NEWSLETTER

OF THE EUROPEAN MATHEMATICAL SOCIETY

September 2012
Issue 85
ISSN 1027–488X

European
Mathematical
Society
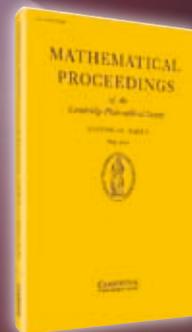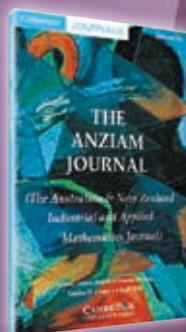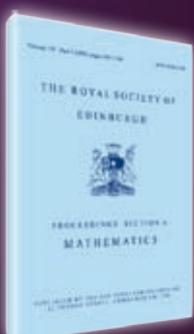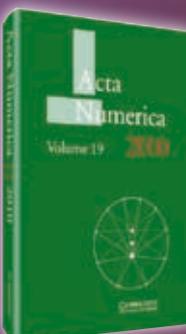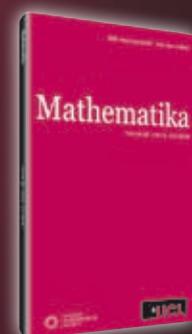
Editorial Team

Editors-in-Chief

Vicente Muñoz (2005–2012)
Facultad de Matematicas
Universidad Complutense
de Madrid
Plaza de Ciencias 3,
28040 Madrid, Spain
e-mail: vicente.munoz@mat.ucm.es

Lucia Di Vizio (2012–2016)
Université de Versailles-
St Quentin
Laboratoire de Mathématiques
45 avenue des États-Unis
78035 Versailles cedex, France
e-mail: divizio@math.cnrs.fr

Associate Editors

Vasile Berinde (2002–2012)
Department of Mathematics
and Computer Science
Universitatea de Nord
Baia Mare
Facultatea de Stiinte
Str. Victoriei, nr. 76
430072, Baia Mare, Romania
e-mail: vberinde@ubm.ro

Krzysztof Ciesielski (1999–2012)
(Societies)
Mathematics Institute
Jagiellonian University
Łojasiewicza 6
PL-30-348, Kraków, Poland
e-mail: Krzysztof.Ciesielski@im.uj.edu.pl

Martin Raussen (2003–2012)
Department of Mathematical
Sciences
Aalborg University
Fredrik Bajers Vej 7G
DK-9220 Aalborg Øst,
Denmark
e-mail: raussen@math.aau.dk

Robin Wilson (1999–2012)
Pembroke College,
Oxford OX1 1DW, England
e-mail: r.j.wilson@open.ac.uk

Copy Editor

Chris Nunn
119 St Michaels Road,
Aldershot, GU12 4JW, UK
e-mail: nunn2quick@qmail.com

Editors

Mariolina Bartolini Bussi
(2005–2012)
(Math. Education)
Dip. Matematica – Universitá
Via G. Campi 213/b
I-41100 Modena, Italy
e-mail: bartolini@unimo.it

Chris Budd (2005–2012)
Department of Mathematical
Sciences, University of Bath
Bath BA2 7AY, UK
e-mail: cjb@maths.bath.ac.uk

Jorge Buescu (2009–2012)
(Book Reviews)
Dep. Matemática, Faculdade
de Ciências, Edifício C6,
Piso 2 Campo Grande
1749-006 Lisboa, Portugal
e-mail: jbuescu@ptmat.fc.ul.pt

Eva-Maria Feichtner
(2012–2015)
Department of Mathematics
University of Bremen
28359 Bremen, Germany
e-mail: emf@math.uni-bremen.de

Eva Miranda (2010–2013)
Departament de Matemàtica
Aplicada I
EPSEB, Edifici P
Universitat Politècnica
de Catalunya
Av. del Dr Marañon 44–50
08028 Barcelona, Spain
e-mail: eva.miranda@upc.edu

Mădălina Păcurar (2008–2015)
(Personal Column)
Department of Statistics,
Forecast and Mathematics
Babeş-Bolyai University
T. Mihaili St. 58–60
400591 Cluj-Napoca, Romania
e-mail: madalina.pacurar@econ.ubbcluj.ro;
e-mail: madalina_pacurar@yahoo.com

Frédéric Paugam (2005–2012)
Institut de Mathématiques
de Jussieu
175, rue de Chevaleret
F-75013 Paris, France
e-mail: frederic.paugam@math.jussieu.fr

Ulf Persson (2005–2012)
Matematiska Vetenskaper
Chalmers tekniska högskola
S-412 96 Göteborg, Sweden
e-mail: ulfp@math.chalmers.se

Themistocles M. Rassias
(2005–2012)
(Problem Corner)
Department of Mathematics
National Technical University
of Athens
Zografou Campus
GR-15780 Athens, Greece
e-mail: trassias@math.ntua.gr.

Erhard Scholz (2009–2012)
(History)
University Wuppertal
Department C, Mathematics,
and Interdisciplinary Center
for Science and Technology
Studies (IZWT),
42907 Wuppertal, Germany
e-mail: scholz@math.uni-wuppertal.de

Olaf Teschke (2010–2013)
(Zentralblatt Column)
FIZ Karlsruhe
Franklinstraße 11
D-10587 Berlin, Germany
e-mail: teschke@zentralblatt-math.org

European Mathematical Society

Newsletter No. 85, September 2012

The views expressed in this Newsletter are those of the authors and do not necessarily represent those of the EMS or the Editorial Team.

EMS Executive Committee

President

Prof. Marta Sanz-Solé
(2011–2014)
University of Barcelona
Faculty of Mathematics
Gran Via de les Corts
Catalanes 585
E-08007 Barcelona, Spain
e-mail:ems-president@ub.edu

Vice-Presidents

Prof. Mireille Martin-Deschamps
(2011–2014)
Département de Mathématiques
Bâtiment Fermat
45, avenue des Etats-Unis
F-78030 Versailles Cedex
France
e-mail: mmd@math.uvsq.fr

Dr. Martin Raussen
(2011–2012)
Department of Mathematical
Sciences, Aalborg University
Fredrik Bajers Vej 7G
DK-9220 Aalborg Øst
Denmark
e-mail: raussen@math.aau.dk

Secretary

Dr. Stephen Huggett
(2011–2014)
School of Computing and
Mathematics
University of Plymouth
Plymouth PL4 8AA, UK
e-mail: s.huggett@plymouth.ac.uk

Treasurer

Prof. Jouko Väänänen
(2011–2014)
Department of Mathematics
and Statistics
Gustaf Hällströmin katu 2b
FIN-00014 University of Helsinki
Finland
e-mail: jouko.vaananen@helsinki.fi
and
Institute for Logic, Language
and Computation
University of Amsterdam
Plantage Muidergracht 24
1018 TV Amsterdam
The Netherlands
e-mail: vaananen@science.uva.nl

Ordinary Members

Prof. Zvi Artstein
(2009–2012)
Department of Mathematics
The Weizmann Institute of
Science
Rehovot, Israel
e-mail: zvi.artstein@weizmann.ac.il

Prof. Franco Brezzi
(2009–2012)
Istituto di Matematica Applicata
e Tecnologie Informatiche del
C.N.R.
via Ferrata 3
27100, Pavia, Italy
e-mail: brezzi@imati.cnr.it

Prof. Rui Loja Fernandes
(2011–2014)
Departamento de Matematica
Instituto Superior Tecnico
Av. Rovisco Pais
1049-001 Lisbon, Portugal
e-mail: rfern@math.ist.utl.pt

Prof. Igor Krichever
(2009–2012)
Department of Mathematics
Columbia University
2990 Broadway
New York, NY 10027, USA
and
Landau Institute of
Theoretical Physics
Russian Academy of Sciences
Moscow
e-mail: krichev@math.columbia.edu

Prof. Volker Mehrmann
(2011–2014)
Institut für Mathematik
TU Berlin MA 4–5
Strasse des 17. Juni 136
D-10623 Berlin, Germany
e-mail: mehrmann@math.TU-Berlin.DE

EMS Secretariat

Ms. Terhi Hautala
Department of Mathematics
and Statistics
P.O. Box 68
(Gustaf Hällströmin katu 2b)
FI-00014 University of Helsinki
Finland
Tel: (+358)-9-191 51503
Fax: (+358)-9-191 51400
e-mail: ems-office@helsinki.fi
Web site: http://www.euro-math-soc.eu

EMS Publicity Officer

Dmitry Feichtner-Kozlov
FB3 Mathematik
University of Bremen
Postfach 330440
D-28334 Bremen, Germany
e-mail: dfk@math.uni-bremen.de

EMS Agenda

2012

27–29 October
Executive Committee Meeting, Helsinki
Stephen Huggett: s.huggett@plymouth.ac.uk

4–8 November
Kristian Seip, EMS Lectures, Tel Aviv University, Israel
"Selected problems in operator-related function theory and
harmonic analysis"
www.euro-math-soc.eu/node/2471

24–25 November
Raising Public Awareness Committee meeting, Berlin
Ehrhard Behrends: behrends@mi.fu-berlin.de

3–7 December
Kristian Seip, EMS Lectures, Saint Petersburg, Russia
"Selected problems in operator-related function theory and
harmonic analysis"
www.euro-math-soc.eu/node/2471

2013

22–23 March
ERCOM meeting, Luminy

22–23 March
Ethics Committee meeting, London
Arne Jensen: matarne@math.aau.dk

5–7 April
Joint EMS-DMF Mathematical Weekend, Aarhus, Denmark

6–7 April
Meeting of Presidents, Aarhus, Denmark
Stephen Huggett: s.huggett@plymouth.ac.uk

12–13 April
Committee for Developing Countries meeting, Linköping,
Sweden
Tsou Sheung Tsun: tsou@maths.ox.ac.uk

25–26 May
Raising Public Awareness Committee meeting, Tycho
Brahe's Island Hven, Sweden
Ehrhard Behrends: behrends@mi.fu-berlin.de

20–25 July
29th European Meeting of Statisticians, Budapest
www.ems2013.eu

2–6 September
16th Conference of Women in Mathematics, Bonn

A Dozen Facts About the 6th European Congress of Mathematics

Stefan Jackowski (University of Warsaw, Poland)

A year ago I tried to convince readers of the EMS Newsletter (Issue 81, September 2011, pp 3–4) to attend the 6th European Congress of Mathematicians by listing a dozen reasons to come to Kraków. Today, three weeks after the Congress, which ended on 7 July 2012, I present a short report on the 6ECM, organised in 12 points.

1. The 6ECM received much attention from important Polish politicians. The President of Poland was the honorary patron of the 6ECM. The members of the Honorary Committee were the Minister of Science and Higher Education, the Governor of the Małopolska Region, the Marshall of the Małopolska Region (Chair of the local parliament) and the Mayor of Kraków. Professor Jacek Majchrowski, the Mayor of Kraków, gave a banquet for members of the EMS Executive Council and invited speakers. Professor Barbara Kudrycka, the Minister of Science and Higher Education, delivered a speech at the 6ECM opening ceremony. The President of the EMS Marta Sanz-Solé opened the congress and presented the EMS prizes (see the list of EMS prizes on page 6). A member of the Board of the National Bank of Poland Professor Eugeniusz Gatnar congratulated the winners and presented them with silver, collectible 10 Polish Zloty coins commemorating Stefan Banach. All 6ECM participants received bronze 2 Zloty coins from the same series.

Photo from www.nbp.pl

2. The congress was well attended. There were 980 registered participants and 126 registered accompanying persons. Many mathematicians not registered for the congress attended individual congress lectures and participated in the Satellite Thematic Sessions (see point 5). Thus, the total number of mathematicians participating in the activities of the congress well exceeded 1,000. Out of the 3,000 individual EMS members, 5% came to Kraków. The gender imbalance was typical for mathematical conferences: among the registered participants, only 24% were women. A panel discussion was devoted to redressing the gender imbalance in mathematics.

3. The largest national group of participants was Polish – 30%. Among them, half were members of the Polish Mathematical Society. Ten or more participants came from 16 countries. The largest groups (greater than 4%) were from the UK, the USA, France, Germany, Ukraine and Spain. Several small but strong mathematical communities were well represented.
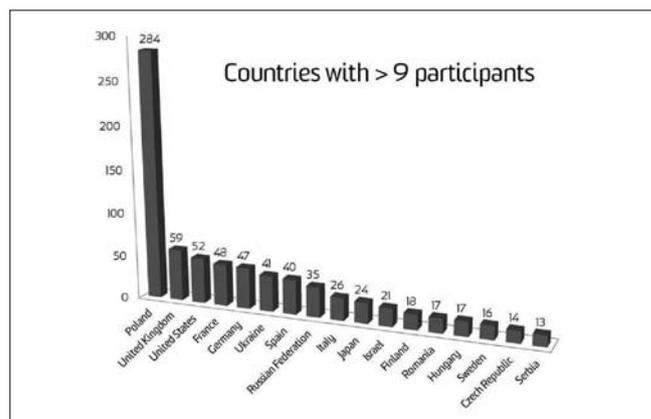
4. The scientific programme of the 6ECM as planned by the scientific committee consisted of 10 plenary sessions, 33 invited lectures and 24 mini-symposia (with about 100 speakers). Two special plenary lectures were invited by the EMS and one by the Polish Mathematical Society. Among the plenary and invited speakers, 18% came from the UK, 18% from France, 13% from the USA and 11% from Israel. The countries equally represented among speakers and participants (each less than 5%) included Germany, Russia, Spain and Finland.

All photos in this article (except the first) by Ada Pałka.

5. As a response to several requests for more opportunities for participants to speak during the 6ECM, the local organisers proposed holding Satellite Thematic Sessions (STS) during the congress. There were 15 STS held during the 6ECM (over 150 talks); some of them were a continuation of mini-symposia and some covered fields modestly represented in the official programme.

6. From over 300 proposals, the poster committee selected 186 posters to be displayed during the 6ECM. A jury chose 10 posters to receive prizes, which were presented during the closing ceremony. Prizes were funded by publishers who exhibited their publications during the congress.

7. The mathematical interests of the registered participants were quite evenly distributed. Two of the largest groups were PDEs (12%) and probability theory and stochastic processes (7%). Respectively, 20% and 16% of the plenary and invited lectures were devoted to these fields. Five percent of participants declared differential geometry as their field of interest, whereas only 2% of lectures were devoted to it.
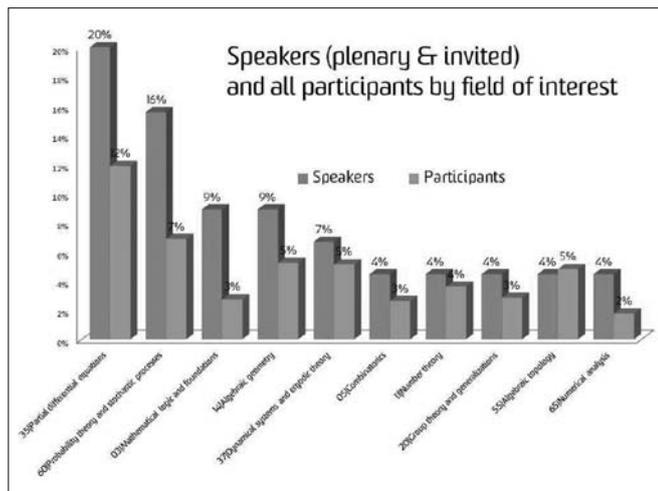
8. There were six panel discussions devoted to the broader social context of mathematics. Hot topics included financing for mathematical research, open access to publications, gender imbalance, mathematics education and mathematics in developing countries.

9. A memorial session was devoted to Friedrich Hirzebruch, the first president of the EMS, who died shortly before the congress (see page 12). Andrzej Pelczar (who died in 2010) was honoured by a special lecture as an initiator and the first organiser of the 6ECM in Kraków, a former vice-president of the EMS and the former rector of the Jagiellonian University in Kraków (an article in remembrance of Andrzej Pelczar appeared in EMS Newsletter Issue 77, September 2010, pp 12–13).

10. The annual prizes of the Polish Mathematical Society were presented at a special session, following the Andrzej Pelczar Memorial Lecture. Among them was the International Prize for a Doctoral Dissertation in Mathematical Sciences, funded by the Kraków based telecommunication software company Ericpol Sp. z o.o. The prize, with a monetary value of 20,000 PLN (almost 5,000 €) went this year to a Hungarian mathematician; four other young mathematicians from Hungary, Finland, Norway and Poland were nominated for the prize. For more details see banachprize.org.

11. A rich social, cultural and tourist programme was offered to the 6ECM participants and their companions. The Jagiellonian Library displayed old mathematical books and, for one day, an original Copernicus manuscript. Several guided walks around Kraków, showing the most spectacular historic sites, were given every day. The conference banquet was held in the medieval Franciscan convent, just a few steps from the Main Market Square. Tickets to an underground archaeological museum under the Main Market Square, which usually require an advance reservation online, were offered to participants. Two films about mathematicians Werner Doeblin and Yuri Manin were presented by their authors. (Yuri Manin spoke at two mini-symposia!) There were several art exhibitions showing mathematical motivations in paintings and installations by contemporary abstract artists in Kraków.

12. Over ten dozen volunteers were involved in the organisation of the 6ECM. Apart from a half dozen members of the executive organising committee – senior representatives of the Jagiellonian University and the Polish Mathematical Society – successful organisation of the 6ECM was possible thanks to the involvement of young mathematicians from the Jagiellonian University and the AGH University in Kraków as well as many students and doctoral students from both institutions.

For more information about the 6ECM please visit www.6ecm.pl. Programme, titles and files of presentations of most of the plenary and invited lectures, a gallery of research posters and a gallery of photos taken during the congress can be found there. One can download three congress brochures, the 6ECM poster and other interesting materials. Registered participants will have free access to the articles submitted to the 6ECM proceedings, which will be published by the EMS Publishing House.

Stefan Jackowski
President of the Polish Mathematical Society
Chair of the 6ECM Executive Organising Committee

Opening Ceremony of the 6ECM Kraków, 2 July 2012

Marta Sanz-Solé, President of the European Mathematical Society

Rector Magnificus; Minister of Science and Higher Education of the Republic of Poland; distinguished guests, ladies and gentlemen.

It is my privilege to welcome you all to the 6th ECM. This is one of the largest events in mathematics in the world and the most important scientific activity of the European Mathematical Society.

We express our sincere thanks to the Jagiellonian University for hosting the congress and for its generous support. We also thank the distinguished guests. With your presence, you are showing a much appreciated interest and support to mathematics. The invitation to Kraków was made by an honourable member of this university, the former professor and rector, and also former vice-president of the European Mathematical Society Andrzej Pelczar. Let me take this opportunity to honour his memory and to pay tribute to his devoted work for the society.

Mathematics has a strong tradition of volunteer work: running mathematical societies, organising scientific events, publishing journals and books, and organising activities to attract talented young students are among the very many examples.

Poland, with its longstanding and solid mathematical tradition, and outstanding mathematicians, has been among the most generous in this respect.

Let me mention a very few but illustrative cases:

- In 1929, only ten years after its foundation, the Polish Mathematical Society organised the First Congress of Mathematicians of the Slavic countries.
- Poland was the organiser of the International Congress of Mathematicians (ICM) in August 1983. To put this event into better context, let us recall that between December 1981 and July 1983, this country was under martial law, in an attempt to crush political opposition. These were extremely difficult times for most of the citizens of this country.
- Mathematics institutions in Poland, and in particular the Banach Centre, have been instrumental in providing conditions for interaction and collaboration of mathematicians across Europe. This has been extremely valuable, especially for those coming from Eastern European countries in a period where crossing borders was extremely difficult if not impossible.
- The last example is of special significance for the history of the EMS, since it constitutes its public debut.

Our society was founded on 28 October 1990, in a residence of the Polish Academy of Sciences in Mądralin (near Warsaw). Bogdan Bojarski, on behalf of the Polish Academy of Sciences, and Andrzej Pelczar, President of the Polish Mathematical Society, were the hosts of this important event.

We are just about to enjoy a great feast of mathematics in Europe. This is made possible thanks to the devoted efforts of very many people and institutions that deserve our gratitude. Let me mention them:

- the members of the scientific committee for their excellent work in putting together the programme of lectures;
- the members of the three prize committees – the EMS Prize, the Felix Klein Prize and the Otto Neugebauer Prize – for their difficult task in selecting the awardees among a large number of remarkable nominations;
- the organising committee. Thanks to its tremendous and brilliant work, we will all be able to savour an unforgettable event. This is yet another example of the generous service to mathematics of the Polish mathematical community;
- the sponsors of the congress: all the funding agencies, universities from Kraków, Warsaw and other cities, and private and public organisations;
- the sponsors of the prizes: Foundation Compositio Mathematica, the Institute for Industrial Mathematics in Kaiserslautern and Springer Verlag.

Why ECMs?

Like many other disciplines, mathematics has reached a degree of extreme specialisation. Nevertheless, there remains a need for keeping its unity as a scientific discipline, for resisting fragmentation and for maintaining and even increasing fluid communication between its domains. An holistic structure will better contribute to genuine progress of scientific knowledge.

As for other theoretical or experimental areas (scientific, social or humanistic) the most significant mathematical advances and breakthroughs involve a complex and sophisticated combination of ingredients, expertise and techniques from different fields.

By keeping our minds wide open and nurturing the desire of exploring beyond the boundaries of one's specific research speciality, we will have a better chance to be at the forefront of the scientific advances in our discipline.

Events like the European Congresses of Mathematics provide a very suitable stage and good conditions for these practices. An ECM is a forum for sharing mathematical knowledge and experience

with mathematicians interested in different subjects, including those at the crossroads of the discipline. It is also a forum for discussion of many aspects of the profession, a place for networking and for establishing bonds of solidarity, for becoming more aware of the importance of mathematics for the world, for feeling the need of coming closer to society and explaining the usefulness of mathematics to the public.

We are in an ancient and beautiful city of Europe, located in a splendid region full of historical monuments. Those who enjoy nature and landscape will have the opportunity to navigate along the Vistula River or to hike in the Tatra Mountains. If you would prefer peace and time for meditation, you will find shelter in the omnipresent, magnificent Krakovian churches. And on the streets, be surprised! You will see that mathematics is the cultural protagonist in the city throughout this week.

On behalf of the European Mathematical Society, I would like to thank all those who helped bring 6ecm to fruition and I wish you all a rewarding and enjoyable congress.

I declare the 6ECM open.

Thank you very much.

Prizes

EMS Prizes
10 EMS prizes were awarded to young researchers not older than 35 years, of European nationality or working in Europe, in recognition of excellent contributions in mathematics. The prize winners were selected by a committee of around 15 internationally recognized mathematicians covering a large variety of fields and chaired by Prof. Frances Kirwan (Oxford, UK). Funds for this prize have been endowed by the Foundation Compositio Mathematica.

List of Prize winners
Simon Brendle, 31 years old, received his PhD from Tübingen University in Germany under the supervision of Gerhard Huisken. He is now a Professor of mathematics at Stanford University, USA. An EMS prize is awarded to him *for his outstanding results on geometric partial differential equations and systems of elliptic, parabolic and hyperbolic types, which have led to breakthroughs in differential geometry including the differentiable sphere theorem, the general convergence of Yamabe flow, the compactness property for solutions of the Yamabe equation, and the Min-Oo conjecture.*

Emmanuel Breuillard, 35 years old, graduated in mathematics and physics from Ecole Normale Superieure (Paris); then he pursued graduate studies in Cambridge (UK) and Yale (USA) where he obtained a PhD in 2004. He is currently a professor of mathematics at Universite Paris-Sud, Orsay. He receives an EMS prize *for his important and deep research in asymptotic group theory, in particular on the Tits alternative for linear groups and on the study of approximate subgroups, using a wealth of methods from very different areas of mathematics, which has already made a long lasting impact on combinatorics, group theory, number theory and beyond.*

Alessio Figalli, 28 years old, graduated in mathematics from the Scuola Normale Superiore of Pisa (2006) and he received a joint PhD from the Scuola Normale Superiore of Pisa and the Ecole Normale Supérieure of Lyon (2007). Currently he is a professor at the University of Texas at Austin. An EMS prize goes to him *for his outstanding contributions to the regularity theory of optimal transport maps, to quantitative geometric and functional inequalities and to partial solutions of the Mather and Mañé conjectures in the theory of dynamical systems.*

Adrian Ioana, 31 years old, obtained a bachelor of Science from the University of Bucharest (2003) and received his Ph.D. from UCLA in 2007 under the direction of Sorin Popa. Currently, he is an assistant professor at the University of California at San Diego. An EMS prize is awarded to him *for his impressive and deep work in the field of operator algebras and their connections to ergodic theory and group theory, and in particular for solving several important open problems in deformation and rigidity theory, among them a long standing conjecture of Connes concerning von Neumann algebras with no outer automorphisms.*

Mathieu Lewin, 34 years old, studied mathematics at the École Normale Supérieure (Cachan), before he went to the university of Paris-Dauphine where he got his PhD in 2004. He currently occupies a full-time CNRS research position at the University of Cergy-Pontoise, close to Paris. He receives an EMS prize *for his ground breaking work in rigorous aspects of quantum chemistry, mean field approximations to relativistic quantum field theory and statistical mechanics.*

Ciprian Manolescu, 33 years old, studied mathematics at Harvard University; he received his PhD in 2004 under the supervision of Peter B. Kronheimer. He worked for three years at Columbia University, and since 2008 he is an Associate Professor at UC in Los Angeles. An EMS prize goes to him *for his deep and highly influential work on Floer theory, successfully combining techniques from gauge theory, symplectic geometry, algebraic topology, dynamical systems and algebraic geometry to study low-dimensional manifolds, and in particular for his key role in the development of combinatorial Floer theory.*

Grégory Miermont received his education at Ecole Normale Supérieure in Paris during 1998–2002. He defended his PhD thesis, which was supervised by Jean Bertoin, in 2003. Since 2009 he is a professor at Université Paris-Sud 11 (Orsay). During the academic year 2011–2012 he is on leave as a visiting professor at the University of British Columbia (Vancouver). An EMS prize is awarded to him *for his outstanding work on scaling limits of random structures such as trees and random planar maps, and his highly innovative insight in the treatment of random metrics.*

Sophie Morel, 32 years old, studied mathematics at the École Normale Supérieure in Paris, before earning her PhD at Université Paris-Sud, under the direction of Gerard Laumon. Since December 2009, she is a professor at Harvard University. She receives an EMS prize *for her deep and original work in arithmetic geometry and automorphic forms, in particular the study of Shimura varieties, bringing new and unexpected ideas to this field.*

Tom Sanders studied mathematics in Cambridge; he received his PhD in 2007 under the supervision of William T. Gowers. Since October 2011, he is a Royal Society University Research Fellow at the University of Oxford. An EMS prize goes to him *for his fundamental results in additive combinatorics and harmonic analysis, which combine in a masterful way deep known techniques with the invention of new methods to achieve spectacular applications.*

Corinna Ulcigrai, 32 years old, obtained her diploma in mathematics from the Scuola Normale Superiore in Pisa (2002) and defended her PhD in mathematics at Princeton University (2007), under the supervision of Ya. G. Sinai. Since August 2007 she is a Lecturer and a RCUK Fellow at the University of Bristol. An EMS prize is awarded to her *for advancing our understanding of dynamical systems and the mathematical characterizations of chaos, and especially for solving a long-standing fundamental question on the mixing property for locally Hamiltonian surface flows.*

Felix Klein Prize
The Felix Klein prize, endowed by the Institute for Industrial Mathematics in Kaiserslautern, is awarded to a young scientist (normally under the age of 38) for using sophisticated methods to give an outstanding solution, which meets with the complete satisfaction of industry, to a concrete and difficult industrial problem. The Prize Committee that selected the winner consisted of six members, chaired by Prof. Wil H.A. Schilders from Eindhoven in the Netherlands.

Emmanuel Trélat, 37 years old, obtained his PhD at the University of Bourgogne in 2000. Currently he is a full professor at the University Pierre et Marie Curie (Paris 6), France, and member of the Institut Universitaire de France, since 2011. He receives the Felix Klein Prize *for combining truly impressive and beautiful contributions in fine fundamental mathematics to understand and solve new problems in control of PDE's and ODE's (continuous, discrete and mixed problems), and above all for his studies on singular trajectories, with remarkable numerical methods and algorithms able to provide solutions to many industrial problems in real time, with substantial impact especially in the area of astronautics.*

Otto Neugebauer Prize
For the first time ever, the newly established Otto Neugebauer Prize in the History of Mathematics will be awarded for a specific highly influential article or book. The prize winner was selected by a committee of five specialists in the history of mathematics, chaired by Prof. Jeremy Gray (Open University, UK). The funds for this prize have been offered by Springer-Verlag, one of the major scientific publishing houses.

Jan P. Hogendijk obtained his PhD at Utrecht University in 1983 with a dissertation on an unpublished Arabic treatise on conic sections by Ibn al-Haytham (ca. 965-1041). He is now a full professor in History of Mathematics at the Mathematics Department of Utrecht University. He is the first recipient of the Otto Neugebauer Prize *for having illuminated how Greek mathematics was absorbed in the medieval Arabic world, how mathematics developed in medieval Islam, and how it was eventually transmitted to Europe.*


From left to right: Jan P. Hogendijk, Emmanuel Trélat, Corinna Ulcigrai, Tom Sanders, Grégory Miermont, Marta Sanz-Solé (EMS President), Mathieu Lewin, Ciprian Manolescu, Adrian Ioana, Alessio Figalli, Emmanuel Breuillard, Simon Brendle. Photo by Ada Pałka.

Meeting of the Editorial Board of the EMS Newsletter in Kraków

The Editorial Board of the EMS Newsletter met at the 6ecm venue on Monday 2 July 2012, with more than half the editors present. This was an invaluable opportunity to meet in person and exchange ideas. The new editor-in-chief Lucia Di Vizio also joined the party via Skype. The last meeting of the Editorial Board of the EMS Newsletter took place eight years ago in Uppsala.

Editors of the Newsletter at the meeting in Kraków. From left to right: Robin Wilson, Eva Maria Feichtner, Krzysztof Ciesielski, Eva Miranda, Vicente Muñoz, Ulf Persson, Olaf Teschke, Martin Raussen and Jorge Buescu.
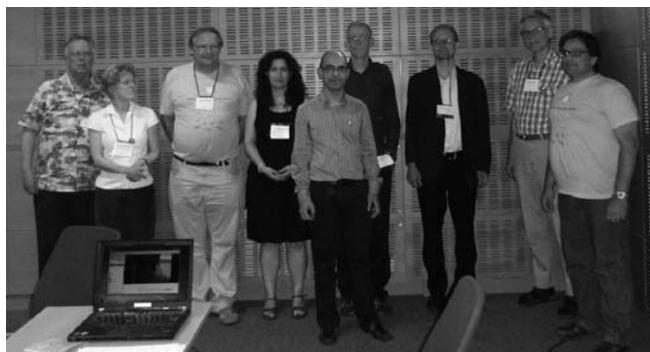
EMS Executive Committee Meeting in Florence, 25–27 November 2011

Stephen Huggett (University of Plymouth, UK)

Preliminaries
We noted the following agreements reached since the last Executive Committee (EC) meeting:

- ERCOM's remit was changed.
- The agreement with the International Association for Mathematical Physics was signed.
- Mireille Chaleyat-Maurel was appointed editor of the e-Newsletter.
- A reduced membership fee of 5 euros was agreed for individual members in developing countries.

Membership
The EC noted that the DMV will be applying for a change of class at the council meeting in Kraków but that this would not take effect until the following council meeting in 2014. Fourteen societies have nominated corresponding members. The EC was pleased to approve the long list of new individual members. Five applications for new institutional members were approved.

EMS website
Martin Raussen demonstrated the website, noting that there was a steady flow of both conferences and jobs, and that the e-Newsletter and the agenda were now on the site. Furthermore, committee webpages can now be embedded within the main EMS site.

The old book reviews are nearly ready to be made available on the site. It was time-consuming but worth-while to allocate the MSC classifications and make the text searchable.

6th European Congress of Mathematics
Stefan Jackowski presented a report on the preparations for the 6th European Congress of Mathematics.

Marta Sanz-Solé reported that there had been a healthy number of nominations for EMS prizes and the Klein and Neugebauer prizes.

The total amount of money available to support people attending was 25,000 euros from the Committee for the Support of East European Mathematicians and another 25,000 euros from various Polish sources, allowing for between one and two hundred grants.

Standing Committees
Mario Primicerio proposed Maria Esteban and Helge Holden, respectively, as the new chair and vice-chair of the Applied Mathematics Committee, and the EC agreed. It was also agreed that the Applied Mathematics Committee should be granted 5,000 euros for the Summer Schools in Applied Mathematics.

Mireille Martin-Deschamps reported that the Committee for Developing Countries had prepared a list of countries whose mathematicians would be eligible for reduced fees, and she proposed that Michel Waldschmidt be elected for his second term as vice-chair, which was approved by the EC. Then the EC focused on the request from the CDC for 15,000 euros in sup-

port of three scholarships for the Emerging Regional Centre of Excellence in Lahore. Martin Raussen argued that we should regard this as seed money, while we look for other sources of support for the ERCE programme. This was agreed.

Martin Raussen gave a brief report on the work of the Raising Public Awareness Committee and then considered the proposals for four new members. The EC agreed to appoint Sara Santos, Steve Humble and Jorge Buescu.

Zvi Artstein introduced the proposals for new members of the Women and Mathematics Committee. The EC agreed to appoint Christine Bessenrodt, Lisbeth Fajstrup and Alice Rogers. It was also agreed to appoint Caroline Series as the new chair.

The EC agreed to appoint Rui Loja Fernandes as the EC member responsible for the Meetings Committee and agreed with him that the committee needed a well defined and short list of tasks and must have a physical meeting. A long list of suggestions for chair were discussed.

Encyclopedia of Mathematics wiki

Rui Loja Fernandes reported that a very good Scientific Committee is now being formed for the Encyclopedia of Mathematics wiki. A big challenge is to change the old pages, in which the mathematics appears as images, into new ones, in which the mathematics is rendered by MathJax. Possibilities include dividing the EoM into subfields and using research students. There will be an official launch by Springer soon. It was agreed that there should be an article in the *Newsletter*.

Publishing

Suggestions for members of the editorial board of the new book series *EMS Symposia in Mathematics and its Applications* were sought. The jury for the new prize for a monograph was agreed. It will be renewed every two years.

Martin Raussen presented the report from the Editor-in-Chief of the Newsletter. The EC agreed to appoint Raquel Díaz as Head of the Team of Reviewers for the new book reviews and that the president would ask Thomas Hintermann to investigate the possibility of improving the *Newsletter* production process. Finally, the EC agreed that the terms of office of editors would be as follows: initial appointments would be for four years and the total term could not exceed eight years. It was also agreed that this system would be introduced gradually if necessary, in consultation with the Editor-in-Chief.

Marta Sanz-Solé reported on the recent meeting of the Zentralblatt Coordinating Committee, which was positive. Gert-Martin Greuel is the new director and among other things he has plans to develop the author profile.

Relations with Funding Organisations and Political Bodies

In June or July 2012 the final version of "Horizon 2020" will be published, to be approved by the end of the year.

From left to right: Ari Laptev, Mario Primicerio, Rui Loja Fernandes, Mireille Martin-Deschamps, Martin Raussen, Marta Sanz-Solé, Terhi Hautala, Jouko Väänänen, Franco Brezzi, Stephen Huggett, Volker Mehrmann and Zvi Arstein.

ScienceEurope is setting up its committees and we will need to suggest names.

The new structure of the ISE was described by Marta Sanz-Solé: it has an Assembly and a Managing Committee and a key person is the Executive Co-ordinator, who is a mathematician from Austria called Wolfgang Eppenchwandter. He is currently working on the ISE's position on Horizon 2020 and the latest paper will be circulated to the EC.

Thanks

The Executive Committee again expressed its thanks to the local organiser of the EC meeting, Mario Primicerio, and to the excellent hospitality of the two societies, SIMAI and UMI.

Stefan Dodunekov 1945–2012

Professor Stefan Dodunekov passed away on August 5. Dodunekov, a specialist in coding theory, was the president of the Union of Bulgarian mathematicians and the director of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences. He had recently been elected president of the Academy of Sciences. Dodunekov played a very important role in the development of the Mathematical Society of South-Eastern Europe (MASSEE).

EMS Executive Committee Meeting in Ljubljana, 17–19 February 2012

Stephen Huggett (University of Plymouth, UK)

Finances and Membership

The treasurer presented his financial report, noting in particular that for 2010 and 2011 there was a small overall surplus. It was noted that the cost of Executive Committee (EC) travel has increased because of the significantly increased activity of the EC.

In separate business, the treasurer reported on his discussions with Wolfram. It was agreed that he should continue to negotiate with them, seeking a non-exclusive agreement with significant benefits to the society. He would consult the EC again before any final agreement.

It was agreed that the council would need an unambiguous list of those full members who have not paid their dues. It was also agreed that before any waivers were recommended, clear proof of existence of the society in question would be needed.

EMS on the Internet

Martin Raussen reported on the website, noting that all the old book reviews are now on the site. More job announcements are coming in and they may need to be put into separate lists by length of contract. It was agreed that the site was now so large and complex that a team of people is needed to look after it.

Separately, the EC agreed to try to identify a small team of people to initiate a blog. This would be discussed at the Meeting of Presidents and former delegates of individual members would be approached.

Scientific Meetings and Activities

Stefan Jackowski presented a report on the preparations for the 6th European Congress of Mathematics. The programme was discussed and it was agreed that the opening ceremony should come first. There were several detailed questions on the budget.

Marta Sanz-Solé and Martin Raussen would soon be visiting Berlin, the proposed site of the 7th European Congress of Mathematics. It was agreed to prepare a document to be signed by the President and the Chair of the Organising Committee, listing the commitments agreed to by the Organising Committee.

The EC discussed a paper from Maria Esteban on general principles governing our response to requests for support for conferences. It was agreed that we should in general publish open calls inviting such requests, according to clear criteria, one of which would certainly be that the conference should be pan-European. However, we would allow for the possibility of exceptional cases. The autumn meeting of the EC would address these questions in detail.

Committees

Joan Porti was appointed Chair of the Meetings Committee and it was also agreed that a desirable feature of the membership of the Meetings Committee will be breadth of subject coverage. The committee will certainly need a physical meeting (in Kraków) and will need to draw up a calendar of actions.

There was a long and detailed discussion of the draft Code of Practice which had been prepared by the Ethics Committee. It was agreed that the Ethics Committee would discuss the Code again in the light of the discussion.

The EC agreed that Jiri Rakosnik would chair the Electronic Publishing Committee from 2013 and that Ulf Rehmann and Jiri Rakosnik would make suggestions for committee membership in time for the autumn meeting of the EC to make appointments.

Publishing

The president reported on the activities of the EMS Publishing House, as follows:

The Editors of *JEMS* and of *Interfaces and Free Boundaries* have been asked to consider procedures for the renewal of membership of their editorial boards and this is now taking place.

The e-books are extremely successful.

The jury for the new book prize has now been assembled. This prize is to celebrate the 10th anniversary of the publishing house and is for 10,000 euros, to be awarded every other year.

Work is proceeding on arranging free online access to *JEMS* for EMS members.

The question was addressed of whether to maintain the position of Associate Editor on the Editorial Board of the *Newsletter* and the EC agreed that from January 2013 this distinction could be removed. Most of the editors of the Newsletter will need renewal after 2012 and it was agreed that the current and new Editors-in-Chief would bring proposals to the autumn meeting of the EC. The terms of office are for four years, with one renewal possible but not automatic. However, this particular transition may need some flexibility. Also, consideration should be given to appointing a deputy to the Editor-in-Chief.

The two candidates for new Editor-in-Chief were discussed at length. It was agreed that they were both very strong but eventually the EC chose Lucia DiVizio. She would be invited to join the Editorial Board with immediate effect and to become Editor-in-Chief in January 2013.

The EC discussed the boycott of Elsevier. It was agreed to be very important to act together with other learned society publishers and perhaps the EMS was in a good position to bring them together. At the very least, a webpage listing them could be set up, making the points that publishing with them keeps the profits in mathematics and leads to more secure archiving.

Relations with Funding Organisations and Political Bodies

A list of influential people to lobby for science has been assembled and the president will pass it on to ISE, although it will also be useful for us. There is an ISE meeting in Barcelona in May, including a workshop on Horizon 2020.

The European Foundation Centre met in Barcelona recently. This is a network of private foundations. An interesting observation was that these people do not use bibliometric data when making appointments. Marta Sanz-Solé would be following up a meeting with a director of a Spanish bank to explore fundraising strategies.

There was some concern about the future budget of the ERC and it was also noted that engineering schools were lobbying for a larger share of the grants. The Board

Around the table. From left to right: Terhi Hautala, Jouko Väänänen, Marta Sanz-Solé, Stephen Huggett, Mireille Martin-Deschamps and Franco Brezzi

of the ERC is being renewed and we can suggest names, either through ISE or directly.

Thanks

The Executive Committee again expressed its thanks to the local organiser of the EC meeting, Tomaz Pisanski, his colleagues and the dean of the faculty for their excellent hospitality.

Friedrich Hirzebruch Memorial Session at the 6th European Congress of Mathematics. Kraków, July 5th, 2012

The first president of the European Mathematical Society and eminent mathematician, Friedrich Hirzebruch, passed away on the 27th of May this year. In this session we will honour and celebrate his life and achievements.

I invite the audience to stand up and to hold one minute's silence in his memory.

Professor Hirzebruch was one of the most influential mathematicians of the 20th century. His early work on the signature theorem and on the high-dimensional Riemann-Roch problem paved the way for important advances such as Atiyah-Singer index theory and Grothendieck's work in algebraic geometry. He was himself one of the contributors to these developments, as well as many others in the fields of topology and geometry. For this, he received awards, like the Wolf Prize for Mathematics, the Lobachevski Prize and the Albert Einstein and Georg Cantor Medals, among others.

His activity was not solely concentrated on his own scientific production but also to set up and to develop suitable structures – both physical and social – for the development of mathematical activity.

Hirzebruch contributed in an essential way to the reconstruction of German mathematical research after World War II. He is the founding director of the Max-Planck Institute for Mathematics in Bonn, an outstanding mathematical centre that, since its creation, has provided excellent conditions for international contacts and collaborations between researchers across the world, independently of their origin and gender.

He served as president of the German Mathematical Society for two different terms and, as has been mentioned earlier, he was the first president of the European Mathematical Society.

My last contact with him was related with a modest initiative I undertook at the beginning of my appointment in 2011. It consisted of editing a "gallery" of past-presidents on the EMS website. We immediately got his kind collaboration. Later, I was able to appreciate from the inside his enormous contributions to the society. Starting from scratch, he built the basic structure that underpins the society nowadays: its basic committees and editorial activity, the contacts with the EU political bodies, the ECMs, the EMS Prizes and an incredible network of collaborations across Europe, in a period where the continent was still politically split into two blocs.

I was privileged to be a member of the Scientific Council of the Banach Centre when he was the chair. During meetings, I always enjoyed his insight, his perceptiveness and his friendly and constructive style.

On behalf of the EMS, I express heartfelt thanks to all the contributors of this memorial, colleagues and friends who will evoke different aspects of his life: Christian Bär, Jean-Pierre Bourguignon, Stanislaw Janeczko, Yuri Manin and, in absence, Sir Michael Atiyah and Gert-Martin Greuel. Our thanks go also to all the participants in the audience, for joining us in this well-deserved tribute to such a relevant mathematician and wonderful human being.

Marta Sanz-Solé
President of the EMS

It is sad that the Congress in Kraków follows the death of Fritz Hirzebruch last month. He was the first President of the EMS at a time when the new Europe was emerging and he played a key role in ensuring that mathematicians from all over Europe were able to participate in the new society.

I was a close friend and collaborator of Fritz for over 50 years so I got to know him very well both as a mathematician and as a person. In fact, his personal qualities were crucial to his achievements. He was kind and considerate to all, young and old, and he was able to handle difficult issues with skill and finesse.

Fritz was the outstanding figure of German mathematics in the post-war world and he was the person who rebuilt mathematics in his country after the terrible years of the Nazi regime. Through his mathematical and personal leadership, Bonn became the centre that Gottingen and Berlin had been before. The Max Planck Institute that he founded in Bonn attracted scholars from all over the world and had a major impact on countries as varied as Japan and the (former) Soviet Union.

The *Arbeitstagung* that he organised on an annual basis for over 20 years was typical of his style. The meetings were informal, with no set programme, and moved with the times, following the most exciting developments in various fields. His own mathematical taste affected the atmosphere and over the years covered areas in all branches of geometry stretching from number theory to physics.

He was a remarkably lucid thinker, speaker and writer. His lectures were beautifully planned, his papers were a joy to read and his theorems were works of art. He was a virtuoso with algebraic formulae but he always integrated these into a grander design.

His legacy, in Europe and beyond, has many dimensions. Besides the institutions that he shaped such as the EMS and the MPI, he had many students and others

whom he profoundly influenced. His own mathematical contributions are deep and varied and have left their mark on our science. But, above all, we will remember him with warmth and affection as a friend.

Sir Michael Atiyah
University of Edinburgh, UK

On 27 May, Friedrich Hirzebruch died at the age of 84.

His death came as a surprise not only to his colleagues but also to his family. We were all shocked when we heard the sad news.

Every mathematician anywhere in the world knows Hirzebruch's name. This is due to his outstanding contributions to our science. Hirzebruch's scientific œuvre consists of about 140 publications including several very influential books. His work contains:

- The signature theorem for differentiable manifolds and a proof of the Riemann–Roch theorem for algebraic varieties of arbitrary dimension.
- Integrality results for characteristic numbers of differentiable manifolds as opposed to topological manifolds.
- The complete theory of characteristic classes of homogeneous spaces of compact Lie groups (with Armand Borel).
- Complex topological K-theory and applications to geometry (with Michael Atiyah).
- Relations between differential topology and algebraic number theory, in particular a proof of the Dedekind reciprocity theorem through 4-manifold theory.
- Hilbert modular-forms and -surfaces and their relations to class numbers.

The list shows that Hirzebruch covered many fields in mathematics including topology, differential and algebraic geometry, and number theory.

This is certainly not the right occasion to give a mathematical talk but I would like to illustrate the importance of his work by an example.

To put things into perspective let me remind you of the classical *Gauss–Bonnet* theorem. It says

$$2\,\pi\,\chi(M) = \int_M K\, dA,$$

where $\chi(M)$ is the Euler-Poincaré characteristic of M which can be computed by counting vertices, edges and triangles of a triangulation of M. It is a topological invariant. On the right side we find the integral of the curvature of M.

This classical result can be generalised to higher dimensions. In odd dimensions the Euler-Poincaré characteristic vanishes but in even dimensions we have the *Gauss–Bonnet–Chern* theorem:

$$\chi(M) = (2\pi)^{-n/2} \int_M Pf(R),$$

where the integrand on the right side is the Pfaffian of the curvature matrix.

Hirzebruch has two important results of a similar flavour. The first one is the *signature theorem*

$$\mathrm{sign}(M) = \int_M L(R),$$

where the signature is another topological invariant of closed oriented manifolds of dimension divisible by 4 and $L(R)$ is the L-polynomial evaluated on the curvature matrix.

The second one is a generalisation of the classical Riemann-Roch theorem for Riemann surfaces to higher complex dimensions, the *Hirzebruch–Riemann–Roch* theorem:

$$\chi(M,E) = \int_M \mathrm{Td}(M) \cdot \mathrm{ch}(E).$$

Here $\chi(M,E)$ is the holomorphic Euler number of the holomorphic vector bundle E, $\mathrm{Td}(M)$ is an expression in the curvature of M and $\mathrm{ch}(E)$ an expression in the curvature of E.

A full proof is contained in Hirzebruch's habilitation thesis which appeared as a book: *Neue topologische Methoden in der algebraischen Geometrie*. Let me cite from a book review by Chern:

The book uses many of the deep results in different branches of mathematics, and may cause difficulty even to readers with a good background. One should realize, however, that this is essentially an original paper. For such the introductory material is ample; it is also well written. If the reader succeeds in reaching the summit, the panorama is highly recommendable.

These two theorems are not only central results with many applications; they were also crucial in paving the way for one of the most exciting developments in the mathematics of the 20th century. To understand this let us reinterpret the signature theorem: Classical Hodge theory tells us that the signature of M is the Fredholm index of a certain elliptic first order operator D acting on differential forms on M, $\mathrm{sign}(M) = \mathrm{index}(D)$. Chern–Weil theory tells us that the curvature integral on the right side is the evaluation of a characteristic class built out of the Pontryagin classes of M. Hence Hirzebruch's signature theorem now reads:

$$\mathrm{index}(D) = \langle L(p_1,\dots,p_k), [M] \rangle.$$

A similar discussion applies to the Hirzebruch–Riemann–Roch theorem. Both theorems were a strong indication that the analytic Fredholm index of elliptic first order operators can be expressed in terms of topological characteristic numbers. Indeed, Atiyah and Singer found the general index theorem which finally explained this phenomenon.

So much for the mathematics. Hirzebruch was extraordinary in various other respects. He not only did wonderful mathematics himself; he also supported mathematics and other mathematicians as much as he could. For instance, he founded the Max-Planck Institute for

Mathematics in Bonn that many of you probably know from your own visits. He started the famous series of conferences known as "Arbeitstagung". He was the first president of the European Mathematical Society. We will hear more about this from the other speakers.

He was President of the German Mathematical Society in 1962 and in 1990. Let me say a few words about this. Hirzebruch was elected as president in 1961 at the last joint meeting of German mathematicians in Halle in East Germany. He was a young man in his early 30s and had to manage a very difficult situation. Shortly before the meeting, the wall in Berlin had been erected and the division of Germany and of Europe had been finalised. Due to travel restrictions, it was no longer possible for all members of the executive committee to meet in the same place. Hirzebruch arranged for the executive committee to meet twice, once in West Berlin and once in East Berlin. In 1962, political pressure became so strong that finally the Mathematical Society of the German Democratic Republic was founded and East German mathematicians were no longer allowed by their government to be members of the German Mathematical Society. Nonetheless, Hirzebruch made a big effort to stay in contact with East German colleagues in the years that followed.

In 1990, the German Mathematical Society would celebrate its 100th birthday and Hirzebruch had agreed several years before to serve as president in that special year. History held an unexpected surprise for him. In 1989 the Berlin wall came down and Germany was reunited the following year. Hirzebruch found himself having to organise the reunification of the two mathematical societies. It was debated whether there should be any checks on whether the members of the Eastern Society had been collaborating with Stasi before allowing them to become members of the newly reunited society. It was finally agreed that everybody could become members of the German Mathematical Society if they wanted. Political checking was left to employers. Hirzebruch's personality was crucial in finding a solution that would, to the extent possible, avoid hurting people. He also played an important role in restructuring the mathematics departments in former East Germany including the one I am now working at: the University of Potsdam. Hirzebruch received the last medal of merit of the Mathematical Society of the GDR.

The newly united society immediately started to prepare the invitation to host the ICM in Berlin in 1998. Hirzebruch was honorary president of the organising committee and managed to obtain a special postage stamp on the occasion.

There was another concern about which he felt very strongly. He got engaged with the Minerva Foundation which provides stipends and exchange programs for German and Israeli scientists. Encouraging cooperation between Jewish and German mathematicians was very important to Hirzebruch. He visited Israel almost every year.

For both his mathematical work and his other activities Hirzebruch received many honours. He was elected member to at least 23 academies, he was awarded doctorates from at least 15 universities including mine and he won many prestigious prices including the Wolf Prize for Mathematics, the Lomonosov Gold Medal of the Russian Academy of Sciences and the Stefan Banach Medal of the Polish Academy of Sciences.

Let me conclude with a few personal words. I had the privilege to meet Professor Hirzebruch when I was a student in Bonn. He was not my supervisor but I attended several of his lecture courses and seminars. We, the students, loved his enthusiasm, his great sense of humour and his ability to explain mathematics in such a way that it appeared totally natural and crystal clear. When a very technical proof had to be carried out, however, it would often happen that he was travelling and his assistent had to deal with the technical details. I once mentioned this to him and he assured me that this was pure coincidence. But he would say it with a twinkle in his eyes.

Thank you very much!

Christian Bär
President of the
Deutsche Mathematiker Vereinigung, Germany

Personally, over the last 40 years, I owe a lot to Friedrich Hirzebruch, for his unfailing support and the continuous inspiration. I met him in Bonn in 1970, while I was visiting Wilhelm Klingenberg as a very young researcher in differential geometry. At this time, French mathematics was strongly dominated by algebraic geometry "à la Grothendieck" and in Bonn, although Friedrich Hirzebruch was also an algebraic geometer, I could feel a more open attitude towards other sorts of mathematics.

The *Arbeitstagung*, a major mathematical event that he organised with his Bonn colleagues for more than 30 years, offered each year in June a broad overview of the most exciting mathematics of the time. It was an exceptional place to meet mathematicians of all sorts, famous and less famous, senior or just beginning. As like many young mathematicians, I have benefited a lot from it, directly through the new perspectives gained by listening to the lectures and indirectly through the great number of encounters, some of which had a great impact on my professional life.

It is really during the academic year 1976–1977, spent in Bonn with my family as guest of the SonderForschungsBereich 40, that I got to know him better. I would also meet there Jacques TITS, whom he attracted to Bonn.

He was always curious to know what kind of mathematics was on your mind and showed special interest in young mathematicians. Note should be made also of his determined, proactive attitude towards women mathematicians at a time where gender equality was not given much priority. Several women colleagues consider that they owe him a lot because of his continued support.

The numerous encounters with him that followed the wonderful year in Bonn gave me ample opportunity to witness his many talents: as an outstanding mathematician of course but also as a remarkably clear lecturer, an

efficient communicator and an exceptionally talented manager. Some of them were quite unexpected for me, such as accompanying him to a press conference with German journalists to discuss the development of mathematics in his country. We also had rather intense discussions when, as Chairman of the Programme Committee for the International Congress of Mathematicians 1986 to be held in Berkeley, he supervised me as I was in charge of the geometry section, most likely due to his support. The establishment of the *Max-Planck Institut für Mathematik in den Wissenschaften* in Leipzig was yet another occasion for extensive exchanges.

He has been a great supporter of the collaboration between the Institut des Hautes Études Scientifiques (IHÉS) and the Max-Planck Gesellschaft (MPG). He represented the MPG on the Board of Directors of IHÉS for several years. He, as director of the Max-Planck Institut für Mathematik, and Sir Michael Atiyah, as founding director of the Isaac Newton Institute in the Mathematical Sciences, endorsed immediately the idea of the European Post-Doctoral Institute (EPDI) that I proposed in the Autumn of 1994, shortly after becoming the director of IHÉS. Already in 1995, the three institutions would join forces to get young post-docs to move around Europe. For the inaugural ceremony in Bures-sur-Yvette, he gave a very inspiring speech on the role of institutes in mathematics.

Friedrich Hirzebruch and International Relations

Very early in his career, Friedrich Hirzebruch had an international dimension to his professional life:

- He visited Heinz Hopf at the ETH in Zurich in the early 1950s and visited the US, where he stayed at the Institute for Advanced Study and the University in Princeton later in the 1950s.
- His involvement in the EUROMAT project as early as 1956 and his leading role in the attempts at broadening the 'Oberwolfach Mathematisches Institut' into a Max-Planck Institute, as discussed in the contribution by Gert-Martin Greuel.

Friedrich Hirzebruch held many responsibilities at the international level during his career. He chaired many evaluation committees, was an editor of several scientific journals and was an active member of numerous scientif-

Chern Shiing-Shen, Samuel Eilenberg, Friedrich Hirzebruch in the early 1950s in the US.

ic committees for conferences of all sorts. The extraordinary number of distinctions and honours that he received shows the very high level of recognition that he enjoyed in many countries, with a special mention for Japan and Israel, where his action was particularly appreciated.

He was of course instrumental in bringing the International Congress of Mathematicians (ICM) back to Germany in 1998, exactly at the time of the 100th anniversary of what has now become the major *rendez-vous* of the international mathematical community and 94 years after the third ICM was held in Heidelberg. It was therefore natural that he be declared Honorary President of the ICM'98 Organising Committee.

Friedrich Hirzebruch's Special Relation to Henri Cartan

All through his career, Friedrich Hirzebruch had a lot of interactions with Henri Cartan: his first interaction was in relation to Cartan's efforts to renew contact between German and French mathematicians. Indeed, as early as November 1946, Henri Cartan lectured in the Lorenzenhof in Oberwolfach. Henri Cartan's long friendship with Heinrich Behnke was the context in which Friedrich Hirzebruch met him.

In this connection, Friedrich Hirzebruch wrote the following:

> *"The 'Association Européenne des Enseignants' ('European Association of Teachers') was founded in Paris in 1956. Henri Cartan was president of the French section. As such he took the initiative to invite participants from eight European countries to a meeting in Paris in October 1960. Emil Artin, Heinrich Behnke and I were the German members. The second meeting of this committee was in Düsseldorf in March 1962. As a result, the Livret Européen de l'Étudiant (European Student's Record) was published and distributed by the Association. The booklet contained a description of minimal requirements for basic courses. It was supposed to increase the mobility of students from one country to another. The professor of one university would mark in the booklet the contents of courses attended by the student. The professor at the next university would then be able to advise the student in which courses to enrol. The booklet was not used very much."*

A lot on their relationship can be learned from reading the letter that Friedrich Hirzebruch wrote in 1994 to Henri Cartan on the occasion of his 90th birthday (see a facsimile of this letter on the next page).

The Beginnings of the European Mathematical Society

The European Council of Mathematics (EMC) opened the way to the European Mathematical Society (EMS). The EMC met regularly in Oberwolfach under the leadership of Sir Michael Atiyah but no Germans were involved in running the EMC.

The foundational meeting of the EMS was held in October 1990 in Madralin and it was not an easy affair, as
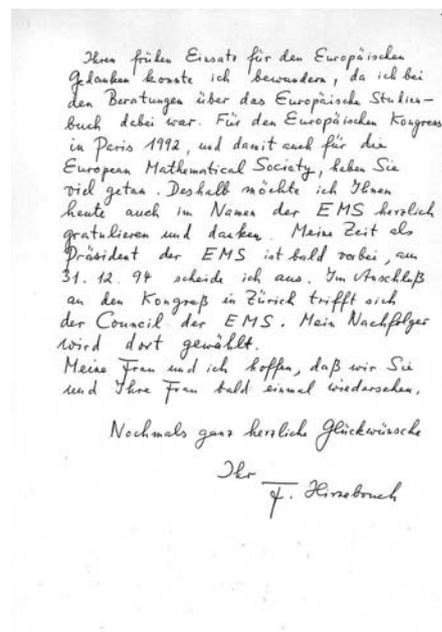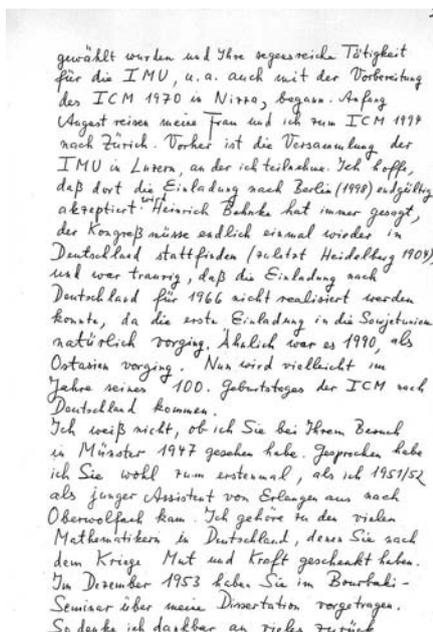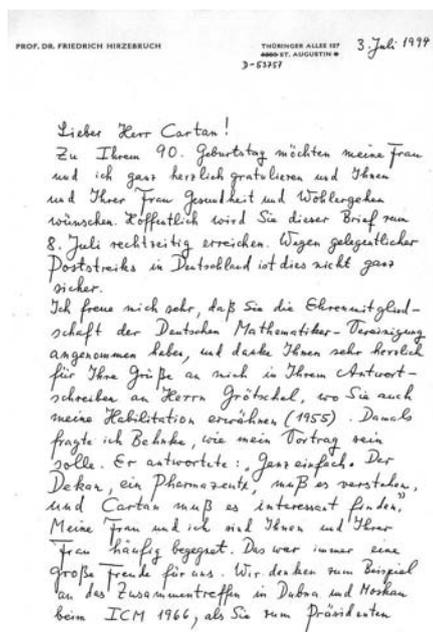
opposite views on the structure of EMS were presented by some delegations. One of the key issues was to decide whether the new society could accept individual members or be only a federation of societies. The first day, while the EMS was not yet in existence, ended in a dangerous situation of tension, with no clear compromise in sight. Friedrich Hirzebruch, who had accepted being considered as the first EMS president, led to success that rather tense meeting held the first evening until late at night behind closed doors, between supporters of the conflicting positions.. As President of the Société Mathématique de France, I was one of the troublemakers on this occasion. The next day, the new society would be created with statutes ensuring a good balance between individual members and member societies, a feature that still remains to this day.

Under Friedrich Hirzebruch leadership, the EMS developed successfully. A lot had to be achieved in a short time to take advantage of the dynamics that accompanied the creation of the society. Among milestones of his mandate, one can single out the setting up of the first European Congress of Mathematics in Paris in 1992 and laying the ground for the creation of the *Journal of the European Mathematical Society* (*JEMS*) that was finally created in 1999.

To my great surprise, he asked me to become his successor as EMS President in 1994, to serve for the second term 1995–1998, another great honour that he bestowed on me.

Friedrich Hirzebruch's Long Friendship with Shiing Shen Chern

Friedrich Hirzebruch shared with Shiing Shen Chern a long friendship. They met in 1953 and had regular and substantial exchanges.

In the book gathering tributes to the late S. S. Chern on the occasion of his centenary, he wrote "Shiing-Shen Chern, one of the greatest mathematicians of the 20th century, was for me a fatherly friend whom I owe very

much. I knew him since 1953 and will always remember our meetings in Chicago, Princeton, Berkeley and Bonn."

He felt that he could not take part in the conferences celebrating the centenary of S. S. Chern held in October 2011 at the Chern Institute in Nankai University, Tianjin, and in November 2011 at the Mathematical Sciences Research Institute in Berkeley but he accepted immediately the invitation I sent him to take part in the more modest part of the celebration held at IHÉS on 17 November 2011. He came with his son and his daughter-in-law. Unfortunately, his beloved wife Inge Hirzebruch, whom I want to thank for her kind friendship and support to my wife and to me over all these years, could not accompany him because of a last minute injury. He lectured brilliantly on "Chern Classes" and could on this occasion meet Mae Chern, the daughter of S. S. Chern. At the end of his lecture, he told me: "I am afraid that this will be my last visit to Paris." It is very sad to remark that he was indeed right.

Jean-Pierre Bourguignon, Director of the
Institut des Hautes Études Scientifiques, France

Friedrich Hirzebruch lecturing on "Chern Classes" at IHÉS on 17 November 2011.

Friedrich Hirzebruch
All photos from the Archives of the Mathematisches Forschungs-institut Oberwolfach

With Friedrich Hirzebruch the mathematical community has lost a great mathematician, a gifted teacher and a wonderful person. Being a student of Egbert Brieskorn, who himself was a student of Hirzebruch, I am mathematically a grandson of Hirzebruch. I have known Hirzebruch since my early days in Bonn, but only during the last years, as the director of the Mathematical Research Institute Oberwolfach, did I have very close contact with Fritz. This article should serve as a reminder of the early activity of Hirzebruch, closely associated to the founding of Oberwolfach of which very little is known.

The following text is mainly drawn and translated into English from the essay of Friedrich Hirzebruch "Euromat, Oberwolfach und ein geplantes Max-Planck-Institut, Erinnerungen an die Jahre 1958–1960", published in the Festschrift on the 60th anniversary of Oberwolfach.[1]

In 1956 Hirzebruch started his professorship at the University of Bonn after he had returned from his stays at the Institute for Advanced Study (IAS) in Princeton (1952–1954) and at Princeton University (1955–1956). He was so impressed by the IAS in Princeton that he immediately thought about a similar institution in Germany. He started to invite guest professors to Bonn, the first being Nicolaas Kuiper, later director of IHES, and the second being Raoul Bott. In 1957 Hirzebruch organised the first Mathematische Arbeitstagung in Bonn. At that time there were not so many conferences as today and the Arbeitstagung in Bonn was an important annual event and it became a tradition continuing to present day.

In April 1958 Hirzebruch was rather unexpectedly invited to a meeting of mathematicians in Brussels, initiated by members of the EURATOM commission, who discussed and prepared a memorandum about the founding of a European Mathematical Institute Euromat within EURATOM. Hirzebruch was chosen to replace Wilhelm Süss, the Rector of the University of Freiburg and Director of Oberwolfach, who was very sick and who died in May 1958. The second representative from Germany in this meeting was Helmuth Kneser. Although the Euromat plan was very promising, it did not work out in the end. One of the reasons was that, in 1958, Léon Motchane, also inspired by the IAS in Princeton, had successfully created a mathematical institute of this sort in Paris, namely the IHÉS.

Hirzebruch realised that, after the creation of IHÉS, it was rather unlikely that Euromat would be created. He therefore developed a plan to make Oberwolfach an Institute for Advanced Study on a smaller scale, so that there would be one in France and one in Germany.

---

[1] The essay by Hirzebruch is written in German. Citations are marked by quotation marks but they are translated into English from the original German text.

The old "castle" where the Oberwolfach workshops took place until 1974.

As one of the authors of the Euromat memorandums and as an intermediary of Helmuth Kneser in Bonn, who had become the successor to Süss as the director of Oberwolfach till 1969, Hirzebruch was already well known in the relevant ministries in Bonn. In July 1958 Hirzebruch wrote a letter to Kneser developing his idea of a kind of Institute for Advanced Study in Oberwolfach. The letter starts:

"Oberwolfach has been taken over by the Federal Ministry of the Interior and will be developed into an institute that could take over a role in mathematics in Germany such as the School of Mathematics of the Institute for Advanced Study in Princeton, NJ, for the United States."
"An annual budget of DM 600 000 will be needed and a one-time amount of DM 1.5 million for additional buildings. Moreover, a building of 20 apartments must be provided."

Hirzebruch already had concrete plans and he was optimistic as only young people can be (in 1958 he was 31 years old). He believed that the project could be realised in 1959. In any case, in order to realise the idea an organisation had to be installed.

Living room in the old building.

In March 1959 the 14 mathematicians R. Baer, H. Behnke, G. Bol, H. Gericke, H. Görtler, F. Hirzebruch, H. Kneser, G. Köthe, W. Maak, Claus Müller, P. Roquette, E. Sperner, K. Stein and K.-H. Weise met in Oberwolfach. (Today you can see the photos of these mathematicians in the big lecture hall in Oberwolfach.)

The minutes of this meeting read as follows:

> *"... the situation requires the creation of an institution at the federal level that takes care of the following tasks:*
> *1) Intensification of mathematical research,*
> *2) Strengthening of the scientific cooperation,*
> *3) Training of young researchers."*

These are basically still the goals of the Oberwolfach Institute today. The minutes continue:

> *"For this purpose it appears to the attendees suitable to create a society for mathematical research (Gesellschaft für Mathematische Forschung e.V.), based at the Mathematical Research Institute Oberwolfach. This institute has already gained, through the care of the scientific cooperation, a strong international reputation and is therefore particularly suited to be the starting point and centre for carrying out the above tasks."*

On 17 June 1959 the formal inaugural meeting of the Society for Mathematical Research took place at the Mathematical Institute in Freiburg. The institute was to be financially supported by the Federal Ministry of the Interior and the Ministry of Education of Baden-Württemberg.

The old building in Oberwolfach

It was clear that Oberwolfach was too isolated a place for an "Institute for Advanced Study" and therefore an extension to a project with Oberwolfach plus Freiburg was discussed. However, how this should be achieved remained unsolved for a while. Finally, the idea came up to create a Max-Planck Institute for Mathematics.

In October 1959 an important meeting took place in Oberwolfach with representatives from the Federal Ministry of the Interior, the Ministry of Education of Baden-Württemberg, the Max-Planck Society and the Gesellschaft für Mathematische Forschung. Hirzebruch himself could not be present because he was on sabbatical at the IAS in Princeton. A commission of the Max-Planck Society (including Werner Heisenberg and Carl-Friedrich von Weizsäcker) was to be created in order to check, together with members of a commission of Oberwolfach, the conditions for founding a Max-Planck Institute of Mathematics at Oberwolfach.

The Max-Planck Society and in particular its president Adolf Butenand, who had just been appointed as successor to Otto Hahn, were very much in favour of this idea. Butenand even pronounced in a press release in May 1960 that a new Max-Planck Institute of Mathematics was to be founded. As a principle of the Max-Planck Society they create their institutes "around a person". The person to become the first director of the new Max-Planck Institute of Mathematics was to be Friedrich Hirzebruch, although he was only 32 years old at that time. Then, the usual examination process including referees was started.

The old castle. The new library and conference building was built there in 1975. Today you can still see the wall from the street.

To cut a long story short: you all know that the creation of a Max-Planck Institute at Oberwolfach failed. The reason is to be found in the referees' reports. There were 11 referees' reports: five from Germany and six from abroad. The names of the referees are known but the content is confidential, except for three reports of which Hirzebruch got a copy: those by Bartel Leendert van der Waerden, Carl Ludwig Siegel and Richard Courant. Van der Waerden praised the workshops in Oberwolfach and the mathematician Hirzebruch. Siegel however denied not only the necessity of a Max-Planck Institute but he was also very sceptical about the "abstract mathematics" of Hirzebruch. He wrote: "I consider it to be possible, even likely, that this whole direction will die out within a few years." Since Siegel was known to be against modern mathematics, his opinion was not crucial.

**Carl Ludwig Siegel**   **Richard Courant**

More decisive was the report by Courant. He criticised the proposal as not well thought out and about Hirzebruch he wrote: "I have always advocated for him and cherish very friendly feelings for him... In my opinion it would be a great injustice to mathematics to tear him out of his productive teaching." He also expressed his concern that Hirzebruch would very visibly symbolise the predominance of the abstract direction.

The plan of a Max-Planck Institute for Mathematics came temporarily to an end.

Hirzebruch finishes his above mentioned essay as follows:

"Today, mathematics in Germany has two Max-Planck Institutes: the Max-Planck Institute for Mathematics in Bonn and the Max-Planck Institute for Mathematics in the Sciences in Leipzig, which was established after reunification. With Oberwolfach and with the two Max-Planck Institutes and the successful participation in the programme of the DFG Collaborative Research Centres, the mathematics in Germany will be very satisfied. Even Courant would be satisfied. (There is no predominance of abstract direction. As he wished, I stayed as a professor at the University of Bonn.)"

"It is a good development that Oberwolfach and the MPIMs with their different tasks are connected in friendship but are separated organisationally."
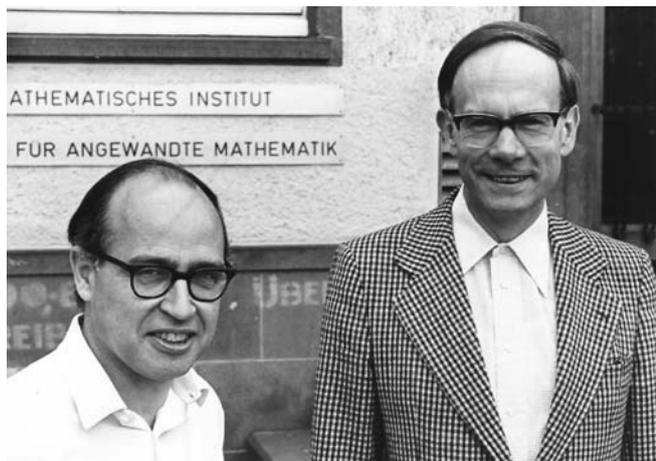
Michael Atiyah and Friedrich Hirzebruch in front of the Mathematical Institute in Bonn, 1977.

Gert-Martin Greuel, Director of the Mathematisches Forschungsinstitut Oberwolfach, Germany

Friedrich Hirzebruch was 18 years old in December 1945 when he started his study at Münster University. Reminiscing about this time in 2009 he wrote:

"Wenn ich damals einen kurzen Lebenslauf abgeben musste, dann enthielt er immer den Satz: 'Von Mitte Januar 1945 bis zum 1. Juli 1945 durchlief ich Arbeitsdienst, Militär und Kriegsgefangenschaft.'"

"In those days, whenever I had to supply a short CV, it always contained the sentence: 'From mid-January 1945 till 1 July 1945, I served fatigue duty, military duty and was detained as a prisoner of war.'"

This statement puts the double distance between the present day and the painful youth of the war years, defies any attempt to express this pain more eloquently and does so by silence.

Settling in Bonn in 1956, Hirzebruch put great efforts into the re-creation of the European mathematical community, destroyed, along with so many other institutions and lives, by the war. The brilliant idea of the annual *Arbeitstagungen* and, later, the founding of the Max-Planck Institute for Mathematics (MPIM) bore rich fruit. Hirzebruch struggled for the new Europe, as did Henri Cartan in France, using all the influence at his disposal as an internationally renowned researcher.

My first close contact with Fritz and Inge Hirzebruch came in 1967. I spent six weeks at the Institut des Hautes Études in Bures-sur-Yvette, where Grothendieck taught me the fresh from the oven project of motivic cohomology. After that I got permission and a German entry visa, which enabled me to visit Bonn and to participate in the *Arbeitstagung* on my way back to Moscow.

The blissful stress of study with Grothendieck and of Paris magic did something to my body but in Bonn Inge and Fritz treated me as their son and helped my healing, and their kindness and generosity forever remained in my memory.

The last two years of the 1960s put an end to these budding direct contacts between mathematicians of Western Europe and their colleagues in the Soviet Union and Eastern Europe. The next generation, coming after Hirzebruch's and then mine, was different. As one of the then young men recalled recently: "We thought it highly likely we would be blown off the planet and that, somehow, it was up to us – children after all – to prevent it."

I had not the least premonition that this epoch would pass as well during my life and that, almost a quarter of a century afterwards, I would meet Fritz again and become a colleague of his at the MPIM. And after 1990 and the fall of the Berlin Wall, Friedrich Hirzebruch helped immensely many mathematicians from East Germany to find jobs and continue their scientific lives in a new environment. Somebody better informed than me should record his efforts and describe his human care.

Mathematics is a *travail de longue haleine*. Leonard Euler (born in Basel and working in St Petersburg), inspired perhaps by the seven bridges of Königsberg (mostly destroyed by bombings in 1944 and 1945), discovered the notion of the *Euler characteristic* of a graph. This notion evolved for two centuries and by the time Friedrich Hirzebruch was maturing as a mathematician, was re-incarnated as an alternating sum of dimensions of cohomology groups of (invertible) sheaves on an algebraic manifold. The celebrated Riemann–Roch–Hirzebruch formula (1954) expressed this number through geometric invariants of the base, crucially using Todd's genus; its discoverer A. J. Todd was born in Liverpool. At the first

Arbeitstagung in 1957, Alexander Grothendieck, son of a Russian anarchist and eternal expatriate in France and everywhere, presented its great generalisation.

Perhaps the Riemann–Roch–Hirzebruch–Grothendieck theorem, which fused and crowned efforts of dozens of great creators from all corners of Europe, deserves to be put on the flag of the United Europe more than any other symbol.

Yuri I. Manin
Max Planck Institut Bonn, Germany

Stanislaw Janeczko, Director of the Banach Centre, Poland, participated in the memorial session giving a presentation in which he focused his speech on the influence of Friedrich Hirzebruch on the Banach Centre; a specific article on this topic will appear soon.

Mathematics in the Streets of Kraków

Ehrhard Behrends (Freie Universität Berlin, Chair of the EMS rpa committee)

The 6th European Congress of Mathematics took place in Kraków, 2–7 July 2012. As a special activity associated with this event the EMS rpa committee organised "Mathematics in the Streets of Kraków".
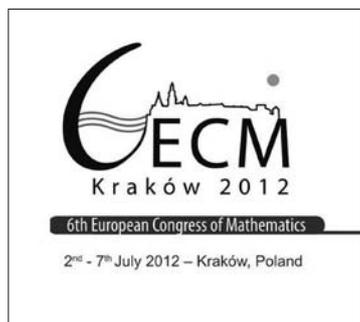

The idea was to increase the profile of mathematics during these days in the city with as many people as possible (inhabitants and tourists) made aware of the fact that an important mathematical congress was taking place in the first few days of July.

The rpa committee decided at its meeting in Bilbao (November 2011) to realise this idea by performing "maths busking", going into the street and interacting with the people directly! Some members of the committee (Franka Brueckler, Croatia, Steve Humble and Sara Santos, UK) have experience with this kind of public awareness activity, and one "only" would have to run a Polish version.

The preparations started early. Krzysztof Ciesielski from Kraków (thank you, Krzysztof!) negotiated with the local authorities and finally we had permission to be active in the central marketplace and in a small place close to the university. He also took care of the numerous preparations: printing of the papers that would be used during the busking project, finding a team of students to help us, production of the special T-shirts, etc.

These T-shirts not only announced the busking activities but also advertised www.mathematics-in-europe.eu, the public awareness webpage of the EMS. (On www.mathematics-in-europe.eu/krakow the English and Polish versions of our presentations were – and still are – available.)
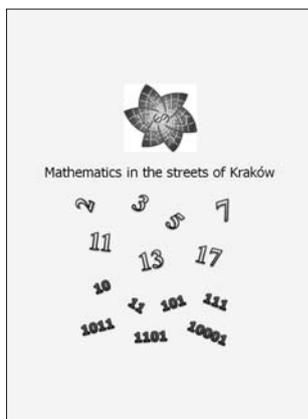
Franka, Sara and Steve had a meeting with the Polish students on Sunday and Monday to prepare the busking activities. These took place on Monday and Tuesday afternoon between 4 and 8 pm. Unfortunately conditions were not optimal: immediately after we started it was necessary to convince the police that we had all necessary permissions, many other (non-mathematical) performances were competing with us and it was very, very hot (about 35 degrees!). Nevertheless, several hundred people participated in our "mathematics in the streets of Kraków".

The police and some of the competitors.

The "buskers" had prepared a large variety of interactive presentations. They performed magical tricks with a mathematical background, provided mathematical riddles with surprising solutions, etc. The Polish students were very helpful in assisting the project, in particular as translators.

The busking team (with Krzysztof in the first row on the right)

Franka presenting a card trick with a mathematical background

Sara prepares a knot trick.

Steve with a performance based on the Kruskal count.

background can be found at www.mathematics-in-europe.eu, "Enjoy maths/recreational mathematics".)

Steve asks a visitor to shuffle a deck of (rather large) cards. Then they are put on the ground face up to form a square pattern. Cards with a picture and the aces count as one; the values of the others are the numbers printed on them. One spectator determines where to start and whether to walk left or right in a snaking pattern down through the grid. The rule of the game: if you are on a card with value i then step i steps forward in the direction determined at the beginning. A number of people start their walk at different starting positions and – big surprise! – all end up on the same card.

"Mathematics in the streets of Kraków" was a very interesting experience and we are sure that many people have seen aspects of mathematics they had never met before.

Ehrhard Behrends is a professor of mathematics at Freie Universität Berlin, working in functional analysis and probability. He is author of several monographs, textbooks and books for the general public. He is also Chair of the RPA Committee of the EMS.

As an example we describe here Steve's variant of the *Kruskal count*, which is explained in a box below. (You are invited to perfom this trick at your next summer party. More examples of magical tricks with a mathematical

---

The Kruskal count

Suppose that you produce n random numbers from the set $\{1,\dots,k\}$, where every i is generated with a positive probability and n is much larger than k. E.g. in the case $k = 6$ one could use an ordinary die; the result could be 3,1,4,5,3,2,2,1,6,5,2,1,3,4,6,1,3,2,4,3,1,1,4,5,2,6.

Such a sequence gives rise to walks: 1. Start at any of the numbers. 2. Proceed by the following rule. If the present number equals i then move i steps to the right (if this is possible). For example, if we use the preceding numbers and start at the second place (the 1) then our steps touch 1,4,2,6,6,1,1,4,4 and here we must stop since only three numbers remain.

There is a very surprising fact in connection with these walks: With an overwhelming probability all end up at the same number, provided that one starts at one of the first numbers. Check it with our example; when starting at 3,4,5,3,2,2,1,6,… the final position is the same 4 as in the case of the first walk.

The explanation is not difficult. Start a walk at the first position and mark the numbers that are visited. Then start at another number. That the final position will be the same (with high probability) is obvious if one combines the following two facts:

- Whenever the new walk touches a number that is already marked the final position will be the same as that of the first walk.
- Suppose that the walk is at a non-marked position. At least one of the next six numbers to the right is marked so that the probability to arrive *not* at a marked one in the next step is at most 5/6. (Here it is of importance that the numbers are generated as an i.i.d. sequence.) Thus, if the total walk has r steps, one touches a marked number with probability at least $1-(5/6)^r$. And since we assumed that n is large when compared with k the number $1-(5/6)^r$ will be close to one.

It should be clear how to generalise this argument to the case of arbitrary k and arbitrary probability distributions on $\{1,\dots,k\}$. Note that in Steve's case the values of the cards are not an i.i.d. sequence so that one has to argue slightly more carefully.

Henri Poincaré's Centenary

Cédric Villani (Institut Henri Poincaré, Paris, France)

A few days ago, we were celebrating the centenary of the death of Henri Poincaré (19 April 1854–17 July 1912). Within the next few months, several institutions, including the Henri Poincaré Institute, will pay tribute to this extraordinary scientist, who was not only an exceptional mathematician but also a physicist, an astronomer, an engineer and an accomplished philosopher, a professor in Sorbonne and a member of both the French Academy of Sciences and the "Académie française". Henri Poincaré personified an ideal of scientific unity; he is said to be the last mathematician who mastered and influenced all branches of his field. Through his immense contribution of more than 500 articles, he has set the basis of whole portions of science, from dynamical systems to automorphic forms to topology.

Let us evoke just two dates in this long career. In 1887, a young Poincaré revolutionised classical mechanics by his study of the three-body problem, a toy model for the solar system in which planets and sun are in interaction. In 1904, a 50 year old Poincaré was no less revolutionary when he completed the founding article on differential topology, stating the famous Poincaré conjecture, which was eventually solved by Grigori Perelman in 2002.

Poincaré made mostly cautious interventions in the public debate but did not try to elude his responsibilities when he had to do so. One of the most important such occasions was his role as a representative of the scientific community to dismiss the absurd "mathematical proofs" set up against Alfred Dreyfus in the famous anti-Semitic affair which poisoned French debates at the turn of the 20th century. His three main works intended for the general public – among which the most well known may be "Science and Hypothesis" – achieved popular fame and had a profound influence on the philosophy of sciences in the 20th century.

For this celebration, the Poincaré100 committee, chaired by Cédric Villani (Professor at Lyon University and Director of the Institute Henri Poincaré), will organise in 2012 a series of events culminating in November with an international conference addressing the main themes which made Poincaré famous. The general audience will be invited to a full day dedicated to the life and work of Poincaré and a medal has been created for the occasion.

- *A Henri Poincaré Exhibition*, "From mathematician to philosopher". Built around 19 panels and profusely illustrated with rare documents (letters, official documents, pictures), this showcase reveals an underestimated image of Poincaré not matching the official portrait.

- A *series of lectures* will take place in Nancy (Poincaré's birthplace), each Thursday from 27 October till 15 November.
- 17 November: A *broad audience day* in the main Sorbonne amphitheatre. A pretext to bring to life for a general audience the history of mathematical and scientific discoveries.
- There will be at the end of the day a *preview* of Philippe Worms's movie "Henri Poincaré" (52 minutes – Produced by Life of High Production, Dominique Garing and France 3). A journey on the road with some enthusiastic "scouts" (Cédric Villani, Étienne Ghys, Tadashi Tokieda, Alberto Verjovsky, Thierry Dauxois, Nicolas Bergeron).
- The publishing of *Sagascience* dedicated to Henri Poincaré. A multimedia file from CNRS (Centre National de la Recherche Scientifique) / SAGASCIENCE available at http://www.cnrs.fr/cw/dossiers/dospoincare/.
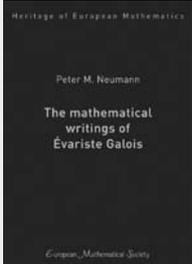
Besides these broad audience events, more specialized events will take place:
- A series of lectures in the École polytechnique.
- An international conference (17–23 November in IHP).
- A special session of the Poincaré seminar (24 November), etc.

More details on www.poincare.fr (in French).

Chair at the Institut Henri Poincaré

On 21 September 2011 the Clay Mathematics Institute (CMI, Cambridge, Massachusetts) and the Institut Henri Poincaré (IHP, Paris) announced at a press conference the establishing of the **Poincaré Chair**, a position for mathematicians of exceptional promise in the early stages of their careers.

Those nominated to the chair will hold their position at the Institut Henri Poincaré for a term of six months to one year. The chair is financed for a period of five years from the Clay Millennium Prize funds for resolution of the Poincaré conjecture.

The conjecture was proved by Grigori Perelman, an achievement for which he was awarded the Millennium Prize in 2010. Dr Perelman subsequently declined to accept the prize. In establishing this chair at IHP, CMI aims to provide an exceptional opportunity for mathematicians of great promise to develop their ideas and pursue their research, just as Grigori Perelman was afforded such an opportunity by a fellowship at the Miller Institute in 1993–95.

Every year, one or two prize winners of the Poincaré Chair will be announced after selection of applications by a special committee of experts recognised around the world. The prize winner will receive an attractive salary, as well as an office and logistic opportunities at IHP. They will not have to teach but we shall offer them the possibility of presenting their research or of other scientific activities. Travelling for the winner(s) within France or abroad will be possible as long as it can serve the project.

http://www.claymath.org/   http://www.ihp.fr/

CALL FOR APPLICATIONS

FSMP
Fondation Sciences Mathématiques de Paris

ACADEMIC YEAR
2013-2014

The Foundation Sciences Mathématiques de Paris is pleased to offer the following opportunities to outstanding scientists and excellent students:

⇨ **Research chairs of excellence**

⇨ **Junior chairs**

⇨ **Postdoctoral positions**

⇨ **Master course scholarships**

How to apply

Procedures of applications available at

www.sciencesmaths-paris.fr

FOUNDERS

CNRS · UPMC · PARIS DIDEROT · Ens

SCIENTIFIC PARTNERS

COLLÈGE DE FRANCE · DAUPHINE UNIVERSITÉ PARIS · PARIS 13 · UNIVERSITÉ PARIS DESCARTES · Inria

PARTNERS

AREVA · MAIRIE DE PARIS · casden · Crédit du Nord
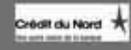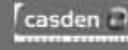
Mathematics at the ESOF2012 in Dublin

Ehrhard Behrends (Freie Universität Berlin, Germany) and Robin Wilson (Pembroke College, Oxford, UK)

The "European Science Open Forum 2012" (ESOF2012) took place in Dublin, Ireland, 11–15 July. It was the third ESOF congress; the first was held in Barcelona in 2008 and the second in Torino in 2010; the next one will take place in Copenhagen in 2014.

The primary aim of these congresses is to bring scientists of different fields together to exchange ideas and information. The second is to use the time of the congress for public awareness activities.

ESOF2012 attracted the attention of more than 5000 participants. The venue of the "scientific part" was the Dublin Convention Centre (DCC), a building with a spectacular design that opened in 2010 by the River Liffey. Some of the public awareness projects took place there whilst others were organised in the streets in the centre of Dublin.

The DCC

On each of the five days there was a rich programme. Among the keynote speakers were Sir Bob Geldof (musician and political activist), Enrico Giusti ("Mathematics at the Museum"), Rolf-Dieter Heuer (Director of CERN), Marcus du Sautoy ("The Secret Mathematicians"), Craig Venter (genomic research) and James Watson (one of the discoverers of the structure of DNA).

Several activities at ESOF2012 were concerned with mathematics. In the public awareness part Steve Humble, a member of the EMS Raising Public Awareness committee, realised a "maths busking" project[1] and some Irish colleagues showed their presentations in the Convention Centre (see www.mathsweek.ie for more details). Besides Giusti's and du Sautoy's talks the scientific part offered a panel discussion on medieval mathematics and a talk by the authors of this article.

The EMS nominated speakers for ESOF2008 and ESOF2010, and in February 2012 we were asked to present a talk for ESOF2012 on the connections between mathematics and music, a subject on which we have both given lectures for many years.

RPA activities (left); the authors (right)

Our presentation took place on Sunday 15 July, the last day of the congress. We were very happy to see that Mathematics you can hear attracted about 200 participants. Here is the summary: "Scales and temperament" (by RW: the mathematical foundation of our scales); "Sine waves – the atoms of sound" (by EB: one can hear the predictions of Fourier analysis); "Symmetry and Music" (by RW: how geometrical ideas were used in classical and contemporary compositions); "Can one hear the shape of a drum?" (by EB: on the Kac problem). At the beginning we gave a short description of the role of the EMS and the efforts towards raising public awareness (mentioning the website www.mathematics-in-europe.eu.

We had an electronic keyboard at our disposal that was used to illustrate the theoretical explanations, as well as many audio files and a film (with Bach's crab canon, a musical version of the Moebius strip).

After the numerous discussions after our lecture it is clear to us that many participants of the congress were very interested to learn more about the connections between mathematics and other fields. We recommend strongly that the EMS be present at the 2014 ESOF congress in Copenhagen.

---

[1] The verb 'to busk' means 'to perform in the streets'.

Heidelberg Laureate Forum
Abel, Fields, and Turing Laureates
Meet the Next Generation

**Winners of the prestigious Abel Prize,
Fields Medal, and Turing Award will meet
ambitious young scientists –
Established by the Klaus Tschira Stiftung**

The Klaus Tschira Stiftung will establish the "Heidelberg Laureate Forum" as an annual meeting bringing together winners of the most prestigious scientific awards in Mathematics (Abel Prize and Fields Medal) and Computer Science (Turing Award) with a select group of highly talented young researchers. The Forum has been initiated by the Heidelberg Institute for Theoretical Studies (HITS), the research institute of the Klaus Tschira Stiftung (KTS) – a German foundation, which promotes Natural Sciences, Mathematics, and Computer Science. The Heidelberg Laureate Forum is modeled after the annual Lindau Nobel Laureate Meetings, established more than 60 years ago to bring forward new ideas. Klaus Tschira, founder and managing partner of the foundation states: "Meeting with the scientific leaders of Mathematics and Computer Science will be extremely inspiring and encouraging for the young scientists."

The agreement on collaborating in the Heidelberg Laureate Forum between the organizers and the award-granting institutions (Norwegian Academy of Science and Letters, International Mathematical Union, and Association for Computing Machinery) was signed in Oslo on May 22nd on the occasion of the 10th Abel Prize Ceremony.

Research in all fields requires both mathematical methods and computational tools, and the results enter all aspects of our daily lives. Mathematics and Computer Science are indispensable foundations of our technological world. The Turing Award has long been recognized as the highest scientific award worldwide in the field of Computer Science, and the same holds for the Fields Medal and the Abel Prize in Mathematics. And yet: While young researchers in Physics, Chemistry, Medicine, and Economics have a chance to closely interact with the Nobel laureates of their fields in Lindau each year, no such opportunity has been existing for Mathematics and Computer Science – until now.

Starting in September 2013, the Heidelberg Laureate Forum will bring together winners of the Abel Prize, the Fields Medal, and the Turing Award with young scientists from all over the world. The Meeting is held in Heidelberg, where the Klaus Tschira Stiftung and the Heidelberg Institute for Theoretical Studies (HITS) are located. It is organized in collaboration with the

Association for Computing Machinery (ACM; Turing Award), the International Mathematical Union (IMU; Fields Medal), and the Norwegian Academy of Science and Letters (DNVA; Abel Prize). The first meeting of the Heidelberg Laureate Forum will take place during September 23–27, 2013.

Signing for the IMU, president Ingrid Daubechies expressed her delight at this initiative of the Klaus Tschira Stiftung, and her hope that this new forum will help foster enthusiasm for mathematics among the next generation. It is important for starting researchers to meet their "scientific heroes" and realize they are not unattainable personalities to keep on pedestals – mathematics, for all its abstractness, is a pursuit by living people, in which all the aspects of human communication and relations can play a role. We need to attract to and retain in mathematical research all the young bright minds that we can reach, eager to work enthusiastically to carry forward the whole living enterprise of mathematics.

The *Klaus Tschira Stiftung (KTS)* is a German foundation promoting the Natural Sciences, Mathematics, and Computer Science. The Heidelberg Institute for Theoretical Studies (HITS) is the research institute of the Klaus Tschira Stiftung. Further information: www.klaus-tschira-stiftung.de; www.h-its.org.

The *Norwegian Academy of Science and Letters (DNVA)* has annually been awarding the Abel Prize for outstanding scientific work in the field of mathematics since 2003. The prize amount is 6 million NOK (800,000 Euro). The Norwegian Academy of Science and Letters, founded in 1857, is a non-governmental, nation-wide, and interdisciplinary body which embraces all fields of learning. The Academy has 895 members, both Norwegian and foreign. Further information: http://english.dnva.no.

More than 70 countries are members of *IMU,* the *International Mathematical Union*. IMU promotes worldwide cooperation in mathematics, organizes the International Congress of Mathematicians (ICM), encourages and supports mathematical activities the world over contributing to the development of mathematical science in any of its aspects, pure, applied, or educational. Further information: www.mathunion.org/.

ACM, the *Association for Computing Machinery*, is the world's largest educational and scientific computing society, uniting computing educators, researchers, and pro-

fessionals to inspire dialogue, share resources, and address the field's challenges. Further information: www.acm.org/.

Download the press release and pictures of the Oslo agreement signing: www.klaus-tschira-stiftung.de/presse/download/2012/signing_event_HLF.jpg.

Press contacts:
Klaus Tschira Stiftung:
   Renate Ries, renate.ries@klaus-tschira-stiftung.de
The Norwegian Academy of Science and Letters:
   Anne-Marie Astad, anne.marie.astad@dnva.no
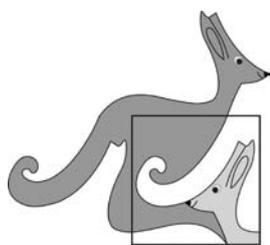International Mathematical Union:
   Martin Grötschel, secretary@mathunion.org
Association for Computing Machinery:
   Virginia Gold, vgold@acm.org

Twenty Years of Mathematical Kangaroo

Gregor Dolinar (University of Ljubljana, Slovenia)

What is the Mathematical Kangaroo?

Every year on the third Thursday in March a huge number of students (this year over six million) from all over the world take part in an important international mathematical event, a competition called the Mathematical Kangaroo.

On an international level there are plenty of mathematical competitions, the most prestigious being the International Mathematical Olympiad (IMO), which has the longest tradition among scientific Olympiads (this year the 53rd IMO was held in Argentina with 548 contestants from exactly 100 countries taking part). But the IMO is only for the six best high school students from each participating country, and these students solve six extremely difficult problems on two consecutive days, for four and a half hours each day. The IMO is very important from many points of view: it helps to find talented students in mathematics; it enables many students to develop proper mathematical thinking at an early age; it is a big challenge and motivation for the best; and it opens the doors of the world's most prestigious universities to the best contestants. However, it clearly influences only a small proportion of students.

The Mathematical Kangaroo is a very different competition from the IMO – in many ways they are exact opposites. It is more of a game than an uncompromising competition. In contrast to the IMO, students of all ages (from 7 to 18) take part, in six different age categories, solving 24 or 30 relatively easy multiple-choice questions in 90 minutes. But perhaps the most obvious difference is that the Kangaroo contest is not just for the best mathematically talented students. Instead it aims to attract as many students as possible, with the purpose of showing them that mathematics can be interesting, useful and even fun. Though, sadly, it has generally become accepted that mathematics is difficult, very abstract and not approachable by the vast majority of people, the number of contestants in the Mathematical Kangaroo proves that this need not be the case. With more than six million competitors in 2012, and with a very high proportion of the student population solving the problems (for example, in Slovenia more than three quarters of students aged 7–10), the Kangaroo contest helps to eradicate such prejudice towards mathematics.

History

At the end of the last century, many countries considered the idea of using mathematical competitions to popularise mathematics among a wide circle of students. In 1991, André Deledicq and Jean Pierre Boudine were inspired by the Australian mathematical competition to start a similar contest in France, which they named the Mathematical Kangaroo. The contest, consisting of mostly easy and attractive multiple-choice problems, was a great success. As a result, in 1993 a meeting was organised in Paris, at which it was proposed to several European countries that they should jointly organise a European Kangaroo contest. The idea was well received and in June 1994, at the European Council in Strasbourg, representatives from 10 European countries established the Association Kangourou Sans Frontières (AKSF). This association, which is responsible for organising the Kangaroo contest, was officially established and registered on 17 January 1995 in Paris, with André Deledicq as its first president.

Present and future

Every year since 1993, in October or November, representatives from all member countries gather at an annual meeting, at which the problems for the next year are chosen. After the meeting, representatives from each country translate the problems into their own language, adapt the questions (for example, changing the name John

Andrew Jobbings chairing the selection of Kangaroo problems for 13–15 years old students at the KSF meeting in Borovets, Bulgaria, 2005.

to Johann) and then use the selected problems in their own countries. The results of the students from different countries are not compared to each other; this would be against the spirit of the Kangaroo, which is intended to be an individual contest, not the basis for international comparisons. So the problems and rules of the contest are international but the contest in each country is organised independently and each country has its own winners. However, many countries organise joint summer camps for the students (for example, Poland, Germany, Romania) or even some additional joint competitions (for example, Austria, Germany and Switzerland). Countries also cooperate in many other fields, for example, publishing materials or buying prizes for the students or even working together on EU projects.

At the moment the AKSF has 52 member countries and this year at the annual meeting in Cyprus three new members (Ghana, Panama and Peru) will join. Since so many countries from all over the world organise the contest, a lot of freedom is given, though the same mathematical problems are used. More precisely, each country may organise the contest however they wish, provided they follow a few rules set down by the AKSF. For example, countries are allowed to organise the contest later than the third Thursday in March (for example, owing to school holidays) but never before that day. That is the reason why the Kangaroo problems may be published on the internet no earlier than one month after the official date of the contest. Also, owing to the very diverse curricula in different countries, each country is allowed to change some of the chosen problems or use fewer than the original number (for example, 24 problems instead of 30). However, the entry fee is another matter, being entirely within the control of each country.

Even though the contest is organised in such a decentralised way, there are many new challenges ahead for the AKSF, especially with more and more new countries wanting to join. One issue is the security of the problems, an issue which is made more difficult because participating countries come from many different continents with many different time zones but one which is growing in urgency as students become more proficient with modern communications technology.

Nevertheless, the Mathematical Kangaroo has managed to bound over many difficult barriers in the last twenty years and there is no doubt that it will be able, if necessary, to overcome more in the next twenty. In any case, the Kangaroo contest is certain to fulfil its primary role, that of popularising mathematics all over the world, especially among students who may not become mathematicians.

52 current members of the Association KSF:
Armenia, Austria, Belarus, Belgium, Brazil, Bulgaria, Canada, Catalonia-Spain, Colombia, Costa Rica, Côte d'Ivoire, Croatia, Cyprus, Czech Republic, Ecuador, Estonia, Finland, France, Georgia, Germany, Greece, Hungary, Indonesia, Iran, Italy, Kazakhstan, Kyrgyzstan, Lithuania, Macedonia, Mexico, Moldova, Mongolia, Netherlands, Norway, Pakistan, Paraguay, Poland, Portugal, Puerto Rico, Romania, Russia, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Tunisia, Ukraine, United Kingdom, United States, Venezuela.

Current board of the Association KSF:
Gregor Dolinar (Slovenia), Gregory Makrides (Cyprus), Andrew Jobbings (United Kingdom), Marta Berini (Catalonia-Spain), Jean-Phillipe Deledicq (France), Robert Geretschläger (Austria), Monika Noack (Germany).

Some recent and future annual meetings of the Association KSF:
Barcelona (2006), Graz (2007), Berlin (2008), Minsk (2009), Tbilisi (2010), Bled (2011), Protaras (2012), UK (2013), Puerto Rico (2014), Sweden (2015).

Numbers of contestants from 1995 till 2011 (see graph):
780,443; 991,201; 1,236,298; 1,315,969; 1,465,514; 1,788,280; 2,239,248; 2,565,451; 2,855,989; 3,186,493; 3,449,737; 3,933,935; 4,504,202; 5,106,709; 5,571,560; 5,840,684; 5,967,277.
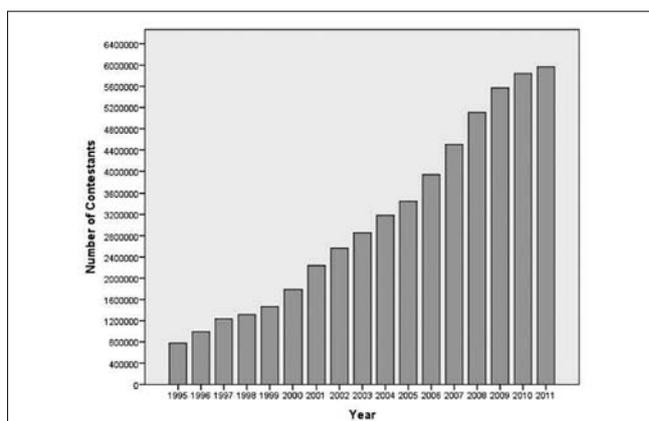
Some additional information about Mathematical Kangaroo can be found at http://www.math-ksf.org/index.php.

Gregor Dolinar is a professor of mathematics at the University of Ljubljana, Slovenia. He is the current president of the Association KSF. He is also the secretary of the IMO Advisory Board.

Marco Brunella 1964–2012

Alberto Verjovsky (UNAM, México)
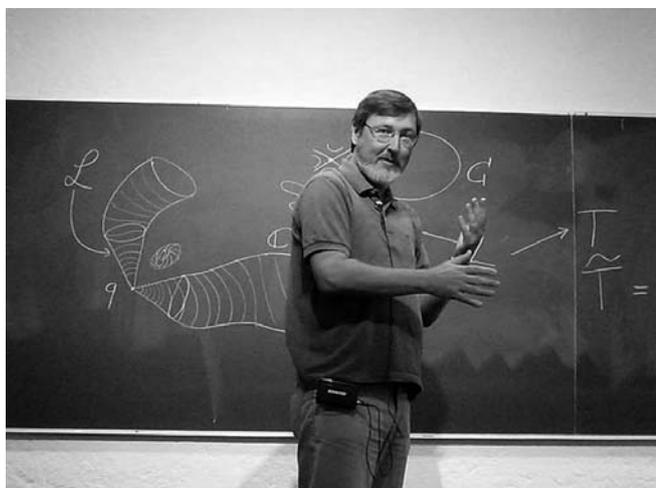
Marco delivering a beautiful lecture (Photo by L. Brambila)

Marco Brunella was born on 28 September 1964 in the city of Varese, Lombardy, Northern Italy, and died in Rio the Janeiro, Brazil, in January 2012. It would be difficult to do justice to the extent of Marco's contributions and qualities as a mathematician in this short note thus I have chosen to touch on the ones considered to be the most outstanding. His mathematics had the qualities of simplicity, elegance and beauty; he was a mathematician of great depth and originality, with a wide spectrum of knowledge. Marco Brunella was a sweet, shy, gentle soul of incredibly high moral standards and decency; he shunned all prizes and artificial recognitions but was vastly generous with his ideas and his influence on several mathematicians and groups of mathematicians around the world was enormous.

At the time of his death Marco was *chargé de recherches* of the CNRS with a position in the Institut de Mathématiques de Bourgogne in Dijon, France. This position started in 1995. Before that he was *ricercatore* at the Università di Bologna in Italy for the years 1992–1994. In 1992 he was awarded a PhD degree at the International School for Advanced Studies (SISSA) at Trieste, Italy. His thesis *Expansive Flows on Three-Manifolds* contains fundamental results on expansive flows and, in particular, it gives sufficient conditions for two expansive flows to be topologically equivalent. These conditions are in terms of Birkhoff's surfaces of sections, their corresponding Poincaré maps and the topology of the singular locus of the associated stable and unstable foliations.

I was extremely lucky and honoured that he asked me to be his thesis advisor. His first published paper [1] was written in 1988 for his *Laurea*, before his PhD, and deals with a generalisation of the Hartman-Grobman Theorem. In a second paper [2] written in collaboration with Massimo Miari, they give conditions for the topological equivalence of a plane vector field to its principal part (a notion depending on the Newton polyhedron introduced in the paper). They use very interesting blowing-up techniques which are suitable for such vector fields.

Marco initially studied physics, a subject which he always loved. Thus, even before he finished his PhD, he wrote three very good papers on symplectic geometry [6],[7],[8]. He was always fascinated by the beautiful geometric ideas involved in contact and symplectic geometry, especially those in the works of Arnold, Eliashberg and Gromov. In a series of four papers Marco continued the ideas and projects initiated in his thesis and made fundamental contributions to the classification of expansive and Anosov flows on 3-manifolds [9],[10],[11],[12]. These papers are standard references on those subjects. In this respect, let me mention that J. Franks and R. Williams presented examples of non-transitive Anosov flows on certain 3-manifolds. There exists transitive Anosov flows that are not topologically equivalent to the geodesic flow of a 3-manifold with a Riemannian metric of strictly negative curvature (or to finite covers of such flows), which makes the classification very difficult but Marco's results opened a road towards that goal.

Almost immediately after he obtained his PhD he wrote, in collaboration with Étienne Ghys, a beautiful result about umbilical foliations and transversally holomorphic codimension one foliations [3]. The class of foliations which are umbilical for a certain metric on a Riemannian manifold M coincides with the class of transversally holomorphic foliations. In this paper Étienne Ghys and Marco classify such foliations. Under a rationality condition Marco gave a complete classification of transversally holomorphic flows on 3-manifolds [4] and in [5] he proves a global stability theorem for transversely holomorphic foliations.

Perhaps the most striking contributions of Marco are those related to the theory of holomorphic foliations and complex geometry: transversely holomorphic flows, vector fields on the complex plane and on the complex projective plane, birational classification of foliations on complex surfaces, uniformisation of the leaves of a holomorphic foliation, subharmonic and plurisubharmonic variation of Poincaré's metric on leaves of a foliation, entire (non constant) maps tangent to a foliation among other themes. These contributions are interdisciplinary and of such depth and originality that they became standard references on all such subjects. He used complex analytic ideas and methods (pseudoconvexity, plurisubharmonic functions, harmonic measures and currents, for instance) but he also used the methods of algebraic geometry, topology and differential geometry.

A holomorphic (possibly singular) foliation on a complex manifold M is given by a subsheaf F of the tangent bundle TM which is closed under Lie bracket. In other words it satisfies the Frobenius integrability criterion. On a complex surface, the only interesting case is when F has rank one. A rank one subsheaf of the tangent bundle always determines a (possibly singular) foliation whose regular leaves are Riemann surfaces. For an algebraic surface the set of points where the rank-one subsheaf is locally-free is a Zariski open set $\mathcal{U}$ and the subsheaf defines on $\mathcal{U}$ a line bundle tangent to the man-

ifold and thus a foliation by Riemann surfaces on this open set. This holomorphic line bundle $T_{\mathcal{F}}$ is in fact the restriction of a holomorphic line bundle defined on all of M. The complement of $\mathcal{U}$ is the set of singularities of the foliation and for a complex surface one can assume without loss of generality that the set of singularities consists of a finite set of points. A foliation on a complex surface can be "lifted" to a foliation on a surface obtained by blow ups of the given surface. Therefore, after a finite sequence of blow-ups, one can assume by a theorem of Seidenberg that the singularities are reduced. A fundamental problem would be to classify such foliations. If $(M_1, \mathcal{F}_1)$ and $(M_2, \mathcal{F}_2)$ are two complex surfaces with corresponding holomorphic foliations $\mathcal{F}_1$ and $\mathcal{F}_2$ there is a natural notion of birational equivalence. For non-singular holomorphic foliations on compact complex surfaces Marco wrote the definitive paper on the subject [14]. This remarkable paper is very well-written and full of new ideas. He gives essentially a complete classification of non-singular holomorphic foliations first using the index theory of P. Baum and R. Bott and the corresponding formulae and then making extensive use of the Enriques-Kodaira classification of complex surfaces. He also uses the fact that "most" surfaces of non-general type admit (possibly singular) fibrations whose generic fibers are rational or elliptic curves. He then compares in a very clever way the given non-singular foliation with the singular foliation given by the fibration. For surfaces of general type Marco shows that a non-singular foliation admits a "singular" transverse distance. Then he proves that this metric can actually be chosen to be smooth and thus the foliation is a Riemannian foliation. These results have the following amazing consequences:

(i) The only non-minimal surface which admits a non-singular foliation is $P_{\mathbb{C}}^2$ blown-up at a point (a Hirzebruch surface) and this foliation is the blow-up of the radial foliation centered at this point.

(ii) If a surface X admits a regular foliation then its signature is non-negative, i.e., $\frac{1}{3}(c_1^2(X) - 2c^2(X)) \geq 0$. In particular, the only complete intersection which admits a non-singular foliation is the Klein quadric $P_{\mathbb{C}}^1 \times P_{\mathbb{C}}^1$.

The works of the great Italian geometers Enriques, Castelnuovo and Severi on the classification of surfaces played a very important role in the works of Marco. Also, the works of the great Italian complex analysts Andreotti and Vesentini had a strong influence on him. It is therefore natural that Marco studied the birational classification of holomorphic foliations on complex surfaces using algebraic geometry and complex analytic methods. He introduced a notion of minimal model and he classified those foliations which do not have such a minimal model in their birational class. He then gives an application to the dynamical study of polynomial diffeomorphisms of $\mathbb{C}^2$ [13]. Very roughly speaking, minimal models are pairs $(M, \mathcal{F})$ where M is a (possibly singular) projective surface and $\mathcal{F}$ is a rank one subsheaf of the tangent sheaf and the line bundle associated to the foliation $\mathcal{F}$ has non-negative degree on every curve.

In a seminal paper [15] M. McQuillan obtained a classification of foliated projective surfaces in the spirit of the Castelnuovo-Enriques-Severi and Kodaira classification of algebraic surfaces. This important paper relies on earlier fundamental work of Marco [18]. Marco considers a foliation

Marco Brunella (first on the left in the third row) during the workshop "Complex Analysis and Geometry" organised by the European Network in Cortona (Italy), 10–13 October 2000

$\mathcal{F}$ on an algebraic surface M having at most Hirzebruch-Jung type singularities (i.e., quotient singularities of the form $\mathbb{B}^2/\Gamma$, where $\mathbb{B}^2$ is the unit disc in $\mathbb{C}^2$ and Γ a finite subgroup of $SL(2, \mathbb{C})$). The model of the foliation in a neighbourhood of the singularities of the surface is required to be like the image on $\mathbb{B}^2/\Gamma$ of a regular foliation on a ball $\mathbb{B}$. Thus the leaves through the singularities of the surface are two dimensional orbifolds. Canonical examples of such foliations are the Hilbert modular foliations. For such a foliation in M he introduces a canonical (singular) leafwise metric which he calls the *Poincaré metric*. This is the metric obtained by the uniformisation theorem on the regular and orbifold leaves. He then studies the subharmonic variation of this metric. As an application Marco proves, amongst many other things, that the only *nef* foliations with reduced singularities and of Kodaira dimension -1 are the Hilbert modular foliations. In a very important paper [17] which is written as lecture notes but contains many new ideas and results of Marco himself he gives an introduction to M. McQuillan's theory but compliments this theory, extending the classification results of M. McQuillan and L. G. Mendes and Y. Miyaoka [15],[22],[24]. This paper is exceptionally beautifully written and it is a smooth ride through the topics of birational classification of surfaces and holomorphically foliated surfaces. It is indeed the best reference on the birational classification of holomorphic foliations. The main protagonist in order to understand a birational classification of foliated surfaces is the canonical bundle of the foliation, namely the dual $K_{\mathcal{F}} := T_{\mathcal{F}}^*$ of the line bundle $T_{\mathcal{F}}$ tangent to the leaves of the foliation. This is a genuine line bundle in all of M even if M has singularities. Again Marco defines a canonical Poincaré metric and proves positivity of the curvature on $K_{\mathcal{F}}$ as well as positivity in the transverse direction (he supposes that the foliation is *nef* which is the only interesting case since Y. Myaoka and M. McQuillan have shown that otherwise the foliation is a fibration with fibres $P_{\mathbb{C}}^1$). It is important to remark that one can define foliated versions of Kodaira dimension and numerical Kodaira dimension [24],[15],[16],[17]. Marco proved the invariance of the Kodaira dimension of a holomorphic foliation under

deformations [20]. After this paper he improves and extends his results in a series of beautiful papers [16],[17],[18],[19]. One of the themes is the study of non-constant entire curves $\phi : \mathbb{C} \to M$ which are tangent to a holomorphic singular foliation $\mathcal{F}$ of M. In this respect he improves on a theorem of M. McQuillan who obtained a striking proof of the Green-Griffiths conjecture using holomorphic foliations [27],[28].

Another important topic in which Marco made substantial contributions was the study of the simultaneous uniformisation of the leaves of a holomorphic foliation by Riemann surfaces. He wrote several excellent papers and lecture notes on this topic. He studied the regularity of the function $k(p)$ whose value at a point p in a nonsingular leaf of a holomorphic foliation by Riemann surfaces on a complex manifold is the value of the natural Gaussian curvature obtained by the uniformisation theorem of the leaf through p [21],[23],[25],[26].

I am, like so many others, deeply touched and saddened by his untimely death. Like all of his collaborators around the world, I will miss him and his constant influence on different fields and on so many people. I am, however, heartened by this recollection of some of his work, and by seeing how ample, deep and beautiful his contribution was in the time he was with us. *Tutta la nostra gratitudine, Marco.*

Bibliography

[1] Brunella, Marco. *A generalization of the Grobman-Hartman theorem for plane vector fields, through Newton polyhedra.* Nonlinear dynamics (Bologna, 1988), 320—321, World Sci. Publ., Teaneck, NJ, 1989.

[2] Brunella, Marco; Miari, Massimo *Topological equivalence of a plane vector field with its principal part defined through Newton polyhedra.* J. Differential Equations 85 (1990), no. 2, 338—366.

[3] Brunella, Marco; Ghys, Étienne. *Umbilical foliations and transversely holomorphic flows.* J. Differential Geom. 41 (1995), no. 1, 1—19.

[4] Brunella, Marco. *On transversely holomorphic flows.* I. Invent. Math. 126 (1996), no. 2, 265—279.

[5] Brunella, Marco. *A global stability theorem for transversely holomorphic foliations.* Ann. Global Anal. Geom. 15 (1997), no. 2, 179—186.

[6] Brunella, Marco. *Sur les familles génératrices des sous-variétés legendriennes.* C. R. Acad. Sci. Paris Sér. I Math. 314 (1992), no. 13, 1049—1052.

[7] Brunella, Marco. *Decomposition of Lagrangian intersections under symplectic isotopy.* Boll. Un. Mat. Ital. A (7) 5 (1991), no. 3, 353—361.

[8] Brunella, Marco. *On a theorem of Sikorav.* Enseign. Math. (2) 37 (1991).

[9] Brunella, Marco. *Separating the basic sets of a nontransitive Anosov flow.* Bull. London Math. Soc. 25 (1993), no. 5, 487—490.

[10] Brunella, Marco. *Expansive flows on Seifert manifolds and on torus bundles.* Bol. Soc. Brasil. Mat. (N.S.) 24 (1993), no. 1, 89—104.

[11] Brunella, Marco. *On the topological equivalence between Anosov flows on three-manifolds.* Comment. Math. Helv. 67 (1992), no. 3,

[12] Brunella, Marco. *Surfaces of section for expansive flows on three-manifolds.* J. Math. Soc. Japan 47 (1995), no. 3, 491—501.

[13] Brunella, Marco. *Minimal models of foliated algebraic surfaces.* Bull. Soc. math. France, 127, 1999, p. 289—305.

[14] Brunella, Marco. *Feuilletages holomorphes sur les surfaces complexes compactes.* Ann. Sci. École Norm. Sup. (4) 30 (1997), no. 5, 569–594.

[15] McQuillan, Michael. *An introduction to non-commutative Mori theory.* European Congress of Mathematics, Vol. II (Barcelona, 2000), 47–53, Progr. Math., 202, Birkhäuser, Basel, 2001.

[16] Brunella, Marco. *Foliations on complex projective surfaces.* Dynamical systems. Part II, 49–77, Pubbl. Cent. Ric. Mat. Ennio Giorgi, Scuola Norm. Sup., Pisa, 2003.

[17] Brunella Marco. *Birational geometry of foliations.* First Latin American Congress of Mathematicians, IMPA (2000).

[18] Brunella Marco. *A subharmonic variation of the leafwise Poincaré metric.* Invent. Math. 152 (2003), 119–148.

[19] Brunella, Marco. *On the plurisubharmonicity of the leafwise Poincaré metric on projective manifolds.* J. Math. Kyoto Univ. 45 (2005), 381–390.

[20] Brunella, Marco. *Invariance par déformations de la dimension de Kodaira d'un feuilletage sur une surface.* (French) Essays on geometry and related topics, Vol. 1, 2, 113–132, Monogr. Enseign. Math., 38, Enseignement Math., Geneva, 2001.

[21] Brunella, Marco. *Foliations on the complex projective plane with many parabolic leaves.* Ann. Inst. Fourier (Grenoble) 44 (1994), no. 4, 1237–1242.

[22] Miyaoka, Yoichi. *Deformations of a morphism along a foliation and applications.* Algebraic geometry, Bowdoin, 1985 (Brunswick, Maine, 1985), 245–268, Proc. Sympos. Pure Math., 46, Part 1, Amer. Math. Soc., Providence, RI, 1987.

[23] Brunella, Marco. *Nonuniformisable foliations on compact complex surfaces.* Mosc. Math. J. 9 (2009), no. 4, 729–748, 934.

[24] Mendes, L. Gustavo. *Kodaira dimension of holomorphic singular foliations.* Bol. Soc. Brasil. Mat. (N.S.) 31 (2000), no. 2, 127–143.

[25] Brunella, Marco. *Uniformisation of foliations by curves.* Holomorphic dynamical systems, 105–163, Lecture Notes in Math., 1998, Springer, Berlin, 2010.

[26] Brunella, Marco. *Uniformisation de feuilletages et feuilles entières.* To appear in Panoramas & Synthèses, SMF

[27] Brunella, Marco. *Courbes entières dans les surfaces algébriques complexes (d'après McQuillan, Demailly-El Goul, et al.)* Séminaire Bourbaki, Vol. 2000/2001. Astérisque No. 282 (2002), Exp. No. 881, vii, 39–61.

[28] Brunella, Marco. *Courbes entières et feuilletages holomorphes.* (French) [Entire curves and holomorphic foliations] Enseign. Math. (2) 45 (1999), no. 1–2, 195–216.

Some Reflections on Alan Turing's Centenary

P. D. Welch (University of Bristol)

We review two of Alan Turing's chief publications in mathematical logic: the classic 1936 paper On Computable Numbers [9] *and the less well known paper* Systems of Logic based on Ordinals [10]. *Whilst the former has rightly received enormous attention the latter is really only known amongst logicians. We outline some of the history and background to the first, whilst emphasising a viewpoint often forgotten in discussions of the so-called 'Church-Turing thesis'; we sketch the development of the second paper and see why its results were equivocal and perhaps somewhat disappointing to Turing.*

Early Life

Alan Mathison Turing was born on 23 June 1912 in London to parents of whom his biographer Andrew Hodges [7] aptly conjectures the English novelist George Orwell would have described as "lower upper middle class", his father holding a position in the Indian Civil Service (ICS). This meant that Turing, like many boys of this time and status, would be educated in England either living with relatives or at boarding school. His father eventually retired from the ICS at a relatively senior position in the Presidency of Madras but then for tax reasons continued to live in France.

Turing was thus sent to Sherborne School from the age of 13, which, whilst not Eton or Harrow, would have provided the required respectable education. He seems to have shown early interest in all matters mechanical, chemical and biological and this persisted throughout his life. He showed strong promise in mathematics and a strong ease and facility but without any Gauss-like precocity. His mathematical abilities won him a Scholarship to King's College, Cambridge, which he entered in the Autumn of 1931.

Alan Mathison Turing (1912 London – 1953 Manchester, England)

The intellectual atmosphere in Cambridge at that time, at least in the areas of interest to Turing, would have been dominated by G. H. Hardy and A. Eddington. Of his own peer group he became friends with the future economist David Champernowne. At Sherborne he had read Eddington's "*Nature of the physical world*" and at Cambridge Hardy and also von Neumann's "*Mathematische Grundlagen der Quantum Mechanik*".

He attended Eddington's lectures entitled *"The distribution of measurements in Scientific Experiments"* and this must have engaged him as he found for himself a mathematical problem to work on, leading him to rediscover and prove the Central Limit Theorem in February 1934. It seems to have been typical of him to work things out for himself from first principles and he was thus quite unaware that this had already been proven in a similar form by Lindeberg in 1922.

Notwithstanding this his tutor, the group theorist Philip Hall, encouraged him to write up this work as a Fellowship Dissertation for the King's College competition in 1935, which was done, being entitled *On the Gaussian Error Function*. This was accepted 16 March 1936, Hall arguing that the rediscovery of a known theorem was a significant enough sign of Turing's strength (which he argued had not yet achieved its full potential). Turing thus won a three-year fellowship, renewable for another three, with £300 *per annum* with room and board. He was 22 years old.

His first published work was in group theory and was finished in March 1935,[1] this being a contribution to the theory of almost periodic functions, improving a result of von Neumann. By coincidence von Neumann arrived the very next month in Cambridge and proceeded to lecture on this subject, and they must have become acquainted from this time.

Probably more decisive than meeting von Neumann was his contact with Max Newman. In Spring 1935 he went on a Part III course of Newman's on the Foundations of Mathematics. (Part III courses at Cambridge were, and are, of a level beyond the usual undergraduate curriculum but preparatory to undertaking a research career.) Newman was a topologist and interested in the theory of sets. Newman attended Hilbert's lecture at the 1928 International Congress of Mathematicians. Logic at this time had disappeared at Cambridge: Russell was no longer present, having left in 1916, and Frank Ramsey had died in 1931. Wittgenstein had moved on from his logical atomistic days and the concerns of the Tractatus to other things (although Turing did attend a Wittgenstein seminar series and conversed with him). Hence Newman was more influenced by Hilbert and Göttingen.

Hilbert had worked on foundational matters for the previous decades and would continue to do so. His aim to obtain a secure foundation for mathematics by finding proofs of

consistency of large parts (if not all) of mathematics by a process of systematic axiomatisation, and then showing that these axiomatisations were safe by providing finite consistency proofs, looked both reasonable and possible. By systematic effort Hilbert and his school had reduced the questions of the consistency of geometry to analysis. There seemed reasonable hope that genuinely finitary methods of proof could render arithmetic provably consistent within finite arithmetical means.

The address that Hilbert gave at the 1928 Congress (when Germany had been re-admitted to the International Congress of Mathematicians after being denied this in 1924) not only gave a plea for the internationalist, apolitical nature of mathematical research but also formulated several important questions for this foundational project.

Hilbert's programme and the *Entscheidungsproblem*

– (I. Completeness) His dictum, concerning the belief (engraved as the famous *non ignorabimus* on his gravestone) that any mathematical problem was in principle solvable, can be restated as the belief that mathematics was *complete*. That is, given any properly formulated mathematical proposition *P*, either a proof of *P* could be found or a disproof.
– (II. Consistency) The question of *consistency* – given a set of axioms for, say, arithmetic, such as the Dedekind-Peano axioms, PA, could it be shown that no proof of a contradiction can possibly arise? Hilbert stringently wanted a proof of consistency that was finitary, that made no appeal to infinite objects or methods.
– (III. Decidability – the *Entscheidungsproblem*) Could there be a finitary process or algorithm that would *decide* for any properly formulated proposition *P* whether it was derivable from axioms or not?

Of course the main interest was consistency but there was hope (discernible from some of the writings of the Göttingen group) that there was such a process and therefore a positive solution to the *Entscheidungsproblem*. From others came expressions that it was not:

Hardy:
"There is of course no such theorem and this is very fortunate, since if there were we should have a mechanical set of rules for the solution of all mathematical problems, and our activities as mathematicians would come to an end." [6]

von Neumann:
"When undecidability fails, then mathematics as it is understood today ceases to exist; in its place there would be an absolutely mechanical prescription with whose help one could decide whether any given sentence is provable or not." [12]

Gödel's Incompleteness Theorems block Hilbert's programme

Theorem 1. (Gödel-Rosser First Incompleteness Theorem – 1931) *For any theory T containing a moderate amount of arithmetical strength, with T having an effectively given list of axioms, then:*

if T is consistent then it is incomplete, that is, for some proposition neither T ⊢ P nor T ⊢ ¬P.

The theorem is, deliberately, written out in a semi-modern form. Here, it suffices that *T* contain the Dedekind-Peano axioms, PA, to qualify as having a 'moderate amount of arithmetical strength'. The axioms of PA can be written out as an 'effectively given' list, since although the axioms of PA include an infinite list of instances of the Induction Axiom, we may write out an effective prescription for listing them. Hence PA satisfies the theorem's hypothesis. Gödel had used a version of the system of *Principia Mathematica* of Russell and Whitehead but was explicit in saying that the theorem had a wide applicability to any sufficiently strong "formal system" (although without being able to specify completely what that meant).

This immediately established that PA is incomplete, as is *any* theory containing the arithmetic of PA. This destroys any hope for the full resolution of Hilbert's programme that he had hoped for.

However in a few months there was more to come:

Theorem 2. (Gödel's Second Incompleteness Theorem – 1931) *For any consistent T as above, containing the axioms of* PA*, the statement that 'T is consistent' (when formalised as* 'Con$_T$'*) is an example of such an unprovable sentence.*
Symbolically:

$$T \nvdash \text{Con}_T$$

The first theorem thus demonstrated the incompleteness of any such formal system, and the second the impossibility of demonstrating the consistency of the system by the means of formal proofs within that system. The first two of Hilbert's questions were thus negatively answered. What was left open by this was the *Entscheidungsproblem*. That there might be some effective or finitary process is not ruled out by the Incompleteness Theorems. But what could such a process be like? How could one *prove* something about a putative system that was not precisely described, and certainly not *mathematically* formulated?

Church and the λ-calculus

One attempt at resolving this final issue was the system of functional equations called the "λ-calculus" of Alonzo Church. He had obtained his thesis in 1927 and, after visiting Amsterdam and Göttingen, was appointed an assistant professor in Princeton in 1931. The λ-calculus gave a strict, but rather forbidding, formalism for writing out terms defining a class of functions from base functions and a generalised recursion or induction scheme. Church had only established that the simple number successor function was "λ-definable" when his future PhD student Stephen Cole Kleene arrived in 1931; by 1934 Kleene had shown that all the usual number theoretic functions were also λ-definable. They used the term "effectively calculable" for the class of functions that could be computed in the informal sense of effective procedure or algorithm alluded to above.

Church ventured that the notion of λ-definability should be taken to coincide with "effectively calculable".

Church's Thesis (1934 – first version, unpublished) *The effectively calculable functions coincide with the λ-definable functions.*

At first Kleene tried to refute this by a diagonalisation argument along the lines of Cantor's proof of the uncountability of the real numbers. He failed in this but instead produced a theorem: the *Recursion Theorem*. Gödel's view of the suggestion contained in the thesis when Church presented it to him was that it was "thoroughly unsatisfactory".

Gödel meanwhile had formulated an expanded notion of primitive recursive functions that he had used in his Incompleteness papers; these became known as the *Herbrand-Gödel general recursive functions*. He lectured on these in 1934 whilst visiting the IAS, Princeton.

Church and Kleene were in the audience and seem to have decided to switch horses. Kleene:

"I myself, perhaps unduly influenced by rather chilly receptions from audiences around 1933–35 to disquisitions on λ-definability, chose, after [Herbrand-Gödel] general recursiveness had appeared, to put my work in that format . . . "

Preliminary solutions to the *Entscheidungsproblem*

By 1935 Church could show that there was no λ-formula "A conv B" iff the λ-terms A and B were convertible to each other within the λ-calculus. Moreover, mostly by the work of Kleene, they could show the λ-definable functions were co-extensive with the general recursive functions. Putting this "non-λ-definable-conversion" property together with this last fact, there was therefore a problem which, when coded in number theory, could not be solved using general recursive functions. This was published by Church [2]. Another thesis was formulated:

Church's Thesis (1936 – second version) *The effectively calculable functions coincide with the [H-G] general recursive functions.*

Gödel still indicated at the time that the issue was unresolved and that he was unsure that the general recursive functions captured all informally calculable functions.

"On Computable Numbers"

Newman and Turing were unaware of these developments in Princeton. The first subject of Turing's classic paper is ostensibly 'Computable Numbers' and is said to be only "with an application to the *Entscheidungsproblem*". He starts by restricting his domain of interest to the natural numbers, although he says it is almost as easy to deal with computable functions of computable real numbers (but he will deal with integers as being the 'least cumbrous'). He briefly initiates the discussion calling computable numbers those 'calculable by finite means.'

In the first section he compares a man computing a real number to a machine with a finite number of states or 'm-configurations' $q_1, \ldots, q_R$. The machine is supplied with a 'tape' divided into cells capable of containing a single symbol from a finite alphabet. The machine is regarded as scanning, and being aware of, only the single symbol in the cell being

King's College Rowing Team 1935 (2nd from the left, rear row) after his election to a Fellowship

viewed at any moment in time. The possible behaviour of the machine is determined only by the current state q_n and the current scanned symbol S_r which make up the current configuration of the machine. The machine may operate on the scanned square by erasing the scanned symbol or writing a symbol. It may move one square along the tape to the left or to the right. It may also change its m-configuration.

He says that some of the symbols written will represent the decimal expansion of the real number being computed, and others (subject to erasure) will be for scratch work. He thus envisages the machine continuously producing output, rather than halting at some stage. It is his contention that "these operations include all those which are used in the computation of a number". His intentions are often confused with statements such as 'Turing viewed any machine calculation as reducible to one on a Turing machine' or some thesis of this form. Or that he had 'distilled the essence of machine computability down to that of a Turing machine'. He explicitly warns us that no "real justification will be given for these definitions until Section 9".

In Section 2 he goes on to develop a theory of his machines giving and discussing some definitions. He also states:

"If at each stage the motion of the machine is *completely* determined by the configuration, we shall call the machine an 'automatic' or *a*-machine."

"For some purposes we may use machines whose motion is only partly determined. When such a machine reaches one of these ambiguous configurations, it cannot go on until some arbitrary choice has been made . . . "

Having thus in two sentences prefigured the notion of what we now call a *non-deterministic Turing machine* he says that he will stick in the current paper only to *a*-machines, and will drop the '*a*'. He remarks that such a non-deterministic machine 'could be used to deal with axiomatic systems'. (He is probably thinking here of the choices that need to be made when developing a proof line-by-line in a formal system.) The succeeding sections develop the theory of the machines. The theory of a *"universal machine"* is explicitly described, as

is in particular the conception of program as input or stored data and the mathematical argument using Cantor's diagonalisation technique, to show the impossibility of determining by a machine, whether a machine program was 'circular' or not. (Thus, as he does not consider a complete computation as a halted one, he instead considers first the problem of whether one can determine a looping behaviour.)

Section 9 "The extent of the computable numbers" is in some ways the heart of the paper, in particular for later discussions of the so-called 'Turing' or 'Church-Turing' theses. It is possibly of a unique character for a paper in a purely mathematical journal of that date (although perhaps reminiscent of Cantor's discussions on the nature of infinite sets in *Mathematische Annalen*). He admits that any argument that any calculable number (by a human) is "computable" (i.e., in his machine sense) is bound to hang on intuition and so be mathematically somewhat unsatisfactory. He argues that the basis of the machine's construction earlier in the paper is grounded on an analysis of what a human computer does when calculating. This is done by appealing to the obvious finiteness conditions of human capabilities: the possibilities of surveying the writing paper and observing symbols together with their writing and erasing.

It is important to see that this analysis should be taken *prior to* the machine's description. (Indeed one can imagine the paper re-ordered with this section placed at the start.) He had asked:

> "*What are the possible* processes *which can be carried out in computing a real number.*" [Author's emphasis]

It is as if the difference between the Princeton approach and Turing's is that the former appeared to be concentrating on discovering a definition whose extension covered in one blow the notion of effectively calculable, whereas Turing concentrated on process, the very act of calculating.

According to Gandy [5] Turing has in fact proved a *theorem* albeit one with unusual subject matter. What has been achieved is a complete analysis of human computation in terms of finiteness of the human acts of calculation broken down into discrete, simple and locally determined steps. Hence:

> **Turing's Thesis: Anything that is humanly calculable is computable by a Turing machine.**

(i)   Turing provides a philosophical paradigm when defining "effectively calculable", in that a vague intuitive notion has been given a unique meaning which can be stated with complete precision.

(ii) He also makes possible a completely precise understanding of what is a 'formal system' thereby making an exact statement of Gödel's results possible (see the quotation below). He claims to have a machine that will enumerate the theorems of predicate calculus. This also makes possible a correct formulation of Hilbert's 10th problem. It is important to note that Turing thus makes expressions along the lines of "such and such a proposition is undecidable" have mathematical content.

(iii) In the final four pages he gives his solution to the *Entscheidungsproblem*. He proves that there is no machine that will decide of any formula φ of the predicate calculus whether it is derivable or not.

He was 23. His mentor and teacher Max Newman was astonished and at first reacted with disbelief. He had achieved what the combined mental resources of Hilbert's Göttingen school and Princeton had not, and in the most straightforward, direct, even simple manner. He had attended Newman's Part III course on the Foundations of Mathematics in Spring 1935 and within 14 months had solved the last general open problem associated with Hilbert's programme.

However, this triumph was then tempered by the arrival of Church's preprint of [1] which came just after Turing's proof was read by Newman. The latter however convinced the London Mathematical Society that the two approaches were sufficiently different to warrant publication; this was done in November 1936, with an appendix demonstrating that the machine approach was co-extensional with the λ-definable functions, and with Church as referee.

Gödel again:[2]

> "When I first published my paper about undecidable propositions the result could not be pronounced in this generality, because for the notions of mechanical procedure and of formal system no mathematically satisfactory definition had been given at that time … The essential point is to define what a procedure is."

> "That this really is the correct definition of mechanical computability was established beyond any doubt by Turing."

Turing's "Ordinal logics"

In 1937 Turing went to Princeton but was somewhat dismayed to find only Church and Kleene there. He first asked von Neumann for a problem, and von Neumann passed on one from Ulam concerning the possibility of approximating continuous groups with finite ones which Turing soon answered negatively.

With this and some other work he published two papers on group theory (described in a letter to Philip Hall as 'small papers, just bits and pieces'; nevertheless they appeared in Compositio and Annals of Mathematics).

He stayed on in Princeton on a Procter Fellowship (of these there were three, one each for candidates from Cambridge, Oxford and the Collège de France). He decided to work towards a PhD under Church. He still had a King's Fellowship and thus a PhD would not have been of great use to him in the Cambridge of that day. He completed his thesis in two years (even whilst grumbling about Church's "suggestions which resulted in the thesis being expanded to appalling length" – it is 106 pages). The topic (probably suggested by Church) concerned trying to partially circumvent incompleteness of formal theories T by adding as axioms statements to the effect that the theory was consistent.

To illustrate the thesis problem with an example (where we may think of T_0 as PA again) set:

$$T_1 : T_0 + \mathrm{Con}(T_0)$$

where "$\mathrm{Con}(T_0)$" is some expression arising from the Incompleteness Theorems expressing that "T_0 is a consistent system"; as $\mathrm{Con}(T_0)$ is not provable from T_0, this is a deductively stronger theory; continuing:

$$T_{k+1} : T_k + \mathrm{Con}(T_k) \text{ for } k < \omega, \quad \text{and then:} \quad T_\omega = \bigcup_{k<\omega} T_k.$$

Presumably we may still continue:

$$T_{\omega+1} = T_\omega + \text{Con}(T_\omega) \text{ etc.}$$

We thus obtain a transfinite hierarchy of theories. As would occur to many people who have spent even a moderate amount of time pondering the Second Incompleteness Theorem, one could ask of this sequence of theories of increasing deductive strength, what can one in general prove from a theory in this sequence? (Indeed this is just one question one can see about the incompleteness results that bubble up from time to time on MathOverflow.)

Turing called these theories "Logics" and used the letter "L" but I shall use the modern convention. He was thus investigating the question as to what extent such a sequence could be 'complete':

Question: Can it be that for any problem A there might be an ordinal α so that T_α proves A or $\neg A$?

Actually he was aiming at a more restricted question, namely what he called *number theoretic problems* which are those that can be expressed in an '$\forall\exists$' form (the twin primes conjecture comes to mind). He does not clarify why he alights on this particular form of the question.

There are several items that must be discussed first, in order to give this sketch of a progression of theories even some modicum of precision. To formally write down in the language of PA a sentence that says "Con(PA)" one really needs a formula $\varphi_0(v_0)$ that defines for us the set of Gödel code numbers n of instances of the axiom set $T_0 = \text{PA}$. There are infinitely many such formulae but we choose one which is both simple (it is Σ_1, meaning definable using a single existential quantifier) and *canonical* in that it simply defines the axiom numbers in a straightforward manner. Assuming we have a φ_0, we then may set $\varphi_{k+1}(\bar{n}) \longleftrightarrow \varphi_k(\bar{n}) \vee \text{Con}(\varphi_k)$ where $\text{Con}(\varphi_k)$ expresses in a Gödelian fashion the consistency of the axiom set defined by φ_k.

But what to do at stage ω? How you choose a formula for a limit stage depends on how you approach that stage, but the problem even occurs for stage ω: how do you define a formula that uniformly depends on the previous stages so that you can express the "union" set of axioms correctly?

Notation and progressions

Turing solved this and devised a method for assigning sets of sentences, so theories, to all constructive (also called *recursive* or *computable*) ordinals by the means of *notations*. In essence a notation for an ordinal is merely some name for it but a system of notations (which Turing used) was invented by Kleene using the λ-calculus. Nowadays we also use the idea of being able to name the ordinal α by the natural number index e of a computable function $\{e\}$ which computed the characteristic function of a well-order of $\mathbb{N}$ of order type α.

This essentially yields a tree order with infinite branching at all and only constructive ordinal limit points.

The set of notations $O \subset \mathbb{N}$ thus forms a tree order, with $n <_O m \leftrightarrow |n| < |m|$, where $|\cdot|$ is the ordinal rank function (defined by transfinite recursion along $<_O$) satisfying:

$$|0| = 0; \quad |2^a| = |a| + 1; \quad |3^e| = \lim_{n\to\infty}|\{e\}(n)|.$$

However O is a *co-analytic* set of integers and is thus highly complex. Let $suc(a) =_{df} 2^a$ and let $lim(e) =_{df} 3^e$.

Definition 1. *A progression based on a theory T is a primitive recursive mapping $n \longrightarrow \varphi_n$ where φ_n is an $\exists$ formula such that* PA *proves:*
(i) $T_0 = T$;
(ii) $\forall n(T_{\text{suc}(n)} = T_n + \text{Con}(\varphi_n))$;
(iii) $T_{\lim(n)} = \bigcup_m T_{\{n\}(m)}$.

Thus one attaches *in a uniform manner* formulae φ_a to define theories T_a to *every* $a \in \mathbb{N}$ of the form $suc(a)$, $lim(a)$. This does not tell us how to build progressions, which however can be justified by the Recursion Theorem.

An *explicit consistency sequence* is then defined to be the restriction of a progression to a path through O.

With these tools Turing proved a form of an enhanced Completeness Theorem.

Theorem 3 (Turing's Completeness Theorem). *For any true $\forall$ sentence of arithmetic, ψ, there is a $b = b(\psi) \in O$ with $|b| = \omega + 1$, so that $T_b \vdash \psi$. The map $\psi \rightarrowtail b(\psi)$ is given by a primitive recursive function.*

Thus we may for any true ψ find a path through O of length $\omega + 1$,

$$T = T_0, T_1, \ldots, T_{\omega+1} = T_b$$

with the last proving ψ. At first glance this looks like magic: how does this work, and can we use it to discover more $\forall$-facts about the natural number system?

However, there is a trick here. As Turing readily admits, what one does is construct for *any* $\forall$ sentence ψ an extension $T_{b(\psi)}$ proving ψ with $|b(\psi)| = \omega + 1$. Then *if ψ is true* we deduce that $T_{b(\psi)}$ is a consistent extension in a proper consistency sequence (notice that conditional in the antecedent of the theorem's statement). However *if ψ is false* $T_{b(\psi)}$ turns out to be merely inconsistent, and so proves anything. In general it is harder to answer $?b \in O?$ than the original $\forall$ question and so we have gained no new arithmetical knowledge. The outcome of the investigation is thus somewhat equivocal: we *can* say that some progressions of theories will produce truths of arithmetic but we cannot determine which ones they will turn out to be.

He regarded the results as somewhat disappointing. He had only succeeded in proving a theorem for '$\forall$' problems and not for his chosen 'number theoretic problems'. He had, moreover, proven another theorem that stated that there would be $b, c \in O$, with, for example, $|b| = |c| = \omega + 1$, such that T_b and T_c would prove different families of sentences. Thus *invariance* would fail even for theories of the same "depth".

It does contain a remarkable aside however. Almost as a throw-away comment he introduces what has come to be called a *relativised Turing Machine* or (as he called it) an *oracle machine*. This machine is allowed an instruction state that permits it to query an 'oracle' (considered perhaps as an infinite bit-stream of information about the members of $B \subseteq \mathbb{N}$ written out on a separate tape) whether $?n \in B?$ An answer is received and computation continues. With this one can develop the idea of 'relative computability' – whether membership of m in set A can be determined from knowledge of finitely many membership questions about set B. This notion is central to modern computability theory. However, Turing
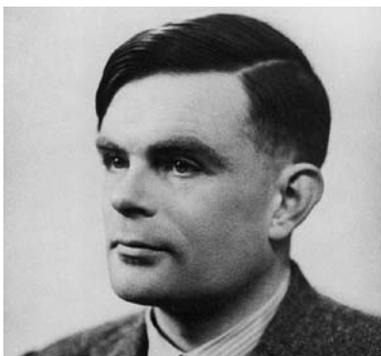
Royal Society Election portrait 1951

introduces the concept, (dubbing it an 'oracle' or *o*-machine) and uses it somewhat unnecessarily to prove the point that there are arithmetic problems that are not in his sense number theoretic problems. And then ignores it for the rest of the paper; it is unused in the sequel.

The paper, duly published in 1939, lay somewhat dormant until taken up by Spector and Feferman some 20 years later. Feferman did a far reaching analysis of the notion of general progressions, using not just formalised consistency statements as Turing had done but also other forms that, roughly speaking, ensured the preservation of truth. Note that a *general* consistency sequence step will not necessarily preserve truth of even say existential statements. However, a properly formulated 'existential soundness' statement – that existential sentences provable from the theory are true – when iterated or progressed in the above manner, can result in a '∀∃'-completeness statement of the Turing kind. Indeed, it can be shown that there are paths through O along which *all* true sentences of arithmetic are provable. However, finding a path through O is no simpler than determining whether a single b is in O, so again there is this equivocal feeling to the results. It is compounded by the fact that there are also paths through O, as Spector and Feferman found, which do *not* establish all truths of arithmetic.

The photograph shows Turing on his election as Fellow of the Royal Society in London in 1951 with a citation for "On Computable Numbers"; he was not the youngest at an age of 38 (Hodges notes that Hardy had been elected at 33 and Ramanujan at 30). Three further small articles appeared on the lambda-calculus, but otherwise Turing published nothing further on mathematical logic.

This article does not aim to discuss his contributions to the wartime decoding effort, the development of actual computers or to morphogenesis but in all these areas he displayed an open mind to ideas no matter whence they came and a startlingly fresh, lucid, when not even slyly mischievous, writing style that is exemplified by his *Mind* paper [11]. He had an ability to get to the heart of a problem and express it in simple, clear terms. Robin Gandy told an anecdote of Turing entering the room where two engineers were laboriously testing the permeability of the cores in certain transformers of radio receivers. Robin marvelled to see Turing take a clean piece of paper, write at the top Maxwell's equations and then proceed to derive what they wanted *ab ovo*.

I'll conclude on a more visionary note with a quotation from an interview he gave following a discussion of a famous British neuroscientist's well publicised lecture on the impossibility of the brain being a mere machine. It shows that he was indeed visionary in what computers would be capable of.

Whilst reports to the US Government or military at about this time supposedly emphasised the rarefied nature of the new or even nascent machines, that they would only be used in university (or presumably government) laboratories or that "five or six machines would suffice for the whole country", Turing's view could not have been more different: he suggested that computers would permeate all walks of life and that in 100 years a machine would pass what has come to be called the "Turing Test."

"This is only a foretaste of what is to come, and only the shadow of what is going to be. We have to have some experience with the machine before we really know its capabilities. It may take years before we settle down to the new possibilities, but I do not see why it should not enter any of the fields normally covered by the human intellect and eventually compete on equal terms."

(Press Interview with *The Times*, June 1949)

Notes

1. *Equivalence of left and right almost periodicity*, J. of the London Math. Society, 10, 1935.
2. There are several approving quotes from Gödel; this is taken from an unpublished (and ungiven) lecture in the *Nachlass* Gödel, Collected Works, Vol. III, p. 166–168.

Bibliography

[1] A. Church. A note on the *Entscheidungsproblem*. *Journal of Symbolic Logic*, 1:40–41, Correction 101–102, 1936.
[2] A. Church. An unsolvable problem of elementary number theory. *American Journal of Mathematics*, 58, 1936.
[3] M. Davis. *The Undecidable*. Raven Press, 1965.
[4] T. Franzen. *Inexhaustibility: A non-exhaustive treatment*, volume 16 of *Lecture Notes in Logic*. ASL/A.K.Peters, 2004.
[5] R.O. Gandy. The confluence of ideas in 1936. In Herken, editor, *The Universal Turing machine: a half century survey*, pages 55–111. Oxford University Press, Oxford, 1988.
[6] G.H. Hardy. Mathematical Proof. *Mind*, 38:1–25, 1929.
[7] A. Hodges. *Alan Turing, the Enigma*. Simon and Schuster, New York, 1983.
[8] R. I. Soare. Turing oracle machines, online computing, and three displacements in computability theory. *Annals of Pure and Applied Logic*, 160:368–399, 2009.

[9] A.M. Turing. On Computable Numbers with an application to the *Entscheidungsproblem*. *Proceedings of the London Mathematical Society*, 42(2), 1936.

[10] A.M. Turing. Systems of Logic based on Ordinals. *Proceedings of the London Mathematical Society*, 45:161–228, 1939.

[11] A.M. Turing. Computing machinery and intelligence. *Mind*, 59(236):433–460, 1950.

[12] J. von Neumann. Zur Hilbertsche Beweistheorie. *Mathematische Zeitschrift*, 26:1–46, 1927.

Philip Welch [p.welch@bristol.ac.uk] is Professor of mathematical logic at the University of Bristol. From 1997 to 2000 he was Professor of mathematical logic at the graduate school, Kobe University, setting up a new research group in set theory. He has held a Mercator Professorship at Bonn University and is a regular visitor at the Kurt Gödel Research Center in Vienna.

Interview with Abel Laureate Endre Szemerédi

Martin Raussen (Aalborg University, Denmark) and Christian Skau (Norwegian University of Science and Technology, Trondheim, Norway)
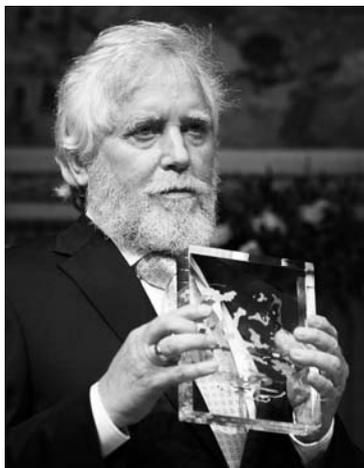
Endre Szemerédi
(Photo: www.abelprize.no)

Professor Szemerédi, first of all we would like to congratulate you as the 10th Abel Prize recipient! You will receive the prize tomorrow from His Majesty, the King of Norway.

Youth

You were born in Budapest, Hungary, in 1940 during the Second World War. We have heard that you did not start out studying mathematics; instead, you started in medical school and only later on shifted to mathematics. Were you nevertheless interested in mathematical problems as a child or teenager? Did you like to solve puzzles?

I have always liked mathematics and it actually helped me to survive in a way: When I was in elementary school, I was very short and weak and the stronger guys would beat me up. So I had to find somebody to protect me. I was kind of lucky, since the strongest guy in the class did not understand anything about mathematics. He could never solve the homework exercises, let alone pass the exam. So I solved the homework exercises for him and I sat next to him at the exam. Of course, we cheated and he passed the exam. But he was an honest person and he always protected me afterwards from the other big guys; so I was safe. Hence my early interest in mathematics was driven more by necessity and self-interest than by anything else. In elementary school I worked a lot with mathematics but only on that level, solving elementary school exercises.

In high school, I was good at mathematics. However, I did not really work on specific problems and, if I remember correctly, I never took part in any competitions. In Hungary there are different kinds of competitions. There

is also a monthly journal *KöMaL*, where you may send in solutions to problems that are posed. At the end of the year the editors will add up points you get for good solutions.

I never took part in this, the main reason being that my father wanted me to be a physician. At the time, this was the most recognised profession, prestigiously and also financially. So I studied mainly biology and some physics but I always liked mathematics. It was not hard for me to solve high school exercises and to pass the exams. I even helped others, sometimes in an illegal way, but I did not do more mathematics than that.

My education was not the usual education you get in Hungary if you want to be a mathematician. In Hungary we have two or three extremely good elite high schools. The best is Fazekas, in Budapest; they produce every year about five to ten mathematicians who, by the time they go to the university, know a lot. I was not among those. This is not a particular Hungarian invention; also in the US, there are special schools concentrating on one subject.

I can name a lot of mathematicians that are now considered to be the best ones in Hungary. Most of them (90%) finished the school at Fazekas. In Szeged, which is a town with about 200,000 inhabitants, there are two specialist schools also producing some really good mathematicians. One of those mathematicians was a student of Bourgain at the Institute for Advanced Study in Princeton, who just recently defended his thesis with a stunning result. But again, I was not among those highly educated high school students.

Is it correct that you started to study mathematics at age 22?

Well, it depends on how you define "started". I dropped out of medical school after half a year. I realised that, for several reasons, it was not for me. Instead I started to work at a machine-making factory, which actually was a very good experience. I worked there slightly less than two years.

In high school my good friend Gábor Ellmann was by far the best mathematician. Perhaps it is not proper to say this in this kind of interview but he was tall. I was very short in high school – at least until I was seventeen. I am not tall now but at the time I was really short and that actually has its disadvantages. I do not want to elaborate. So I admired him very much because of his mathematical ability and also because he was tall.

It was actually quite a coincidence that I met him in the centre of the town. He was to date a girlfriend but he was 15 minutes late so she had left. He was standing there

and I ran into him and he asked me what I was doing. Gábor encouraged me to go to Eötvös University and he also told me that our mathematics teacher at high school Sándor Bende agreed with his suggestion. As always, I took his advice; this was really the reason why I went to university. Looking back, I have tried to find some other reason but so far I have not been successful.

At that time in Hungary you studied mathematics and physics for two years, and then one could continue to study physics, mathematics and pedagogy for three years in order to become a maths-physics teacher. After the third year they would choose 15 out of about 200 students who would specialise in mathematics.

Turán and Erdős

We heard that Paul Turán was the first professor in mathematics that made a lasting impression on you.
That's true. In my second year he gave a full-year lecture on number theory which included elementary number theory, a little bit of analytic number theory and algebraic number theory. His lectures were perfect. Somehow he could speak to all different kinds of students, from the less good ones to the good ones. I was so impressed with these lectures that I decided I would like to be a mathematician. Up to that point I was not sure that I would choose this profession, so I consider Paul Turán to be the one who actually helped me to decide to become a mathematician.

He is still one of my icons. I have never worked with him; I have only listened to his lectures and sometimes I went to his seminars. I was not a number theorist and he mainly worked in analytic number theory.

By the way, Turán visited the Institute for Advanced Study in Princeton in 1948 and he became a very good friend of the Norwegian mathematician Atle Selberg.
Yes, that is known in Hungary among the circle of mathematicians.

May we ask what other professors at the university in Budapest were important for you; which of them did you collaborate with later on?
Before the Second World War, Hungarian mathematics was very closely connected to German mathematics. The Riesz brothers, as well as Haar and von Neumann and many others actually went to Germany after they graduated from very good high schools in Hungary. Actually, my wife Anna's father studied there almost at the same time as von Neumann and, I guess, the physicist Wigner. After having finished high school he, and also others, went to Germany. And after having finished university education in Germany, most of them went to the US. I don't know the exact story but this is more or less the case. After the Second World War, we were somehow cut off from Germany. We then had more connections with Russian mathematics.

In the late 50s, Paul Erdős, the leading mathematician in discrete mathematics and combinatorics – actually, even in probability theory he did very good and famous work – started to visit Hungary, where his mother lived.

We met quite often. He was a specialist in combinatorics. At the time combinatorics had the reputation that you didn't have to know too much. You just had to sit down and meditate on a problem. Erdős was outstanding in posing good problems. Well, of course, as it happens to most people he sometimes posed questions which were not so interesting. But many of the problems he posed, after being solved, had repercussions in other parts of mathematics – also in continuous mathematics, in fact. In that sense Paul Erdős was the most influential mathematician for me, at least in my early mathematical career. We had quite a lot of joint papers.

Twenty-nine joint papers, according to Wikipedia…
Maybe, I'm not sure. In the beginning I almost exclusively worked with Paul Erdős. He definitely had a lasting influence on my mathematical thinking and mathematical work.

Was it usually Erdős who posed the problems or was there an interaction from the very start?
It was not only with me, it was with everybody. It was usually he who came up with the problems and others would work on them. Probably for many he is considered to be the greatest mathematician in that sense. He posed the most important problems in discrete mathematics which actually affected many other areas in mathematics. Even if he didn't foresee that solving a particular problem would have some effect on something else, he had a very good taste for problems. Not only the solution but actually the methods used to obtain the solution often survived the problem itself and were applied in many other areas of mathematics.

Random methods, for instance?
Yes, he was instrumental in introducing and popularising random methods. Actually, it is debatable who invented random methods. The Hungarian mathematician Szele used the so-called random method – it was not a method yet – to solve a problem. It was not a deterministic solution. But then Paul Erdős had a great breakthrough result when he gave a bound on the Ramsey number, still the central problem in Ramsey Theory. After that work there has been no real progress. A little bit, yes, but nothing really spectacular. Erdős solved the problem using random methods. Specifically, he proved that by 2-colouring the edges of a complete graph with n vertices randomly, then almost certainly there will not be more than $2 \log n$ vertices so that all the connecting edges are of the same colour.

In the US, where I usually teach undergraduate courses, I present that solution. The audience is quite diverse; many of them do not understand the solution. But the solution is actually simple and the good students do understand it. We all know it is extremely important – not only the solution but the method. Then Erdős systematically started to use random methods. To that point they just provided a solution for a famous problem but then he started to apply random methods to many problems, even deterministic ones.

Abel Laureate Endre Szemerédi interviewed by Christian Skau and Martin Raussen. (Photo: Eirik Furu Baardsen)

And, of course, his collaboration with Rényi on the random graph is a milestone in mathematics; it started almost everything in random graph theory.

And that happened around 1960?
Yes. It was in the 60s and it is considered to be the most influential paper in random graph theory. Their way of thinking and their methods are presently of great help for many, many mathematicians who work on determining the properties of real-life, large-scale networks and to find random methods that yield a good model for real-life networks.

Moscow: Gelfond and Gelfand

You did your graduate work in Moscow in the period 1967–1970 with the eminent mathematician Israel Gelfand as your supervisor. He was not a specialist in combinatorics. Rumours would have it that you, in fact, intended to study with another Russian mathematician, Alexander Gelfond, who was a famous number theorist. How did this happen and whom did you actually end up working with in Moscow?
This can be taken, depending how you look at it, as a joke or it can be taken seriously. As I have already told you, I was influenced by Paul Turán, who worked in analytic number theory. He was an analyst; his mathematics was much more concrete than what Gelfand and the group around him studied. At the time, this group consisted of Kazhdan, Margulis, Manin, Arnold and others, and he had his famous Gelfand seminar every week that lasted for hours. It was very frightening sitting there and not understanding anything. My education was not within this area at all. I usually had worked with Erdős on elementary problems, mainly within graph theory and combinatorics; it was very hard for me!

I wanted to study with Gelfond but by some unfortunate misspelling of the name I ended up with Gelfand. That is the truth.

But why couldn't you swap when you realised that you had got it wrong?
I will try to explain. I was a so-called candidate student. That meant that you were sent to Moscow – or to Warsaw for that matter – for three years. It had already been decided who would be your supervisor and the system was quite rigid, though not entirely. I'm pretty sure that if you put a lot of effort into it, you could change your supervisor, but it was not so easy. However, it was much worse if you decided after half a year that it was not the right option for you, and to go home. It was quite a shameful thing to just give up. You had passed the exams in Hungary and kind of promised you were going to work hard for the next three years. I realised immediately that this was not for me and Gelfand also realised it and advised me not to do mathematics anymore, telling me: "Just try to find another profession; there are plenty in the world where you may be successful." I was 27 years old at the time and he had all these star students aged around 20; and 27 was considered old!

But in a sense, I was lucky: I went to Moscow in the Fall of 1967 and, in the Spring next year, there was a conference on number theory in Hungary – in Debrecen, not Budapest. I was assigned to Gelfond; it was customary that every guest had his own Hungarian guide. I had a special role too, because Gelfond was supposed to buy clothes and shoes which were hard to get in Russia at the time for his wife. So I was in the driving seat because I knew the shops pretty well.

You spoke Russian then?
Well, my Russian was not that good. I don't know if I should tell this in this interview but I failed the Russian exam twice. Somehow I managed to pass the final exam and I was sent to Russia. My Russian was good enough for shopping but not good enough for having more complex conversations. I only had to ask Gelfond for the size of the shoes he wanted for his wife and then I had a conversation in Hungarian with the shopkeepers. I usually

don't have good taste but because I had to rise to the occasion, so to say, I was very careful and thought about it a lot. Later Gelfond told me that his wife was very satisfied. He was very kind and said that he would arrange the switch of supervisors!

This happened in the Spring of 1968 but unfortunately he died that summer of a heart attack, so I stayed with Gelfand for a little more than a year after that. I could have returned to Hungary but I didn't want that; when I first agreed to study there, I felt I had to stay. They, i.e. Gelfand and the people around him, were very understanding when they realised that I would never learn what I was supposed to. Actually my exam consisted of two exercises about representation theory taken from Kirillov's book, which they usually give to third-year students. I did it but there was an error in my solution. My supervisor was Bernstein, as you know a great mathematician and a very nice guy, too. He found the error in the solution but he said that it was the effort that I had put into it that was important, rather than the result – and he let me pass the exam.

To become a candidate you had to write a dissertation and Gelfand let me write one about combinatorics. This is what I did. So, in a way, I finished my study in Moscow rather successfully. I did not learn anything but I got the paper showing that I had become a candidate.

At this time there was a hierarchy in Hungary: doctorate of the university, then candidate, doctorate of the academy, then corresponding member of the academy and then member of the academy. I achieved becoming a candidate of mathematics.

You had to work entirely on your own in Moscow?
Yes, since I worked in combinatorics.

Gelfond must have realised that you were a good student. Did he communicate this to Gelfand in any way?
That I don't know. I only know that Gelfand very soon realised my lack of mathematical education. But when Gelfand came to Hungary, he talked to Turán and Erdős and also to Hungarian number theorists attending that meeting, and they were telling him: "Here is this guy who has a very limited background in mathematics." This may be the reason why Gelfand agreed to take me as his student. But unfortunately he died early.

Hungarian mathematics

We would like to come back to Hungarian mathematics. Considering the Hungarian population is only about ten million people, the list of famous Hungarian mathematicians is very impressive. To mention just a few, there is János Bolyai in the 19th century, one of the fathers of non-Euclidean geometry. In the 20th century there is a long list, starting with the Riesz brothers, Frigyes and Marcel, Lipót Fejér, Gábor Szegő, Alfréd Haar, Tibor Radó, John von Neumann, perhaps the most ingenious of them all, Paul Turán, Paul Erdős, Alfréd Rényi, Raoul Bott (who left the country early but then became famous in the United States). Among those still alive, you have Peter Lax, who won the Abel

Prize in 2005, Bela Bollobás, who is in Great Britain, László Lovász and now you. It's all very impressive. You have already mentioned some facts that may explain the success of Hungarian mathematics. Could you elaborate, please?
We definitely have a good system to produce elite mathematicians, and we have always had that. At the turn of the century – we are talking about the 19th century and the beginning of the 20th century – we had two or three absolutely outstanding schools, not only the so-called Fasori where von Neumann and Wigner studied but also others. We were able to produce a string of young mathematicians, some of whom later went abroad and became great mathematicians – or great physicists, for that matter. In that sense I think the educational system was extremely good. I don't know whether the general education was that good but definitely for mathematics and theoretical physics it was extremely good. We had at least five top schools that concentrated on these two subjects; and that is already good enough to produce some great mathematicians and physicists.

Back to the question of whether the Hungarians are really so good or not. Definitely, in discrete mathematics there was a golden period. This was mainly because of the influence of Erdős. He always travelled around the world but he spent also a lot of time in Hungary. Discrete mathematics was certainly the strongest group.

The situation has changed now. Many Hungarian students go abroad to study at Princeton, Harvard, Oxford, Cambridge or Paris. Many of them stay abroad but many of them come home and start to build schools. Now we cover a much broader spectrum of mathematics, like algebraic geometry, differential geometry, low-dimensional topology and other subjects. In spite of being myself a mathematician working in discrete mathematics who practically doesn't know anything about these subjects, I am very happy to see this development.

You mentioned the journal KöMaL that has been influential in promoting mathematics in Hungary. You told us that you were not personally engaged, but this journal was very important for the development of Hungarian mathematics; isn't that true?
You are absolutely right. This journal is meant for a wide audience. Every month the editors present problems, mainly from mathematics but also from physics. At least in my time, in the late 50s, it was distributed to every high school and a lot of the students worked on these problems. If you solved the problems regularly then by the time you finished high school you would almost know as much as the students in the elite high schools. The editors added the points you got from each correct solution at the end of the year, giving a bonus for elegant solutions. Of course, the winners were virtually always from one of these elite high schools.

But it was intended for a much wider audience and it helped a lot of students, not only mathematicians. In particular, it also helped engineers. People may not know this but we have very good schools for different kinds of engineering, and a lot of engineering students-to-be

actually solved these problems. They may not have been among the best but it helped them to develop a kind of critical thinking. You just don't make a statement but you try to see connections and put them together to solve the problems. So by the time they went to engineering schools, which by itself required some knowledge of mathematics, they were already quite well educated in mathematics because of *KöMaL*.

KöMaL plays an absolutely important role and, I would like to emphasise, not only in mathematics but more generally in natural sciences. Perhaps even students in the humanities are now working on these problems. I am happy for that and I would advise them to continue to do so (of course not to the full extent because they have many other things to study).

Important methods and results

We would now like to ask you some questions about your main contributions to mathematics.

You have made some groundbreaking – and we don't think that this adjective is an exaggeration – discoveries in combinatorics, graph theory and combinatorial number theory. But arguably, you are most famous for what is now called the Szemerédi theorem, the proof of the Erdős-Turán conjecture from 1936.

Your proof is extremely complicated. The published proof is 47 pages long and it has been called a masterpiece of combinatorial reasoning. Could you explain first of all what the theorem says, the history behind it and why and when you got interested in it?

Yes, I will start in a minute to explain what it is but I suspect that not too many people have read it. I will explain how I got to the problem. But first I want to tell how the whole story started. It started with the theorem of van der Waerden: you fix two numbers, say five and three. Then you consider the integers up to a very large number, from 1 to n, say. Then you partition this set into five classes, and then there will always be a class containing a three-term arithmetic progression. That was a fundamental result of van der Waerden, of course not only with three and five but with general parameters.

Later, Erdős and Turán meditated over this result. They thought that maybe the reason why there is an arithmetic progression is not the partition itself; if you partition into five classes then one class contains at least one fifth of all the numbers. They made the conjecture that what really counts is that you have dense enough sets.

That was the Erdős-Turán conjecture: if your set is dense enough in the interval 1 to n – we are of course talking about integers – then it will contain a long arithmetic progression. Later Erdős formulated a very brave and much stronger conjecture: let's consider an infinite sequence of positive integers, $a_1 < a_2 < \ldots$ such that the sum of the inverses $\{1/a_i\}$ is divergent. Then the infinite sequence contains arbitrarily long arithmetic progressions. Of course, this would imply the absolutely fundamental result of Green and Tao about arbitrarily long arithmetic progressions within the primes because for the primes we know that the sum of the inverses is divergent.

That was a very brave conjecture; it isn't even solved for arithmetic progressions of length $k = 3$. But now, people have come very close to proving it: Tom Sanders proved that if we have a subset between 1 and n containing at least n over $\log n (\log \log n)^5$ elements then the subset contains a 3-term arithmetic progression. Unfortunately, we need a little bit more but we are getting close to solving Erdős's problem for $k = 3$ in the near future, which will be a great achievement. If I'm not mistaken, Erdős offered 3,000 USD for the solution of the general case a long time ago. If you consider inflation, that means quite a lot of money.

Erdős offered 1,000 USD for the problem you solved, and that's the highest sum he ever paid, right?
Erdős offered \$1,000 as well for a problem in graph theory that was solved by V. Rödl and P. Frankl. These are the two problems I know about.

Let us get back to how you got interested in the problem.
That was very close to the Gelfand/Gelfond story, at least in a sense. At least the message is the same: I overlooked facts. I tried to prove that if you have an arithmetic progression then it cannot happen that the squares are dense inside of it; specifically, it cannot be that a positive fraction of the elements of this arithmetic progression are squares. I was about 25 years old at the time and at the end of my university studies. At that time I already worked with Erdős. I very proudly showed him my proof because I thought it was my first real result. Then he pointed out two, well not errors but deficiencies in my proof. Firstly, I had assumed that it was known that $r_4(n) = o(n)^1$, i.e. that if you have a set of positive upper density then it has to contain an arithmetic progression of length four, or for that matter of any length. I assumed that that was a true statement. Then I used that there are no four squares that form an arithmetic progression. However, Erdős told me that the first statement was not known; it was an open problem. The other one was already known to Euler, which was 250 years before my time. So I had assumed something that is not known and, on the other hand, I had proved something that had been proven 250 years ago!

The only way to try to correct something so embarrassing was to start working on the arithmetic progression problem. That was the time I started to work on $r_4(n)$ and, more generally, on $r_k(n)$. First I took a look at Klaus Roth's proof from 1953 of $r_3(n)$ being less than n divided by $\log \log n$. I came up with a very elementary proof for $r_3(n) = o(n)$ so that even high school students could understand it easily. That was the starting point. Later I proved also that $r_4(n) = o(n)$.

Erdős arranged for me to be invited to Nottingham to give a talk on that result. But my English was virtually non-existent. Right now you can still judge that there is room for improvement of my English, but at the time it was almost non-existent. I gave a series of lectures; Peter

---

[1] $r_k(n)$ denotes the proportion of elements between 1 and n that a subset must contain in order for it to contain an arithmetic progression of length k.

Elliot and Edward Wirsing, both extremely strong mathematicians, wrote a paper based almost entirely on my pictures on the blackboard. Perhaps they understood some easy words in English that I used. Anyway, they helped to write up the paper for me. A similar thing happened when I solved $r_k(n) = o(n)$ for general k. Then my good friend András Hajnal helped me to write up the paper. That is actually an understatement. The truth is that he listened to my explanations and he then wrote up the paper. I am very grateful to Peter Elliot, Edward Wirsing and to my good friend András for their invaluable help.

When did all this happen?
It was in 1973. The paper appeared in *Acta Arithmetica* in 1975. There is a controversial issue – well, maybe controversial is too strong a word – about the proof. It is widely said that one of the main tools in the proof is the so-called regularity lemma, which is not true in my opinion. Well, everybody forgets about the proofs they produced 30 years ago. But I re-read my paper and I couldn't find the regularity lemma. There occurs a lemma in the proof which is similar to the regularity lemma, so maybe that lemma, which is definitely not the regularity lemma, inspired me later to prove the regularity lemma.

The real story is that I heard Bollobás' lectures from 1974 about strengthening the Erdős–Stone theorem. The Erdős–Stone theorem from the 40s was also a breakthrough result but I don't want to explain it here. Then Bollobás and Erdős strengthened it. I listened to Bollobás' lectures and tried to improve their result. Then it struck me that a kind of regularity lemma would come in handy and this led me to proving the regularity lemma. I am very grateful to Vasek Chvatal who helped me to write down the regularity paper. Slightly later the two of us gave a tight bound for the Erdős-Stone theorem.

I've seen that people refer to it in your proof of the Erdős-Turán conjecture as a weakened form of the regularity lemma.
Yes, weaker; but similar in ideology, so to speak.

Connections to ergodic theory

Your proof of the Szemerédi Theorem is the beginning of a very exciting story. We have heard from a reliable source that Hillel Furstenberg at the Hebrew University in Jerusalem first learned about your result when somebody gave a colloquium talk there in December 1975 and mentioned your theorem. Following the talk, there was a discussion in which Furstenberg said that his weak mixing of all orders theorem, which he already knew, would prove the ergodic version of the Szemerédi Theorem in the weak mixing case. Since the Kronecker (or compact) case is trivial, one should be able to interpolate between them so as to get the full ergodic version. It took a couple of months for him to work out the details which became his famous multiple recurrence theorem in ergodic theory.
We find it very amazing that the Szemerédi Theorem and Furstenberg's Multiple Recurrence Theorem are

equivalent, *in the sense that one can deduce one theorem from the other. We guess it is not off the mark to say that Furstenberg's proof gave a conceptual framework for your theorem. What are your comments?*
As opposed to me, Furstenberg is an educated mathematician. He is a great mathematician and he already had great results in ergodic theory; he knew a lot. He proved that a measure-preserving system has a multiple recurrence property; this is a far-reaching generalisation of a classical result by Poincaré. Using his result, Furstenberg proved my result on the k-term arithmetic progressions. So that is the short story about it. But I have to admit that his method is much stronger because it could be generalised to a multi-dimensional setting. Together with Katznelson he proved that in 1978. They could actually also prove the density Hales–Jewett theorem but it took more than ten years. Then Bergelson and Leibman proved a polynomial version of the arithmetic progression result, much stronger than the original one. I doubt that you can get it by elementary methods but that is only my opinion. I will bet that they will not come up with a proof of the polynomial version within the next ten years by using elementary methods.

But then very interesting things happened. Tim Gowers started the so-called Polymath Project: many people communicated with each other on the internet and decided that they would try to give a combinatorial proof of the Hales–Jewett density theorem using only elementary methods. After two months, they come up with an elementary proof. The density Hales–Jewett theorem was considered to be by far the hardest result proved by Furstenberg and Katznelson and its proof is very long. The elementary proof of the density Hales–Jewett theorem is about 25 pages long.

There is now a big discussion among mathematicians whether one can use this method to solve other problems. Joint papers are very good, when a small group of mathematicians cooperate. But the Polymath Project is different: hundreds of people communicate. You may work on something your whole life, then a hundred people appear and many of them are ingenious. They solve your problem and you are slightly disappointed. Is this a good thing? There is a big discussion among mathematicians about this method. I am for it. I will soon turn 72 years old, so I believe I can evaluate it without any self-interest.

Still, all this started with your proof of the Erdős-Turán conjecture. You mentioned Green-Tao. An important ingredient in their proof of the existence of arithmetic progressions of arbitrary length within the primes is a Szemerédi-type argument involving so-called pseudo-primes, whatever that is. So the ramifications of your theorem have been impressive.
In their abstract they say that the three main ingredients in their proof are the Goldstone–Yıldırım result which gives an estimate for the difference of consecutive primes, their transference principle and my theorem on arithmetic progressions.

By the way, according to Green and Tao one could have used the Selberg sieve instead.

You are right. However, in my opinion the main revolutionary new idea is their transference principle that enables us to go from a dense set to a sparse set. I would like to point out that later, while generalising their theorem, they did not have to use my theorem. Terry Tao said that he read all the proofs of the Szemerédi theorem and compared them, and then he and Ben Green meditated on it. They were probably more inspired by Furstenberg's method, the ergodic method. That is at least my take on this thing but I am not an expert on ergodic theory.

But Furstenberg's theorem came after and was inspired by yours. So however you put it, it goes back to you.
Yes, that is what they say.

We should mention that Tim Gowers also gave a proof of the Szemerédi Theorem.
He started with Roth's method, which is an estimation of exponential sums. Roth proved in his paper that $r_3(n)$ is less than n divided by *log log n*. Tim Gowers' fundamental work did not only give an absolutely strong bound for the size of a set A in the interval [1,*n*] not containing a *k*-term arithmetic progression; he also invented methods and concepts that later became extremely influential. He introduced a norm (actually, several norms), which is now called the Gowers norm. This norm controls the randomness of a set. If the Gowers norm is big, he proved that it is correlated with a higher order phase function, which is a higher order polynomial. Gowers, and independently Rödl, Naegle, Schacht and Skokan, proved the hypergraph regularity lemma and the hypergraph counting lemma, which are main tools in additive combinatorics and in theoretical computer science.

We should mention that Gowers received the Fields Medal in 1998 and that Terence Tao got it in 2006. Also, Roth was a Fields Medal recipient back in 1958.

Random graphs and the regularity lemma

Let's get back to the so-called Szemerédi regularity theorem. You have to explain the notions of random graphs and extremal graphs because they are involved in this result.
How can we imagine a random graph? I will talk only about the simplest example. You have n points and the edges are just the pairs, so each edge connects two points. We say that the graph is complete if you include all the edges, but that is, of course, not an interesting object. In one model of the random graph, you just close your eyes and with probability ½, you choose an edge. Then you will eventually get a graph. That is what we call a random graph, and most of them have very nice properties.

You just name any configuration – like 4-cycles C_4 for instance, or the complete graph K_4 – then the number of such configurations is as you would expect. A random graph has many beautiful properties and it satisfies almost everything. Extremal graph theory is about finding a configuration in a graph. If you know that your graph is a random graph, you can prove a lot of things.

The regularity lemma is about the following. If you have any graph – unfortunately we have to assume a dense graph, which means that you have a lot of edges – then you can break the vertex set into a relatively small number of disjoint vertex sets, so that if you take almost any two of these vertex sets, then between them the so-called bipartite graph will behave like a random graph. We can break our graph into not too many pieces, so we can work with these pieces and we can prove theorems in extremal graph theory.

We can also use it in property testing, which belongs to theoretical computer science and many other areas. I was surprised that they use it even in biology and neuroscience but I suspect that they use it in an artificial way – that they could do without the regularity lemma. But I am not an expert on this so I can't say this for sure.

The regularity lemma really has some important applications in theoretical computer science?
Yes, it has; mainly in property testing but also in constructing algorithms. Yes, it has many important applications. Not only the original regularity lemma but, since this is 30 years ago, there have appeared modifications of the regularity lemma which are more adapted for these purposes. The regularity lemma is for me just a philosophy. Not an actual theorem. Of course, the philosophy is almost everything. That is why I like to say that in every chaos there is an order. The regularity lemma just says that in every chaos there is a big order.

Do you agree that the Szemerédi theorem, i.e. the proof of the Erdős–Turán conjecture, is your greatest achievement?
It would be hard to disagree because most of my colleagues would say so. However, perhaps I prefer another result of mine with Ajtai and Komlós. In connection with a question about Sidon sequences we discovered an innocent looking lemma. Suppose we have a graph of *n* vertices in which a vertex is connected to at most *d* other vertices. By a classical theorem of Turán, we can always find at least *n*/*d* vertices such that no two of them are connected by an edge. What we proved was that under the assumption that the graph contains no triangle, a little more is true: one can find *n*/*d* times *log d* vertices with the above property.

I am going to describe the proof of the lemma very briefly. We choose *n*/2*d* vertices of our graph randomly. Then we omit all the neighbours of the points in the chosen sets. This is, of course, a deterministic step. Then in the remaining vertex set we again choose randomly *n*/2*d* vertices and again deterministically omit the neighbours of the chosen set. It can be proved that this procedure can be repeated log d times and in the chosen set the average degree is at most 2. So in the chosen sets we can find a set of size at least *n*/4*d* such that no two points are connected with an edge.

Because of the mixture of random steps and deterministic steps we called this new technique the "semirandom method".

Historically, the first serious instance of a result of extremal graph theory was the famous theorem of Ramsey,

and, in a quantitative form, of Erdős and Szekeres. This result has also played a special role in the development of the "random method". Therefore it has always been a special challenge for combinatorialists to try to determine the asymptotic behaviour of the Ramsey functions $R(k,n)$[2], as n (or both k and n) tend to infinity. It can be easily deduced from our lemma that $R(3,n) < cn^2/\log n$, which solved a longstanding open problem of Erdős. Surprisingly, about 10 years later, Kim proved that the order of magnitude of our bound was the best possible. His proof is based on a brilliant extension of the "semirandom method".

The "semirandom method" has found many other applications. For instance, together with Komlós and Pintz I used the same technique to disprove a famous geometric conjecture of Heilbronn. The conjecture dates back to the 40s. The setting is as follows: you have n points in the unit square and you consider the triangles defined by these points. Then the conjecture says that you can always choose a triangle of area smaller than a constant over n^2. That was the Heilbronn conjecture. For the bound $1/n$, this is trivial, and then Klaus Roth improved this to 1 over $n(\log\log n)^{1/2}$. Later Wolfgang Schmidt improved it further to 1 over $n(\log n)^{1/2}$. Roth, in a very brilliant and surprising way, used analysis to prove that we can find a triangle of area less than 1 over $n^{(1+\alpha)}$, where α is a constant.

We then proved, using the semi-random method, that it is possible to put down n points such that the smallest area of a triangle is at least $\log n$ over n^2, disproving the Heilbronn conjecture. Roth told us that he gave a series of talks about this proof.

Further research areas

It is clear from just checking the literature and talking with people familiar with graph theory and combinatorics, as well as additive number theory, that you – sometimes with co-authors – have obtained results that have been groundbreaking and have set the stage for some very important developments. Apart from the Szemerédi theorem and the regularity lemma that we have already talked about, here is a short list of important results that you and your co-authors have obtained:

(i) The Szemerédi-Trotter theorem in the paper "Extremal problems in discrete geometry" from 1983.

(ii) The Erdős–Szemerédi theorem on product-sum estimates, in the paper "On sums and products of integers" from 1983.

(iii) The results obtained by AKS, which is the acronym for Miklós Ajtai, János Komlós and Endre Szemerédi. The "sorting algorithm" is among the highlights.

Could you fill in some details, please?

(i) Euclid's system of axioms states some of the basic facts about incidences between points and lines in the

plane. In the 1940s, Paul Erdős started asking slightly more complicated questions about incidences that even Euclid would have understood. How many incidences can occur among m points and n lines, where an "incidence" means that a line passes through a point? My theorem with Trotter confirmed Erdős' rather surprising conjecture: the maximal number of incidences is much smaller in the real plane than in the projective one – much smaller than what we could deduce by simple combinatorial considerations.

(ii) Together with Paul Erdős, we discovered an interesting phenomenon and made the first non-trivial step in exploring it. We noticed, roughly speaking, that a set of numbers may have nice additive properties or nice multiplicative properties but not both at the same time.

This has meanwhile been generalised to finite fields and other structures by Bourgain, Katz, Tao and others. Their results had far-reaching consequences in seemingly unrelated fields of mathematics.

(iii) We want to sort n numbers, that is, to put them in increasing order by using comparisons of pairs of elements. Our algorithm is non-adaptive: the next comparison never depends on the outcome of the previous ones. Moreover, the algorithm can efficiently run simultaneously on cn processors such that every number is processed by only one of them at a time. Somewhat surprisingly, our algorithm does not require more comparisons than the best possible adaptive non-parallel algorithm. It is well known that any sorting algorithm needs at least $n \log n$ comparisons.

What are, in your opinion, the most interesting and important open problems in combinatorics and graph theory?

I admit that I may be somewhat conservative in taste. The problem that I would like to see solved is the very first problem of extremal graph theory: to determine the asymptotic behaviour of the Ramsey functions.

Combinatorics compared to other areas of mathematics

It is said, tongue in cheek, that a typical combinatorialist is a bright mathematician with an aversion to learning or embracing abstract mathematics. Does this description fit you?

I am not sure. In combinatorics we want to solve a concrete problem, and by solving a problem we try to invent new methods. And it goes on and on. Sometimes we actually borrow from so-called well established mathematics. People in other areas of mathematics often work in ways that are different from how we do in combinatorics.

Let's exaggerate somewhat: they have big theories and they find sometimes a problem for the theory. In combinatorics, it is usually the other way around. We start with problems which actually are both relevant and necessary; that is, the combinatorics itself requires the solution of the problems; the problems are not randomly chosen. You then have to find methods which you apply to solve the problems and sometimes you might create

---

[2] $R(k,n)$ denotes the least positive integer N such that for any (red/blue)-coloring of the complete graph K_N on N vertices, there exists either an entirely red complete subgraph on k vertices or an entirely blue complete subgraph on n vertices.

theory. But you start out by having a problem; you do not start by having a theory and then finding a problem for which you can apply the theory. Of course, that happens from time to time but it is not the major trend.

Now, in the computer era, it is unquestionable that combinatorics is extremely important. If you want to run programs efficiently, you have to invent algorithms in advance and these are basically combinatorial in nature. This is perhaps the reason why combinatorics today is a little bit elevated, so to say, and that mathematicians from other fields start to realise this and pay attention. If you look at the big results, many of them have big theories which I don't understand, but at the very root there is often some combinatorial idea. This discussion is a little bit artificial. It's true that combinatorics was a second rated branch of mathematics 30 years ago but hopefully not any longer.

Do you agree with Bollobás who in an interview from 2007 said the following:

"The trouble with the combinatorial problems is that they do not fit into the existing mathematical theories. We much more prefer to get help from "mainstream" mathematics rather than to use "combinatorial" methods only, but this help is rarely forthcoming. However, I am happy to say that the landscape is changing."
I might agree with that.

Gowers wrote a paper about the two cultures within mathematics. There are problem solvers and there are theory builders. His argument is that we need both. He says that the organising principles of combinatorics are less explicit than in core mathematics. The important ideas in combinatorial mathematics do not usually appear in the form of precisely stated theorems but more often as general principles of wide applicability.
I guess that Tim Gowers is right. But there is interplay between the two disciplines. As Bollobás said, we borrow from the other branches of mathematics if we can, when we solve concrete discrete problems, and vice versa. I once sat in class when a beautiful result in analytic number theory was presented. I understood only a part of it. The mathematician who gave the talk came to the bottleneck of the whole argument. I realised that it was a combinatorial statement and if you gave it to a combinatorialist, he would probably have solved it. Of course, one would have needed the whole machinery to prove the result in question but at the root it actually boiled down to a combinatorial argument. A real interplay!

There is one question that we have asked almost all Abel Prize recipients; it concerns the development of important new concepts and ideas. If you recollect: would key ideas turn up when you were working hard at your desk on a problem or did they show up in more relaxed situations? Is there any pattern?
Actually, both! Sometimes you work hard on a problem for half a year and nothing comes out. Then suddenly you see the solution, and you are surprised and slightly ashamed that you haven't noticed these trivial things which actually finish the whole proof, and which you did

not discover for a long time. But usually you work hard and step-by-step you get closer to the solution. I guess that this is the case in every science. Sometimes the solution comes out of the blue but sometimes several people are working together and find the solution.

I have to tell you that my success ratio is actually very bad. If I counted how many problems I have worked on and in how many problems I have been successful, the ratio would be very bad.

Well, in all fairness this calculation should take into consideration how many problems you have tried to solve.
Right, that is a nice remark.

You have been characterised by your colleagues – and this is meant as a huge compliment – as having an "irregular mind". Specifically, you have been described as having a brain that is wired differently than most mathematicians. Many admire your unique way of thinking, your extraordinary vision. Could you try to explain to us how you go about attacking problems? Is there a particular method or pattern?
I don't particularly like the characterisation of having an "irregular mind". I don't feel that my brain is wired differently and I think that most neurologists would agree with me. However, I believe that having unusual ideas can often be useful in mathematical research. It would be nice to say that I have a good general approach of attacking mathematical problems. But the truth is that after many years of research I still do not have any idea what the right approach is.

Mathematics and computer science

We have already talked about connections between discrete mathematics and computer science – you are in fact a professor in computer science at Rutgers University in the US. Looking back, we notice that for some important mathematical theorems, like the solution of the four-colour problem for instance, computer power has been indispensable. Do you think that this is a trend? Will we see more results of this sort?
Yes, there is a trend. Not only for this but also for other types of problems as well where computers are used extensively. This trend will continue, even though I am not a computer expert. I am at the computer science department but fortunately nobody asked me whether I could answer email, which I cannot! They just hired me because so-called theoretical computer science was highly regarded in the late 80s. Nowadays, it does not enjoy the same prestige, though the problems are very important, the P versus NP problem, for instance. We would like to understand computation and how fast it is; this is absolutely essential mathematics, and not only for discrete mathematics. These problems lie at the heart of mathematics, at least in my opinion.

May we come back to the P versus NP problem which asks whether every problem whose solution can be verified quickly by a computer can also be solved quickly by a computer. Have you worked on it yourself?

I am working on two problems in computer science. The first one is the following: assume we compute an n-variable Boolean function with a circuit. For most of the n-variable Boolean functions the circuit size is not polynomial. But to the best of my knowledge, we do not know a particular function which cannot be computed with a Boolean circuit of linear size and depth $\log n$. I have no real idea how to solve this problem.

The second one is the minimum weight spanning tree problem; again, so far I am unsuccessful.

I have decided that now I will, while keeping up with combinatorics, learn more about analytic number theory. I have in mind two or three problems, which I am not going to tell you. It is not the Riemann hypothesis; that I can tell.

The P versus NP conjecture is on the Clay list of problems, the prize money for a solution being one million USD, so it has a lot of recognition.
Many people believe that the P versus NP problem is the most important one in current mathematics, regardless of the Riemann hypothesis and the other big problems. We should understand computation. What is in our power? If we can check easily that something is true, can we easily find a solution? Most probably not! Almost everybody will bet that P is not equal to NP but not too much has been proved.

Soccer

You have described yourself as a sport fanatic.
Yes, at least I was. I wanted to be a soccer player but I had no success.

We have to stop you there. In 1953, when you were 13 years old, Hungary had a fantastic soccer team; they were called "The Mighty Magyars". They were the first team outside the British Isles that beat England at Wembley, and even by the impressive score of six to three. At the return match in Budapest in 1954 they beat England seven to one, a total humiliation for the English team. Some of these players on the Hungarian team are well known in the annals of soccer, names like Puskás, Hidegkuti, Czibor, Bozsik and Kocsis.
Yes. These five were world class players.

We have heard that the Hungarian team, before the game in Budapest, lived at the same place as you did. Bozsik watched you play soccer and he said that you had real talent. Is this a true story?
Yes, that is true except that they did not live at the same place. My mother died early; this is why we three brothers lived at a boarding school. That school was very close to the hotel where the Hungarian team lived. They came sometimes to our soccer field to relax and watch our games, and one time we had a very important game against the team that was our strongest competitor. You know, boarding schools were competing like everyone else.

I was a midfielder like Bozsik. I was small and did not have the speed but I understood the Hungarian team's strategy. They revolutionised the soccer game, foreshadowing what was later called "Total Football". They did not pass the ball to the nearest guy but rather they aimed the ball to create space and openings, often behind the other team's defence. That was a completely different strategy than the standard one and therefore they were extremely effective.

I studied this and I understood their strategy and tried to imitate it. Bozsik saw this and he understood what I was trying to do.

You must have been very proud.
Yes, indeed I was very proud. He was nice and his praise is still something which I value very much.

Were you very disappointed with the World Cup later that year? As you very well know, the heavily favoured Hungarian team first beat West Germany eight to three in the preliminary round but then they lost two to three in the final to West Germany.
Yes. It was very unfortunate. Puskás was injured, so he was not at his best, but we had some other problems, too. I was very, very sad and for months I practically did not speak to anybody. I was a real soccer fan. Much later, in 1995, a friend of mine was the ambassador for Hungary in Cairo and I visited him. Hidegkuti came often to the embassy because he was the coach for the Egyptian team. I tried to make him explain to me what happened in 1954 but I got no answer.

By the way, to my big surprise I quite often guess correct results. Several journalists came to me in Hungary for an interview after it was announced that I would receive the Abel Prize. The last question from one of them was about the impending European Cup quarter final match between Barcelona and Milan. I said that up to now I have answered your questions without hesitation but now I need three minutes. I reasoned that the defence of Barcelona was not so good (their defender Puyol is a bit old) but their midfield and attack is good, so: 3 to 1 to Barcelona. On the day the game was played, the paper appeared with my, as it turned out, correct prediction. I was very proud of this and people on these blogs wrote that I could be very rich if I would enter the odds prediction business!

We can at least tell you that you are by far the most sports interested person we have met so far in these Abel interviews!

On behalf of the Norwegian, Danish and European mathematical societies, and on behalf of the two of us, thank you very much for this most interesting interview.
Thank you very much. I am very happy for the possibility of talking to you.

Martin Raussen is an associate professor of mathematics at Aalborg University, Denmark.
Christian Skau is a professor of mathematics at the Norwegian University of Science and Technology, Trondheim, Norway.

Speech at Abel Prize Ceremony

Ivar Ekeland (Université Paris-Dauphine, France)

Your majesty,
Excellencies,
Professor Szemerédi,
Distinguished guests.

Once upon a time, there was a little boy named Nils. He had five siblings and when the father died the family was very poor. So Nils would go out into the forest to find food to eat and wood to burn. One day he got lost. He found himself by a lake, where seven beautiful maidens were swimming. When they saw him, they took fright and shouted: "A human! Let us fly back to Soria Moria!" They turned into swans, took off and Nils saw them disappear towards the south.

He decided to follow them. He walked south and eventually he reached Christiania. At the gate of the city stood a small troll, who asked him: "What is the algebraic formula for solving equations of the fifth degree?" Nils answered: "There is no algebraic formula for solving equations of the fifth degree.

What is the way to Soria Moria?" The troll said: "I don't know but go to Berlin. You will find my brother there; he is older and bigger than I am. Perhaps he knows the way to Soria Moria."

Nils then went to Berlin and there he met the second troll, who was uglier and meaner than the first one. The troll asked him: "Is there anything more beautiful than the trigonometric functions? " Nils answered: "Yes, the elliptic functions because they are periodic on a lattice. What is the way to Soria Moria?" The troll said: "I don't know but go to Paris. You will find my brother there; he is older and bigger than I am. Perhaps he knows the way to Soria Moria."

Nils then went to Paris and there he found the third troll, who was much uglier and meaner than the other two put together. The troll asked him: "Can you divide the arc of the lemniscate?" Nils answered: "Yes, I can even do it for any curve of degree four. What is the way to Soria Moria?"

By now, I think you all have recognised the story of Nils Henrik Abel but I hope some of you have recognised another one, for it is also the story of Askeladden. Askeladden, for those who don't know him, is the hero of many Norwegian folk tales. Typically, he is the youngest of three brothers and they set out to marry the king's daughter. There are three impossible tasks to fulfil and whoever succeeds will get the princess. Aske-

ladden's brothers fail miserably because they are strong and arrogant and they quickly find out that strength is not enough. It is Askeladden who wins the princess because he is smart and helpful; he helps people along the way so he gets help in return.

There is an Askeladden in every mathematician. We do not need expensive equipment or time-consuming experiments to practise our craft; brain power is enough. Niels Henrik Abel on his own, a poor student in Norway, did better than all these famous professors in Berlin or Paris. And yet, our collective efforts have significantly contributed to shaping the modern world. Endre Szemerédi pointed out this morning that the internet and medical imaging rely on discrete mathematics. There is another lesson that we learned from Askeladden: give and you will be given back much more than you gave. Don't stay in your office: share your ideas with other mathematicians; go out of your way to talk to physicists, to biologists, to economists, to managers; find out about their problems. This is how mathematics will progress; this is also how science will progress.

There is even another twist to the Askeladden story: strength and wealth will turn to ashes; what is important is to be smart. If you worry about the future, don't invest in gold and oil, invest in education and research. However, I would not presume to give lessons to Government so I will get back to my story. The Paris troll told Nils the way to Soria Moria; he went there and met the princesses again. He married the youngest one and they lived happily ever after.

Well, perhaps it is not the way it ended but this is the way it should have ended.
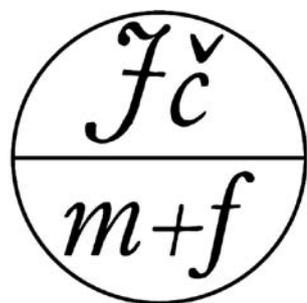
Thank you very much.

Ivar Ekeland,
Akershus Castle, 22 May 2012

150 Years of the Union of Czech Mathematicians and Physicists

Jiří Rákosník (Institute of Mathematics AS CR, Prague, Czech Republic)

On Wednesday 28 March 2012 the historic hall of Charles University in Prague hosted a special ceremony – the Union of Czech Mathematicians and Physicists celebrated 150 years of uninterrupted activity in the field of education, research and popularisation of mathematics and physics in the region of what is today the Czech Republic.

The ceremonial atmosphere was underlined by the participation of numerous distinguished representatives from education, research and politics. The host, Rector of Charles University Václav Hampl, opened the festive assembly by reminding people that the foundations of the union were laid on the floor of Charles University (at that time called Charles-Ferdinand University). President of the Union Josef Kubát recalled the long history of the union and its permanent readiness to affect education in schools of all types. He reminded everyone that the first meeting of the society was held exactly 150 years ago, on 28 March 1862, and that, symbolically, this was the date when the great humanist and educator Jan Amos Komenský (Comenius) was born 420 years ago. President of the Czech Republic Václav Klaus criticised the currently decreasing level of educational standards in schools, emphasised the importance of mathematics as a training tool for thinking and abstraction and acknowledged the important work of the union in this field. A series of further distinguished guests paid tribute to the union including President of the EMS Marta Sanz-Solé, President of the Academy of Sciences of the Czech Republic Jiří Drahoš, President of the sisterly Slovak Union of Mathematicians and Physicists Martin Kalina, Rector of the Czech Technical University Václav Havlíček, Vice-President of the Senate of the Czech Republic Alena Gajdůšková and Dean of the Faculty of Mathematics and Physics of Charles University Zdeněk Němeček. The ceremony was accompanied by an exhibition about the history and activities of the union [1].

The programme continued with a press conference devoted to the current problems of mathematics and physics education in Czech high schools and, in particular, the arguable state organisation of graduation exams. Marta Sanz-Solé participated in the discussion and reflected her experience from Spain and Europe. The quality of education in mathematics and physics and the purpose and organisation of graduation exams was also a hot topic in the subsequent public panel discussion.

The jubilee provided an occasion for coining a new commemorative medal. Its front shows various Prague motifs connected with mathematics and physics; the reverse presents faces of four mathematicians and physicists from Czech history: Tadeáš Hájek z Hájku (1525–1600), Jan Marek Marci (1595–1667), Bernard Bolzano (1781–1848) and František Josef Studnička (1836–1903).

The jubilee day was closed with a chamber concert in the Bethlehem Chapel, the 14th century preaching place of Jan Hus and today's festive hall of the Czech Technical University.

The Czech Mathematical Society used the anniversary to invite presidents of EMS member mathematical societies for a meeting which took place in the Villa Lanna of the Academy of Sciences on 31 March and 1 April [2].

There was a very good reason for such celebration, indeed. The Union of Czech Mathematicians and Physicists is not only the eldest and one of the largest learned societies in the Czech Republic. Since its foundation it has also been highly recognised for its broad ranging and important activities.

The rich history of the union has already been described in the EMS newsletter issue of March 2002 [3] and in the surveys [4,5] by M. Bečvářová. Here we recollect only a few basic facts. The foundations of the union were laid in 1862 when four university students of mathematics and physics established the *Society for Open Lectures in Mathematics and Physics* for the purpose of promoting research, improving education and supporting young high school teachers in these fields.

Meeting of presidents of EMS member societies.

The Czech Post issued a special post stamp …
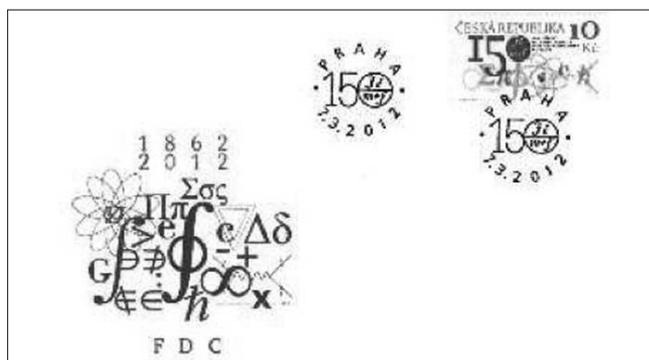

… and the first day cover.

The membership grew, their activities got the support of university professors and in 1869, when the absolutism period in the Austro-Hungarian Empire ended, the society changed the statutes adopting the new name of *Jednota českých matematiků* (*Union of Czech Mathematicians*). Soon after, important publishing activities started. In 1872, the first issue of the journal *Časopis pro pěstování mathematiky a fysiky* (*Journal for Cultivation of Mathematics and Physics*) was published and a year later the union started to publish textbooks in mathematics and physics written according to good foreign models. The importance of the union was continuously increasing and when the independent Czechoslovakia was founded in 1918, the union became a centre of mathematics and physics and a respected partner of the government.

Its influence unfortunately decreased after World War II when the communist regime confiscated its property, including buildings, library and the successful company Fysma, which produced instruments and tools for research and education. Despite the unfavourable circumstances the union remained an isle of free thinking and social engagement of its members – researchers, teachers, students and other persons interested in mathematics and physics during the whole period till the Velvet Revolution in 1989.

Luckily, the buildings had been assigned to the newly founded Institute of Mathematics of the Academy of Sciences, as well as the valuable, voluminous library, which formed a basis of the largest public mathematical library in the Czech Republic. Today the union is back to the original address where the Institute of Mathematics provides the necessary space for activities of the society.

Organisation

The union has a twofold structure. Each of its 2800 members, half them being primary and high school teachers, belongs to one of 15 regional branches located usually in university centres. These branches organise workshops and seminars for teachers, competitions for students, lectures for the public and various activities popularising mathematics and physics. The other structure reflects the members' specialisation; each member can also be engaged in any of the four sections: the Czech Mathematical Society, the Czech Physical Society, the Society of Mathematics Teachers and the Physical Pedagogical Society.

The union is governed by the committee elected for a four year term by the general assembly. The committee meets twice a year to discuss current important questions, to specify tasks for the next period and to approve budgets. In the meantime the tasks are fulfilled by the board consisting of the president, two vice-presidents representing mathematics and physics, a secretary, a treasurer and four chairs of the sections. Several specialised committees are appointed to deal with topics like teaching mathematics and physics in technical universities, further education of teachers, talented students, Czech terminology in mathematics and physics and publicity.

Supporting talents

One of the most important fields where the synergy of mathematicians and physicists, researchers and teachers proves successful is in searching and supporting talented students. Competitions for students, beginning teachers and young researchers were already being organised in the 19th century (see the journal *Časopis pro pěstování mathematiky a fysiky*). Today, members of the union are involved in the organisation of numerous competitions and correspondence seminars for students of schools of all types and degrees, from primary school pupils to PhD students and young researchers. The most important are the Mathematical Olympiad, the Olympiad in Informatics, the Physical Olympiad and the Tournament of Young Physicists.

The union has been involved in the Mathematical Olympiad since 1951 and in the Physical Olympiad since 1959. Every year a great number of its members organise competitions at the school, regional and national levels for several thousands of contestants as well as seminars and training camps for students and their teachers including the preparation of the Czech team in the International Mathematical Olympiad. The total number of contestants at all levels of the Mathematical Olympiad in the Czech Republic in recent years amounts to around six thousand. They also prepare participation of the Czech teams in the International Olympiads. This altogether represents immense voluntary work. Condemnably, state support for these competitions has been constantly decreasing in recent years and so in addition to the huge

effort of its members the union has to support these activities with its own members' fees. It is rewarding that despite the unsatisfactory attention of the state authority our students are remarkably successful in international competitions winning, e.g., two Silver Medals, four Bronze Medals and six Honourable Mentions in the International Mathematical Olympiad in 2011 and 2012. It is not surprising that many of the present researcher and university staff can be found on the list of Mathematical Olympiad winners.

Supporting teachers

A great attention is traditionally paid to problems of teaching mathematics and physics at all levels of schools. There are regular workshops and conferences where high school teachers can share their experiences and improve their knowledge and skills. It is important that many of these events are attended by researchers lecturing about recent developments in mathematics and physics.

The union is involved in publishing three journals for teachers and students. One of them *Rozhledy matematicko-fyzikální* (*Mathematical and Physical Horizons*) has been established since 1922. Printing books had to be stopped years ago but the union is a partner in the publishing house Prometheus specialising in high school and primary school textbooks for mathematics and physics; many of its members continue writing textbooks, reviews and opinions on new textbooks for the Ministry of Education, Youth and Sports.

Even though the union is not a teaching profession organisation, it is a respected body with a rather strong voice in important public discussion concerning organisation and quality of teaching; for instance, there is the current agitated debate about the purpose and system of final high school exams which has recently seen huge problems.

The Czech Mathematical Society

In the period of certain political relaxation in the 1960s it became clear that the structure of the regional branches did not meet the needs of ever-growing specialisation of the union's members and their intensified international contacts. The first component of a new type, the *Physical Research Section* (today the Czech Physical Society), was formed in 1968. Four years later, the Czech Mathematical Society (CMS) was established under the name Mathematical Research Section of the Union, gathering researchers, university students and other union members interested in mathematics. In the following we shall focus on the CMS only.

The mission of the Czech Mathematical Society is to organise and support the organisation of workshops and conferences, to stimulate research activities of university students and young colleagues and to promote international cooperation in mathematics. Every year since 2000, the CMS has been organising research competitions for undergraduate students, which start in individual universities and culminate in a final national research conference where around 60 students qualifying

from university competitions present their results in several specialised sections. Since 2003 this very successful competition has been organised in cooperation with the Slovak Mathematical Society. Every four years up to four young mathematicians are awarded the CMS Prize for Young Mathematicians for their research publications.

The CMS implements international cooperation of the union in the field of mathematics. It is a member organisation of the EMS and is involved in several cooperation agreements with national mathematical societies. Members of the CMS elect the National Committee representing the Czech Republic in the International Mathematical Union. The CMS was the second society to organise the Joint EMS Weekend in 2002 [6]. The interrelations with other mathematical societies resulted in a successful series of Joint Mathematical Conferences CSASC organised alternately by each of the "small" mathematical societies: Czech, Slovak, Austrian, Slovenian and Catalan [7].

Electronic information and publishing is another important domain where the CMS is quite active. It is supervising the Zentralblatt Prague Editorial Group which has been successfully working since 1996. In 2005 the CMS instigated the project of the Czech Digital Mathematics Library. DML-CZ is a freely accessed archive of the major part of the mathematical literature produced in the region of the Czech Republic since the 19th century [8]. It offers free access to more than 32,000 digital documents, journal papers, conference proceedings and books including the collection of 25 writings by Bernard Bolzano. DML-CZ is an active partner of the emerging European Digital Mathematics Library EuDML [9].

To some the word "Union" in a society's name may sound a little outdated. It has, however, quite a symbolic meaning. It is the conjunction of teaching and research expertise and experience, enforced by the institutional membership of universities, research institutes and high schools that enables the continuation of such broad specialised activities for 150 years.

References

[1] http://jcmf.cz/node/296.
[2] http://www.mp2012.zcu.cz.
[3] Luboš Pick: Union of Czech Mathematicians and Physicists. *EMS Newsletter* 43, March 2002, 20–21.
[4] Martina Bečvářová: Union of Czech Mathematicians and Physicists. *Notices from the ISMS*, January 2011, 1–9. http://www.jams.or.jp/notice/Notices1101.pdf.
[5] Martina Bečvářová: The history of mathematics in the Czech Republic. *EMS Newsletter* 77, September 2010, 37–39.
[6] http://cms.jcmf.cz/emsweekend.
[7] http://cms.jcmf.cz/csasc.
[8] http://dml.cz.
[9] http://eudml.eu.

Speech of the EMS President at the 150th Anniversary of the Union of Czech Mathematicians and Physisists

Mr. President,
Distinguished authorities,
Dear colleagues and friends,
Ladies and gentlemen.

It gives me great pleasure to speak to you in my role as President of the European Mathematical Society, to celebrate the 150th anniversary of the Union of Czech Mathematicians and Physicists.

By founding the Union in 1862, you were among the pioneers of several successful initiatives in Europe to promote and to disseminate scientific knowledge. In so doing, you preceded the London Mathematical Society founded in 1865, the Société Mathématique de France in 1872 and the Deutsche Mathematiker Vereinigung in 1890, just to mention a few.

Searching on the internet, I learnt that the Union began as an association for free lectures in mathematics and physics. This description immediately caught my attention and won my affection.

What could be a more laudable objective than removing every possible border to the mind, to have learned people communicate and discuss their findings and conjectures openly and in freedom?

Creativity is stifled by restrictive boundaries and significant scientific advances and breakthroughs are vastly more likely to occur with broad communication and large interaction. Thus, I consider your strategy of keeping mathematics and physics united under the flag of one single learned society a very wise one.

Mathematics and physics, like many other disciplines, have reached a degree of extreme specialisation. The figure of the universal scientist, as in the Renaissance, no longer exists. Theories and techniques have become extremely sophisticated and focused.

Nevertheless, there remains a need for communication between disciplines, for resisting fragmentation, for bringing and keeping pieces together, in order to contribute to genuine progress of knowledge.

Scientific societies play a key role in this process. By providing a solid structure for collaboration, stable mechanisms for cooperation and channels for participation, they are one of the essential pillars underpinning scientific communities and one of the most effective advisors of policymakers on cultural and scientific issues.

I feel impressed being in a town that has hosted historical figures from science such as Johannes Kepler, Tycho Brahe, Christian Doppler and Albert Einstein among others, and in the country that has produced relevant mathematicians like Bernard Bolzano, Eduard Cech and Vojtech Jarnik.

It is a fascinating experience to be a witness to the good health of your prestigious academic institution, to the drive of its members, to your active presence and involvement in international institutions and to your solid roots in your country.

The European Mathematical Society is very proud of having The Union as one of its full members and feels very honoured to be able to be here with you for such a major and well-deserved celebration.

Long life to the Union of Czech Mathematicians and Physicists.

Marta Sanz-Solé
Prague, 28 March 2012

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences

Péter Pál Pálfy (Hungarian Academy of Sciences, Budapest, Hungary)

On the occasion of the Abel Prize being awarded to Endre Szemerédi, here is an update of the information on the Rényi Institute published in the March 2007 issue of the EMS Newsletter.

The institute was founded in 1950. Its first director was Alfréd Rényi (1921–1970); the institute now bears his name. In 2000 the Rényi Institute was awarded the prestigious title "Centre of Excellence" by the European Commission. Recently, the high level of mathematical activity in the institute has been recognised by three Advanced Grants from the European Research Council: for János Pintz in number theory, for András Stipsicz in topology and for Imre Bárány in geometry. The institute was also successful in obtaining significant grants from the Hungarian Academy of Sciences "Momentum" programme, supporting outstanding Hungarian mid-career scientists. Since the launching of this programme in 2009, three research groups have been supported: Gábor Tardos in cryptography, András Stipsicz in topology and Miklós Abért in group theory.

As the most outstanding results of Endre Szemerédi (his theorem on arithmetic progressions and the regularity lemma) were obtained in the 1970s when he was a full-time member of the institute, the Rényi Institute is justly proud of his Abel prize. In the history of Hungarian science it can only be compared to the Nobel Prize in physiology for Albert Szent-Györgyi (University of Szeged) in 1937. Other Nobel Prize winners of Hungarian origin achieved their most important work abroad.

Recently another member of the Rényi Institute, Katalin Marton, has been named as the recipient of an important international prize: she will receive the 2013 Claude E. Shannon Award of the IEEE Information Theory Society.

The institute's members do not have teaching duties. However, many of them teach at various Hungarian universities. The Rényi Institute runs a joint international English-language PhD program in collaboration with the Mathematics Department of the Central European University in Budapest. Over the past five years 13 students (from the USA, Switzerland, Germany, Bulgaria, Slovakia and Hungary) have obtained a PhD there under the guidance of a supervisor from the Rényi Institute.

In 2011 the Rényi Institute hosted nine visitors for 2–4 months and about 130 for shorter periods. The institute runs a wide-ranging conference program that includes meetings on Higher Order Fourier Analysis and on Infinite and Finite Sets, the Summer Symposium in Real Analysis XXXV, the Beyond Next Generation Sequencing Workshop (in bioinformatics), the Memphis-Budapest Summer School in Combinatorics, the Paul Turán

Memorial Conference and the EuroComb'11 conference in 2011, the Motivic Donaldson–Thomas Theory and Singularity Theory workshop of the American Institute of Mathematics, the Contact and Symplectic Topology Summer School and Conference and the First International Conference on Logic and Relativity in 2012. Next year the Rényi Institute will be the main organiser of the Erdős Centennial Conference (1–5 July 2013).

In 2012 the Rényi Institute hosted the annual meeting of ERCOM.

For further information please visit the website www.renyi.mta.hu. (MTA is the Hungarian acronym for Hungarian Academy of Sciences.)

Péter Pál Pálfy is the Director of the Alfréd Rényi Institute of Mathematics.

ICMI Column

Mariolina Bartolini Bussi (Università di Modena e Reggio Emilia, Reggio Emilia, Italy)

On 8–12 July the 12th International Congress on Mathematical Education was held in Seoul (South Korea) under the auspices of the International Commission on Mathematical Instruction (www.icme12.org). The ICMI Executive Committee (chaired by the president Bill Barton) had meetings before and after the conference, with the participation of the president of the International Mathematical Union (Ingrid Daubechies), the secretary-general (Martin Grötschel) and other members of the IMU Executive Committee (Cheryl Praeger, Manuel De León).

On 8 July the General Assembly of the ICMI took place with the election of the new Executive Committee starting from 1 January 2013 for four years:

President: Ferdinando Arzarello (Italy);
Secretary-General: Abraham Arcavi (Israel);
Vice-Presidents (2): Cheryl Praeger (Australia), Angel Ruiz (Costa Rica);
Memers at large (5): Yuriko Baldin (Brazil), Jean Luc Dorier (Switzerland), Zahra Gooya (Iran), Roger Howe (USA), Catherine Vistro-Yu (Philippines).

The conference was chaired by Professor Sung Je Cho and was held at COEX Seoul Convention Center with the collaboration of dozens of colleagues and volunteers. It gathered more than 3600 participants from all over the world (84 countries). As usual in the ICMEs, most participants were from the neighbouring region (about 70%) with large communities from South Korea (1641), China (324), Japan (236), Thailand (111), Singapore (66), India (31), Philippines (21), Malaysia (20) and also Nepal (6) and Cambodia (5). A large group came from the US (354) and Canada (33). Eighty-three participants were from Latin America. The less affluent continent was Africa with 26 participants from South Africa and about 30 from other nations: in most cases only one representative from each nation was present, thanks to the solidarity fund for delegates from non-affluent countries.

Europe was represented in an unequal way as the following table shows.

Germany	95	Netherlands	17	Belgium	2
Sweden	83	Italy	10	Switzerland	1
Portugal	34	Finland	10	Slovenia	1
Spain	32	Slovak		Romania	1
Israel	32	Republic	5	Macedonia	1
Denmark	29	Austria	5	Iceland	1
France	28	Czech		Hungary	1
Norway	27	Republic	4	Holland	1
United		Bulgaria	3	Cyprus	1
Kingdom	26	Latvia	2	Bosnia and	
Turkey	21	Greece	2	Herzegovina	1

The important role of mathematics education in Korea was highlighted by both the President of the Republic, who sent a video message for the opening ceremony, and the Ministry of Education, who took part in the opening ceremony.

Four plenary speeches were organised:
- *Don Hee Lee (Korea): Mathematics Education in the national curriculum system*. The speaker offered some reflections on the national curriculum and on mathematics education in accordance with the tradition of liberal education.
- *Bernard Hodgson (Canada): Whither the mathematics/didactics interconnection? Evolution and challenges of a kaleidoscopic relationship as seen from an ICMI perspective.* The speaker, having been for nine years the secretary-general of the ICMI, reconstructed the links between mathematics on the one hand and the didactics of mathematics on the other, each being considered as a scientific discipline in its own right. He illustrated the specificity and complementarity of the roles incumbent upon mathematicians and upon mathematics educators and examined possible ways of fostering their collaboration and making it more productive. He enriched his presentation with reference to the relationship between different cultural traditions, showing beautiful examples of Korean art.
- *Etienne Ghys (France): The butterfly effect.* The speaker reconstructed the history of chaos theory, popularised by Lorenz's butterfly effect: 'Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?' from the origin dating back to Poincaré and Hadamard to Smale and others. He enriched his presentation with beautiful images and clips from a forthcoming movie about chaos
- *Werner Blum (Germany): Quality teaching of mathematical modelling – what do we know, what can we do?* The speaker presented some of the most important criteria for choosing modelling activities and showed some examples (at the secondary level) of how teachers have successfully implemented such criteria in their classrooms and students have subsequently improved their modelling competency.

Three plenary panels were held:
- The first, chaired by *Konrad Kreiner (Austria)* reported about the international project TEDS-M 2008 which involved 17 countries: Teacher education and development study: learning to teach mathematics.
- The second, chaired by *Frederick Leung (Hong Kong)* on Math education in East Asia (Korea-China-Japan) reported about the social and educational context, teacher education and development and classroom practices of East Asian countries.
- The third, chaired by *Gila Leder (Australia)* reported about Gender and mathematics education with input from different countries (Australia, Sweden, India, Morocco, US, Mexico).

The cultural perspective for explaining achievement and classroom practices and practical implications and

for suggesting education policies and practices in East Asian countries and beyond pervaded the whole conference. The countries which belong to the so-called CHC (Confucian Heritage Culture), e.g. Korea, China, Japan, are home to an ancient and still extant acknowledgement of the importance of education (and mathematics education) and of the high status of teachers. As Sūnzǐ (544 a.C.–496 a.C.) wrote: "Teachers are respected when a country is to be prosperous. Teachers are belittled when a country is to decline." The Korean colleagues proudly quote King Sejong the Great (1397–1450), who invented Hangul, the phonetic Korean Alphabet, a scientific and easy-to-learn alphabet, with a specific (and anachronistic at that time) aim, i.e. to teach the people: "The spoken language of our country is different from that of China and does not suit the Chinese characters. Therefore among uneducated people there have been many who, having something they wish to put into words, have been unable to express their feelings in writing. I am greatly distressed because of this, and so I have made 28 new letters. Let everyone practise them at their ease, and adapt them to their daily use (1443)."

The conference also hosted *national presentations* from Korea, Singapore, India and USA and an additional one on Spanish cultural heritage (including the cooperation between Spanish and Latin American mathematicians).

About 70 *regular lectures* were given by prominent mathematics educators from different parts of the world, including the four awardees of 2009 and 2011: Alan Schoenfeld, 2011 Felix Klein medal; Luis Radford, 2011 Hans Freudenthal medal; Gilah Leder, 2009 Felix Klein medal; Yves Chevallard, 2009 Hans Freudenthal medal. For more information and citations, see http://www.mathunion.org/icmi/news/ and the issues 76 and 84 of the EMS Newsletter.

Thirty-seven Topic Study Groups were offered, covering all school levels (from pre-school to university), contexts (mathematics in workplaces), gifted students and students with special needs, subject areas (algebra, geometry, probability, statistics, calculus), processes (proving, problem solving, visualisation, modelling, cognition), technology, history of mathematics, research on classroom practice, teacher education and development, motivation, beliefs and attitudes, language and communication, gender issues, task design and analysis, curriculum development, assessment, mathematical competitions, history of mathematics education, ethnomathematics and theoretical issues.

Also 17 Discussion Groups, proposed by the participants themselves, were held on different topics (see www.icme12.org for details) and 41 Workshops & Sharing Groups were given rooms to convene for small group discussions. About 500 posters were on show for the whole conference and attached to the other activities.

The ICMI Affiliated Organisations present at the conference were: *CIAEM* (Inter-American Committee on Mathematics Education); *CIEAEM* (International Commission for the Study and Improvement of Mathematics Teaching); *ERME* (European Society for Research in

Mathematics Education); *MERGA* (Mathematics Education Research Group of Australasia); *HPM* (The International Study Group on the Relations between the History and Pedagogy of Mathematics); *ICTMA* (The International Study Group for Mathematical Modelling and Applications); *IOWME* (The International Organization of Women and Mathematics Education); *MCG* (The International Group for Mathematical Creativity and Giftedness); *PME* (The International Group for the Psychology of Mathematics Education); *WFNMC* (The World Federation of National Mathematics Competitions). In addition, the most recent ICMI Studies were presented and the Klein Project was reported. All the above organisations and activities have been briefly described in past issues of this newsletter.

A rich *Mathematical Carnival* was offered to the participants and to thousands of secondary school students from Seoul, with sections on mathematical manipulatives and art and workshops held by university students on different topics. In the Mathematical Plaza workshops were offered mainly by researchers from Eastern Countries with televising from classrooms in Japan, Hong Kong and China.

ICME Conferences are held every four years in different parts of the world. ICME 11 was held in Mexico in 2008 (issue 69 of the newsletter). ICME 13 will be held in Hamburg in 2016.

The *ERME column* is not published in this issue. CERME 8 is to be held in Antalya (Turkey), 6–10 February 2013 (http://www.cerme8.metu.edu.tr/). The deadline for submitting papers is 15 September 2012.

The continuing paper by the Education Committee of the European Mathematics Society (Solid findings of research on mathematics learning and teaching) will be published again as from next issue.

EuDML: The Prototype and Further Development

Jiří Rákosník (Institute of Mathematics AS CR, Prague, Czech Republic) and Olaf Teschke (FIZ Karlsruhe, Berlin, Germany)

Abstract: The EuDML project (introduced in detail in Newsletter 76, June 2010) has released the first public version of the European Digital Mathematics Library. As a unique infrastructure to access Europe's mathematical treasures, its achievements and perspectives have been intensely discussed at the 6ECM in Kraków.

Release 1.3: 230,000 articles since 1777 accessible online

The EuDML project [1] was introduced in detail right after its start by Thierry Bouche in a newsletter article in 2010 [2]. Two years later, the joint effort of the 14 partners is taking shape, as the first public beta version of the European Digital Mathematics Library was released in May 2012. By that time, 230,000 open access articles from 223 collections have already been included in the Library and made accessible via a unified platform [3]
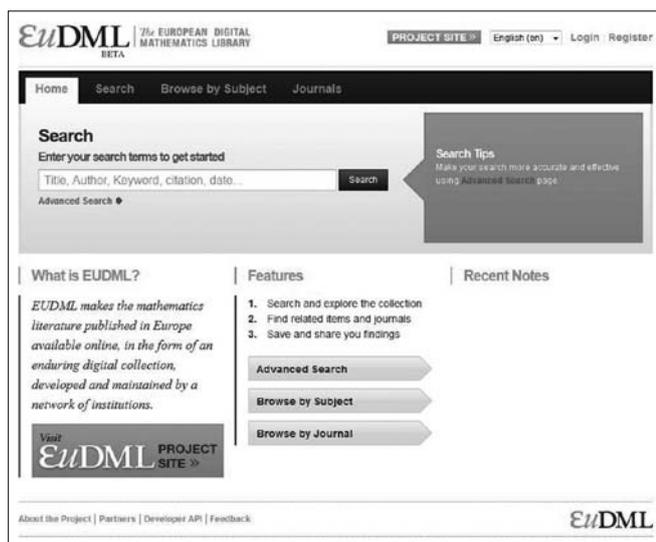
EuDML homepage

Gathering and unifying resources from such diverse sources and times (the oldest entries date back to 1777, while more than 30,000 articles from the 21st century are already included) obviously marks a huge effort. Despite being still far from complete (compared to, e.g. 3.1 million entries in Zentralblatt MATH since 1826), the figures show that on the European level a large proportion of the mathematical treasures have already been sustainably preserved in the public domain.

At this stage, EuDML is already explicit about its extensive links to other resources and its interoperability. This comes with the very nature of EuDML, as a service connecting different digital libraries. Moreover, it provides and maintains widespread links to review databases not just for the article itself but also for references, thereby opening the view far beyond the current content. Again, the present figures are quite impressive: in addition to the 60,000 links for references pointing to the EuDML content, there exist more than 1,000,000 external links to Zentralblatt MATH and Mathematical Reviews, connecting the content with a huge part of the existing literature.

The tools allowing this rely much on the achieved homogenisation of the data. They also allow for many more developed tools: features like subject and journal browsing and first versions of LaTeX formula search and similarity tools are integrated. A simple registration allows the user to use the implemented annotation service. Many more services are yet to come but even now you may start to explore. Certainly, your feedback is appreciated!
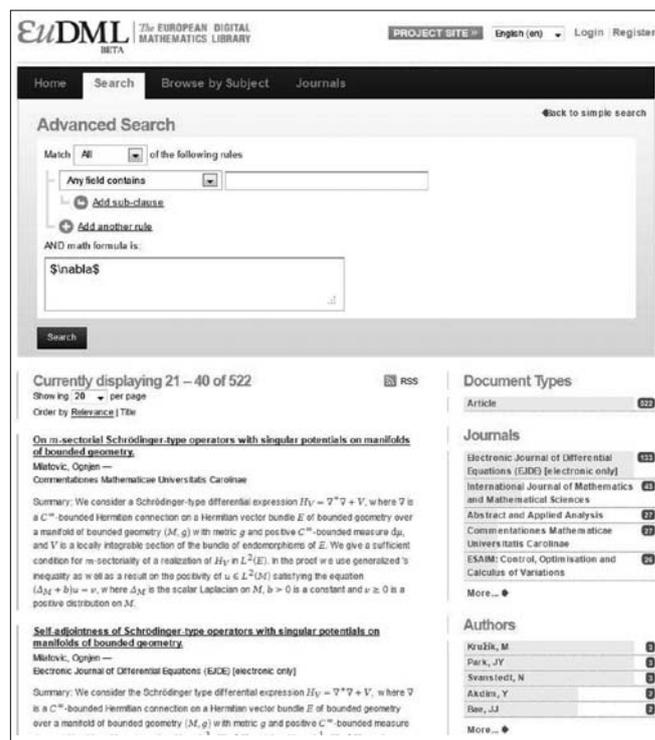
Just some features of the interface: LaTeX search, MathML presentation, journal and author filters

From EuDML towards a global DML

The sustainability and further development of the EuDML is a very important issue. Most of the project partners are willing to continue their effort after the project funding ends in January 2013. Suitable models are under discussion, where the EMS as one of the

EuDML associated partners plays a very important role. The current EuDML content already represents a critical mass but a sustained further extension will be crucial. Services and enhancements should be attractive even for commercial publishers to provide their digital content to the EuDML, under well defined conditions with commitment to eventual open access, which is one of fundamental goals of the EuDML.

All these questions were subjects of the panel discussion organised at the 6th European Congress of Mathematics in Kraków [4]. Four panellists M. Niezgódka (ICM, University of Warsaw), L. Guillopé (Université Nantes), O. Teschke (FIZ Karlsruhe/Zentralblatt MATH, Berlin) and J. Rákosník (Institute of Mathematics AS CR, Praha) presented the EuDML project, its aims and status, as well as their visions for the future. The discussion of about 50 congress participants focused not only on the EuDML but also on more general questions of a global DML and the not fully foreseen development in the domain of publishing and archiving mathematical literature and making it accessible. The presence of representatives of the EMS Publishing House and Springer-Verlag allowed some issues to be tackled that may disquiet commercial publishers, particularly Open Access, which was advocated by the mathematicians present. Even though there was no space for specific arrangements, the participants left with the belief that a constructive dialogue will proceed.
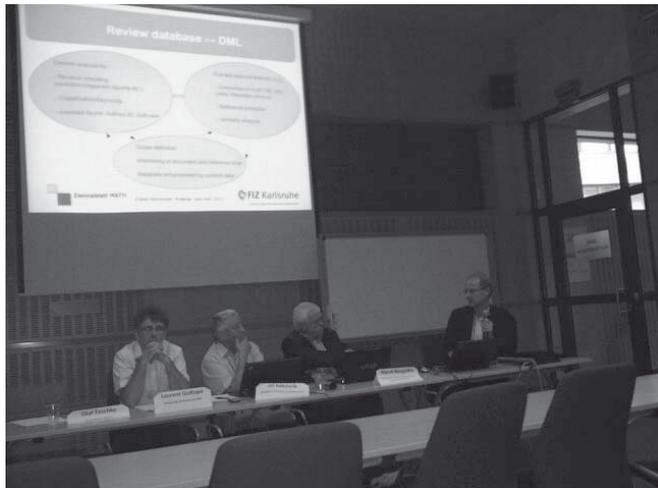
Left to right: L. Guillopé, J. Rákosník, M. Niezgódka, O. Teschke

The EuDML was also often mentioned in the recent workshop on The Future World Heritage Digital Mathematics Library held at the National Academy of Sciences in Washington, DC [5]. It was stated that the EuDML can represent a prototype for building a truly global DML. Merging and extending the EuDML with yet unrealised US-DML and other DMLs, bringing in expertise from the EuDML and using the accomplishments would be important components of the eventual WDML.

References

[1] EuDML project webpage: http://project.eudml.org.
[2] EMS Newsletter 76, June 2010, 11–16.
[3] EuDML Release 1.3, http://eudml.org.
[4] EuDML: Accessing Europe's mathematical treasures. Panel discussion at the 6th European Congress of Mathematics, Kraków, 2 July 2012, http://project.eudml.org/news/eudml-6th-european-congress-mathematics.
[5] The Future World Heritage Digital Mathematics Library. Workshop held in Washington, 1–3 June 2012, http://ada00.math.uni-bielefeld.de/dml/index.php/Main_Page.

Jiří Rákosník [rakosnik@math.cas.cz] is a researcher in the Department of Topology and Functional Analysis in the Institute of Mathematics in Prague. His research interest is the theory of function spaces, in particular spaces with variable exponents, and their use in partial differential equations. He is also active in the field of digitization and digital mathematical libraries.

Olaf Teschke [teschke@zblmath.fiz-karlsruhe.de] is member of the Editorial Board of the EMS Newsletter, responsible for the Zentralblatt Column.

Book Reviews

Simon Donaldson

Riemann Surfaces

Oxford University Press, 2011
256 pages
ISBN: 978-0198526391

Reviewer: Carlos Florentino

The theory of Riemann surfaces, introduced by the mathematical genius G. B. Riemann in his PhD thesis dated 1851 and developed by him and many others, may be considered one of the remarkable achievements of 19th century mathematics.

On one hand, this theory furnishes a geometric interpretation and counterpart to deep analytical and number theoretic work of Abel and Jacobi. But it can also be argued that it represents a turning point in the history of algebraic geometry, as well as a significant contribution to the development of entirely new areas of mathematics (for instance algebraic topology and functional analysis), representing a noteworthy example of the unity of mathematics.

Because of its richness and great influence in much of the research of the 20th century (and continuing into the 21st century), many treatises have been written on the theme, aimed at different levels, each one with its own balance on the many facets of the theory (see, for example, [F, FK, K, M]). Recall that, since the study of compact Riemann surfaces is, in a different language, essentially the same as the study of projective one-dimensional non-singular varieties over the complex number field, some texts typically rely on one or the other kind of language.

Therefore, it is with some surprise that one encounters, in this new book authored by Simon Donaldson, a manifestly original and modern approach, emphasising the special features of Riemann surface theory in the broader context of multidimensional complex and global analysis, and also its connections to many diverse topics of current interest. Being developed over the years as Donaldson's own perspective on a subject he helped raise to a new level of significance, this book will certainly become a central reference in the field.

The main novelty of the author's approach is the crucial use of PDE methods for differential forms on surfaces in the derivation of the main technical results of the theory: the existence of meromorphic functions and the Uniformization Theorem. This method is well aligned with the outstanding contributions Donaldson has made

to research problems in geometry. Another distinguishing feature is the emphasis on connections to algebraic and differential topology, manifold theory and mathematical physics, as the author clearly and concisely touches on many diverse and interesting topics whose relations to the main stream of Riemann surface theory are well known by experts but rarely seen in textbooks.

The monograph is divided into four parts. In Part I, we find a short introductory chapter, motivating the idea of a Riemann surface from the concept of analytic continuation, which appears naturally when trying to globalize local solutions of linear differential equations. Chapter 2 presents an informal treatment of the classification of topological surfaces (including the non-orientable case), a sketch of a proof of the well known classification of "closed" surfaces using ideas from Morse theory but leaving the precise definitions and methods to the reader. The chapter finishes with a discussion of the first advanced topic: the mapping class group.

The five chapters of Part II are designed as a quick first course on Riemann surface theory. Chapters 3 and 4 deal with the main definitions and examples of Riemann surfaces, algebraic curves and maps between them. Donaldson's treatment of the charts in algebraic curves (both affine and projective), and the usual quotient constructions of Riemann surfaces from discrete subgroups of the automorphism group of their universal cover, is efficient and elegant. Note also the concise presentation of the degree of a map between surfaces, the coverings constructed from monodromy, the compactification and normalization of algebraic curves, and the relation to Puiseaux expansions.

Chapter 5 is included as a short introduction to calculus on surfaces, intended for readers without a strong background in differentiable manifolds, tangent spaces and differential forms. It also sketches the relevant algebraic topology, focusing on de Rham cohomology, cohomology with compact support and Poincaré duality. Then, a short treatment is given of the basic properties of holomorphic and meromorphic differential forms, as well as harmonic forms and the Dirichlet norm.

Chapter 6 is an interesting summary of the classic topic of elliptic curves, functions and integrals. Here, the Weierstrass p-function and theta functions are studied and, assuming the existence of a nowhere vanishing holomorphic one-form, the classification of elliptic curves is obtained through the j invariant. The next chapter provides a well written, but at some points informal, presentation of many aspects related to the Euler characteristic, such as the Riemann-Hurwitz formula and the degree-genus formula. Applications are given to the study of real structures on a given Riemann surface, including the proof of Harnack's bound, and to the computation of the genus of modular curves.

Part III is the central part of the book, where the fundamental analytic results for the study of compact Riemann surfaces are shown and where Donaldson's approach is rooted. Motivated by previous examples where the existence of global meromorphic functions or holomorphic forms is found to be the main issue, the author

states in Chapter 8 what he calls the "Main theorem for compact Riemann surfaces" – an existence and uniqueness result for solutions of the non-homogeneous Laplace equation. As almost immediate consequences, one then obtains the classification of low genus Riemann surfaces and the Riemann–Roch formula.

Chapter 9 is fully dedicated to the proof of the Main Theorem, using the Riesz representation theorem and going through Weyl's lemma. The following chapter states and proves the uniformization theorem. One sees that this latter theorem follows, in a relatively easy way, from an analogue of the Main Theorem for simply connected non-compact Riemann surfaces.

Finally, Part IV "Further developments", which amounts to almost half of the monograph, motivates and studies a set of topics which naturally reflect the preferences of the author and are very well balanced (despite some absent common topics such as Bezout's theorem, the Weierstrass gap theorem and Riemann's bilinear relations). It starts in Chapter 11 with some algebraic aspects of the theory. One finds here a discussion of the fields of meromorphic functions on compact surfaces, a proof of the bijection between a surface and the valuations on it and a short digression into algebraic number theory. Also in this chapter is an elegant treatment of hyperbolic surfaces, which directly links to the usual concepts in differential geometry, such as geodesics, curvature and the Gauss–Bonet theorem.

Chapter 12 treats the classical theme of divisors, line bundles and the Abel–Jacobi theorem. It begins with a crash course on sheaves and cohomology. Then, Serre duality is presented and Donaldson indicates how to approach the proof that "would fit best into the general scheme" of the book. Unfortunately, the proof is not completed, probably because it would be a longer than wanted digression. Projective embeddings are then considered.

Chapters 13 and 14 provide an introduction to moduli spaces and deformations, one of the striking features of algebraic geometry but something that is usually more difficult for the starting student. The approach is through almost complex structures and Beltrami differentials. The diffeomorphisms of the plane and Dehn twists are studied and applications are given to hyperbolic geometry and compactification of moduli spaces. Finally, coming back to the hypergeometric equation hinted at in the very beginning, the book ends in Chapter 15 with a discussion of periods of holomorphic forms and the Gauss-Manin connection.

Let me say a few words about the prerequisites and assumed background. The informal style of the book may lead some to think this is a gentle introduction to Riemann surfaces. But after reading the book, one concludes that the level cannot be considered elementary, even for a typical graduate course. In fact, the stronger emphasis on explaining the main ideas, rather than laying out definitions, statements and proofs (as would be the case in a "Bourbaki style" book) is a great choice for someone with some familiarity of geometry or topology but may present serious difficulties for the starting stu-

dent who is used to understanding new mathematics by training through computations, algorithms and problem solving. Also, many of the discussions and digressions in the book will require a great level of mathematical maturity from the reader.

Another aspect I would like to mention is that, although some exercises have been included, the text would greatly benefit from expanding these problem sets. There are also a few unfortunate typographical errors (some in important formulae, such as the Riemann-Hurwitz formula, in Section 7.2, page 101), which will certainly be corrected in a second printing. Personally, I think it would be great if a new edition provides a full proof of Serre's duality theorem, which seems to be another great argument for using the "Main theorem".

To conclude, I would say that the book is an extraordinarily clear and insightful monograph on a classical theme and is manifestly original. As was mentioned above, if the book is not complemented with other texts, it may be not so easy to follow for the average student. On the other hand, it provides a handful of deep and useful ideas that will certainly be welcome by researchers in neighbouring fields and by those bright students eager to start their own path into research.

In my opinion, being certainly a novel and modern treatment, the approach taken in this book looks surprisingly close to Riemann's own eclectic point of view, as a fascinating and delicate blend of analysis and geometry, with mathematical physics lying in the background.

Bibliography

[FK] H. Farkas, I. Kra, Riemann surfaces. Second edition. Graduate Texts in Mathematics, 71. Springer-Verlag, New York, 1992. ISBN: 0-387-97703-1

[F] O. Forster, Lectures on Riemann surfaces. Graduate Texts in Mathematics, 81. Springer-Verlag, New York, 1991. ISBN: 0-387-90617-7

[K] F. Kirwan, Complex algebraic curves. London Mathematical Society Student Texts, 23. Cambridge University Press, Cambridge, 1992. ISBN: 0-521-41251-X

[M] R. Miranda, Algebraic curves and Riemann surfaces. Graduate Studies in Mathematics, 5. American Mathematical Society, Providence, RI, 1995. ISBN: 0-8218-0268-2

Carlos Florentino [carlos.florentino@math. ist.utl.pt] obtained his PhD at Stony Brook, USA, in 1997 and is currently a professor of mathematics and researcher at the Instituto Superior Técnico, Lisbon, Portugal. His research interests include moduli spaces of vector bundles over Riemann surfaces, geometric quantisation of algebraic varieties, geometric invariant theory and character varieties.
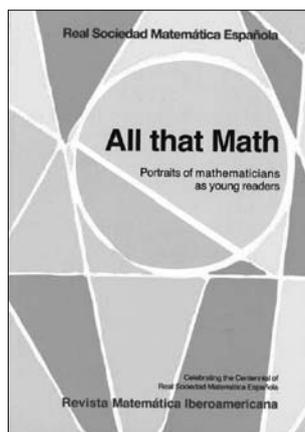
Antonio Córdoba,
José L. Fernández,
Pablo Fernández (editors)

All that Math
Portraits of mathematicians
as young readers

Revista Matemática Ibero-
americana, Madrid, 2011.
Paperback, x+346 pages
ISBN: 978-84-615-2900-1.

Reviewer: Vagn Lundsgaard Hansen

Most people remember their first day in school, their first love and other important moments in their lives. Like other scientists, research mathematicians also remember when they first experienced the very special excitement of making a (mathematical) discovery leading to publishing their first scientific paper. Quite often they also clearly remember the events that brought them on the track to their discoveries. In many cases it was reading a paper, in other cases it was attending a lecture and very often it was stimulated by mathematical discussions with fellow mathematicians. We all have our small stories to tell about how we got our first original idea and what happened in our attempts to develop it. But are there common features in all these personal stories? I personally think so, and I find that the book under review supports this view in an excellent manner by presenting the recollections of their entrance into research by 34 distinguished mathematicians from all over the world.

In 2011, the *Real Sociedad Matemática Española* asked the editors of the journal *Revista Matemática Iberoamericana* (founded by the society) to devote a special issue of the journal to celebrate the society's centennial. The editors are to be commended for the idea of asking a group of authors and members of past and present editorial boards of *Revista* to contribute to this special issue "with an essay about a paper – not necessarily the most important publication in the field – which, in one way or another, had a deep impact on their own mathematical careers, especially at its early stages".

The essays are of a very diverse nature. Some essays are rather demanding from a technical point of view while others are more discursive with emphasis on personal reminiscences about eminent mathematicians from recent times. Of the more technical essays, I personally liked "A random walk in analysis" by the American mathematician Christopher J. Bishop, reporting on important developments in conformal geometry. Of the biographically oriented essays, I found great pleasure in reading "Mathematical encounters" by the Czechoslovakian born mathematician Joseph J. Kohn, which contains interesting and amusing information about great mathematicians such as Norbert Wiener, John F. Nash, Solomon Lefschetz and, not least, Kohn's thesis advisor

Donald C. Spencer. In a very short essay "Some recollections", the American mathematician Stephen Wainger declares his great debt for becoming a mathematician to several books by Eric Temple Bell, in particular his famous book *Men of Mathematics*, thereby demonstrating that good popularization can be of importance in stimulating young people to a career in mathematics.

The group of authors also includes the American Fields Medallist Charles Fefferman, who in the essay "A reminiscence on BMO" describes his reading of a paper by Eli Stein on singular integrals and differentiability properties of functions while a graduate student at Princeton University. Arriving in 1970 at the University of Chicago as a new assistant professor, Fefferman was challenged by a question from Antoni Zygmund to find a characterization of functions in BMO in terms of the Poisson integral. Fefferman realised quickly how he could use techniques acquired by reading the paper by Stein to answer Zygmund's question and solved in fact a more general problem in less than two weeks. Fefferman's final remark about this experience deserves quotation: "For many years, I've worked very hard to prove theorems. With luck, I've found complicated proofs after much suffering. With extraordinary luck, I found simple proofs after even more suffering. To find a simple proof, without suffering for it, is a very rare success. I will always be grateful for my incredible luck in reading Eli's paper and hearing Zygmund's question."

The stimulation you can obtain by attending a general lecture by a real master is vividly recorded in the essay "Olé!" by the Spanish mathematician José L. Fernández. The essay takes the point of departure in a lecture by Mark Kac that the author attended in 1984. It takes us on a tour through random walks, martingales, geometric function theory and, finally, conformal mappings. In the latter area the author singles out a fundamental paper close to his heart published by Nicolai G. Makarov in 1985 in the Proceedings of the London Mathematical Society. The essay ends with a statement about beauty in mathematics: "Appreciation of beauty is a cumulative cultural affair continuously evolving: you have to be prepared and ready to discriminate, to appreciate and to *share* it. But to *create* beauty, well, that is altogether a different matter. *Olé Makarov*!"

During the wonderful "6th European Congress of Mathematicians" in magnificent Kraków, Poland, at the beginning of July 2012, I had many opportunities to think about the proud traditions of Polish mathematics, which become even more impressive taking into account the many dark moments in Polish history. I was therefore happy to find in the book an essay "Let the beauty of Harmonic Analysis be revealed through nonlinear PDEs" by the Polish mathematician Tadeusz Iwaniec. He is obviously a highly spirited person and his essay contains a lot of amusing comments and quotations, among others a self-ironic one by the New Zealand mathematician Gavin Martin, another contributor to the book, who says about Iwaniec: "After all, he has quite a good memory even if it is a bit short." But there is also a short, gloomy paragraph in the essay where Iwaniec says: "As I

share the beauty and joy of mathematics with you I also remember Polish mathematicians whose glorious scientific careers came to a cruel end during Nazi-Soviet occupation. *Józef Marcinkiewicz*, *Stanisław Saks* and *Juliusz Paweł Schauder* were inspirations to me."

Among the many exciting essays in the book, I shall here only mention one more, namely the inspiring essay "Two papers by Alberto P. Calderón" written by the French mathematician Yves Meyer. In the essay, Meyer describes two short papers by Calderón, which were pivotal not only to his own research work but which – in his opinion – completely changed the paths of real analysis, complex analysis and operator theory for the future of mathematics by their elegance, conciseness, profoundness and vision.

The essays in this book describe some of the first experiences with contemporary research that a young mathematician meets when entering a research career in mathematics. The topics cover a wide scope of branches of mathematics and the exposition in many of the essays will therefore, in places, inevitably challenge any reader, especially since the authors of the essays attempt to give a fair description of the scope and depth of the mathematics behind the essays. Nevertheless, most of the essays are in my opinion valuable contributions to understanding the way research mathematicians think and work, in particular, how they get their ideas. I congratulate the *Real Sociedad Matemática Española* on their first hundred years and especially on the publication of this inspiring and valuable commemorative issue of their journal *Revista Matemática Iberoamericana*.

Vagn Lundsgaard Hansen [V.L.Hansen@ mat.dtu.dk] is Professor Emeritus at the Technical University of Denmark, Chairman of the EMS RPA Committee, 2000-2006, an invited speaker at the ICM, Beijing, 2002, and ICME 10, Copenhagen, 2004, President of the Danish Mathematical Society, 2008-12, Knight of the Order of Dannebrog, 1st class 2010 and recipient of the G. A. Hagemann Gold Medal, 2011.
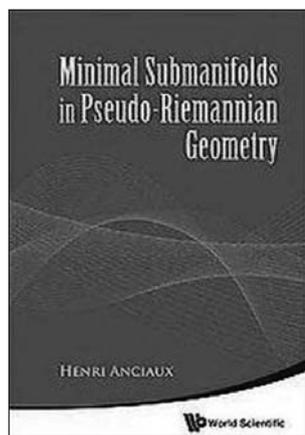
Henri Anciaux

Minimal Submanifolds in Pseudo-Riemannian Geometry

World Scientific Publishing Co., November 2010
184 pages
ISBN: 978-981-4291-24-8

Reviewer: Magdalena Rodríguez

The classical theory of minimal surfaces in Euclidean space, whose roots go back to the beginning of the calculus of variations in the 18th century, while still enjoying a great amount of research effort nowadays, has motivated the emergence of the theory of minimal submanifolds in Riemannian manifolds, an extremely active field of research of its own. And it is raising an interest for generalising such a theory to a wider range of ambient spaces. Minimal submanifold theory involves techniques from different areas, including partial differential equations, complex analysis, algebraic geometry and geometric measure theory. On the other hand, the study of pseudo-Riemannian manifolds was developed in the last century, in part due to its importance as the main tool of the theory of relativity. In this book the author combines an interest in both topics, studying minimal submanifolds in pseudo-Riemannian manifolds (the concept of minimal submanifolds must be understood in a wider sense, which includes also, for instance, maximal hypersurfaces in Minkowski space, largely studied nowadays). This book exploits the fact that the Riemannian hypothesis on the metric of the ambient manifold can be sometimes relaxed, thus generalising some well known results such as the existence of the Weierstrass representation for minimal surfaces. From my point of view, this is what singles out this book from the many other works that focus on the Riemannian case.

This book has its origins in two mini-courses given by the author at the Technische Universität of Berlin and at the Federal University of São Carlos. The content has been completed with detailed proofs of the results presented there and many examples. It only assumes from the reader some basic knowledge about differential geometry. Thus the book is rather self-contained, which makes it ideal as a textbook for an advanced graduate course on the subject. The book is well-structured and easy to read. On the other hand, historical references are often overlooked and the results which are either new in this book or in very recent research are not sufficiently highlighted.

The book is composed of six chapters:
- Submanifolds in pseudo-Riemannian geometry.
- Minimal surfaces in pseudo-Euclidean space.
- Equivariant minimal hypersurfaces in space forms.
- Pseudo-Kähler manifolds.
- Complex and Lagrangian submanifolds in pseudo-Kähler manifolds.
- Minimizing properties of minimal submanifolds.

The first chapter contains definitions and basic results on the theory of submanifolds in pseudo-Riemannian manifolds needed in the book, giving either a sketch of their proof or precise references of where they can be found. This will be very helpful to people not familiar with the theory. The special cases of dimension one (i.e. curves)

and co-dimension one (i.e. hypersurfaces) are studied in more detail. This chapter eventually leads to obtaining the important first variation formula for the volume associated to the variation of a submanifold in this setting, from which we deduce that minimal submanifolds are critical points of the volume variation (i.e. a generalisation of Meusnier's result for surfaces in $\mathbb{R}^3$ to submanifolds in pseudo-Riemannian manifolds). Also deduced in this chapter is the second variation formula of the volume for normal variations of a minimal submanifold.

Besides curves, the simplest submanifolds we can study are surfaces. In this case, one can prove the existence of isothermal coordinates, which simplifies later calculations and constitutes the first result proved in the second chapter. The next result is the derivation of the quasi-linear equations for minimal graphs of Minkowski space defined over an open set of both the horizontal plane $\mathbb{R}^2$ and the vertical plane $\mathbb{R}^{2,1}$. (These equations, together with the classical one for minimal graphs in $\mathbb{R}^3$, are also obtained in the last chapter from a more general point of view.) Apart from the affine solutions (classified by Calabi as the only entire solutions in the first case), the simplest ones are those with a radial dependence, giving rise to rotationally invariant minimal graphs. These examples, as well as ruled minimal surfaces in pseudo-Euclidean space, are classified. In both cases, explicit parameterisations are provided.

A way to characterise minimal surfaces is to say that their coordinate functions are harmonic for the induced metric. This implies that there are no compact minimal surfaces with definite metric and allows us to define a Weierstrass representation for minimal surfaces in pseudo-Euclidean space. This is a generalisation of the classical Weierstrass representation for minimal surfaces in Euclidean space (also generalised for maximal surfaces, which are the minimal space-like surfaces in Minkowski space) and can be found at the end of Chapter 2, together with some examples in both the definite and indefinite case.

Chapter 3 starts by introducing pseudo-Riemannian space forms, defined as non-flat totally umbilic hypersurfaces in pseudo-Euclidean space. They are analogous to the round sphere in Euclidean space and include: the round sphere, hyperbolic space, de Sitter space and anti de Sitter space. Also introduced and classified are equivariant minimal hypersurfaces (with respect to the subgroup of rotations fixing a positive or a negative direction) in pseudo-Euclidean space. When the ambient space is a pseudo-Riemannian space form, both totally umbilic and equivariant hypersurfaces (with respect to certain subgroups of rotations) are classified.

As a particular case of pseudo-Riemannian manifolds with even dimension, we can find the pseudo-Kähler manifolds, a direct generalisation of the Kähler manifolds in the Riemannian case (they are pseudo-Riemannian manifolds of even dimension endowed with a complex and a symplectic structure). Chapter 4 consists of a description of pseudo-Kähler manifolds, using complex pseudo-Euclidean space $\mathbb{C}^n$ as a motivating model. As an illustrative example, it is proved that any oriented Riemannian surface can be equipped with a pseudo-Kähler

manifold structure. The author describes next the pseudo-Kähler structure of the set of positive complex lines of $\mathbb{C}^n$, denoted by $\mathbb{CP}_p^n$, which can be seen as a quotient of a complex space form. This chapter finishes by proving that the tangent bundle of a pseudo-Kähler manifold is a pseudo-Kähler manifold itself.

The fifth chapter is devoted to the description of some examples of minimal submanifolds in pseudo-Kähler manifolds. The first ones are complex submanifolds, defined as submanifolds (of even dimension) for which the complex structure at any point maps the tangent plane to itself. It follows the study of minimal Lagrangian submanifolds in the most symmetric settings: the spaces $\mathbb{C}^n$ and $\mathbb{CP}_p^n$ just introduced in the preceding chapter, including a detailed description of those which are equivariant. This chapter finishes with a local classification of the minimal Lagrangian surfaces in the tangent bundle of an oriented Riemannian surface, a recent result proved by the author in a joint work with Guilfoyle and Romon.

In the last chapter, minimising and maximising properties for the volume of some minimal submanifolds are studied. The first examples considered are the minimal graphs in pseudo-Euclidean space: they are volume minimising in the Riemannian case and volume maximising in the Lorentzian case. The next examples treated are complex submanifolds in a pseudo-Kähler manifold M for which the induced metric on both tangent and normal bundles is definite (the later technical hypothesis is proved to be necessary): they are volume minimising when the metric in M is definite and volume maximising when the metric is indefinite. Finally, it is remarked that Harvey and Lawson's theorem (they proved that any minimal Lagrangian submanifold in complex pseudo-Euclidean space is always volume minimising) fails to hold when the metric is indefinite; but it still holds when we restrict to Lagrangian variations of the submanifold.

In conclusion, this book shows a new point of view in the theory, unifying different research lines. The results are stated in the most general way possible. On the other hand, it only assumes from the reader some basic knowledge about manifolds theory although it reaches recent results of current research interest. The book is well-structured, self-contained and easy to read. I therefore recommend it for any researcher interested in either the theory of minimal submanifolds or in pseudo-Riemannian geometry.

Magdalena Rodríguez [magdarp@ugr.es] is a Ramón y Cajal researcher at the University of Granada. She obtained her PhD in mathematics at the University of Granada in 2005 under the supervision of Joaquín Pérez. She has held postdoctoral positions at the University of Marné-la-Vallée and the Intitute of Mathematics of Jussieu in Paris and teaching positions at the Complutense University in Madrid. She returned to Granada in 2010. Her research activity focuses on geometric analysis and, more specifically, on the theory of minimal and constant mean curvature surfaces.

Solved and Unsolved Problems

Themistocles M. Rassias (Athens, Greece)

Mathematics is an art, and as an art chooses beauty and freedom.
Marston Morse (1892–1977)

I Six new problems – solutions solicited

99. For any positive integer n let x_n be the unique positive root of the polynomial

$$f_n(t) = t^3 + 3t^2 - \frac{12}{n^2}.$$

Compute the limit

$$\lim_{n\to\infty} n(nx_n - 2).$$

(Dorin Andrica, Babes-Bolyai University of Cluj-Napoca, Romania)

100. Let $f : (0,1) \to \mathbb{R}$ be infinitely differentiable (i.e., $C^\infty(0,1)$), with the property that for any $x \in (0,1)$ there exists some $n = n(x)$ nonnegative integer such that $f^{(n)}(x) = 0$. Show that f is a polynomial.

(Francesco Sica, University of Waterloo, Canada)

101. Examine if a two-variate algebraic polynomial $p(x,y)$ such that $p(x,y) \geq 0$ for all $x, y \in \mathbb{R}^2$ always possesses a point of local minimum.

(Vladimir Protasov, Moscow State University, Russia)

102. Find the value of the series

$$\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{1}{n+1} - \frac{1}{n+2} + \frac{1}{n+3} - \cdots\right).$$

(Ovidiu Furdui, Technical University of Cluj-Napoca, Romania)

103. Let $f : [a,b] \to \mathbb{R}$ be a convex function defined on the closed interval $[a,b]$. Prove that for any $x \in (a,b)$ the following holds

$$\frac{1}{2}\left[(b-x)^2 f'_+(x) - (x-a)^2 f'_-(x)\right] \leq \int_a^b f(t)\,dt - (b-a)f(x). \tag{1}$$

The constant $\frac{1}{2}$ on the left side of (1) is sharp in the sense that it cannot be replaced by a larger real constant.

(Sever S. Dragomir, Victoria University, Australia)

104. Let $f : [a,b] \to \mathbb{R}$ be a convex function defined on the closed interval $[a,b]$. Prove that for any $x \in [a,b]$, the following holds

$$\int_a^b f(t)\,dt - (b-a)f(x) \leq \frac{1}{2}\left[(b-x)^2 f'_-(b) - (x-a)^2 f'_+(a)\right]. \tag{2}$$

The constant $\frac{1}{2}$ on the right side of (2) is sharp in the sense that it cannot be replaced by a smaller constant.

(Sever S.Dragomir, Victoria University, Australia)

II Two new open problems

105[*]. Let $A_n = (a_{ij})$ be a real matrix of order n, with

$$a_{ij} = \sum_{k=1}^{j} (-1)^k \cos^{2i}\left(\frac{k}{j+1}\frac{\pi}{2}\right),$$

and let $B_n = (b_{ij})$ be the inverse of A_n. Prove that $b_{ij} = 0$, for $j > i+1$.

(Carlos M. da Fonseca, University of Coimbra, Portugal)

106[*]. Consider a function $P(t) = \mathrm{Re}\left(\sum_{k=1}^{n} p_k e^{a_k t}\right)$, where a_k, p_k are complex numbers, $\mathrm{Re}\,a_k < 0$, $|a_k| \leq 1$, $k = 1, \ldots, n$. Is it true that there exists an absolute constant C such that for every positive integer n and for every such function $P(t)$ the following holds

$$\|P'\|_{C[0,+\infty)} \leq C\,n\,\|P\|_{C[0,+\infty)}$$

Comment. This is a generalisation of the classical Markov inequality for polynomials to quasipolynomials, i.e., to linear combinations of exponential functions. For the moment we can prove this assertion for real a_k. In this case

$$\|P'\|_{C[0,+\infty)} \leq (4n+1)\|P\|_{C[0,+\infty)}$$

and this inequality is asymptotically sharp as $n \to \infty$.

(Vladimir Protasov, Moscow State University, Russia)

III Solutions

91. Let

$$S = \left\{\sum_{n=1}^{\infty} \frac{x_n}{3^n} : x_n = 0 \quad \text{or} \quad 1\right\}.$$

Show that S has Lebesgue measure 0 and find the set $S + S$.

(Wing-Sum Cheung, Department of Mathematics, University of Hong Kong, Hong Kong)

Solution by the proposer.

Observe that for any $x \in S$,

$$x \leq \sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{2}.$$

Thus $S \subset [0, \frac{1}{2}] \subset [0,1)$.

For any $x \in [0,1) \setminus S$, we have

$$x = \sum_{n=1}^{\infty} \frac{x_n}{3^n},$$

where the x_n's satisfy

(i) $0 \leq x_n \leq 2$ for all n,

(ii)   for any $n \in \mathbb{N}$, there exists $m \geq n$ such that $x_m \leq 1$, and

(iii) there exists $k \in \mathbb{N}$ such that $x_k = 2$.

Let
$$n = \min\{k \in \mathbb{N} : x_k = 2\} .$$

Then $0 \leq x_1, \ldots, x_{n-1} \leq 1$ and

$$x \in \left[\frac{x_1}{3} + \cdots + \frac{x_{n-1}}{3^{n-1}} + \frac{2}{3^n}, \frac{x_1}{3} + \cdots + \frac{x_{n-1}}{3^{n-1}} + \frac{3}{3^n} \right) .$$

Therefore, we have

$$[0, 1) \setminus S = \bigcup_{n \in \mathbb{N}} \left\{ \left[\frac{x_1}{3} + \cdots + \frac{x_{n-1}}{3^{n-1}} + \frac{2}{3^n}, \frac{x_1}{3} + \cdots + \frac{x_{n-1} + 1}{3^{n-1}} \right) : \right.$$
$$\left. 0 \leq x_1, \ldots, x_{n-1} \leq 1 \right\} .$$

Hence S is measurable and, since

$$\mu([0, 1) \setminus S) = \sum_{n=1}^{\infty} \sum_{0 \leq x_1, \ldots, x_{n-1} \leq 1} \frac{1}{3^n} = \sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n} = 1 ,$$

we conclude that $\mu(S) = 0$.

Finally, as $S \subset [0, \frac{1}{2}]$, we have $S + S \subset [0, 1]$. Conversely, for any $x \in [0, 1)$, write

$$x = \sum_{n=1}^{\infty} \frac{x_n}{3^n}$$

where the x_n's satisfy

(i) $0 \leq x_n \leq 2$ for all n and

(ii)   for any $n \in \mathbb{N}$, there exists $m \geq n$ such that $x_m \leq 1$.

Define, for any $n \in \mathbb{N}$,

$$y_n := \begin{cases} x_n & 0 \leq x_n \leq 1 \\ 1 & x_n = 2 , \end{cases}$$

$$z_n := \begin{cases} 0 & 0 \leq x_n \leq 1 \\ 1 & x_n = 2 , \end{cases}$$

and

$$y := \sum_{n=1}^{\infty} \frac{y_n}{3^n} ,$$

$$z := \sum_{n=1}^{\infty} \frac{z_n}{3^n} .$$

Then $y, z \in S$ and it is easy to verify that

$$x = y + z .$$

Since we have trivially that

$$1 = \sum_{n=1}^{\infty} \frac{1}{3^n} + \sum_{n=1}^{\infty} \frac{1}{3^n} \in S + S ,$$

we conclude that

$$S + S = [0, 1] .$$

Also solved by W. Fensch (Karlsruhe, Germany), John N. Lillington (Wareham, UK)

---

92. Let $(X, \mathcal{M}, \mu)$ be a measure space with $\mu(X) < \infty$ and $f : X \to \mathbb{R}$ be a measurable function with $|f| < 1$. Show that the limit

$$\lim_{n \to \infty} \int_X \sum_{k=0}^{n} f^k d\mu$$

either exists in $\mathbb{R}$ or equals $+\infty$.

(Wing-Sum Cheung, Department of Mathematics, University of Hong Kong, Hong Kong)

Solution by the proposer.

Write $f = f_+ - f_-$, where, as usual,

$$f_+ = \max\{f, 0\}$$
$$f_- = \max\{-f, 0\} .$$

Then, for any $n \in \mathbb{N}$,

$$f^n = (f_+)^n + (-f_-)^n$$

and so

$$\int \sum_{k=0}^{n} f^k = \int \sum_{k=0}^{n} (f_+)^k + \int \sum_{k=1}^{n} (-f_-)^k$$
$$= \int \frac{1 - (f_+)^{k+1}}{1 - f_+} + \int \frac{(-f_-) - (-f_-)^{k+1}}{1 + f_-} .$$

Since

$$0 \leq \frac{(-f_-) - (-f_-)^{k+1}}{1 + f_-} \leq \frac{-f_-}{1 + f_-} \quad \text{for all} \quad k \in \mathbb{N}$$

and $\frac{-f_-}{1+f_-}$ is integrable, by Lebesgue's Dominated Convergence Theorem, we have

$$\lim_{n \to \infty} \int \frac{(-f_-) - (-f_-)^{k+1}}{1 + f_-} = \int \frac{-f_-}{1 + f_-} < \infty .$$

On the other hand, since

$$0 \leq \frac{1 - (f_+)^{k+1}}{1 - f_+} \uparrow \frac{1}{1 - f_+} ,$$

by Lebesgue's Monotone Convergence Theorem, we have

$$\lim_{n \to \infty} \int \frac{1 - (f_+)^{k+1}}{1 - f_+} = \int \frac{1}{1 - f_+} \in [0, \infty] .$$

So the limit

$$\lim_{n \to \infty} \int \sum_{k=0}^{n} f^k = \int \frac{1}{1 - f_+} + \int \frac{-f_-}{1 + f_-}$$

either exists in $\mathbb{R}$ or equals $+\infty$, depending on whether $\frac{1}{1-f_+}$ is integrable.

Also solved by John N. Lillington (Wareham, UK)

---

93. Find all functions $f : (0, \infty) \to (0, \infty)$ which are differentiable at $x = 1$ and satisfy the property that

$$f(f(x)) = x^2 \quad \text{for every} \quad x \in (0, \infty).$$

(Dorin Andrica, Babes-Bolyai University of Cluj-Napoca, Romania)

Solution by the proposer.

We will show that f satisfies the relation

$$\sqrt{f(x)} = f(\sqrt{x}),$$

for every $x \in (0, \infty)$. Indeed, from the relation in the hypothesis it follows

$$f\big(f(f(x))\big) = f(x^2),$$

hence we get $f^2(x) = f(x^2)$ for every $x \in (0, \infty)$. Replacing x by $\sqrt{x}$ we obtain the desired relation. Now, we proceed in two steps.

1. Considering $x = 1$, it follows that $f^2(1) = f(1)$ and hence $f(1) = 1$. Also, f is continuous at 1 since it is differentiable at 1.

2. By induction, we obtain

$$f(x)^{\frac{1}{2^n}} = f(x^{\frac{1}{2^n}})$$

for every positive integer n and for every $x \in (0, \infty)$. We have

$$f(x) = \lim_{n \to \infty} \left(f(x^{\frac{1}{2^n}}) \right)^{2^n}$$

$$= \lim_{n \to \infty} \left(1 + (f(x^{\frac{1}{2^n}}) - 1) \right)^{2^n}$$

$$= e^{\lim_{n \to \infty} 2^n \left(f(x^{\frac{1}{2^n}}) - 1 \right)}.$$

Denoting $t = x^{\frac{1}{2^n}}$ we get $2^n = \frac{\ln x}{\ln t}$ and the limit at the exponent becomes

$$\lim_{n \to \infty} 2^n \left(f(x^{\frac{1}{2^n}}) - 1 \right) = \lim_{t \to 1} \frac{\ln x}{\ln t} (f(t) - 1)$$

$$= (\ln x) \lim_{t \to 1} \frac{t - 1}{\ln t} \frac{f(t) - f(1)}{t - 1} = f'(1) \ln x.$$

Therefore $f(x) = x^c$, where $c = f'(1)$. Checking the relation in the hypothesis it follows that $x^{c^2} = x^2$ and hence $c = \pm\sqrt{2}$. The desired functions are $f(x) = x^{\pm\sqrt{2}}$.

Also solved by Enrique Macías-Virgós (University of Santiago de Compostela, Spain), W. Fensch (Karlsruhe, Germany)

---

94. (1) Let $f : [0, \infty) \to \mathbb{R}$ be a convex differentiable function with $f(0) = 0$.
Prove that

$$\int_0^x f(t)dt \leq \frac{x^2}{2} f'(x) \text{ for all } x \in [0, \infty).$$

(2) Find all differentiable functions $f : [0, \infty) \to \mathbb{R}$ for which equality holds in the above inequality.

(Dorin Andrica, Babes-Bolyai University of Cluj-Napoca, and Mihai Piticari, National College "Dragos Voda" Campulung Moldovenesc, Romania)

Solution by the proposer.

1. We have

$$\int_0^x f(t)dt = \int_0^x (f(t) - f(0)) \, dt = \int_0^x t f'(c_t)dt,$$

where $c_t \in (0, t)$. Because of the fact f is convex, it follows that f' is increasing and we get

$$\int_0^x f(t)dt = \int_0^x t f'(c_t)dt \leq \int_0^x t f'(x)dt = \frac{x^2}{2} f'(x).$$

2. Let $F(x) = \int_0^x f(t)dt$. Then

$$F(x) = \frac{x^2}{2} F''(x). \tag{1}$$

Let $x = e^t$, $z(t) = F(e^t)$, $t \in \mathbb{R}$, $x > 0$. We have

$$z'(t) = F'(e^t)e^t, \tag{2}$$

hence relation

$$F'(e^t) = e^{-t}z'(t)$$

implies

$$f''(e^t)e^t = e^{-t}z''(t) - e^{-t}z'(t).$$

We obtain

$$F''(e^t) = e^{-2t}(z''(t) - z'(t)). \tag{3}$$

Replacing in (1) gives the follows differential equation

$$z(t) = \frac{1}{2}(z''(t) - z'(t)),$$

which is the second order differential equation with constant coefficients

$$z''(t) - z'(t) - 2z(t) = 0.$$

Its characteristic equation is $r^2 - r - 2 = 0$ with the roots $r_1 = -1, r_2 = 2$.
We get

$$z(t) = C_1 e^{-t} + C_2 e^{2t}, \ C_1, C_2 \in \mathbb{R},$$

hence

$$F(x) = \frac{C_1}{x} + C_2 x^2, \ \forall \ x \in (0, \infty),$$

implying

$$f(x) = F'(x) = -\frac{C_1}{x^2} + 2C_2 x.$$

Because of the fact that f is differentiable at $x = 0$ it follows that $C_1 = 0$, hence we obtain $f(x) = ax$, $a \in \mathbb{R}$. Clearly, all these functions satisfy the desired condition.

Also solved by W. Fensch (Karlsruhe, Germany), P. T. Krasopoulos (Athens, Greece), John N. Lillington (Wareham, UK), Paolo Secchi (University of Brescia, Italy)

---

95. Let $k \geq 2$ and j be such that $0 \leq j \leq k$. Assume that T_j is the multiple series

$$\sum_{n_1, n_2, \ldots, n_k = 1}^{\infty} n_1 n_2 \cdots n_j (n_1 + n_2 + \cdots + n_k - \zeta(2) - \zeta(3) -$$

$$\cdots - \zeta(n_1 + n_2 + \cdots + n_k)),$$

where the product $n_1 \cdots n_j$ disappears when $j = 0$. Prove that

$$T_j = \sum_{m=0}^{j} \binom{j}{m} \zeta(k + 1 + j - m),$$

where ζ denotes the Riemann zeta function.

(Ovidiu Furdui, Technical University of Cluj-Napoca, Romania)

Solution by the proposer.

First, we prove that

$$S_n = \sum_{k=1}^{\infty} \frac{1}{k(k + 1)^n} = n - \zeta(2) - \zeta(3) - \cdots - \zeta(n).$$

We have, since

$$\frac{1}{k(k + 1)^n} = \frac{1}{k(k + 1)^{n-1}} - \frac{1}{(k + 1)^n},$$

that

$$\sum_{k=1}^{\infty} \frac{1}{k(k + 1)^n} = \sum_{k=1}^{\infty} \frac{1}{k(k + 1)^{n-1}} - \sum_{k=1}^{\infty} \frac{1}{(k + 1)^n}$$

and hence

$$S_n = S_{n-1} - (\zeta(n) - 1).$$

Iterating this equality we obtain

$$S_n = S_1 - (\zeta(2) + \zeta(3) + \cdots + \zeta(n) - (n - 1))$$

and, since

$$S_1 = \sum_{k=1}^{\infty} 1/(k(k+1)) = 1,$$

we get that

$$S_n = n - \zeta(2) - \zeta(3) - \cdots - \zeta(n).$$

We have

$$T_j = \sum_{n_1,n_2,\ldots,n_k=1}^{\infty} n_1 n_2 \cdots n_j \left(\sum_{p=1}^{\infty} \frac{1}{p(p+1)^{n_1+n_2+\cdots+n_k}} \right)$$

$$= \sum_{p=1}^{\infty} \frac{1}{p} \left(\sum_{n_1=1}^{\infty} \frac{n_1}{(p+1)^{n_1}} \right) \cdots \left(\sum_{n_j=1}^{\infty} \frac{n_j}{(p+1)^{n_j}} \right)$$

$$\left(\sum_{n_{j+1}=1}^{\infty} \frac{1}{(p+1)^{n_{j+1}}} \right) \cdots \left(\sum_{n_k=1}^{\infty} \frac{1}{(p+1)^{n_k}} \right)$$

$$= \sum_{k=1}^{\infty} \frac{1}{p} \left(\sum_{m=1}^{\infty} \frac{m}{(p+1)^m} \right)^j \left(\sum_{s=1}^{\infty} \frac{1}{(p+1)^s} \right)^{k-j}$$

$$= \sum_{k=1}^{\infty} \frac{1}{p} \left(\frac{p+1}{p^2} \right)^j \left(\frac{1}{p} \right)^{k-j}.$$

Thus

$$T_j = \sum_{k=1}^{\infty} \frac{(p+1)^j}{p^{k+j+1}}$$

$$= \sum_{k=1}^{\infty} \frac{1}{p^{k+j+1}} \left(\sum_{m=0}^{j} \binom{j}{m} p^m \right)$$

$$= \sum_{m=0}^{j} \binom{j}{m} \sum_{p=1}^{\infty} \frac{1}{p^{k+j+1-m}}$$

$$= \sum_{m=0}^{j} \binom{j}{m} \zeta(k+j+1-m)$$

and the problem is solved.

Remark. When $j = 0$ one has that

$$T_0 = \sum_{n_1,n_2,\ldots,n_k=1}^{\infty} (n_1 + n_2 + \cdots + n_k - \zeta(2) - \zeta(3) - \cdots$$

$$- \zeta(n_1 + n_2 + \cdots + n_k)) = \zeta(k+1).$$

When $j = 1$ one has that

$$T_1 = \sum_{n_1,n_2,\ldots,n_k=1}^{\infty} n_1(n_1 + n_2 + \cdots + n_k - \zeta(2) - \zeta(3) - \cdots$$

$$- \zeta(n_1 + n_2 + \cdots + n_k))$$

$$= \zeta(k+2) + \zeta(k+1).$$

When $j = k$ one has that

$$T_k = \sum_{n_1,n_2,\ldots,n_k=1}^{\infty} n_1 \cdots n_k(n_1 + n_2 + \cdots + n_k - \zeta(2) - \zeta(3) - \cdots -$$

$$\zeta(n_1 + n_2 + \cdots + n_k))$$

$$= \sum_{m=0}^{k} \binom{k}{m} \zeta(2k+1-m).$$

Also solved by Sotirios E. Louridas (Athens, Greece)

96. A function $f : (0,1) \to (0,+\infty)$ possesses the following property:

$$\frac{f(x)}{f(y)} \le 1 + \frac{x}{y} \text{ for every } x, y \in (0,1).$$

Prove the existence of a finite limit $\lim_{x \to 0} f(x)$.

(*Vladimir Protasov, Department of Mechanics and Mathematics, Moscow State University, Russia*)

Solution by the proposer.

Take arbitrary $y \in (0,1)$. For every $x \in (0, y)$ we have

$$f(x) \le \left(1 + \frac{x}{y}\right) f(y) \le 2 f(y).$$

Whence, the function $f(x)$ is bounded on the interval $(0, y)$. If this function does not have a limit as $x \to 0$ then there are sequences $\{a_k\}$ and $\{b_k\}$ both converging to zero such that both limits $a = \lim_{k \to \infty} f(a_k)$ and $b = \lim_{k \to \infty} f(b_k)$ exist and are different, i.e. $a \ne b$. Let us fix an arbitrary n. For every k we have

$$f(a_k) \le \left(1 + \frac{a_k}{b_n}\right) f(b_n).$$

Taking the limit as $k \to \infty$ we obtain $a \le f(b_n)$. Taking the limit as $n \to \infty$, we have $a \le b$. Similarly one shows that $b \le a$ and therefore $a = b$. The contradiction concludes the proof.

Also solved by W. Fensch (Karlsruhe, Germany), P. T. Krasopoulos (Athens, Greece), John N. Lillington (Wareham, UK), Paolo Secchi (University of Brescia, Italy)

Open Problem 98*. Let $A_n = \left(a_{i,j}\right)_{1 \le i, j \le n}$ be the square matrix with real entries

$$a_{i,j} = \sum_{k=1}^{j} (-1)^k \cos^{2i}\left(\frac{k}{j+1} \cdot \frac{\pi}{2}\right).$$

Prove that

$$\det A_n = (-1)^n \frac{n!}{2^{n^2}}.$$

Solution.

Let us first analyse the entries of a_{ij} for $j \ge i$. Since

$$\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2},$$

we have

$$a_{ij} = \frac{1}{2^{2i}} \sum_{k=1}^{j} (-1)^k \left(e^{i\frac{k}{j+1}\frac{\pi}{2}} + e^{-i\frac{k}{j+1}\frac{\pi}{2}} \right)^{2i}$$

$$= \frac{1}{2^{2i}} \sum_{k=1}^{j} (-1)^k \sum_{\ell=0}^{2i} \binom{2i}{\ell} e^{-i\frac{k(i-\ell)}{j+1}\pi}$$

$$= \frac{1}{2^{2i}} \sum_{\ell=0}^{2i} \binom{2i}{\ell} \sum_{k=1}^{j} (-1)^k e^{-i\frac{k(i-\ell)}{j+1}\pi}$$

$$= \frac{1}{2^{2i}} \sum_{\ell=0}^{2i} \binom{2i}{\ell} \frac{(-1)^{j+\ell-i} - e^{i\frac{\ell-i}{j+1}\pi}}{1 + e^{i\frac{\ell-i}{j+1}\pi}} \qquad (3)$$

$$= \frac{1}{2^{2i}} \sum_{\substack{\ell=0 \\ \text{odd}}}^{2i} \binom{2i}{\ell} \frac{(-1)^{j+\ell-i} - e^{i\frac{\ell-i}{j+1}\pi}}{1 + e^{i\frac{\ell-i}{j+1}\pi}} + \frac{1}{2^{2i}} \sum_{\substack{\ell=0 \\ \text{even}}}^{2i} \binom{2i}{\ell} \frac{(-1)^{j+\ell-i} - e^{i\frac{\ell-i}{j+1}\pi}}{1 + e^{i\frac{\ell-i}{j+1}\pi}}$$

$$(4)$$

Now, if we assume that $j - i$ is even then we get

$$
\begin{aligned}
a_{ij} &= -\frac{1}{2^{2i}} \sum_{\substack{\ell=0 \\ \text{odd}}}^{2i} \binom{2i}{\ell} + \frac{1}{2^{2i}} \sum_{\substack{\ell=0 \\ \text{even}}}^{2i} \binom{2i}{\ell} \frac{1 - e^{\mathbf{i} \frac{\ell-i}{j+1} \pi}}{1 + e^{\mathbf{i} \frac{\ell-i}{j+1} \pi}} \\
&= -\frac{1}{2^{2i}} 2^{2i-1} + 0 \\
&= -\frac{1}{2}
\end{aligned}
$$

Note that

$$
\frac{1 - e^{\mathbf{i} \frac{(2i-\ell)-i}{j+1} \pi}}{1 + e^{\mathbf{i} \frac{(2i-\ell)-i}{j+1} \pi}} = \frac{1 - e^{-\mathbf{i} \frac{\ell-i}{j+1} \pi}}{1 + e^{-\mathbf{i} \frac{\ell-i}{j+1} \pi}} = -\frac{1 - e^{\mathbf{i} \frac{\ell-i}{j+1} \pi}}{1 + e^{\mathbf{i} \frac{\ell-i}{j+1} \pi}},
$$

i.e.

$$
\frac{1 - e^{\mathbf{i} \frac{\ell-i}{j+1} \pi}}{1 + e^{\mathbf{i} \frac{\ell-i}{j+1} \pi}}
$$

is odd under the exchange $\ell \to 2i - \ell$.

Observe also that the denominator in (3) does not vanish since we are assuming that $j \geq i$. In fact,

$$
|l - i| \leq i \leq j < j + 1
$$

and, consequently,

$$
1 + e^{\mathbf{i} \frac{\ell-i}{j+1} \pi} \neq 0.
$$

If $j - i$ is odd, the analysis is analogous and the conclusion the same, since the first summation in (4) is 0 and the other is $-1/2$.

Thus, to evaluate $\det A_n$ we only need to find the value of $a_{i+1,i}$, for $i = 1, \ldots, n-1$.

$$
\begin{aligned}
a_{i+1,i} &= \frac{1}{2^{2i+2}} \sum_{k=1}^{i} (-1)^k \left(e^{\mathbf{i} \frac{k}{i+1} \frac{\pi}{2}} + e^{-\mathbf{i} \frac{k}{i+1} \frac{\pi}{2}} \right)^{2i+2} \\
&= \frac{1}{2^{2i+2}} \sum_{k=1}^{i} (-1)^k \sum_{\ell=0}^{2i+2} \binom{2i+2}{\ell} e^{-\mathbf{i} \frac{k(i-\ell+1)}{i+1} \pi} \\
&= \frac{1}{2^{2i+2}} \left(2i + \sum_{\ell=1}^{2i+1} \binom{2i+2}{\ell} \sum_{k=1}^{i} (-1)^k e^{-\mathbf{i} \frac{k(i-\ell+1)}{i+1} \pi} \right) \quad (5) \\
&= \frac{1}{2^{2i+2}} \left(2i + \sum_{\ell=1}^{2i+1} \binom{2i+2}{\ell} \frac{(-1)^{\ell+1} + e^{\mathbf{i} \frac{\ell}{i+1} \pi}}{1 - e^{\mathbf{i} \frac{\ell}{i+1} \pi}} \right) \quad (6) \\
&= \frac{1}{2^{2i+2}} \left(2i - 2^{2i+1} + 2 \right) \quad (7) \\
&= \frac{i+1}{2^{2i+1}} - \frac{1}{2}.
\end{aligned}
$$

Note that considering separately the cases $\ell = 0$ and $\ell = 2i + 2$ in (5), we avoid the poles in (6). The procedure for getting (7) follows the same procedure as the previous case.

Therefore

$$
a_{n,n-1} = \frac{n}{2^{2n-1}} - \frac{1}{2}.
$$

Now

$$
A_n = \begin{pmatrix}
-\frac{1}{2} & -\frac{1}{2} & \cdots & -\frac{1}{2} & -\frac{1}{2} \\
a_{21} & -\frac{1}{2} & \cdots & -\frac{1}{2} & -\frac{1}{2} \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \ddots & \ddots & -\frac{1}{2} & -\frac{1}{2} \\
a_{n,1} & \cdots & \cdots & a_{n,n-1} & -\frac{1}{2}
\end{pmatrix}
$$

and applying elementary row operations of A_n (using the first row), we have

$$
\begin{aligned}
\det A_n &= \begin{vmatrix}
-\frac{1}{2} & -\frac{1}{2} & \cdots & -\frac{1}{2} & -\frac{1}{2} \\
\frac{2 \times 2}{2^{2 \times 2}} & 0 & \cdots & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
\vdots & \ddots & \ddots & 0 & 0 \\
* & \cdots & \cdots & \frac{2n}{2^{2n}} & 0
\end{vmatrix} \\
&= (-1)^n \frac{1}{2} \prod_{\ell=2}^{n} \frac{2\ell}{2^{2\ell}} \\
&= (-1)^n \frac{2^{n-1} n!}{2^{n^2+n-1}} \\
&= (-1)^n \frac{n!}{2^{n^2}}.
\end{aligned}
$$

Remark. Problems 83 and 84 were also solved by Francesco Sica (University of Waterloo, Canada). Open problem 89* was also solved by Francesco Sica.

We wait to receive your solutions to the proposed problems and ideas on the open problems. Send your solutions both by ordinary mail to Themistocles M. Rassias, Department of Mathematics, National Technical University of Athens, Zografou Campus, GR 15780, Athens, Greece, and by email to trassias@math.ntua.gr.

We also solicit your new problems with their solutions for the next "Solved and Unsolved Problems" column, which will be devoted to *Real Analysis*.

Koen Thas (Ghent University, Belgium)

A Course on Elation Quadrangles (EMS Series of Lectures in Mathematics)

ISBN 978-3-03719-110-1. 2012. 129 pages. Softcover. 17 x 24 cm. 28.00 Euro

The notion of elation generalized quadrangle is a natural generalization to the theory of generalized quadrangles of the important notion of translation planes in the theory of projective planes. Almost any known class of finite generalized quadrangles can be constructed from a suitable class of elation quadrangles.

In this book the author considers several aspects of the theory of elation generalized quadrangles. Special attention is given to local Moufang conditions on the foundational level, exploring for instance a question of Knarr from the 1990s concerning the very notion of elation quadrangles. All the known results on Kantor's prime power conjecture for finite elation quadrangles are gathered, some of them published here for the first time. The structural theory of elation quadrangles and their groups is heavily emphasized. Other related topics, such as p-modular cohomology, Heisenberg groups and existence problems for certain translation nets, are briefly touched.

The text starts from scratch and is essentially self-contained. Many alternative proofs are given for known theorems. Containing dozens of exercises at various levels, from very easy to rather difficult, this course will stimulate undergraduate and graduate students to enter the fascinating and rich world of elation quadrangles. The more accomplished mathematician will especially find the final chapters challenging.
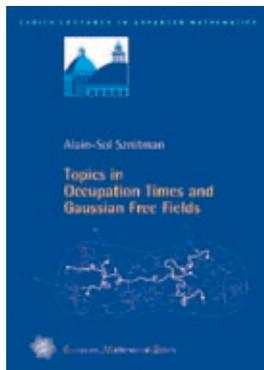
Alain-Sol Sznitman (ETH Zürich, Switzerland)

Topics in Occupation Times and Gaussian Free Fields (Zurich Lectures in Advanced Mathematics)

978-3-03719-109-5. 2012. 122 pages. Softcover. 17 x 24 cm. 28.00 Euro

This book grew out of a graduate course at ETH Zurich during the Spring term 2011. It explores various links between such notions as occupation times of Markov chains, Gaussian free fields, Poisson point processes of Markovian loops, and random interlacements, which have been the object of intensive research over the last few years. These notions are developed in the convenient set-up of finite weighted graphs endowed with killing measures.

The book first discusses elements of continuous-time Markov chains, Dirichlet forms, potential theory, together with some consequences for Gaussian free fields. Next, isomorphism theorems and generalized Ray-Knight theorems, which relate occupation times of Markov chains to Gaussian free fields, are presented. Markovian loops are constructed and some of their key properties derived. The field of occupation times of Poisson point processes of Markovian loops is investigated. Of special interest are its connection to the Gaussian free field, and a formula of Symanzik. Finally, links between random interlacements and Markovian loops are discussed, and some further connections with Gaussian free fields are mentioned.
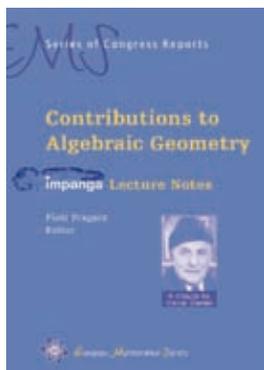
Contributions to Algebraic Geometry. Impanga Lecture Notes (EMS Series of Congress Reports)

Piotr Pragacz (IM PAN, Warsaw, Poland), Editor

ISBN 978-3-03719-114-9. 2012. 516 pages. Hardcover. 17 x 24 cm. 98.00 Euro

The articles in this volume are the outcome of the Impanga Conference on Algebraic Geometry in 2010 at the Banach Center in Będlewo. The following spectrum of topics is covered: K3 surfaces and Enriques surfaces; Prym varieties and their moduli; invariants of singularities in birational geometry; differential forms on singular spaces; Minimal Model Program; linear systems; toric varieties; Seshadri and packing constants; equivariant cohomology; Thom polynomials; arithmetic questions.

The main purpose of the volume is to give comprehensive introductions to the above topics through texts starting from an elementary level and ending with the discussion of current research. The first four topics are represented by the notes from the minicourses held during the conference. In the articles the reader will find classical results and methods, as well as modern ones. The book is addressed to researchers and graduate students in algebraic geometry, singularity theory and algebraic topology. Most of the material exposed in the volume has not yet appeared in book form.
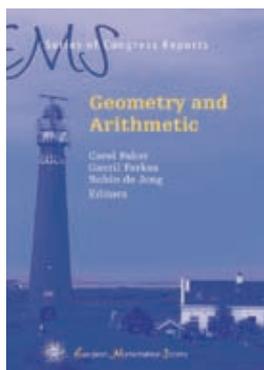
Geometry and Arithmetic (EMS Series of Congress Reports)

Carel Faber (Royal Institute of Technology, Stockholm, Sweden), Gavril Farkas (Humboldt-Universität zu Berlin, Germany) and Robin de Jong (University of Leiden, The Netherlands), Editors,

ISBN 978-3-03719-119-4. 2012. 384 pages. Hardcover. 17 x 24 cm. 78.00 Euro

This volume contains 21 articles written by leading experts in the fields of algebraic and arithmetic geometry. The treated topics range over a variety of themes, including moduli spaces of curves and abelian varieties, algebraic cycles, vector bundles and coherent sheaves, curves over finite fields, and algebraic surfaces, among others.

The volume originates from the conference "Geometry and Arithmetic", which was held on the island of Schiermonnikoog in The Netherlands in September 2010.

Contributors: V. Alexeev, I. Bauer, A. Beauville, F. Catanese, C. Ciliberto, J.I. Cogolludo-Agustín, J.-M. Couveignes, B. Edixhoven, F. Eusen, F. Flamini, A. Gibney, B.H. Gross, G. Harder, J. Heinloth, E.W. Howe, H. Ito, T. Katsura, R. Kloosterman, S. Kondō, S.J. Kovács, H. Lange, K.E. Lauter, P.E. Newstead, R. Pandharipande, D. Petersen, A. Pixton, R. Schoof, F.-O. Schreyer, S. Schröer, D. Swinarski, C. Voisin, Y.G. Zarhin