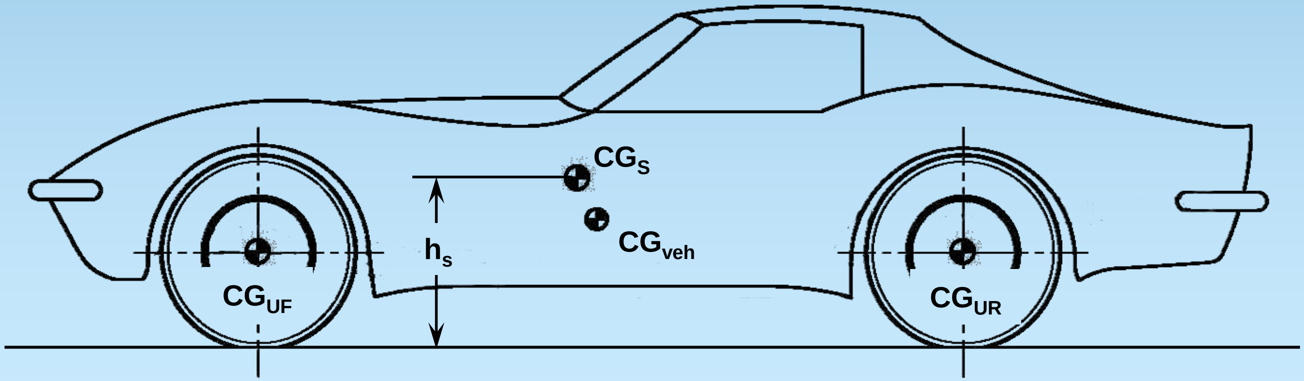
Sprung and Unsprung Weight

- **Sprung Weight CG height**

Example C3 Corvette Upgrade:
h_S = Sprung weight CG height (in)
W_T = 3542lb
h = 17in
W_{UF} = 210lb
RL_F = 12.27in
W_{UR} = 237lb
RL_R = 12.83in
W_S = 3095lb

$$h_S = \frac{(3542 * 17) - (210 * 12.27) - (237 * 12.83)}{3095}$$

$$h_S = 17.64 in$$



Roll Stiffness

- **Roll Stiffness**

  - Roll stiffness is the resistance of the springs within the suspension against the body roll when a vehicle goes through a corner.

  - Roll stiffness is developed by a roll resisting moment of the body (sprung mass) about the roll axis.

  - The roll stiffness in a vehicle is equal to the combined roll stiffness of the front and rear suspension.

  - Roll stiffness expressed in (torque) ft-lb/degree of roll.
    - Example: If a vehicle had a roll stiffness of 600 ft-lb/deg of roll, it would take a torque of 600 ft-lb about the roll axis to move the body 1 degree.

  - Roll stiffness of the complete vehicle is the sum of the separate roll stiffness rates of all vehicle suspensions.

  - Tire deflection rates are included in the front and rear roll stiffness rate values.

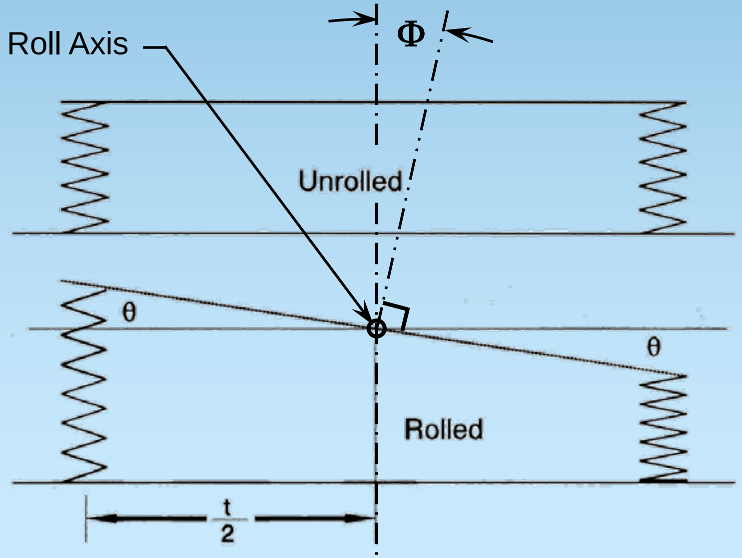
Roll Stiffness

- **Roll Stiffness**
  - Roll stiffness is the torque (T) (moment or roll couple) to rotate the body (sprung weight) about the roll axis is shown in the following equations.

$$T = \left(\frac{t}{2} K_L \frac{t}{2} + \frac{t}{2} K_R \frac{t}{2}\right)\theta$$

Where:
T = Torque
K_L = left vertical spring rate
K_R = right vertical spring rate
t = distance between the springs

$$T = \frac{t^2}{4}(K_L + K_R)\theta$$



For equal spring rates, left and right the above equation reduces to the following:

$$T = \frac{t^2}{2}(K)\theta$$

Roll Stiffness

- **Roll Stiffness**
  - Roll stiffness (K_F) in radians for suspension with equal spring rate either side (symmetric) is shown in the following equation.

$$T = \frac{t^2}{2}(K)\theta = \frac{t^2}{2}(K)\Phi$$

$$K_\Phi = \frac{T}{\theta} = \frac{t^2}{2}(K)$$

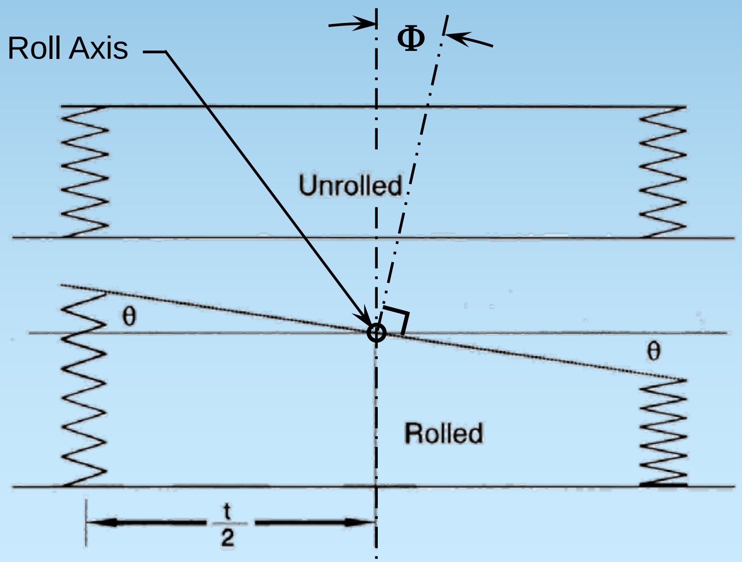
Where:

K_F = Roll Stiffness (Roll Rate)

T = Torque

K = vertical spring rate or wheel rate*

t = distance between the springs



***Solid axles with leaf springs would use vertical spring rates (K).**

***Independent suspensions and anti-roll bars would use the wheel rates (K).**

- **Roll Stiffness**
  - Roll stiffness (KΦ) expressed in metric units (N-m/deg).

$$K_{\Phi} = \frac{T}{\theta} = \frac{t^2}{(2 * \frac{180}{\pi})} K = \frac{t^2}{114.6} K$$

  - Roll stiffness (KF) expressed in English units (ft-lb/deg).

$$K_{\Phi} = \frac{T}{\theta} = \frac{t^2}{(2 * \frac{180}{\pi} * 12)} K = \frac{t^2}{1375} K$$

Assuming original (t) values given in inches and (K) values in lb/in.

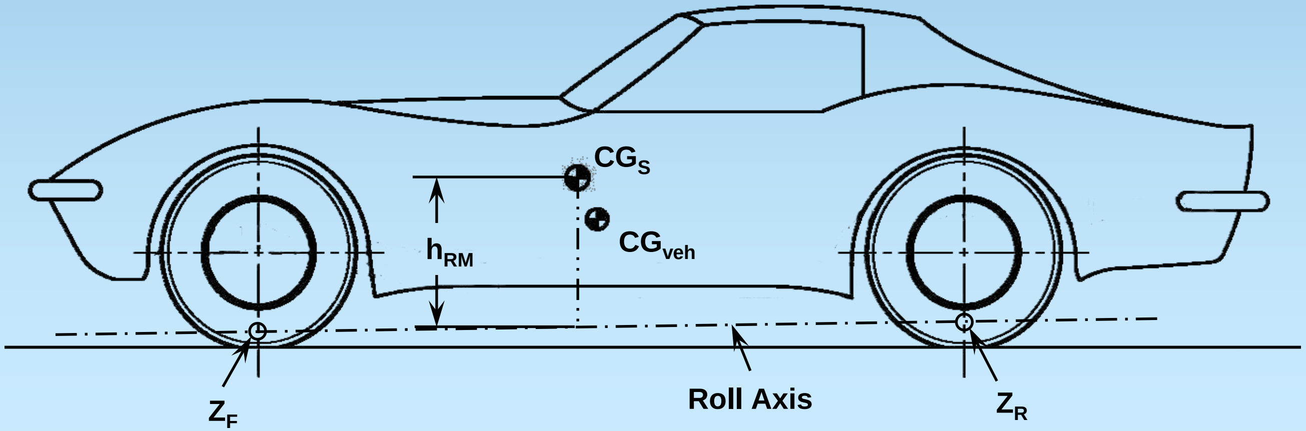
Roll Moment

- **Sprung Weight Roll Moment lever arm**

$$h_{RM} = h_S - \left[Z_F + (Z_R - Z_F) * (1 - \frac{a_S}{\ell}) \right]$$

Where:

h_{RM} = Sprung weight RM lever arm (in)
h_S = Sprung weight CG height (in)
Z_F = Roll Center height front (in)
Z_R = Roll Center height rear (in)
a_S/l = Sprung mass weight dist. front (%)



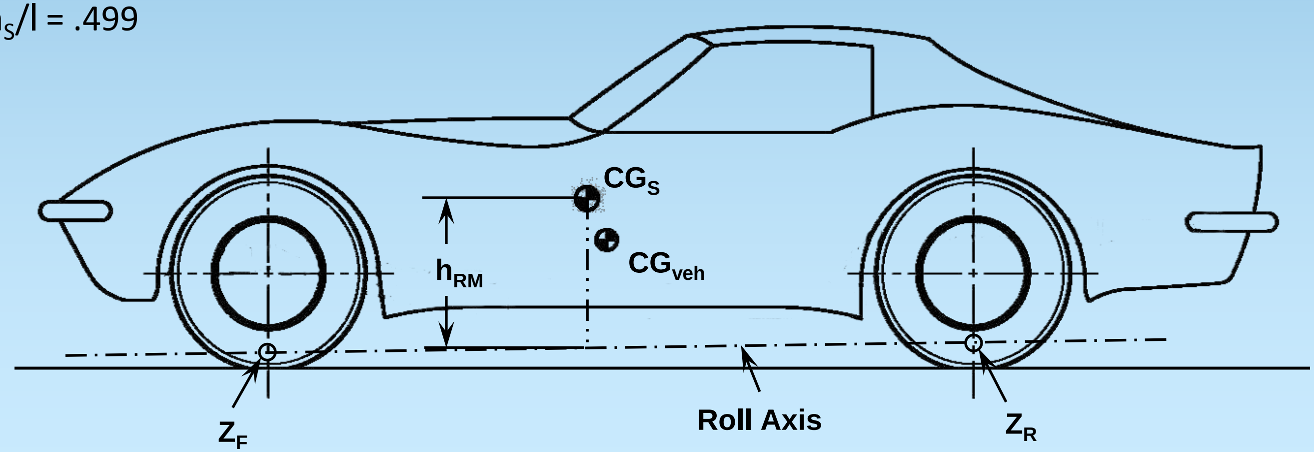
Roll Moment

- **Sprung Weight Roll Moment lever arm**

$$h_{RM} = 17.64 - [.497 + (.896 - .497) * (1 - .499)]$$

$$h_{RM} = 17.19in$$

Example C3 Corvette Upgrade :
h_{RM} = Sprung weight RM lever arm (in)
h_S = 17.64in
Z_F = 0.497in
Z_R = 0.896in
a_S/l = .499

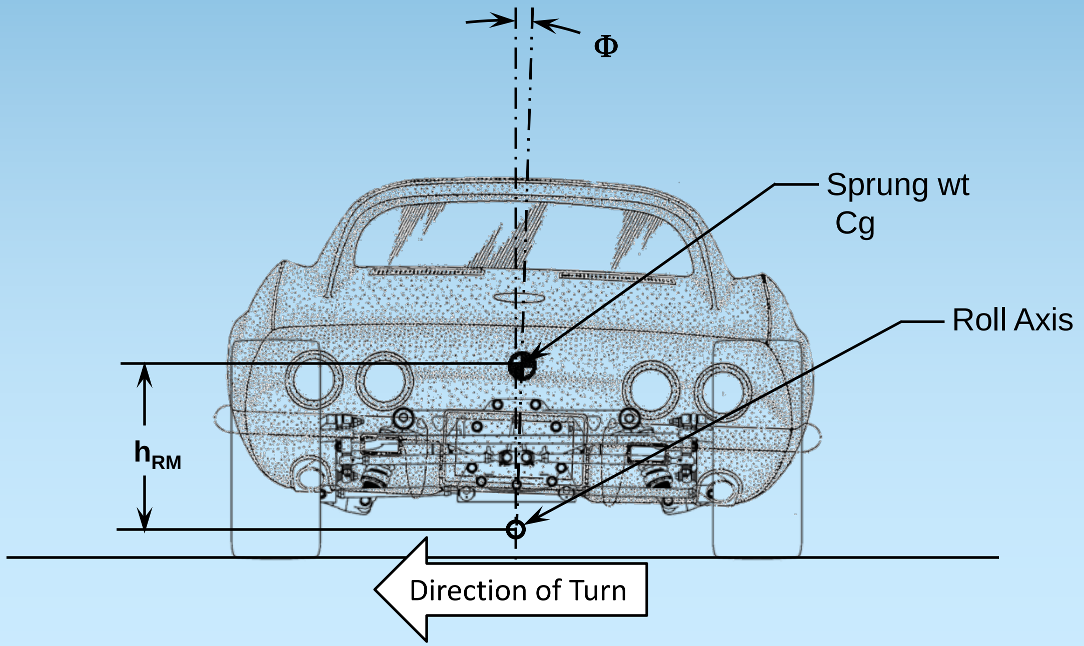


Roll Moment

- **Roll Moment per lateral g acceleration**

$$\frac{M_\Phi}{A_y} = \frac{h_{RM} * W_S}{12}$$

Where:
M_Φ = Roll Moment (ft-lb)
A_y = Lateral acceleration (g)
h_{RM} = Sprung weight RM lever arm (in)
W_S = Sprung weight (lb)

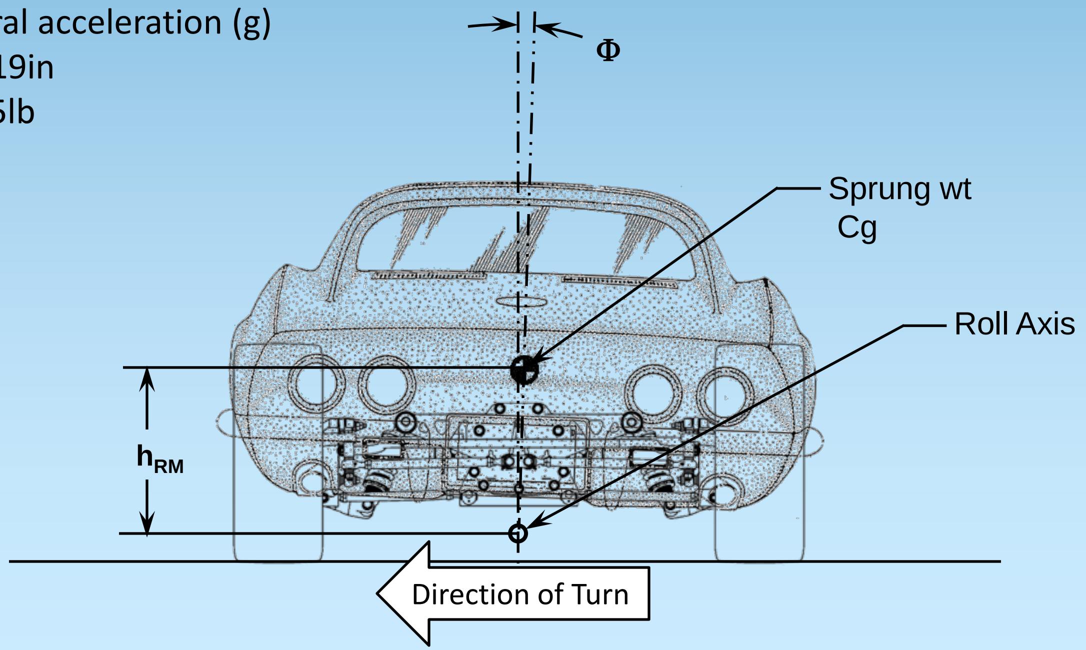


Roll Moment

- **Roll Moment per lateral g acceleration**

$$\frac{M_\Phi}{A_y} = \frac{17.19 * 3095}{12} = 4433.6\, ft - lb / g$$

Example C3 Corvette Upgrade:
M_Φ = Roll Moment (ft-lb)
A_y = Lateral acceleration (g)
h_{RM} = 17.19in
W_S = 3095lb

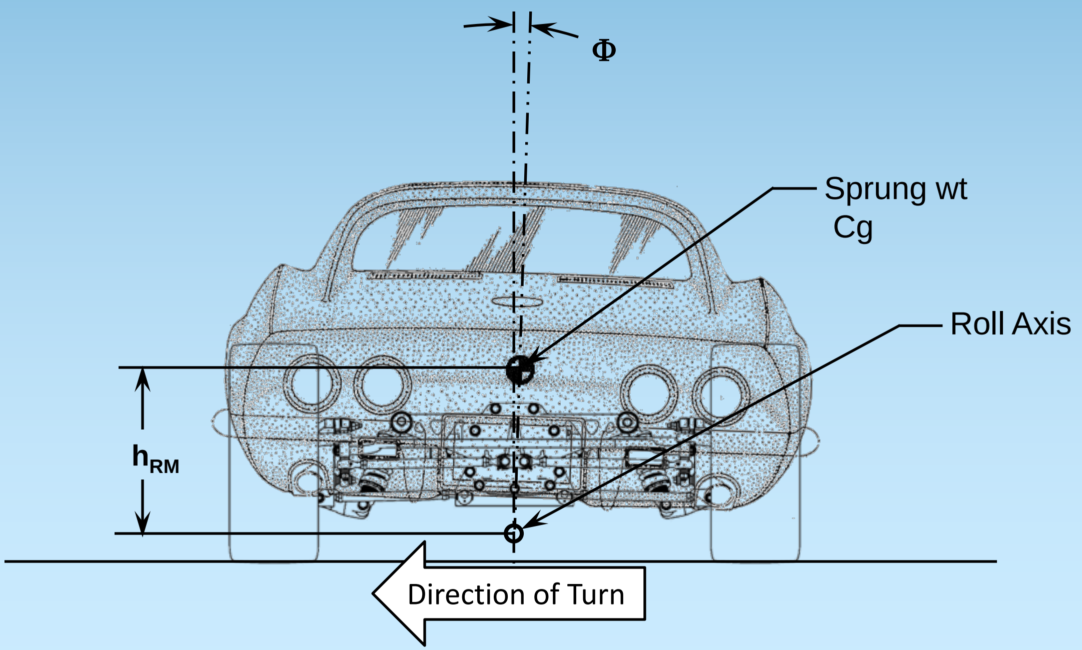


Roll Stiffness

- **Total Roll Stiffness Rate per Roll Gradient**

$$K_\Phi = \frac{M_\Phi / A_y}{RG}$$

Where:
K_Φ = Total Roll Stiffness (ft-lb/deg)
M_Φ = Roll Moment (ft-lb)
A_y = Lateral acceleration (g)
RG = Roll Gradient = 1.5deg/g

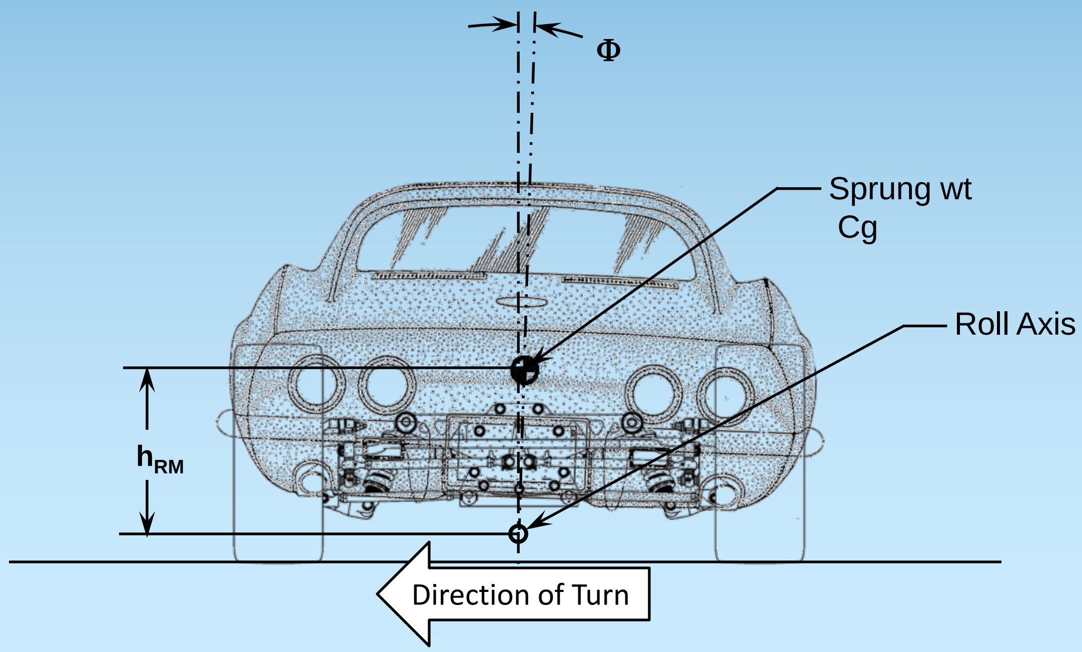


Roll Stiffness

- **Total Roll Stiffness per Roll Gradient**

$$K_\Phi = \frac{4433.6}{1.5} = 2955.7\,ft - lb/\deg$$

Example C3 Corvette Upgrade:
K_Φ = Total Roll Stiffness (ft-lb/deg)
M_Φ = 4433.6ft-lb
A_y = 1g
RG = Roll Gradient = 1.5deg/g



Roll Stiffness

- **Front Roll Stiffness**
  - To calculate the available roll stiffness from the springs alone for an independent suspension:

$$K_{\Phi SF} = \frac{K_{RF} * (T_F)^2}{1375}$$

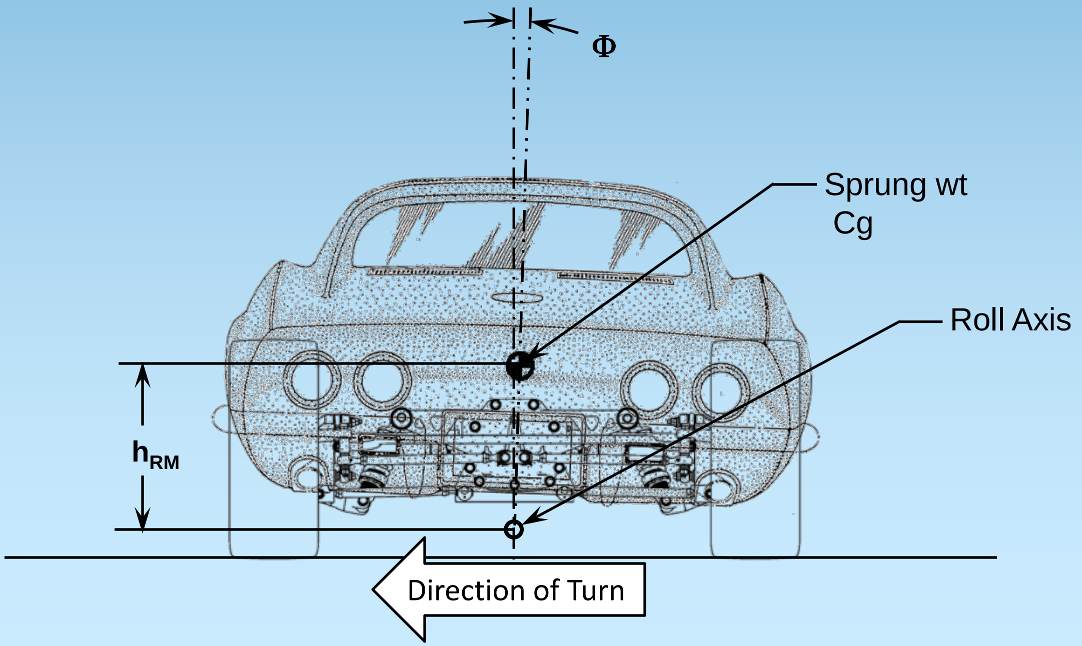
Where:

$K_{\Phi SF}$ = Front Roll Stiffness (ft-lb/deg)

K_{RF} = Front Ride Rate (lb/in)

T_F = Track front (in)

1375 = 2*(180/π)*12



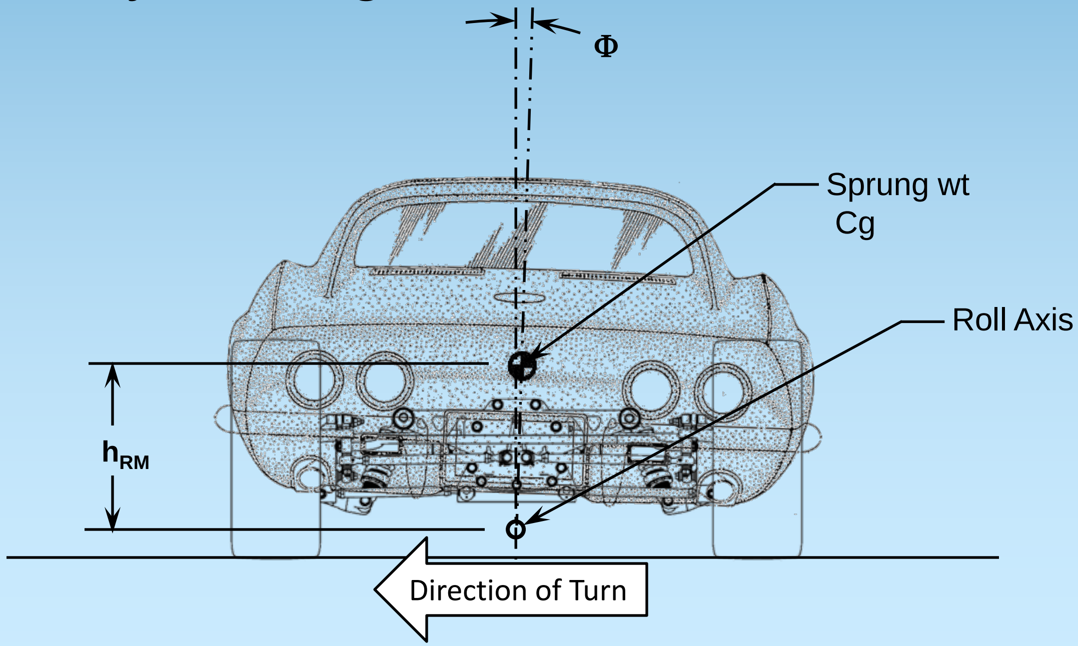
Roll Stiffness

- **Front Roll Stiffness**
  - To calculate the available roll stiffness from the springs alone for an independent suspension:

$$K_{\Phi SF} = \frac{200.77 * (58.66)^2}{1375}$$

$$K_{\Phi SF} = 502.4\, ft - lb / \deg$$

Example C3 Corvette Upgrade:
$K_{\Phi SF}$ = Front Roll Stiffness (ft-lb/deg)
K_{RF} = 200.77lb/in
T_F = 58.66in
1375 = 2*(180/π)*12

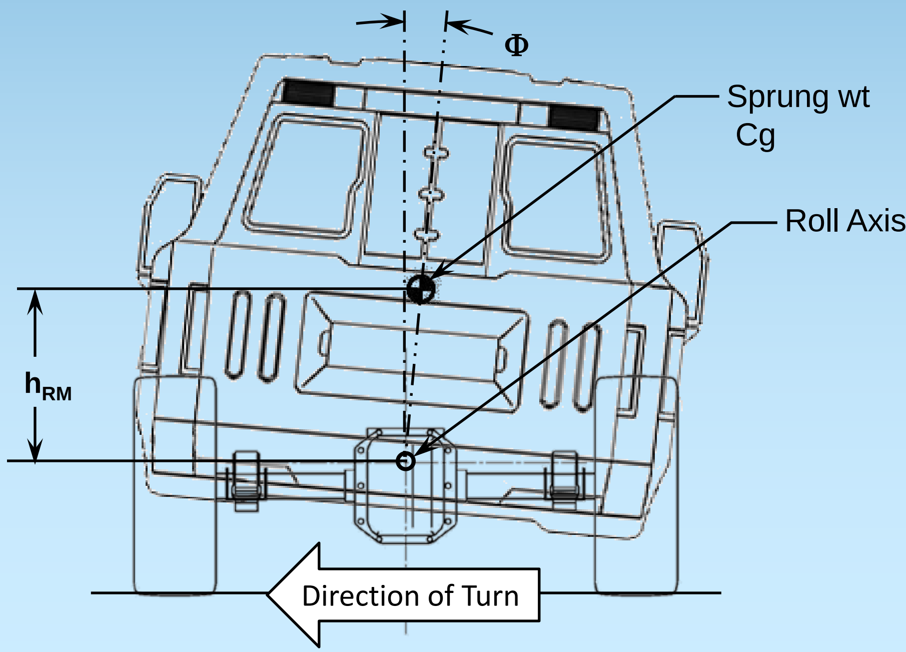


Roll Stiffness

- **Front Roll Stiffness**
  - To calculate the available roll stiffness from the springs alone for an solid (live) axle suspension:

$$K_{\Phi SF} = \frac{(K_{WF} * T_S^2) * (K_T * T_F^2)}{1375 * (K_{WF} * T_S^2) + (K_T * T_F^2)}$$

Where:
$K_{\Phi SF}$ = Front Roll Stiffness (ft-lb/deg)
K_{WF} = Front Wheel Rate (lb/in)
T_S = Spring spacing (in)
K_T = Tire Rate (lb/in)
T_F = Track front (in)
1375 = 2*(180/π)*12



Roll Stiffness

- **Rear Roll Stiffness**
  - To calculate the available roll stiffness from the springs alone for an independent suspension:

$$K_{\Phi SR} = \frac{K_{RR} * (T_R)^2}{1375}$$

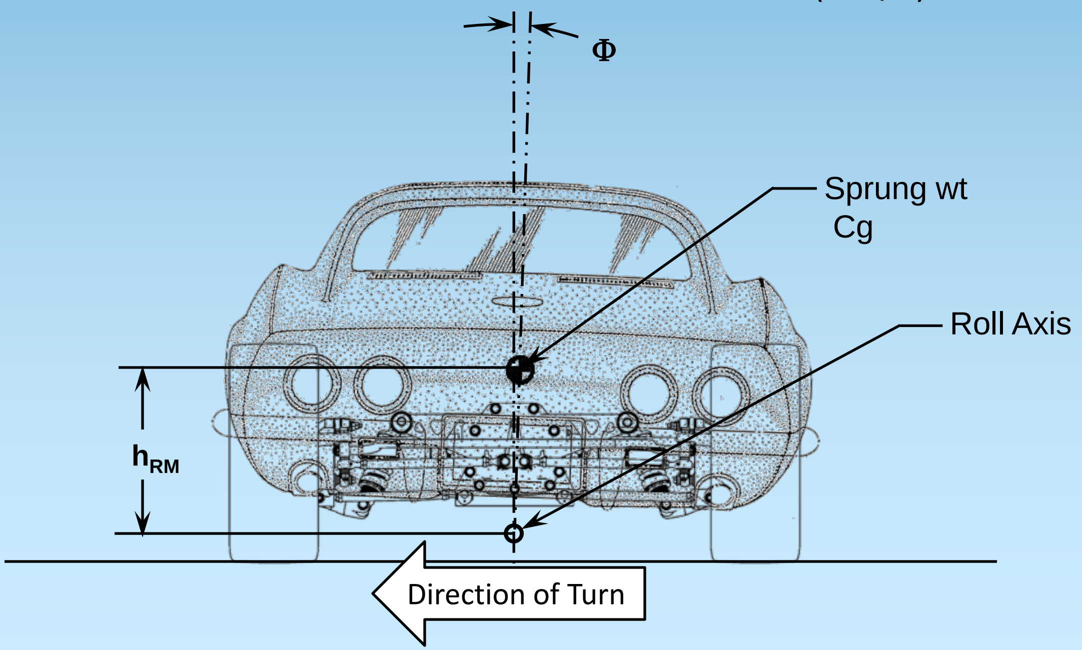
Where:

$K_{\Phi SR}$ = Rear Roll Stiffness (ft-lb/deg)

K_{RR} = Rear Ride Rate (lb/in)

T_R = Track rear (in)

1375 = 2*(180/π)*12



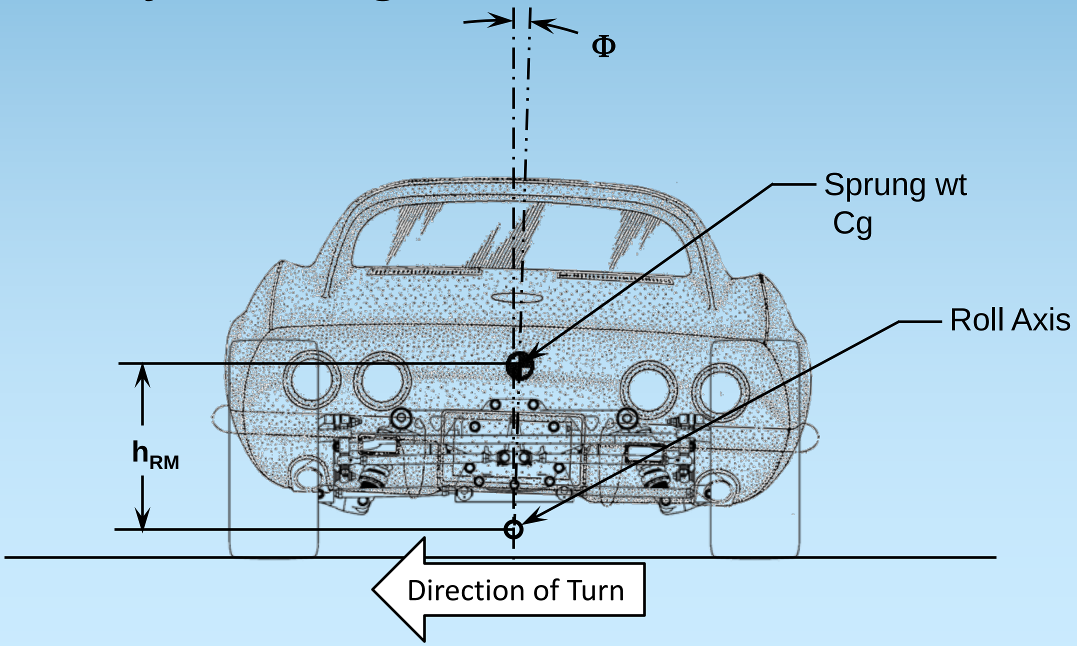
Roll Stiffness

- **Rear Roll Stiffness**
  - To calculate the available roll stiffness from the springs alone for an independent suspension:

$$K_{\Phi SR} = \frac{266.81 * (58.66)^2}{1375}$$

$$K_{\Phi SR} = 667.7\, ft - lb/\deg$$

Example C3 Corvette Upgrade:
$K_{\Phi SR}$ = Rear Roll Stiffness (ft-lb/deg)
K_{RR} = 266.81lb/in
T_R = 58.66in
1375 = 2*(180/π)*12

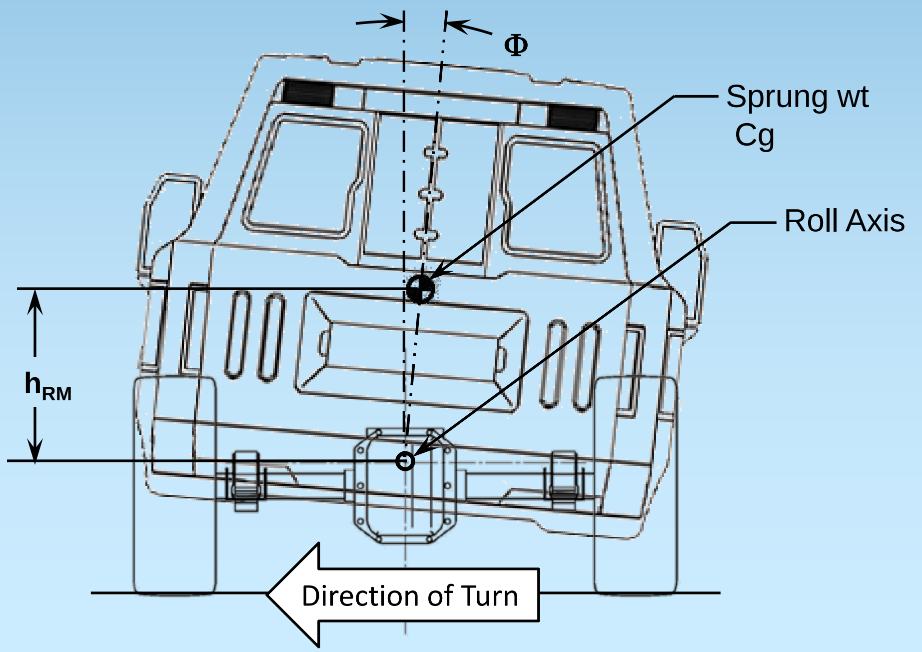


Roll Stiffness

- **Rear Roll Stiffness**
  - To calculate the available roll stiffness from the springs alone for an solid (live) axle suspension:

$$K_{\Phi SR} = \frac{(K_{WR} * T_S^2) * (K_T * T_R^2)}{1375 * (K_{WR} * T_S^2) + (K_T * T_R^2)}$$

Where:
$K_{\Phi SR}$ = Rear Roll Stiffness (ft-lb/deg)
K_{WR} = Rear Wheel Rate (lb/in)
T_S = Spring spacing (in)
K_T = Tire Rate (lb/in)
T_R = Track rear (in)
1375 = 2*(180/π)*12



- **Anti-Roll Bars**
  - Total stiffness due to springs:

$$K_{\Phi S} = K_{\Phi SF} + K_{\Phi SR}$$

Where:
$K_{\Phi S}$ = Total Spring Roll Stiffness (ft-lb/deg)
$K_{\Phi SF}$ = Front Spring Roll Stiffness (ft-lb/deg)
$K_{\Phi SR}$ = Rear Spring Roll Stiffness (ft-lb/deg)

  - Anti-roll bars would then need to provide the difference to equal the Total Roll Stiffness:

$$K_{\Phi B} = K_{\Phi} - K_{\Phi S}$$

Where:
$K_{\Phi B}$ = Total ARB Roll Stiffness (ft-lb/deg)
K_{Φ} = Total Roll Stiffness (ft-lb/deg)
$K_{\Phi S}$ = Spring Roll Stiffness (ft-lb/deg)

Anti-Roll Bars

- **Anti-Roll Bars**

  - Total stiffness due to springs:

$$K_{\Phi S} = 502.4 + 667.7$$

$$K_{\Phi S} = 1170.1\, ft - lb/\deg$$

Example C3 Corvette Upgrade:
$K_{\Phi S}$ = Total Spring Roll Stiffness (ft-lb/deg)
$K_{\Phi SF}$ = 502.4ft-lb/deg
$K_{\Phi SR}$ = 667.7ft-lb/deg

  - Anti-roll bars would then need to provide the difference to equal the Total Roll Stiffness:

$$K_{\Phi B} = 2955.7 - 1170.1$$

$$K_{\Phi B} = 1785.6\, ft - lb/\deg$$

Example C3 Corvette Upgrade:
$K_{\Phi B}$ = Total ARB Roll Stiffness (ft-lb/deg)
K_{Φ} = 2955.7ft-lb/deg
$K_{\Phi S}$ = 1170.1ft-lb/deg

Anti-Roll Bars

- **Anti-Roll Bars**
  - To calculate the requirements of the front and rear anti-roll bars, it is important to know the lateral load transfer distribution per g of acceleration.

$$\frac{TLT}{A_y} = \frac{W_T * h}{T_{ave}}$$

Where:
TLT = Total Load Transfer (lb)
A_y = Lateral acceleration (g)
W_T = Total vehicle weight (lb)
h = vehicle CG height (in)
T_{ave} = Average track width [$(T_F+T_R)/2$] (in)

  - To insure initial understeer of the vehicle, calculate the Front Lateral Load Transfer to be 5% above the total front weight distribution (W_F + 5%).

$$FLT = \left(\frac{TLT}{A_y}\right) * (W_F + 5\%)$$

Where:
FLT = Front Load Transfer (lb)
TLT = Total Load Transfer (lb)
A_y = Lateral acceleration (g)
W_F = Front vehicle weight (lb)

Anti-Roll Bars

- **Anti-Roll Bars**

  - To calculate the requirements of the front and rear anti-roll bars, it is important to know the lateral load transfer distribution per g of acceleration.

$$\frac{TLT}{A_y} = \frac{3542 * 17}{58.66} = 1026.49 lb/g$$

Example C3 Corvette Upgrade:
TLT = Total Load Transfer (lb)
A_y = Lateral acceleration (g)
W_T = 3542lb
h = 17in
T_{ave} = 58.66in

  - To insure initial understeer of the vehicle, calculate the Front Lateral Load Transfer to be 5% above the total front weight distribution (W_F% + 5%).

$$FLT = 1026.49 * (.497 + .05)$$

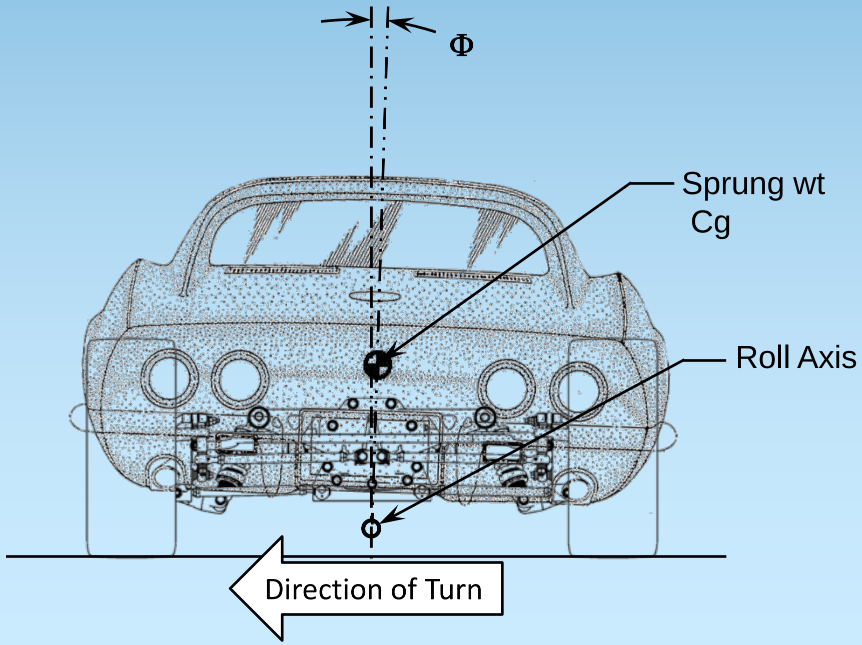
$$FLT = 561.49 lb$$

Example C3 Corvette Upgrade:
FLT = Front Load Transfer (lb)
TLT = 1026.49lb
A_y = 1g
W_F %= 49.7%

Anti-Roll Bars

- **Anti-Roll Bars**
  - To calculate the front body roll stiffness solve for $K_{\phi F}$:

$$\frac{FLT}{A_y} = \frac{12(K_{\Phi F})*\Phi}{T_F} + \frac{(W_{SF}*Z_F)}{T_F} + \frac{(W_{UF}*RL_F)}{T_F}$$



Where:
FLT = Front Load Transfer (lb)
A_y = Lateral acceleration (g)
$K_{\Phi F}$ = Front Roll Stiffness (ft-lb/deg)
Φ = Roll gradient (deg)
W_{SF} = Sprung weight front (lb)
Z_F = Roll center height front (in)
W_{UF} = Unsprung weight front (lb)
RL_F = Tire static load radius front (in)
T_F = Front track width (in)

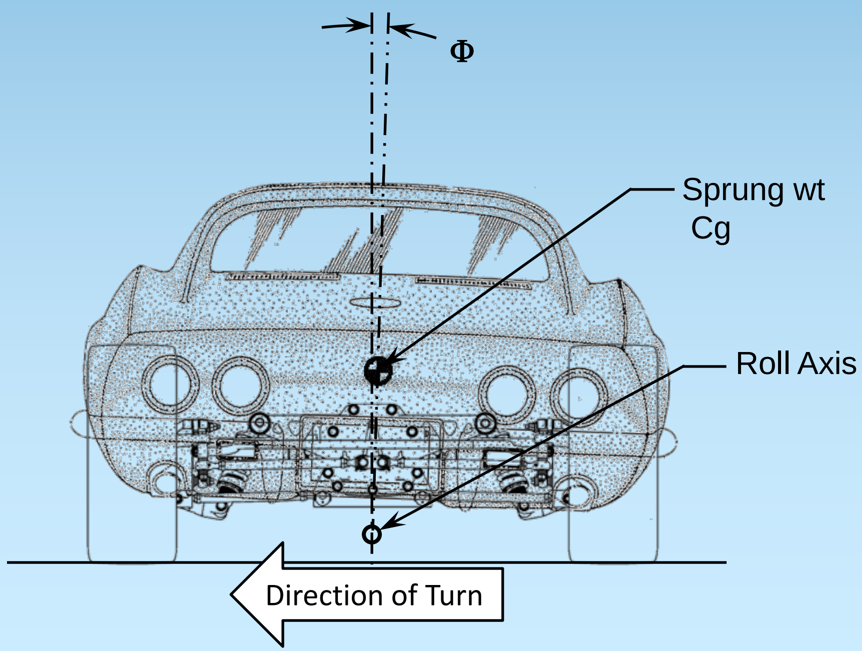
Anti-Roll Bars

- **Anti-Roll Bars**
  - To calculate the front body roll stiffness solve for $K_{\phi F}$:

$$561.49 = \frac{12(K_{\Phi F})*1.5}{58.66} + \frac{(1544.4*.497)}{58.66} + \frac{(210*12.27)}{58.66}$$

$$561.49 = 0.307(K_{\Phi F}) + 13.09 + 43.93$$

$$K_{\Phi F} = 504.47/.307 = 1643.2\,ft - lb/\deg$$



Example C3 Corvette Upgrade:
FLT = 561.49lb
A_y = 1g
$K_{\Phi F}$ = Front Roll Stiffness (ft-lb/deg)
Φ = 1.5deg
W_{SF} = 1544.4lb
Z_F = 0.497in
W_{UF} = 210lb
RL_F = 12.27in
T_F = 58.66in

- **Anti-Roll Bars**

  - To balance the body roll stiffness between the springs and the ARB, the front anti-roll bar stiffness is calculated as:

$$K_{\Phi BF} = K_{\Phi F} - K_{\Phi SF}$$

Where:
$K_{\Phi BF}$ = Front ARB Roll Stiffness Req'd (ft-lb/deg)
$K_{\Phi F}$ = Front Roll Stiffness (ft-lb/deg)
$K_{\Phi SF}$ = Front Spring Roll Stiffness (ft-lb/deg)

  - To balance the body roll stiffness between the springs and the ARB, the rear anti-roll bar stiffness is calculated as:

$$K_{\Phi BR} = K_{\Phi} - K_{\Phi F} - K_{\Phi SR}$$

Where:
$K_{\Phi BR}$ = Rear ARB Roll Stiffness Req'd (ft-lb/deg)
K_{Φ} = Total Roll Stiffness (ft-lb/deg)
$K_{\Phi F}$ = Front Roll Stiffness (ft-lb/deg)
$K_{\Phi SR}$ = Rear Spring Roll Stiffness (ft-lb/deg)

Anti-Roll Bars

- **Anti-Roll Bars**

  - To balance the body roll stiffness between the springs and the ARB, the front anti-roll bar stiffness is calculated as :

$$K_{\Phi BF} = 1643.2 - 502.4$$

$$K_{\Phi BF} = 1140.8\, ft - lb / \deg$$

Example C3 Corvette Upgrade:
$K_{\Phi BF}$ = Front ARB Roll Stiffness Req'd (ft-lb/deg)
$K_{\Phi F}$ = 1643.2ft-lb/deg
$K_{\Phi SF}$ = 502.4ft-lb/deg

  - To balance the body roll stiffness between the springs and the ARB, the rear anti-roll bar stiffness is calculated as :

$$K_{\Phi BR} = 2955.7 - 1643.2 - 667.7$$

$$K_{\Phi BR} = 644.8\, ft - lb / \deg$$

Example C3 Corvette Upgrade:
$K_{\Phi BR}$ = Rear ARB Roll Stiffness Req'd (ft-lb/deg)
K_{Φ} = 2955.7ft-lb/deg
$K_{\Phi F}$ = 1643.2ft-lb/deg
$K_{\Phi SR}$ = 667.7ft-lb/deg

- **Anti-Roll Bars**

  - The front anti-roll bar rate (lb/in) for body roll compensation can be derived from the ARB stiffness as:

$$K_{BF} = \frac{K_{\Phi BF} * 1375}{(T_F)^2}$$

Where:
K_{BF} = Front ARB Body Roll Rate (lb/in)
$K_{\Phi BF}$ = Front ARB Roll Stiffness (ft-lb/deg)
T_F = Front track width(in)
1375 = (2*180/π*12)

  - The rear anti-roll bar rate (lb/in) for body roll compensation can be derived from the ARB stiffness as :

$$K_{BR} = \frac{K_{\Phi BR} * 1375}{(T_R)^2}$$

Where:
K_{BR} = Rear ARB Body Roll Rate (lb/in)
$K_{\Phi BR}$ = Rear ARB Roll Stiffness (ft-lb/deg)
T_R = Rear track width(in)
1375 = (2*180/π*12)

Anti-Roll Bars

- **Anti-Roll Bars**

  - The front anti-roll bar rate (lb/in) for body roll compensation can be derived from the ARB stiffness as:

$$K_{BF} = \frac{1140.8 * 1375}{(58.66)^2}$$

$$K_{BF} = 455.8 lb/in$$

Example C3 Corvette Upgrade:
K_{BF} = Front ARB Body Roll Rate (lb/in)
$K_{\Phi BF}$ = 1140.8ft-lb/deg
T_F = 58.66in
1375 = (2*180/π*12)

  - The rear anti-roll bar rate (lb/in) for body roll compensation can be derived from the ARB stiffness as :

$$K_{BR} = \frac{644.8 * 1375}{(58.66)^2}$$

$$K_{BR} = 257.6 lb/in$$

Example C3 Corvette Upgrade:
K_{BR} = Rear ARB Body Roll Rate (lb/in)
$K_{\Phi BR}$ = 644.8ft-lb/deg
T_R = 58.66in
1375 = (2*180/π*12)

Anti-Roll Bars

- **Anti-Roll Bars**
  - The anti-roll bars must apply their force through their motion ratios. Therefore:
  - The front anti-roll bar rate is calculated as:

$$K_{bf} = \frac{(K_{BF}/2)}{(MR_f)^2}$$

Where:
K_{bf} = Front ARB Roll Rate (lb/in)
K_{BF} = Front ARB Body Roll Rate (lb/in)
MR_f = Front ARB motion ratio (in/in)

  - The rear anti-roll bar rate is calculated as:

$$K_{br} = \frac{(K_{BR}/2)}{(MR_r)^2}$$

Where:
K_{br} = Rear ARB Roll Rate (lb/in)
K_{BR} = Rear ARB Body Roll Rate (lb/in)
MR_r = Rear ARB motion ratio (in/in)

Anti-Roll Bars

- **Anti-Roll Bars**
  - The anti-roll bars must apply their force through their motion ratios. Therefore:
  - The front anti-roll bar rate is calculated as:

$$K_{bf} = \frac{(455.8/2)}{(.643)^2}$$

$$K_{bf} = 551.2 lb/in$$

Example C3 Corvette Upgrade:
K_{bf} = Front ARB Roll Rate (lb/in)
K_{BF} = 455.8lb/in
MR_f = 0.643in/in

  - The rear anti-roll bar rate is calculated as:

$$K_{bf} = \frac{(257.6/2)}{(.75)^2}$$

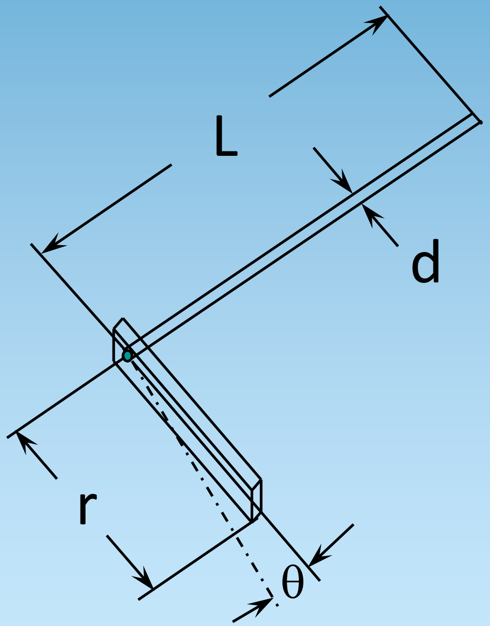
$$K_{bf} = 229 lb/in$$

Example C3 Corvette Upgrade:
K_{br} = Rear ARB Roll Rate (lb/in)
K_{BR} = 257.6lb/in
MR_r = 0.75in/in

Anti-Roll Bars

- **Anti-Roll Bars**
  - Anti-roll bars perform in torsion. The deflection rate at the free end of a torsion spring is:

$$\frac{F}{\delta} = \frac{\pi d^4 G}{32 L r^2} = k$$



Where:
G = Modulus of Rigidity
δ = Deflection
$\theta = \frac{\delta}{r}$

Anti-Roll Bars

- **Anti-roll bars (independent suspension)**
  - The ARB stiffness ($k_{\phi ind\ bar}$) [ft-lb/deg] for this type of anti-roll bar (torsion bar) is calculated as:

$$k_{\phi\ ind\ bar} = \frac{\left[\left(\frac{(500{,}000)OD^4}{(0.4422A^2 * B) + 0.2264C^3}\right) - \left(\frac{(500{,}000)id^4}{(0.4422A^2 * B) + 0.2264C^3}\right) * MR^2 * 2\right] * (t_r)^2}{1375}$$

When one wheel hits a bump the anti-roll bar twists as the wheel is raised, and since the other wheel does not move, the bar twists over its whole length (B). In roll the bar is twisted from both ends so its effective length is half the actual length which **doubles** the one wheel rate of the anti-roll bar.

ARB (anti-roll bar) Rate Calculator

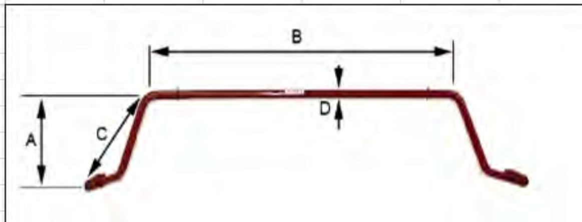
Back to Susp Project Tasks

Front:

Motion Ratio:	0.643	(in/in)
Virtual torque arm (A):	8.9	in
Length of center (B):	32	in
Physical torque arm (C):	10.7	in
OD in millimeters: (D)	30	mm
ID in millimeters: (if hollow)	21	mm

Front:

ABR rate:	546.447	lb/in
ABR effect Wheel Rate (one wheel):	225.928	lb/in
ABR effect Wheel Rate (body roll):	451.856	lb/in



A - Length of end perpendicular to B (torque arm - inches)	
B - Length of center section (inches)	
C - Length of end (inches)	
D - Diameter bar (millimeters)	

Rear:

Motion Ratio:	0.75	(in/in)
Virtual torque arm (A):	8.51	in
Length of center (B):	32.28	in
Physical torque arm (C):	8.59	in
OD in millimeters: (D)	21.5	mm
ID in millimeters: (if hollow)	0	mm

Rear:

ABR rate:	226.021	lb/in
ABR effect Wheel Rate (one wheel):	127.137	lb/in
ABR effect Wheel Rate (body roll):	254.274	lb/in

The diameter inputs are in millimeters since that is how ABRs are usually sold. This calculation does not take into account flex allowed from ABR bushings, which can be very significant when using soft rubber bushings.
See Effects of Bushing Deflection below.

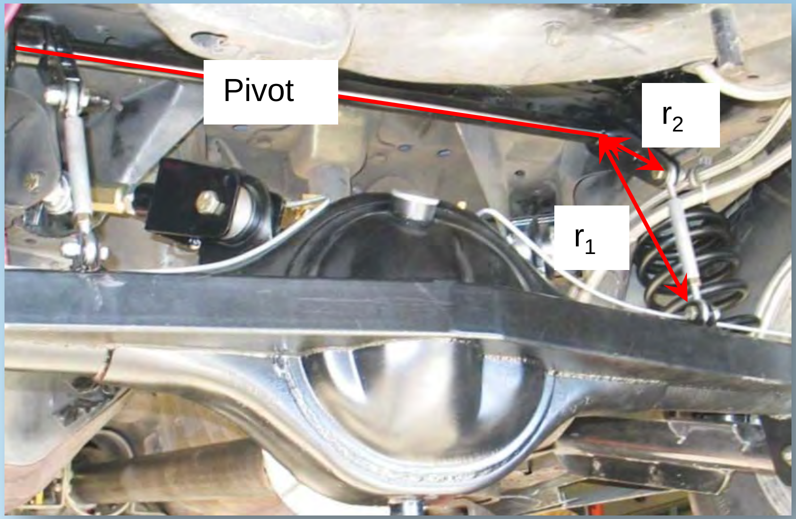
Anti-Roll Bars

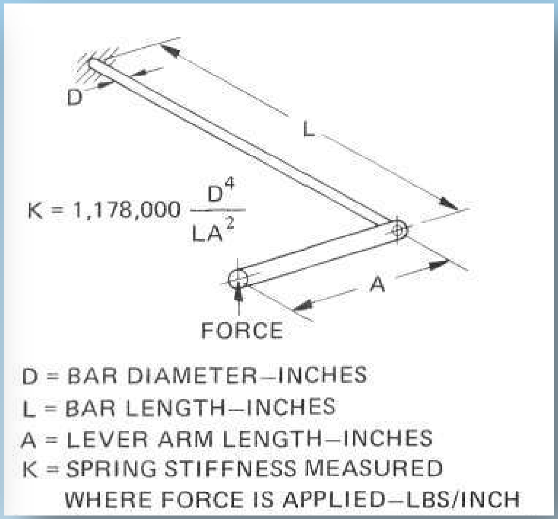
- **Anti-roll bars (solid axle)**
  - The ARB stiffness ($k_{\phi bar}$) [ft-lb/deg] for this type of anti-roll bar (torsion bar) is calculated as:

$$k_{\phi\ sol\ bar} = \frac{\left[\left(1{,}178{,}000\,\dfrac{D^4}{L\,A^2}\right)*2\right]\left(\dfrac{r_2}{r_1}\right)^2 *(t_r)^2}{1375}$$

Where:
t_r = track width (in)
(r_2/r_1) = Motion Ratio

When one wheel hits a bump the anti-roll bar twists as the wheel is raised, and since the other wheel does not move, the bar twists over its whole length (L) . In roll the bar is twisted from both ends so its effective length is half the actual length which **doubles** the one wheel rate of the anti-roll bar.





Cornering Forces

- **Centripetal vs. Centrifugal Force**

  - When the trajectory of an object travels on a closed path about a point -- either circular or elliptical -- it does so because there is a force pulling the object in the direction of that point. That force is defined as the CENTRIPETAL force.

  - CENRTIFUGAL force is a force that operates in the opposite direction as the CENTRIPETAL force. The centripetal force points inward - ***toward*** the center of the turn (circle). The feeling of being "thrown outward“ is due to the inertia of an object. Therefore, the inertial reaction could be considered by some as centrifugal force.

Cornering Forces

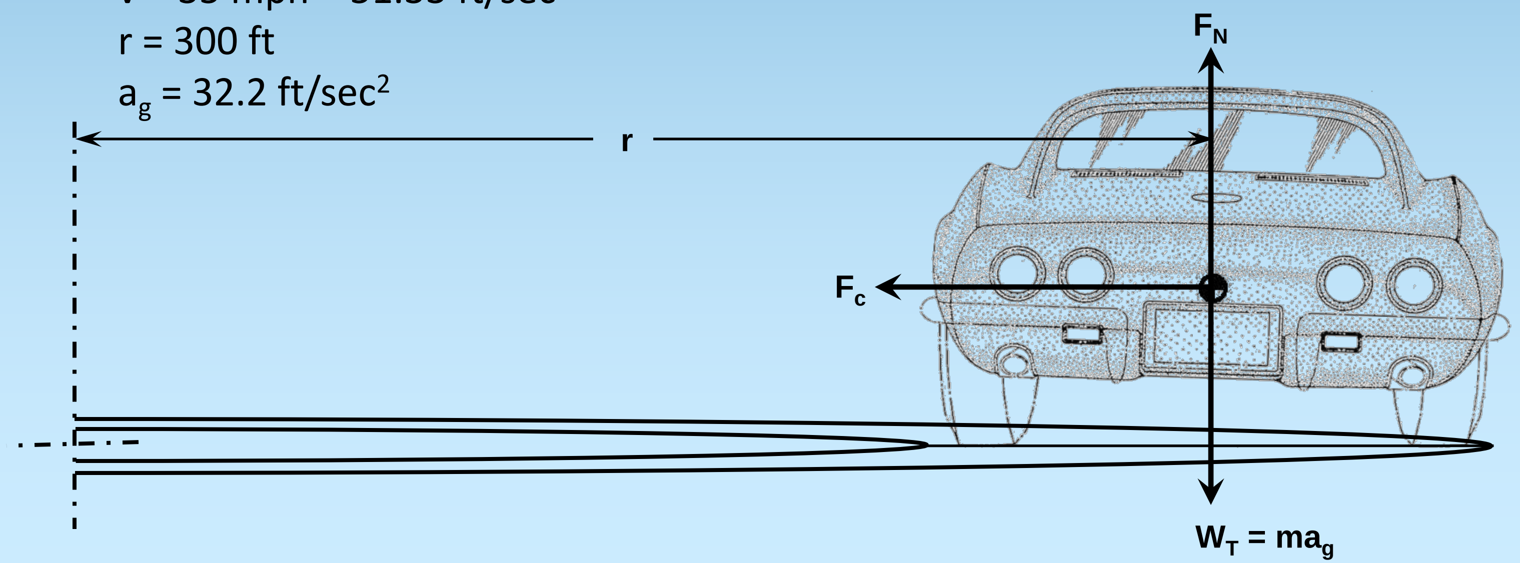
- **Centripetal Force**

$$F_c = ma_c = \frac{W_T}{a_g} * \frac{v^2}{r}$$

$F_N = ma_g = W_T$ (vertical forces cancel)
a_c = centripetal acceleration

$$F_c = \frac{3542 lb}{32.2 ft/\sec^2} * \frac{(51.33 ft/\sec)^2}{300 ft} = 966 lb$$

Example: C3 Corvette Upgrade
W_T = 3542 lb
v = 35 mph = 51.33 ft/sec
r = 300 ft
a_g = 32.2 ft/sec^2



Cornering Forces

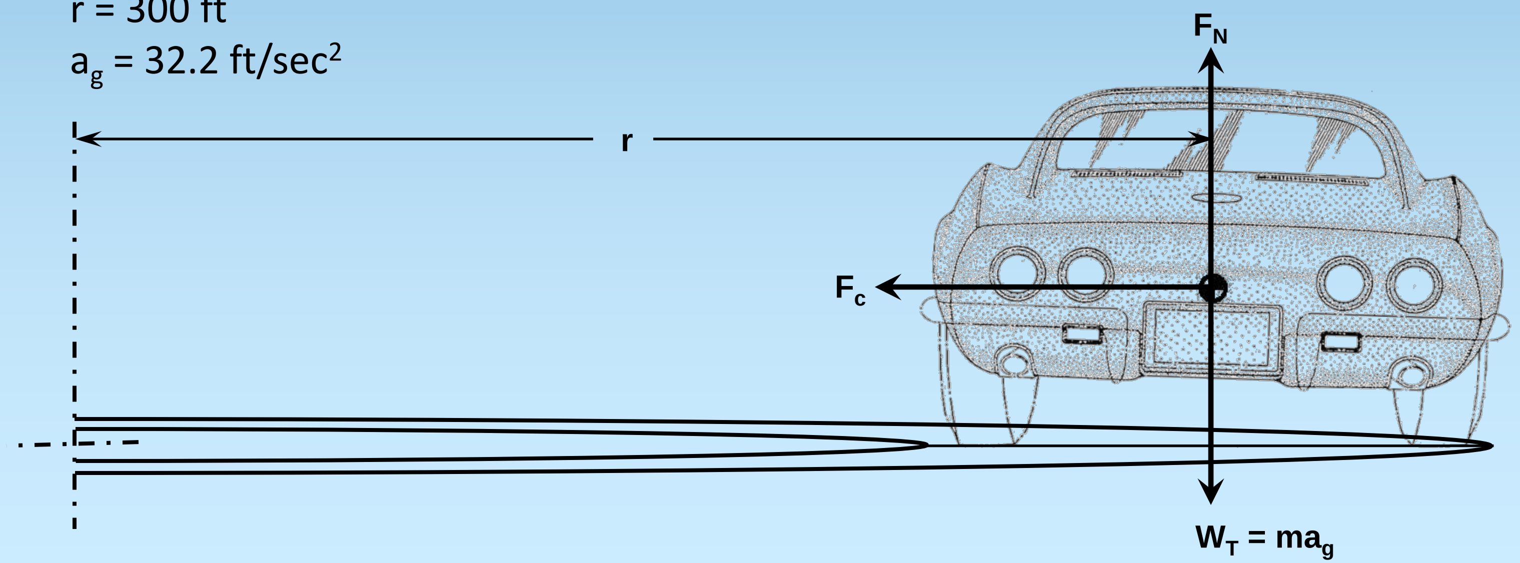
- **Lateral Acceleration (g's)**

$$a_c = \frac{v^2}{r} = \text{as distance/sec}^2 \qquad a_c = \frac{(51.33\,ft/sec)^2}{300\,ft} = 8.78\,ft/sec^2$$

$$a_c = \frac{v^2/r}{a_g} = \text{as g's} \qquad a_c = \frac{(51.33\,ft/sec)^2/300\,ft}{32.2\,ft/sec^2} = .273g's$$

Example: C3 Corvette Upgrade
W_T = 3542 lb
v = 35 mph = 51.33 ft/sec
r = 300 ft
a_g = 32.2 ft/sec^2

$F_N = ma_g = W_T$ (vertical forces cancel)
a_c = centripetal acceleration
a_g = acceleration due to gravity (1g)



Cornering Forces

- **Frictional Force**
  - If frictional force (F_f) is equal to centripetal force (F_c) the vehicle would be at its limit of adhesion to the road.

$$F_c = ma_c = \frac{W_T}{a_g} * \frac{v^2}{r}$$

$$F_f = \mu F_N = \mu m a_g$$

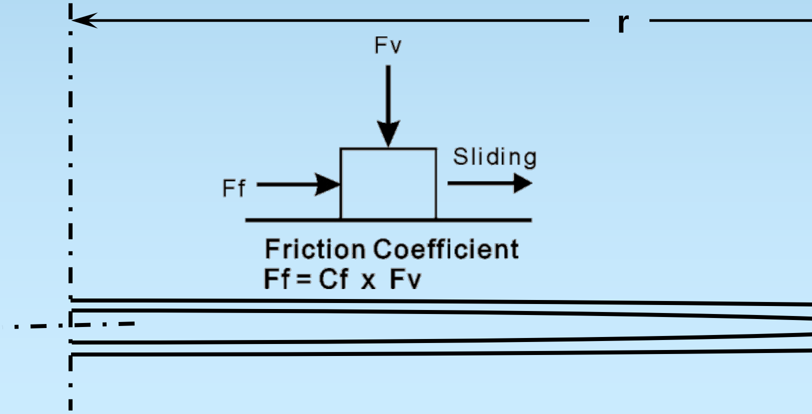
$F_N = ma_g = W_T$ (vertical forces cancel)
a_c = centripetal acceleration
μ = Coefficient of Friction between tires and road
(dry pavement 0.7 – 0.8; wet pavement 0.3 – 0.4)

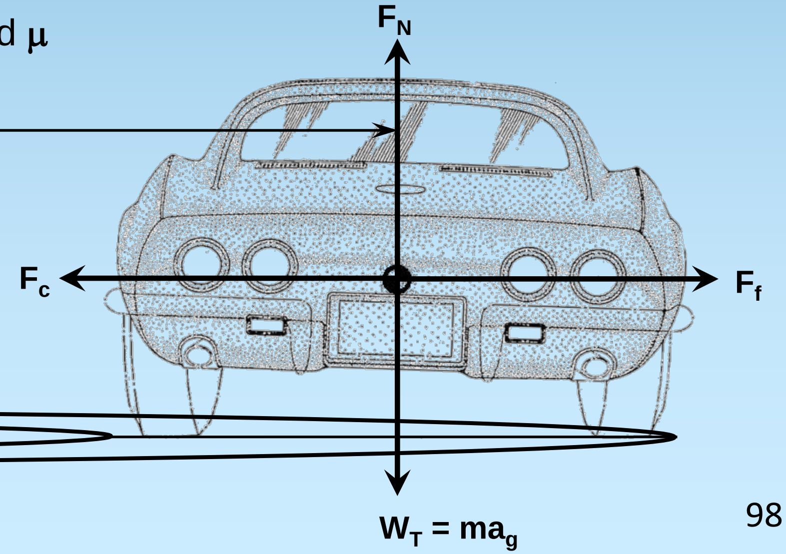
$$\mu a_g = \frac{v^2}{r} \Rightarrow v^2 = \mu a_g r \Rightarrow v = \sqrt{\mu a_g r} =$$

Max velocity for a given radius and μ

$$\mu a_g = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{\mu a_g} =$$

Min radius for a given velocity and μ





Cornering Forces

Frictional Force

- If frictional force (F_f) is equal to centripetal force (F_c) the vehicle would be at its limit of adhesion to the road.

$$F_c = \frac{3542}{32.2} * \frac{87.91^2}{300} = 2833.6lb$$

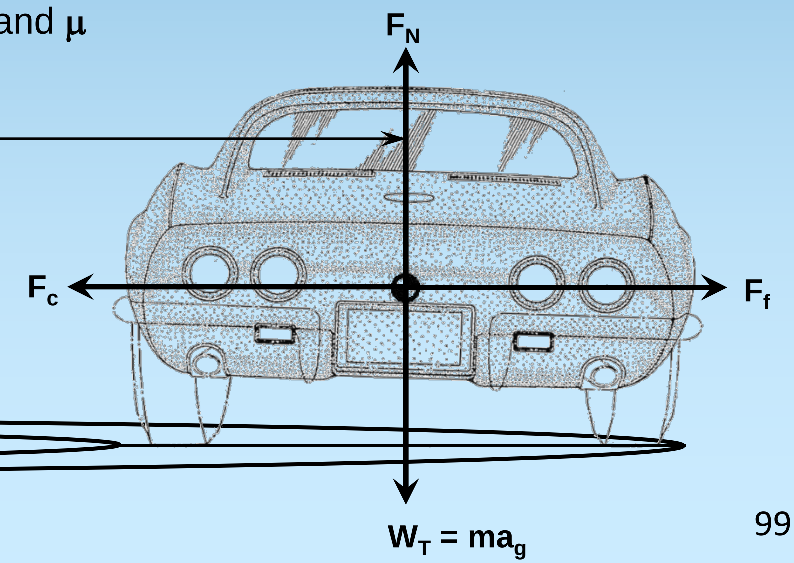
$$F_f = 0.8 * 3542 = 2833.6lb$$

$F_N = ma_g = W_T$ (vertical forces cancel)
a_c = centripetal acceleration
μ = Coefficient of Friction between tires and road
(dry pavement 0.7 – 0.8; wet pavement 0.3 – 0.4)

$$v = \sqrt{0.8 * 32.2 * 300} = 87.91 ft/sec = 59.9 mph =$$ Max velocity for a given radius and μ

$$r = \frac{51.33^2}{0.8 * 32.2} = 102.3 ft =$$ Min radius for a given velocity and μ

Example: C3 Corvette Upgrade
W_T = 3542 lb
v = 35 mph = 51.33 ft/sec
r = 300 ft
a_g = 32.2 ft/sec^2
m = 0.8



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The End

Thank You!

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