Lattice QCD for pedestrians (Lattice QCD for physicists)

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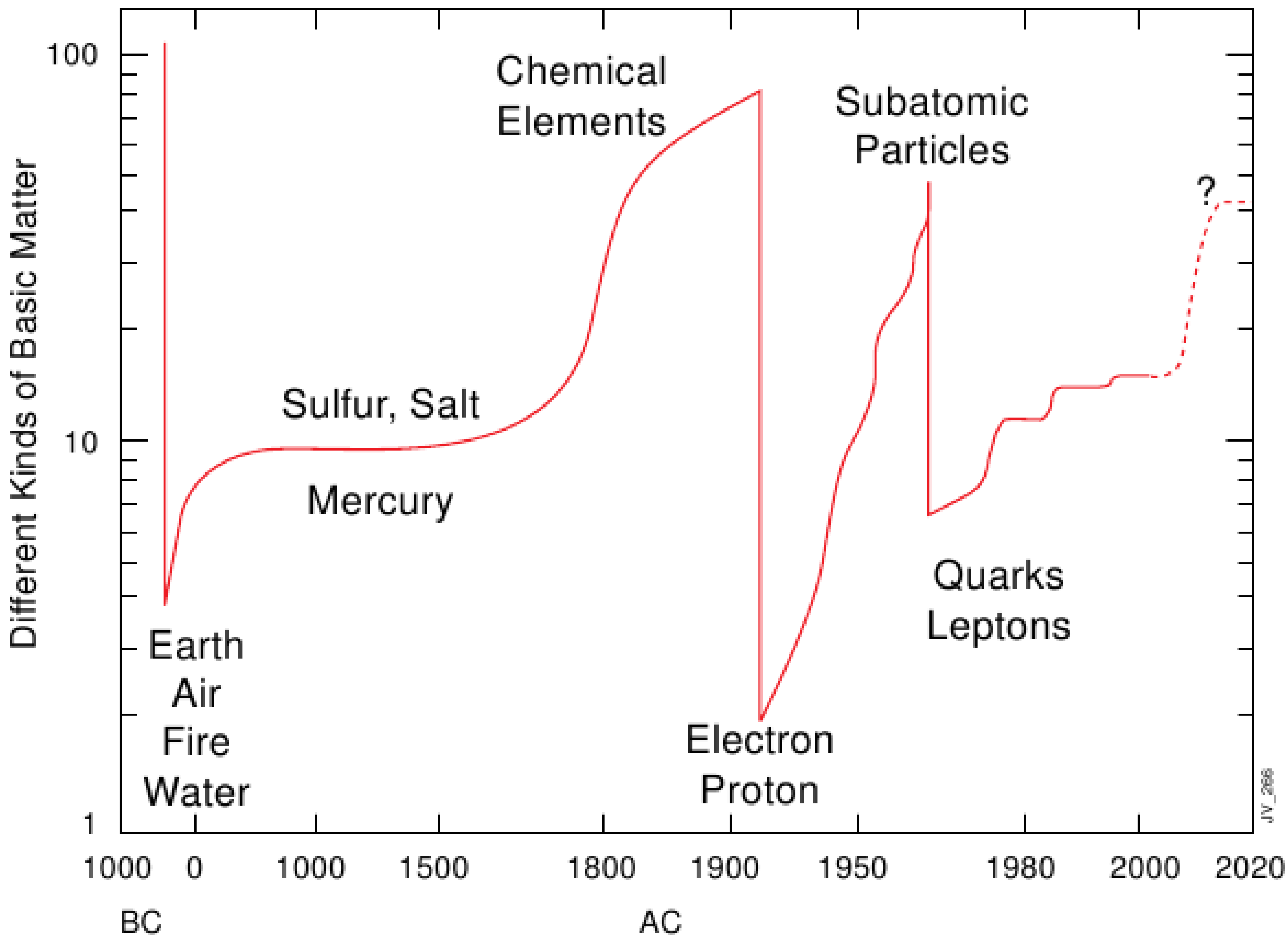
Overview (1): history of “elementary particle physics”



Tejinder S Virdee, Weighty Matters, Inaugural Lecture, 1998

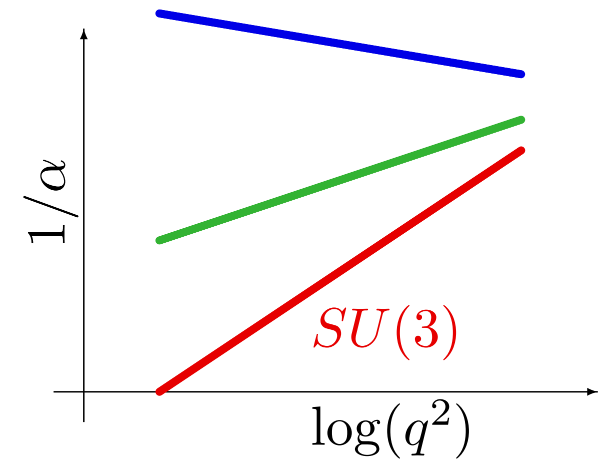
Overview (2): Standard Model with strong interactions

matter:

ν_e	ν_μ	ν_τ
e	μ	τ
u	c	t
d	s	b

forces:

$$\underbrace{U(1)\times SU(2)}_{\text{EW}}\times\underbrace{SU(3)}_{\text{QCD}}$$



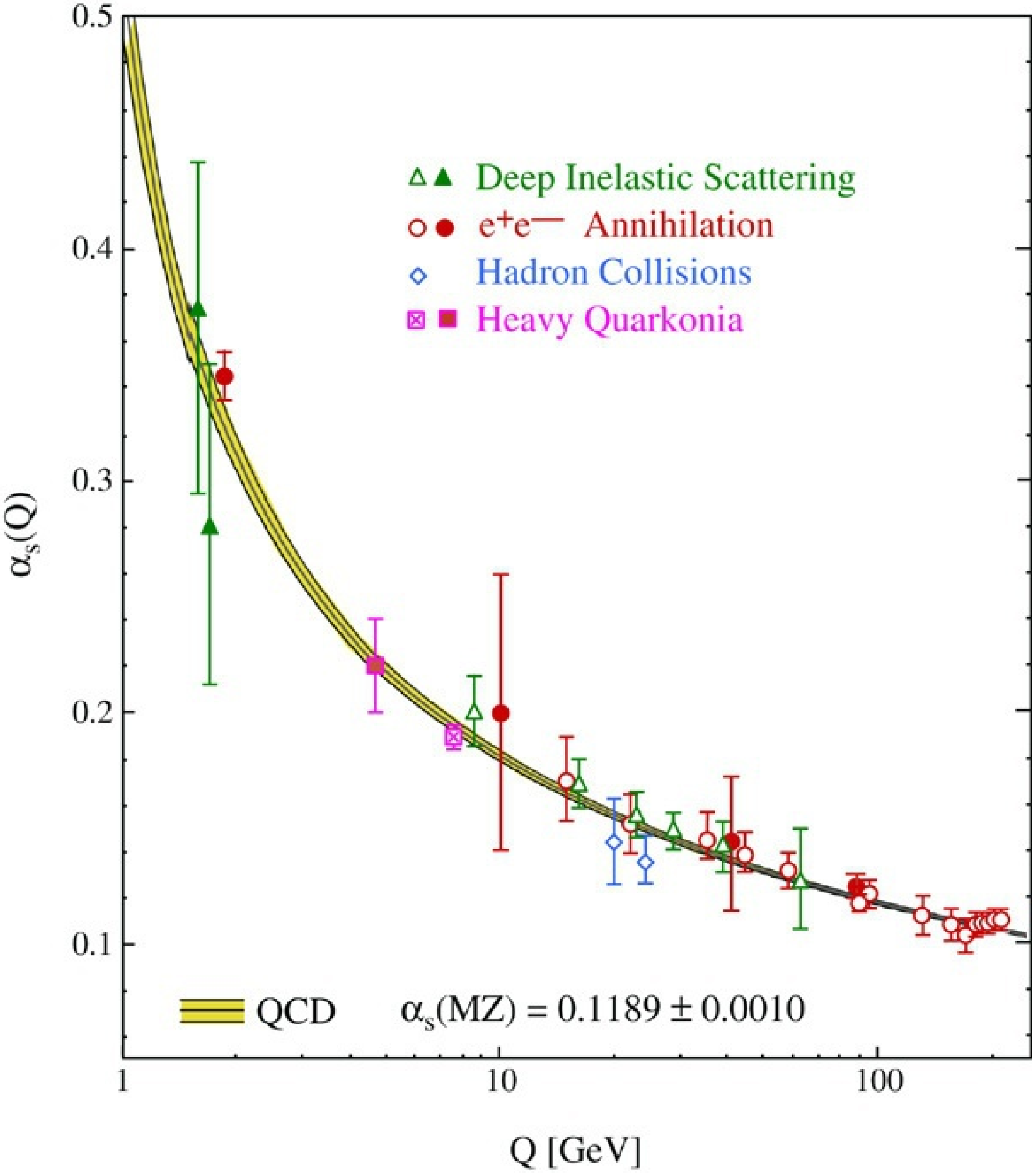
Relevant parameters for strong interaction: $\alpha_{\mathrm{QCD}}, m_{d,u,s,c,\ldots}$ with basic law

$$S_{\mathrm{QCD}} = \frac{1}{2}\mathrm{tr}(G_{\mu\nu}G^{\mu\nu}) + \sum_{q=d,u,s,c\ldots} \bar{q}(D+m_q)q$$

Phenomenology of QCD with $1+N_f$ parameters

- non-Abelian gauge symmetry $\Longrightarrow$ non-linear
- asymptotic freedom $\Longrightarrow$ perturbation theory at high energy
- confinement $\Longrightarrow$ hadrons $\neq$ fundamental degrees of freedom
- spontaneous breaking of chiral symmetry $\Longrightarrow$ $M_\pi \ll 4\pi F_\pi$

Overview (3): QCD at high energies



Asymptotic freedom

[t'Hooft 1972, Gross-Wilczek/Politzer 1973]

$$\frac{\beta(\alpha)}{\alpha} = \frac{\mu}{\alpha}\frac{\partial\alpha}{\partial\mu} = \beta_1\alpha^1 + \beta_2\alpha^2 + \ldots$$

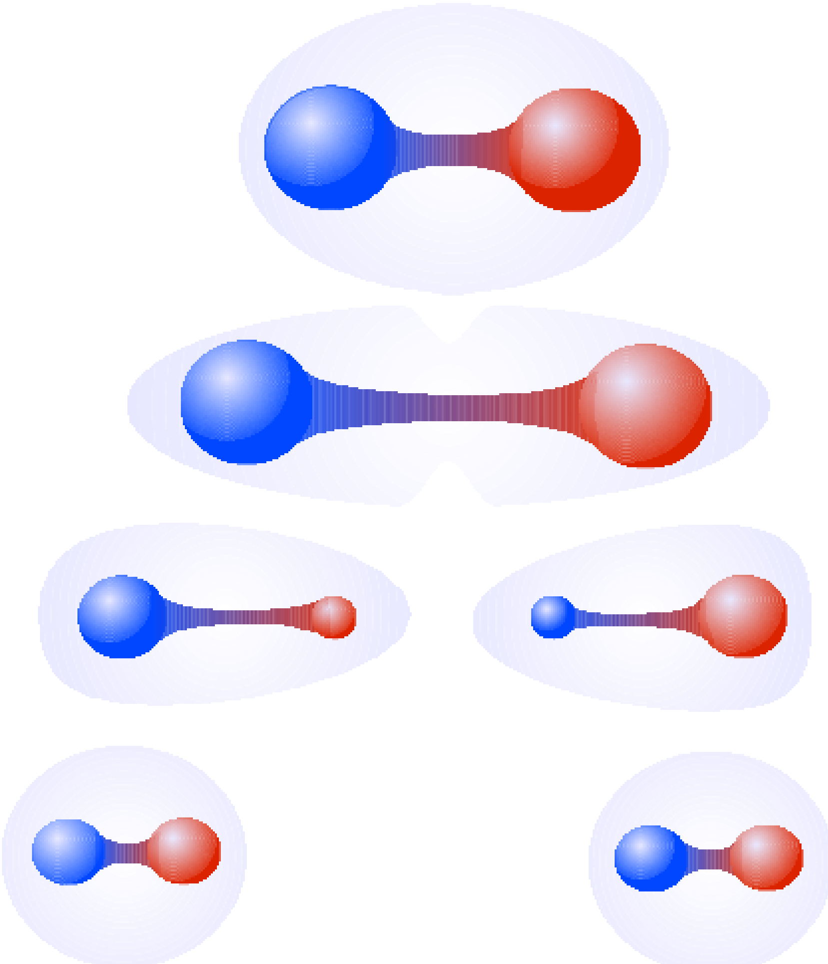
$$\beta_1 = (-11N_c + 2N_f)/(6\pi)$$

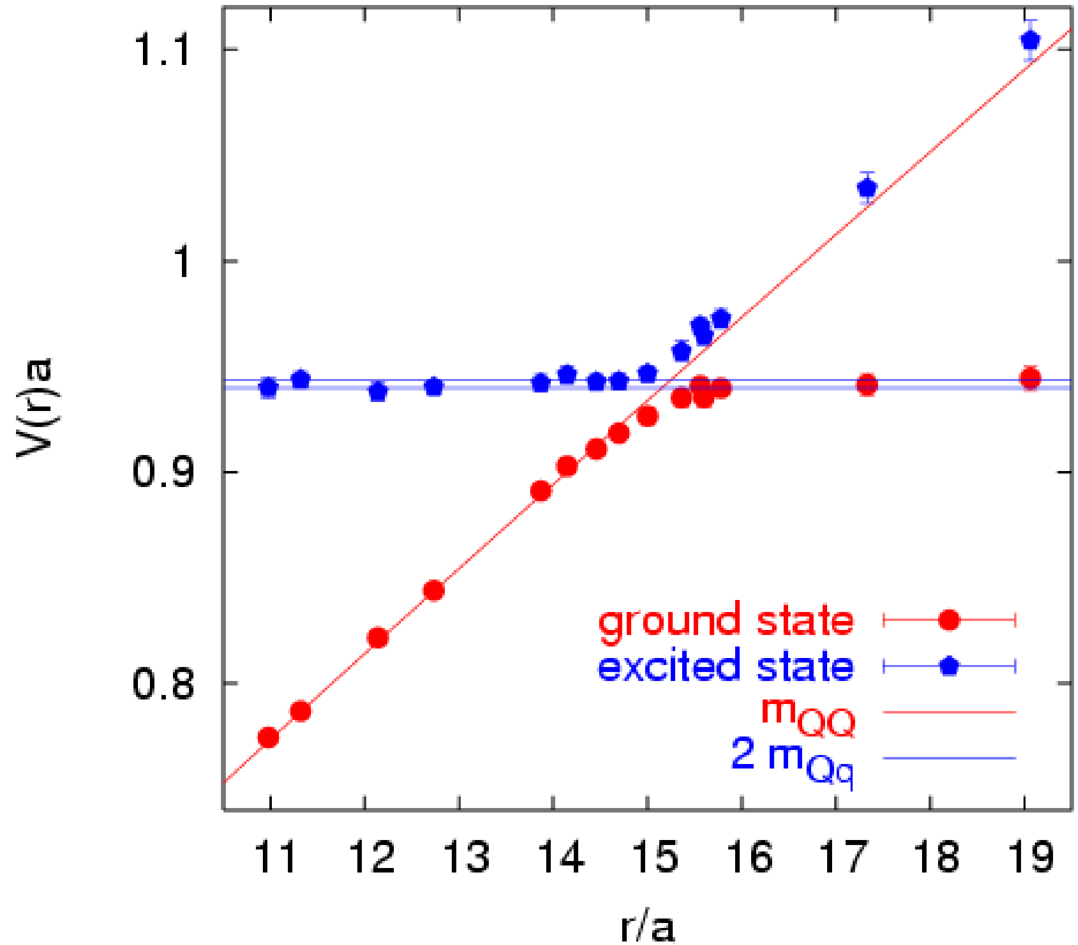
with $N_c\!=\!3$ gives

$\beta_1\!<\!0$ for $N_f\!<\!33/2$

- virtual gluons anti-screen, i.e. they make a static color source appear *stronger* at large distance.
- virtual quarks weaken this effect.

Overview (4): QCD at low energies

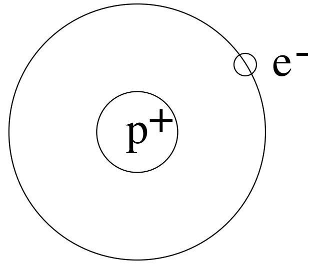




- In quenched QCD the $\bar{Q}Q$ potential keeps growing, $V(r)=\alpha/r+\mathrm{const}+\sigma r$.
- In full QCD it is energetically more favorable to pop a light $\bar{q}q$ pair out of the vacuum, $V(r)\leq\mathrm{const}$. Analysis with explicit $\bar{Q}q\bar{q}Q$ state: Bali et al., PRD 71, 114513 (2005).

Overview (5): QED versus QCD bound state dynamics

QED



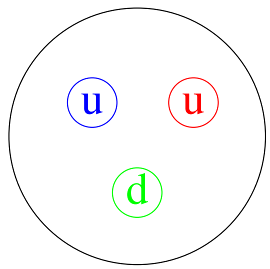
$M_e = 0.5$ MeV

$M_p = 938$ MeV

$E_{binding} = 13.6$ eV

Hydrogen Atom (EM force)

QCD



$M_u \sim 2.2$ MeV

$M_d \sim 4.8$ MeV

$M_p = 938$ MeV

Proton (Strong force)

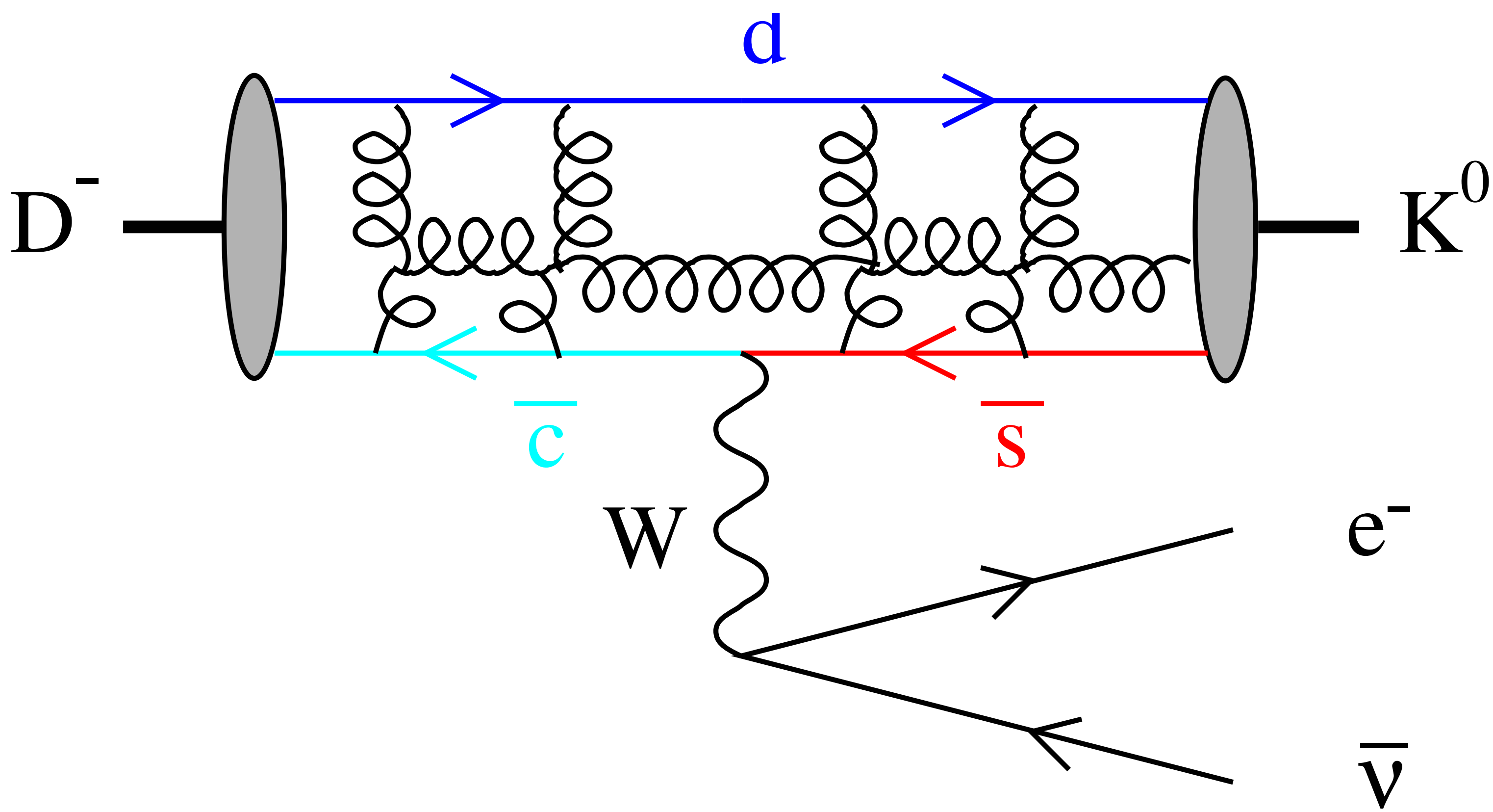
Q0: What is the physical meaning of the "wrong sign" of the proton binding energy if *current quark masses* are used ?

Q1: Do we understand strong dynamics sufficiently well as to postdict the mass of the proton ?

Q2: If so, can we turn the calculation around and determine $m_{ud} = (m_u + m_d)/2$ from first principles ?

Overview (6): separating EW from QCD dynamics

Consider $D^- \to K^0 e^- \bar{\nu}_e$, mediated through flavor changing weak decay $\bar{c} \to \bar{s} W^-$



Experiment: $\Gamma \propto |V_{cs} f_+^{D\to K}(q_*^2)|^2$ and $\Gamma \propto |V_{cs} f_{D_s}|^2$ in semileptonic/leptonic decay

How do we separate QCD "contamination" from EW "vertex" and extract V_{cs} ?

Would QCD result be precise enough to track BSM physics through inconsistencies ?

Talk outline

(1) Lattice Basics

- how to put scalars/gluons/quarks on the lattice

(2) Lattice Spectroscopy

- sea versus valence quarks and (partial) quenching
- spectra of stable versus unstable hadrons

(3) Lattice Techniques

- weak and strong coupling expansion
- numerical aspects, parallel architectures

(4) Lattice Phenomenology

- quark masses: m_d, m_u, m_s, m_c
- decay constants, form factors and CKM-physics
- kaon mixing: B_K, $B_{\rm BSM}$, $K \to 2\pi$ amplitude

(5) Lattice Outreach

- baryon sigma terms, nuclear physics, ...
- QCD thermodynamics at $\mu = 0$ and $\mu > 0$
- large N_c, large N_f, different fermion representations

Lattice Basics

- path-integral and euclidean spacetime
- spin models and Metropolis algorithm
- how to put scalars on the lattice
- how to put gluons on the lattice
- how to put fermions the lattice
- Wilson versus Susskind/staggered fermions

Lattice basics (1): path-integral and euclidean spacetime

QFT: $e^{\mathrm{i}S_\mathrm{M}} = e^{\mathrm{i}\int L_\mathrm{M}\, d^4x_\mathrm{M}}$ $\quad x_\mathrm{M} = (x^0, \mathbf{x}) = (x^0, x^{1/2/3})\,, \quad x^4 \equiv \mathrm{i}x^0$

$$L_\mathrm{M} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V[\phi]\,, \qquad V[\phi(x)] \equiv \frac{m^2}{2}\phi^2(x) + \frac{\lambda}{4!}\phi^4(x)$$

$$\partial_\mu\phi\partial^\mu\phi = (\partial_0\phi)^2 - (\partial_{1/2/3}\phi)^2 = (\frac{\partial\phi}{\partial x^0})^2 - (\frac{\partial\phi}{\partial x^{1/2/3}})^2$$

$$L_\mathrm{E} \equiv -L_\mathrm{S} = (\frac{\partial\phi}{\partial x^{1/2/3}})^2 + (\frac{\partial\phi}{\partial x^4})^2 + \frac{m^2}{2}\phi^2(x) + \frac{\lambda}{4!}\phi^4(x) \quad > 0 \ \ (\text{for } \lambda > 0)$$

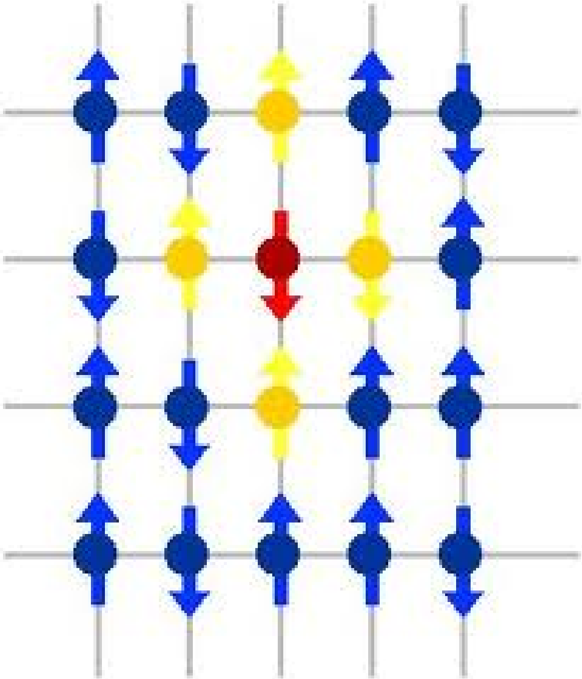
$$\mathrm{i}\int L_\mathrm{M}\, dx^0dx^1dx^2dx^3 = \int L_\mathrm{M}\, dx^1dx^2dx^3dx^4 = -\int L_\mathrm{E}\, dx^1dx^2dx^3dx^4$$

$\Longrightarrow$ euclidean standard is e^{-S_E} with $S_\mathrm{E} = \int L_\mathrm{E} d^4x_\mathrm{E} > 0$

Lorentz symmetry		$O(4)$ symmetry
$(x^0)^2 - \mathbf{x}^2$ invariant	$\longleftrightarrow$	$\mathbf{x}^2 + (x^4)^2$ invariant
$(+---)$ signature		$(++++)$ signature

[box $L^3 \times T$ (lattice spacing $a=1$) contains $N = L^3T$ continuous dofs]

Lattice basics (2): spin models and Metropolis algorithm



Ising model (in $d\!=\!2$ dimensions):

$N = N_1 N_2$ sites
$s_i = \pm 1 \quad \forall i \in \{1, ..., N\}$
toroidal boundary conditions

spin configuration $s = (s_1, ..., s_N)$
energy/Hamiltonian $H(s) = -J \sum_{\langle ij \rangle} s_i s_j - h \sum_k s_k$

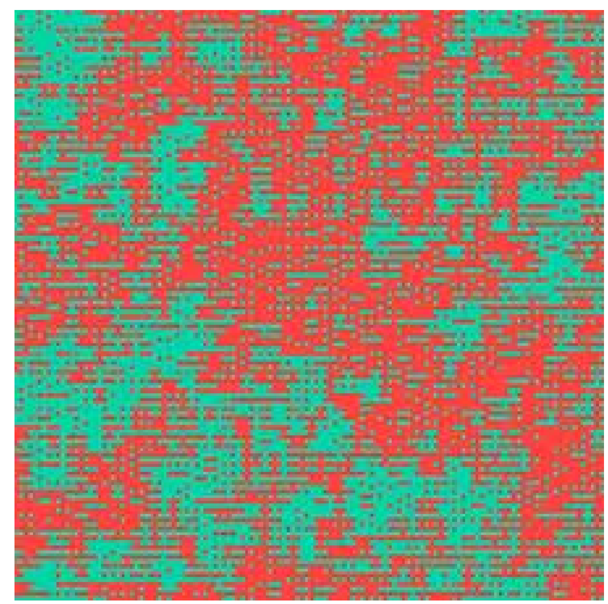
$J\!>\!0$, parallel preferred ("ferromagnetic")
$J\!<\!0$, antipar. preferred ("antiferromag.")

bounded from below, $H(s)\!>\!\mathrm{const}$, as in EQFT

partition function, free energy: $Z = \sum_s e^{-\beta H} \equiv e^{-\beta F}$
inverse temperature $\beta = 1/(kT)$ is external parameter
overall 2^N contributions ("proliferation of states")

Task: $\langle O \rangle = \frac{\sum_s O(s) e^{-\beta H(s)}}{\sum_s e^{-\beta H(s)}}$

Goal: generate sequence of spin configurations in which specific configuration s shows up with probability
$p(s) = \frac{1}{Z} e^{-\beta H(s)} \;, \quad Z \equiv \sum_{s'} e^{-\beta H(s')}$
("Boltzmann distribution", solution MRRTT'53)



Lattice basics (3): how to put scalars on the lattice

$$S_{\mathrm{E}} = a^4 \sum_{x,\mu} \left\{ \frac{1}{2} (\nabla_\mu \phi)(x) (\nabla_\mu \phi)(x) + V[\phi(.)] \right\}$$ [drop "E" henceforth]

$$(\nabla_\mu \phi)(x) \equiv \frac{1}{a} \left[\phi(x+a\hat{\mu}) - \phi(x) \right]$$ ("forward derivative")

$$(\nabla^*_\mu \phi)(x) \equiv \frac{1}{a} \left[\phi(x) - \phi(x-a\hat{\mu}) \right]$$ ("backward derivative")

$$= a^4 \sum_x \left\{ -\frac{1}{2} \phi(x) \triangle \phi(x) + V[\phi(.)] \right\}$$ > 0 (for $\lambda > 0$)

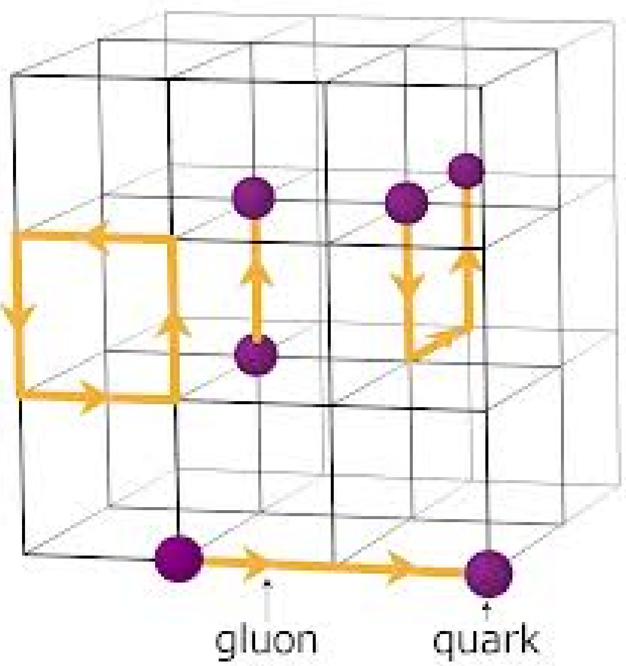
$$(\triangle \phi)(x) = \left\{ \begin{array}{l} (\nabla_\mu \nabla^*_\mu \phi)(x) \\ (\nabla^*_\mu \nabla_\mu \phi)(x) \end{array} \right. = \sum_\mu \frac{\phi(x+a\hat{\mu}) - 2\phi(x) + \phi(x-a\hat{\mu})}{a^2}$$

EQFT/simulation exploits formal analogy to statistical mechanics:

$$Z = \int d\phi(x_1)...d\phi(x_N)\, e^{-S[\phi]} \equiv \int D\phi\, e^{-S[\phi]}$$

$$\underbrace{\langle \phi(x_1)...\phi(x_n) \rangle}_{\text{means } \langle 0|T\{\phi(x_1)...\}|0\rangle\text{, i.e. time–ordered product of } n=2,3,\ldots \text{ fields}} = \underbrace{\frac{1}{Z} \int D\phi\, \phi(x_1)...\phi(x_n)\, e^{-S[\phi]}}_{\text{finite ratio of two high–dimensional integrals, each of the } N=L^3T \text{ fields runs from } -\infty \text{ to } +\infty}$$

Lattice basics (4): how to put gluons on the lattice



Attempts to put gauge fields $A_\mu(x)$ on the lattice break gauge invariance by $O(a)$ effects.

Only path-ordered exponentials $\exp(\mathrm{i}g\int A(s)ds)$ are measurable (Aharonov-Bohm).

Wilson: identify $U_\mu(x) \longleftrightarrow e^{\mathrm{i}g\int_x^{x+\hat{\mu}} A_\mu(\tilde{x})d\tilde{x}}$ and consider $U_\mu(x) \in SU(3)$ fundamental dof.

- $U_\mu(x)$ is parallel transporter from $x+\hat{\mu}$ to x
- cov. derivative $(D_\mu\phi)(x) = U_\mu(x)\phi(x{+}\hat{\mu}) - \phi(x)$
- $U_\mu(x)$ transforms into $g(x)U_\mu(x+\hat{\mu})g^\dagger(x+\hat{\mu})$
- traced closed loops of links are gauge-invariant

Wilson: simplest gauge action involves 1×1 loop ("plaquette")

$\mathrm{Tr}(P_{\mu\nu}) = N_c - \frac{a^4g^2}{2}\mathrm{Tr}(F_{\mu\nu}F_{\mu\nu})$

$\beta \equiv \frac{2N_c}{g^2}$ plays role of J in Ising model

$\beta \ll 1 \longleftrightarrow g^2 \gg 1$ "strong coupling"

$\beta \gg 1 \longleftrightarrow g^2 \ll 1$ "weak coupling"

$$\begin{aligned} S &= a^4 \sum_{x,\mu,\nu} \frac{1}{2}\mathrm{Tr}(F_{\mu\nu}(x)F_{\mu\nu}(x)) \\ &= \frac{1}{g^2}\sum_{x,\mu,\nu}\Big\{N_c - \mathrm{Tr}(P_{\mu\nu}(x))\Big\} \\ &= \frac{2N_c}{g^2}\sum_{x,\mu<\nu(!)}\Big\{1 - \frac{1}{N_c}\mathrm{ReTr}(P_{\mu\nu}(x))\Big\} \end{aligned}$$

Lattice basics (5): how to put fermions on the lattice

Bosons were handy, because they required *second-order* operator:

$$S_B = \frac{a^4}{2} \sum_x \left\{ \phi^\dagger(x)(-\triangle\phi)(x) + m^2 \phi^\dagger(x)\phi(x) \right\}$$

Fourier transform $\hat{p}^2 + m^2$ for $m = 0$ with $\hat{p} \equiv \frac{2}{a}\sin(\frac{ap}{2})$
has only 1 zero in BZ which is $(\,]-\frac{\pi}{a}, \frac{\pi}{a}]\,)^4$

Fermions give troubles, since they require *first-order* operator ("naive fermions"):

$$S_F = a^4 \sum_x \left\{ \bar\psi(x)\gamma_\mu \frac{\nabla_\mu + \nabla_\mu^*}{2} \psi(x) + m\bar\psi(x)\psi(x) \right\}$$

Fourier transform $\mathrm{i}\gamma_\mu \bar{p}_\mu + m$ for $m = 0$ with $\bar{p} \equiv \frac{1}{a}\sin(ap)$
has 16 zeros (one doubling per dim) in BZ
$\longrightarrow$ lift 15 of these to $O(\frac{1}{a})$ [Wilson]

Simulation as in pure YM, but with Grassmann-valued fermions integrated out:

$$\langle O \rangle = \frac{\int DU\; O[U]\; \det^{N_f}(D[U])\; e^{-S_G[U]}}{\int DU\; \det^{N_f}(D[U])\; e^{-S_G[U]}} \quad \text{with} \quad DU \equiv \prod_{\mu=1}^{4} \prod_{x}^{N} \underbrace{dU_\mu(x)}_{\text{Haar measure on SU(3)}}$$

Lattice basics (6): Wilson versus Susskind fermions

Susskind/staggered fermions yield 4 species: $S_\mathrm{S} = \sum_{x,y} \bar{\chi}(x) D_\mathrm{S}(x,y) \chi(y)$ with

$$D_\mathrm{S}(x,y) = \frac{1}{2} \sum_\mu \eta_\mu(x) \Big\{ U_\mu(x) \delta_{x+\hat{\mu},y} - U_\mu^\dagger(x-\hat{\mu}) \delta_{x-\hat{\mu},y} \Big\}$$

Wilson fermions [slower] yield 1 species: $S_\mathrm{W} = \sum_{x,y} \bar{\psi}(x) D_\mathrm{W}(x,y) \psi(y)$ with

$$D_\mathrm{W}(x,y) = \frac{1}{2} \sum_\mu \Big\{ (\gamma_\mu - I) U_\mu(x) \delta_{x+\hat{\mu},y} - (\gamma_\mu + I) U_\mu^\dagger(x-\hat{\mu}) \delta_{x-\hat{\mu},y} + 2\delta_{x,y} \Big\}$$

Overlap construction, traditionally with $X = D_\mathrm{W} - \rho$, makes things even slower:

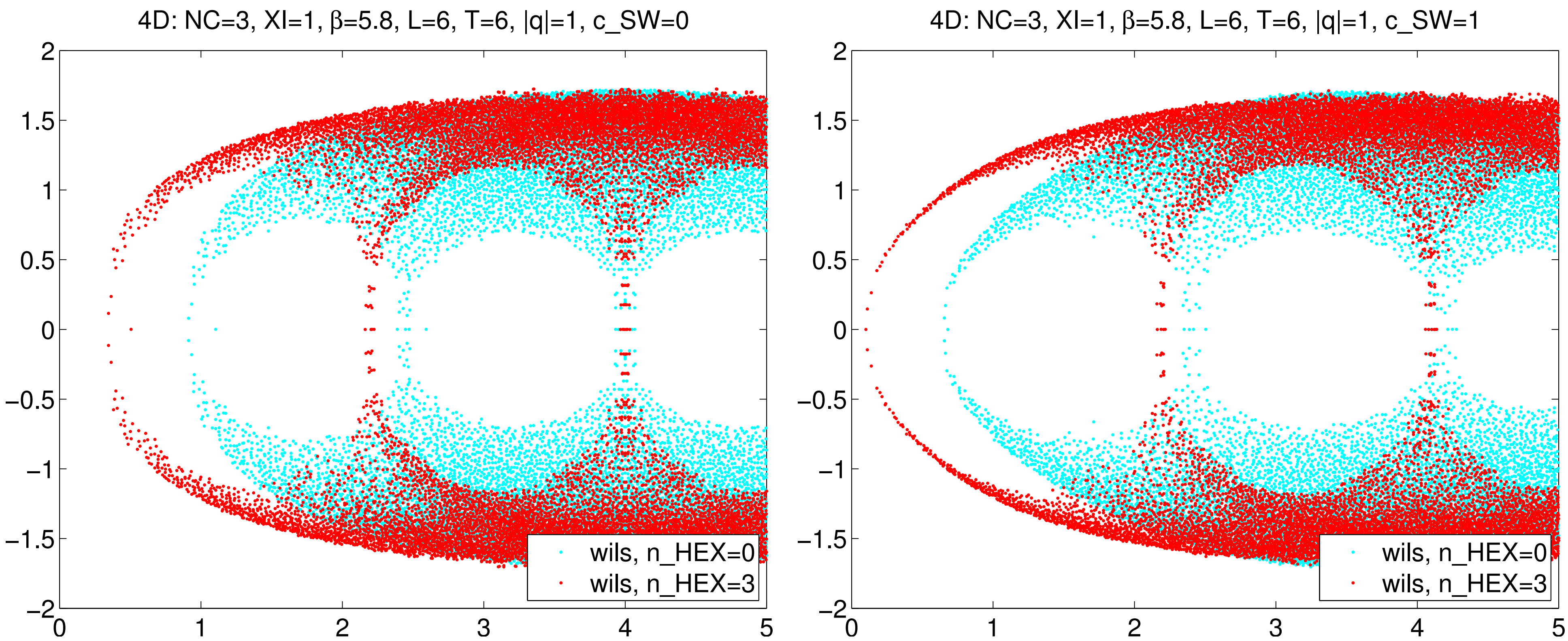
$$D_\mathrm{N}(x,y) = \frac{\rho}{a} \Big(1 + X (X^\dagger X)^{-1/2} \Big) = \frac{\rho}{a} \Big(1 + (X X^\dagger)^{-1/2} X \Big)$$

- main advantage of staggered fermions is their expedience [plus flavored symm.]
- main advantage of Wilson-like fermions is 1-to-1 [latt-cont] flavor identification

Lattice basics (7): rationale for "smearing+clover"

info: staggered D_S has (for $m=0$) EV spectrum on imaginary axis

info: overlap D_N has (for $m=0$) EV spectrum on unit circle around $(1,0)\in\mathbb{C}$



$\longrightarrow$ link smearing in D_W alone does not help on "horizontal jitter"

$\longrightarrow$ Symanzik improvement $c_\mathrm{SW}\simeq 1$ alone does not help much on "mass shift"

$\longrightarrow$ smearing and $c_\mathrm{SW}\simeq 1$ cure "mass shift" and "horizontal jitter" in physical branch

Lattice Spectroscopy

- scale hierarchies in LQCD
- sea quarks versus valence quarks
- terminology: QCD / QQCD / PQQCD
- hadron interpolating fields
- spectroscopy of stable particles
- spectroscopy of scattering states

Lattice spectroscopy (1): scale hierarchies

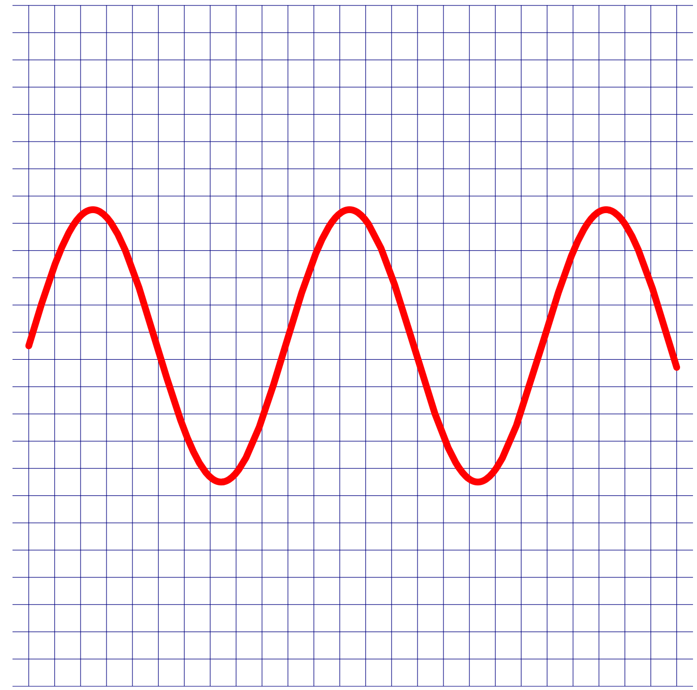
typical spacing: $0.05\,\mathrm{fm} \leq a \leq 0.20\,\mathrm{fm}$

$1\,\mathrm{GeV} \leq a^{-1} \leq 4\,\mathrm{GeV}$

typical boxsize: $2\,\mathrm{fm} \leq L \leq 6\,\mathrm{fm}$

require (UV) : $a\,m_q \ll 1$

require (IR) : $M_\pi L \geq 4$



u, d	c, s	(t), b
work near	interpolate to	extrapolate
$m_{ud} \gtrsim m_{ud}^{\mathrm{phys}}$	$m_s^{\mathrm{phys}}, m_c^{\mathrm{phys}}$	$m_b \to m_b^{\mathrm{phys}}$

For each β (a posteriori lattice spacing a) tune $1/\kappa_{ud,s,c,...}$ such that $\{M_\pi^2, 2M_K^2 - M_\pi^2, M_{\eta_c}^2, ...\}/M_\Omega^2$ assume correct values ("sacrificed observables").

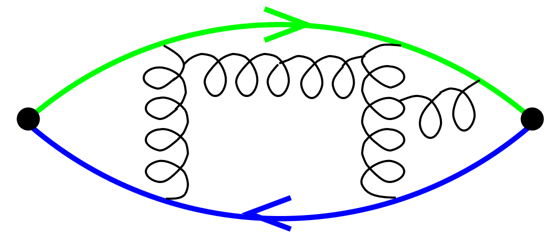
Lattice spectroscopy (2): sea versus valence quarks

Hadronic correlator in $N_f \geq 2$ QCD: $C(t) = \int d^4x\, C(t,\mathbf{x})\, e^{i\mathbf{px}}$ with

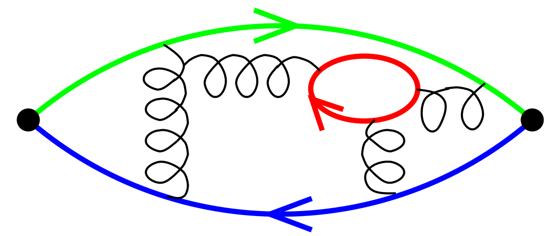
$$C(x) = \langle O(x)\, O(0)^\dagger \rangle \;=\; \frac{1}{Z}\int DU\, D\bar{q} Dq\;\; O(x)\, O(0)^\dagger \;\; e^{-S_G - S_F}$$

where $O(x) = \bar{d}(x)\Gamma u(x)$ and $\Gamma = \gamma_5, \gamma_4\gamma_5$ for $\pi^\pm$ and
$S_G = \beta \sum (1 - \frac{1}{3}\mathrm{ReTr}\, U_{\mu\nu}(x))$, $S_F = \sum \bar{q}(D+m)q$

$$\langle \bar{d}(x)\Gamma_1 u(x)\; \bar{u}(0)\Gamma_2 d(0)\rangle = \frac{1}{Z}\int DU\;\; \det(D+m)^{N_f}\; e^{-S_G}$$
$$\times\; \mathrm{Tr}\Big\{\Gamma_1 (D+m)^{-1}_{x0}\, \Gamma_2\, \underbrace{(D+m)^{-1}_{0x}}_{\gamma_5 [(D+m)^{-1}_{x0}]^\dagger \gamma_5}\Big\}$$



(A) Quenched QCD: quark loops neglected



(B) Full QCD

- Choose $m_u = m_d$ to save CPU time, since isospin SU(2) is a good symmetry.
- In principle $m_{\text{valence}} = m_{\text{sea}}$, but often additional valence quark masses to broaden data base. Note that "partially quenched QCD" is an *extension* of "full QCD".
- $(D+m)^{-1}_{x0}$ for all x amounts to 12 *columns* (with spinor and color) of the inverse.

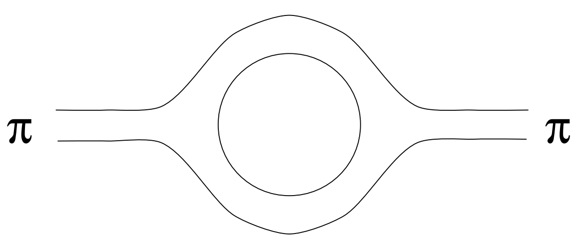
Lattice spectroscopy (3): QCD/QQCD/PQQCD terminology

$N_f = 0$ "QQCD" no dynamical "sea" quarks, only "valence" quarks
$N_f = 2$ 2 dynamical flavors with common mass m_{ud}
$N_f = 2{+}1$ 3 dynamical flavors with masses m_{ud}, m_s
$N_f = 2{+}1{+}1$ 4 dynamical flavors with masses m_{ud}, m_s, m_c
$N_f = 1{+}1{+}1{+}1$ 4 dynamical flavors with masses m_d, m_u, m_s, m_c

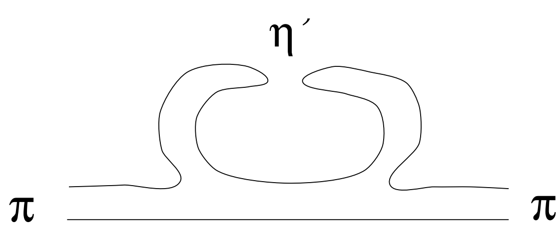
Note: in none of the above cases is $m_q = m_q^{\text{phys}}$ understood (are to be reached a posteriori through interpolations/extrapolations)

Note: "partially quenched" may mean absent in sea (e.g. c in $N_f = 2{+}1$) or present in sea with different mass (i.e. $m_c^{\text{sea}} \neq m_c^{\text{val}}$)

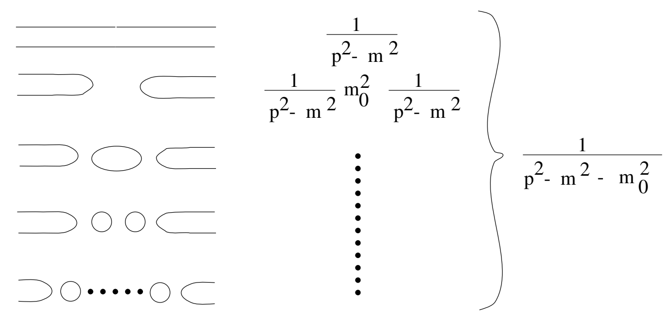
Note: quenching introduces serious artefacts in theory (non-unitarity, as η' has double-pole rather than shifted single-pole), but numerically effects seemed to be small [these days QQCD is gone]



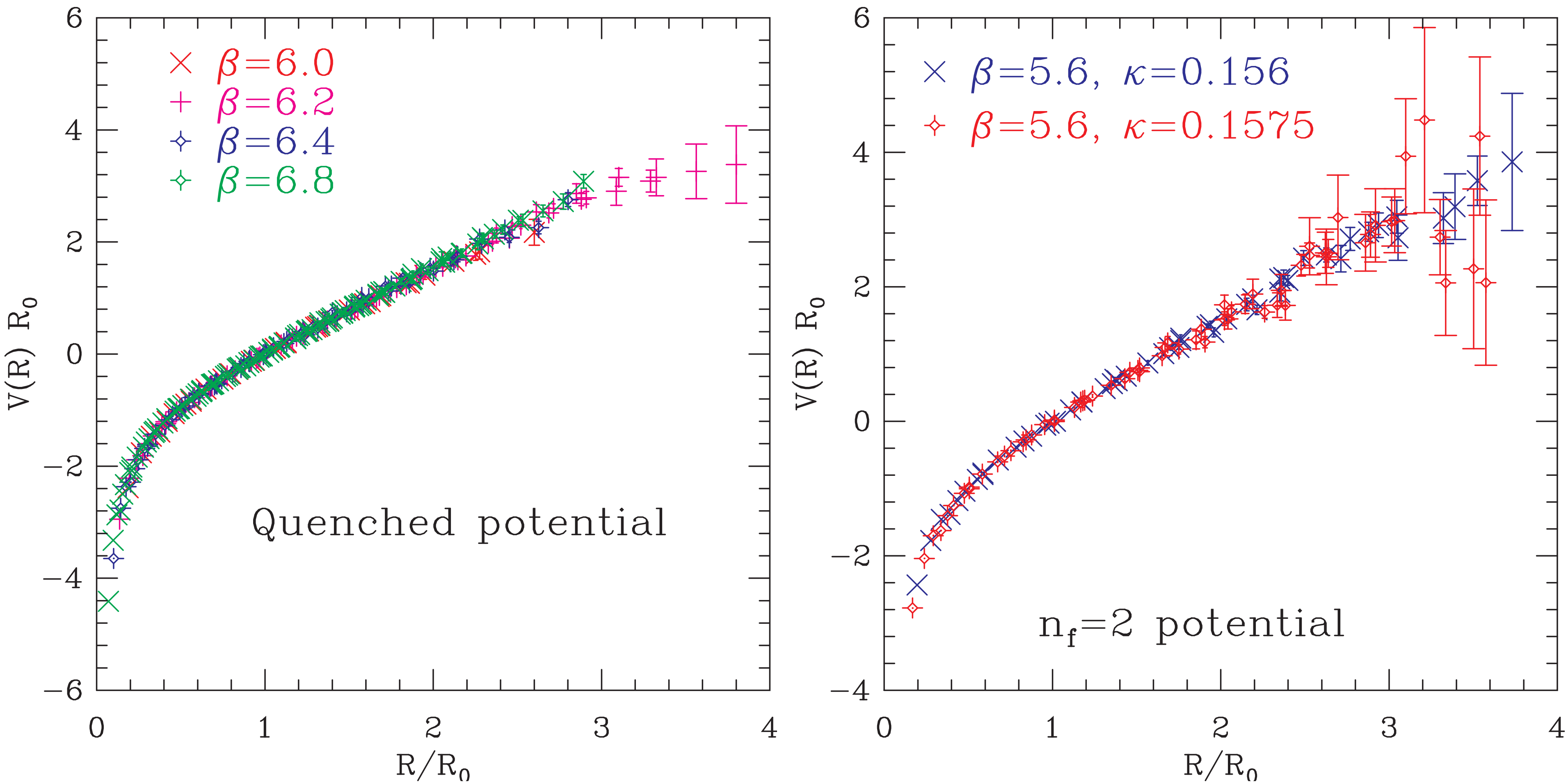
(A)



(B)



Lattice spectroscopy (4): HQ potential in quenched/full QCD



- asymptotic rise $V(r) \propto \sigma r$ in QQCD ("string tension" σ well-defined for $N_f = 0$)
- string breaks in (full) QCD, can be seen with better technique (cf. overview)
- short distance part is $V(r) \propto \frac{\alpha}{r}$ or $V(r) \propto \frac{\alpha(r)}{r}$; this can be used to get $\alpha_V(r)$

Lattice spectroscopy (5): meson/baryon interpolating fields

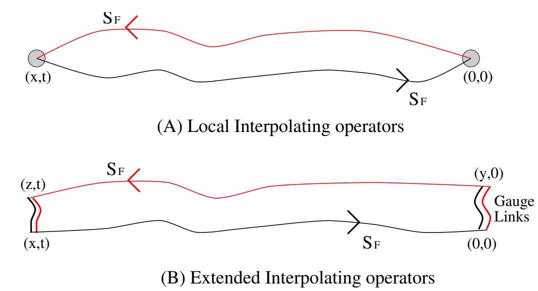
Flavor quantum number is to be kept track of explicitly:

$$O_{\pi^+}(x)=\bar{d}(x)\gamma_5 u(x),\; O_{\pi^0}(x)=\frac{1}{\sqrt{2}}[\bar{u}(x)\gamma_5 u(x)-\bar{d}(x)\gamma_5 d(x)],\; O_{\pi^-}(x)=\bar{u}(x)\gamma_5 d(x)$$

$$\langle O_{\pi^+}(x)\bar{O}_{\pi^+}(y)\rangle = \langle \bar{d}(x)\gamma_5 u(x)\;\bar{u}(y)\,|\,\gamma_5 d(y)\rangle$$

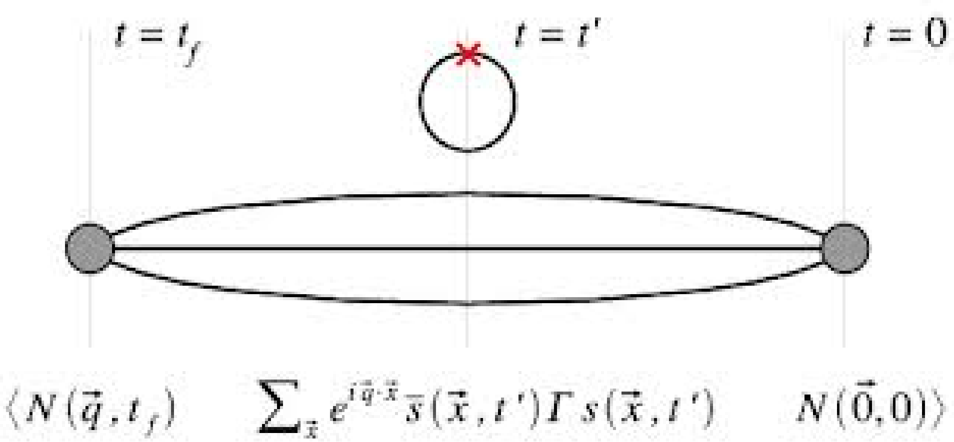
$$= \langle \mathrm{Tr}\{\gamma_5 D_{m_d}^{-1}(y,x)\,\gamma_5 D_{m_u}^{-1}(x,y)\}\rangle$$

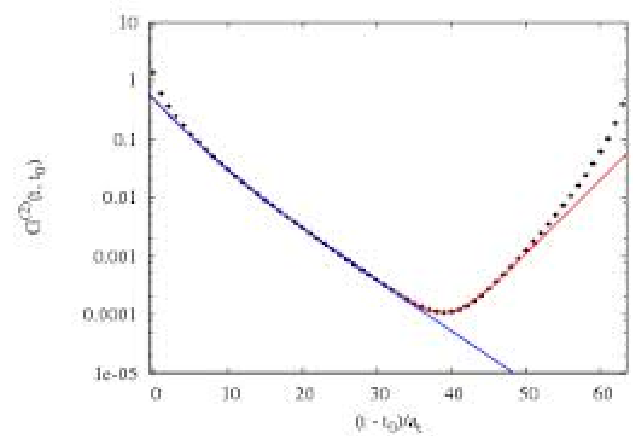
$$= \langle \mathrm{Tr}\{[D_{m_d}^{-1}(x,y)]^\dagger\, D_{m_u}^{-1}(x,y)\}\rangle$$



$\langle O_{\pi^0}(x)\,\bar{O}_{\pi^0}(y)\rangle$ = 6 terms, 2 connected and 4 disconnected, latter cancel for $m_u=m_d$

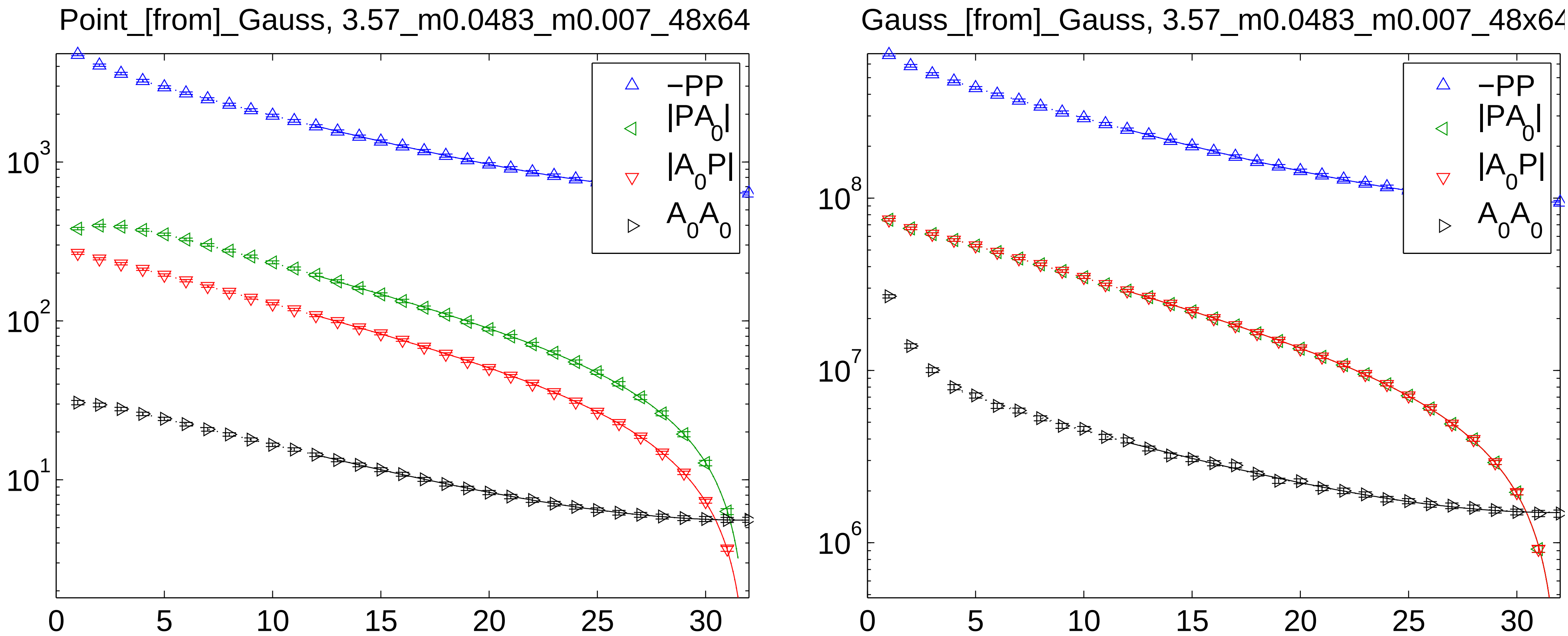
$$\langle O_N(x)\,O_{\bar{N}}(y)\rangle = \langle(\mathrm{contractions})\,u(x)u(x)d(x)\,\bar{u}(y)\bar{u}(y)\bar{d}(y)\rangle$$





Lattice spectroscopy (6): pseudoscalar meson correlators

Excellent data quality even on our lightest ensemble ($M_\pi \simeq 190\,\mathrm{MeV}$ and $L \simeq 4.0\,\mathrm{fm}$):



$\cosh(.)/\sinh(.)$ for $-PP, |PA_0|, |A_0P|, A_0A_0$ with Gauss source and local/Gauss sink

$C_{Xx,Yy}(t) = c_0 e^{-M_0 t} \pm c_0 e^{-M_0(T-t)} + \ldots$ with $X,Y \in \{P, A_0\}$ and $x,y \in \{\mathrm{loc}, \mathrm{gau}\}$

$\longrightarrow$ $c_0 = G\tilde{G}/M_0, G\tilde{F}, F\tilde{G}, F\tilde{F}M_0$ (left) and $c_0 = \tilde{G}\tilde{G}/M_0, \tilde{G}\tilde{F}, \tilde{F}\tilde{G}, \tilde{F}\tilde{F}M_0$ (right)

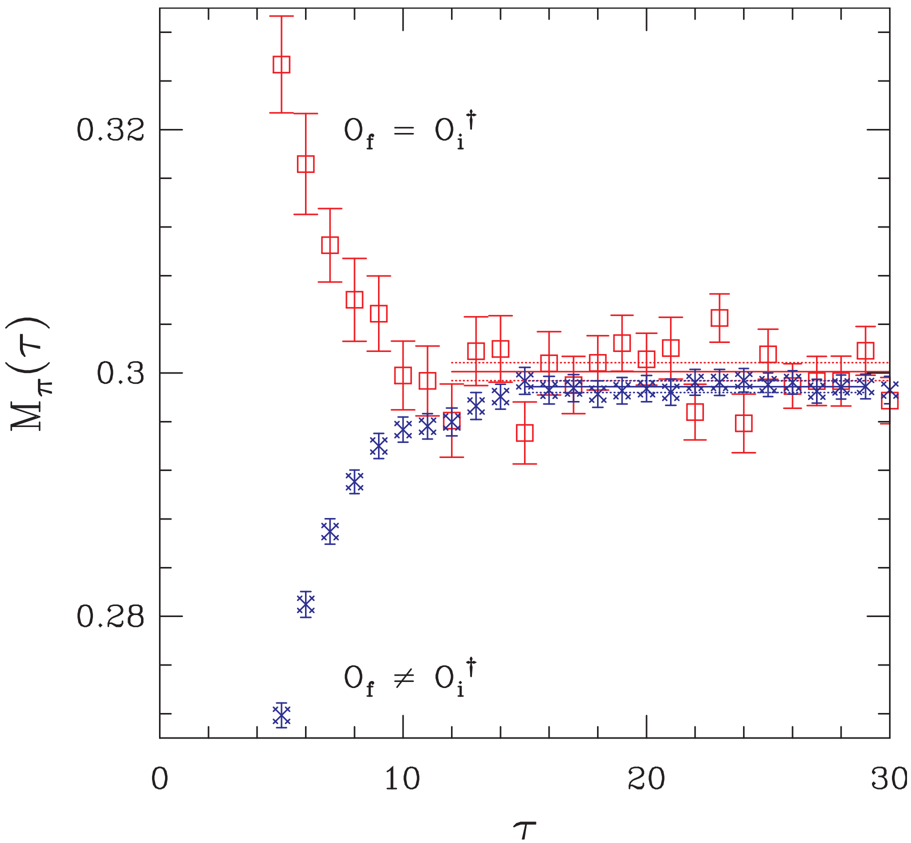
$\longrightarrow$ combined 1-state fit of 8 correlators with 5 parameters yields $M_\pi, F_\pi, m_{\mathrm{PCAC}}$

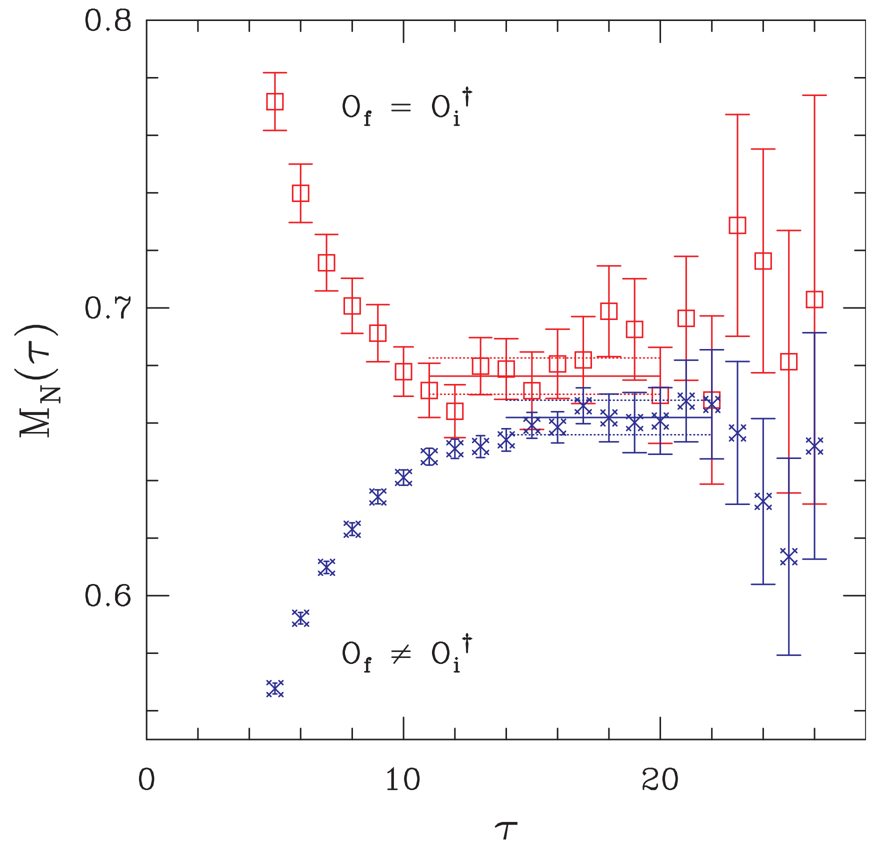
Lattice spectroscopy (7): spectroscopy of stable states

stable states: meaning is *under strong interactions* (example: $\pi, N, ...$)

$$\langle A(x)B(y)\rangle = \sum_{n\geq 0} \frac{1}{2E_n} \langle 0|A(\mathbf{x},0)e^{-E_n x_4}|n\rangle\langle n|e^{+E_n y_4}B(\mathbf{y},0)|0\rangle$$
$$= \sum_{n\geq 0} \frac{1}{2E_n} \langle 0|A(\mathbf{x},0)|n\rangle\langle n|B(\mathbf{y},0)|0\rangle e^{-E_n(x_4-y_4)}$$

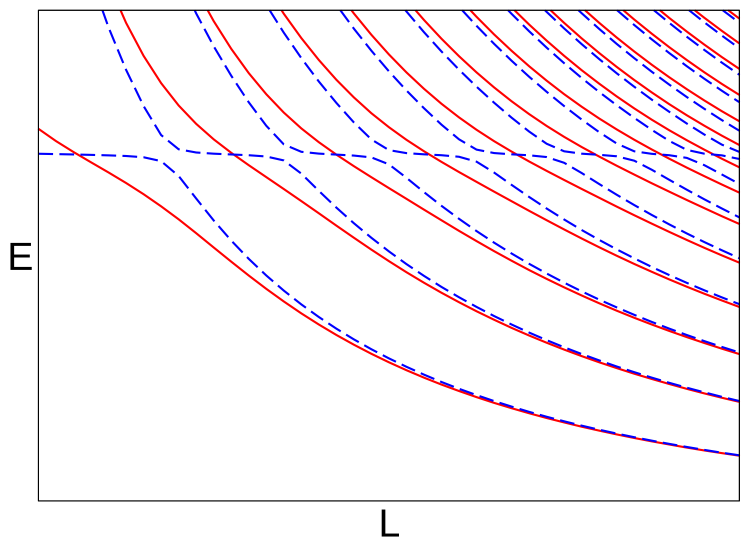
Consider local effective mass $M_{\text{eff}}(t) = \frac{1}{2}\log(\frac{C(t-1)}{C(t+1)})$ and determine plateau value:





Lattice spectroscopy (8): spectroscopy of unstable/mixing states

unstable states: meaning is *under strong interactions* (example: $\rho, \Delta, ...$)



2-particle ($\pi\pi$, πK, KK, πN, NN) states:

Scattering length and phase-shift can be determined in Euclidean space from tower of states in finite volume [Lüscher 1991].

Example: L-dependence of states with $\pi\pi$ or ρ quantum numbers is different for small (dashed blue) versus large (full red) $g_{\pi\pi\rho}$.

Original framework by Lüscher refined in many respects [Rummukainen and Gottlieb, Rusetsky et al] and successfully applied to a variety of systems.

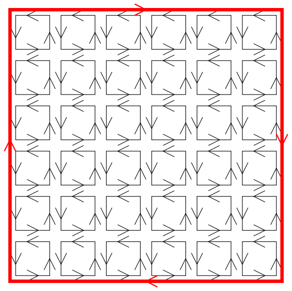
Method in practice rather demanding, since limited number of L values available, and extraction of high-lying states remains a challenge.

Results on $\pi\pi, \pi K, KK, \pi D, \pi N, NN, ...$ from various groups, e.g. Beane/Savage et al [NPLQCD], Dudek et al [HSC], Lang et al, Mohler et al, Aoki et al [HAL-QCD], ...

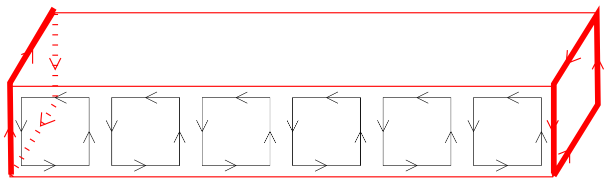
Lattice Techniques

- strong coupling expansion
- weak coupling expansion
- iterative solvers
- CPUs in parallel mode
- GPUs in farming mode
- postprocess: $a \to 0$, $V \to \infty$, $m_q \to m_q^{\mathrm{phys}}$

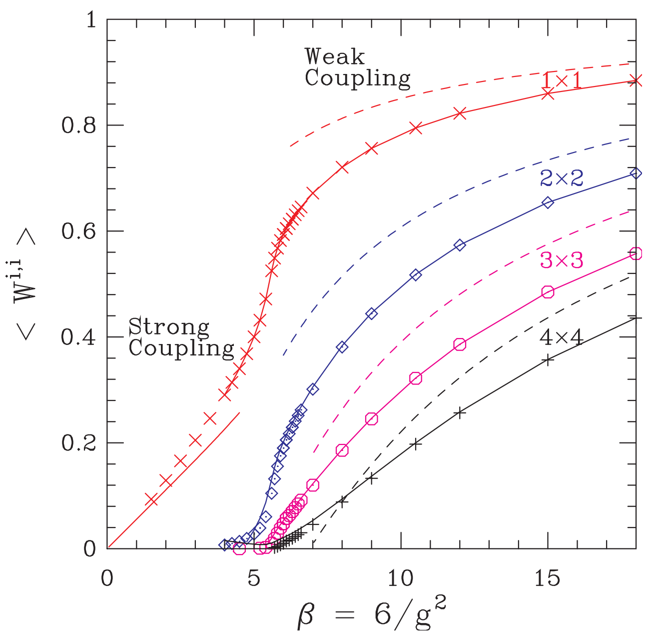
Lattice techniques (1): strong-coupling perturbation theory



(A) Minimum tiling of a 6x6 Wilson loop.



(B) Tiling of one face of a plaquette-plaquette correlation function



Strong coupling PT: expansion in $\beta = 6/g_0^2$; expansion about "disorder", i.e. about rough configurations.

Rather large $O(20)$ orders can be reached by massive amount of computer algebra.

$$W_{1\times1}(r,t) = \left(\frac{\beta}{2N_c^2}\right)^{rt} (1 + O(\beta))$$

$\longrightarrow$ confinement proven to leading order in SCPT

Weak coupling PT: expansion in $g_0^2 = 6/\beta$; expansion about "order", i.e. about smooth configurations.

Already 2-loop computations extremely tedious due to broken Lorentz invariance.

$\longrightarrow$ most successful are "mixed schemes" in which $W_{2\times2}, W_{3\times3}, W_{4\times4}$ are analytically linked to $W_{1\times1}$ and the latter is measured in simulation

Lattice techniques (2): weak-coupling perturbation theory

Z-factors ("renormalization") needed/useful for lattice-to-continum matching; distinguish operators with/without anomalous dimension, beware of mixing:

$$\langle .|O_i^{\text{cont}}(\mu)|.\rangle = \sum_j Z_{ij}(a\mu)\langle .|O_j^{\text{latt}}(a)|.\rangle$$

$$Z_{ij}(a\mu) = \delta_{ij} - \frac{g_0^2}{16\pi^2}(\Delta_{ij}^{\text{latt}} - \Delta_{ij}^{\text{cont}}) = \delta_{ij} - \frac{g_0^2}{16\pi^2} C_F z_{ij}$$

$$Z_S(a\mu) = 1 - \frac{g_0^2}{4\pi^2}\Big[\frac{z_S}{3} - \log(a^2\mu^2)\Big] \qquad Z_V = 1 - \frac{g_0^2}{12\pi^2} z_V$$

$$Z_P(a\mu) = 1 - \frac{g_0^2}{4\pi^2}\Big[\frac{z_P}{3} - \log(a^2\mu^2)\Big] \qquad Z_A = 1 - \frac{g_0^2}{12\pi^2} z_A$$

Generically $[z_P - z_S]/2 = z_V - z_A$, and for a chiral action either side vanishes.

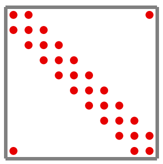
Typically n-loop LPT yields results with leading cut-off effects $O(\alpha^n a)$; usual hope/belief is that with non-perturbative improvement Symanzik scaling window is larger.

Lattice techniques (3): sparse iterative solvers

$$D_{\rm st}(x,y) = \frac{1}{2}\sum_\mu \eta_\mu(x)\Big\{U_\mu(x)\delta_{x+\hat\mu,y} - U_\mu^\dagger(x-\hat\mu)\delta_{x-\hat\mu,y}\Big\} + m\delta_{x,y}$$

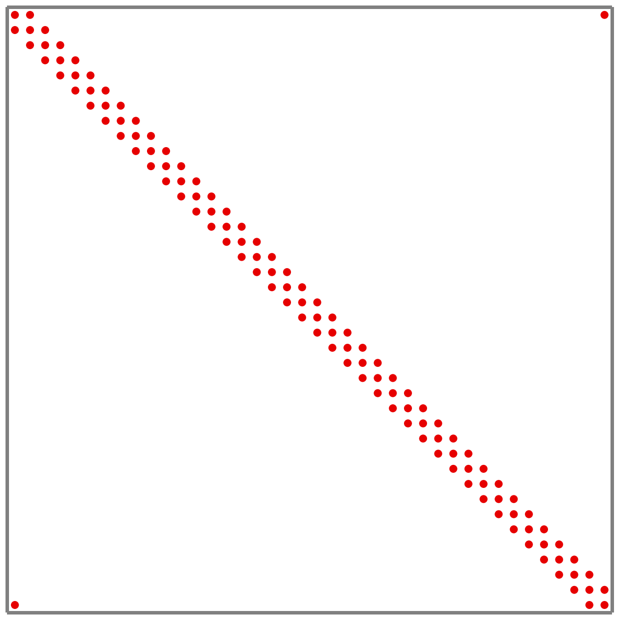
$$D_{\rm W}(x,y) = \frac{1}{2}\sum_\mu \Big\{(\gamma_\mu - I)U_\mu(x)\delta_{x+\hat\mu,y} - (\gamma_\mu + I)U_\mu^\dagger(x-\hat\mu)\delta_{x-\hat\mu,y}\Big\} + (4+m_0)\delta_{x,y}$$

staggered:

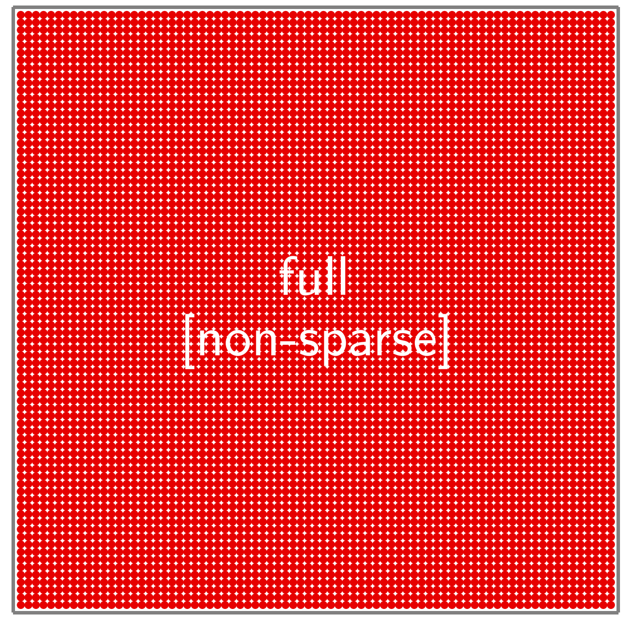


$\eta_\mu(x) = (-)^{\sum_{\nu<\mu} x_\nu}$

Wilson:



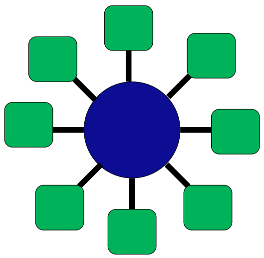
overlap:

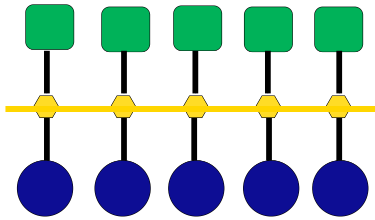


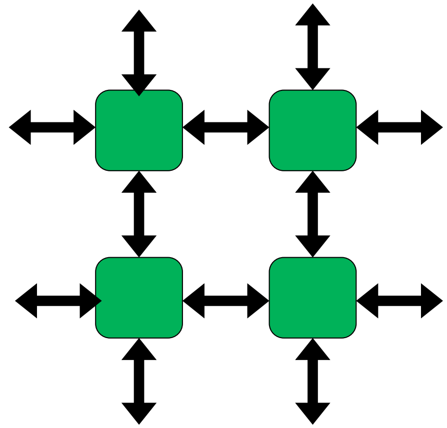
- D is $12N\times12N$ complex sparse matrix, for $N=64^3\times128$ this is $402\,10^6 \times 402\,10^6$
- each line/column contains only $1+3\cdot2\cdot8 = 49$ non-zero entries
- inverse is full [non-sparse], example above would require $2.4\,10^6$ TB of memory
- CG solver yields $D^{-1}\eta \simeq c_0\eta + c_1 D\eta + \ldots + c_n D^n\eta$ with $n^2 \propto \mathrm{cond}(D^\dagger D) = \frac{\lambda_{\max}}{\lambda_{\min}}$

Lattice techniques (4): new CPU packing strategies

SMP versus SIMD:

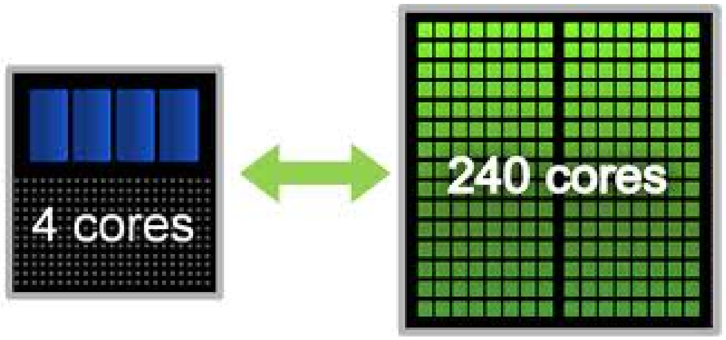






JUQUEEN [IBM BG/Q]	06/2012 - 10/2012	02/2013 - ...
processor type	64-bit PowerPC A2 1.6 GHz (205 Gflops each)	
compute node	16-way SMP processor (water cooled)	
racks, nodes, cores	8, 8'192, 131'072	28, 28'672, 458'752
memory	16 GB per node, aggregate 131 TB	aggregate 448 TB
performance (double)	1678/1380 Teraflops peak/Linpack	5873/4830 Teraflops
power consumption	$<$100 kW/rack, aggregate 0.8 MW	aggregate 2.8 MW
network topology	5D torus among compute nodes (incl. global barriers)	
network bandwidth	40 Gigabyte/s	
network latency	2.5 μsec (light travels 750 meters)	

Lattice techniques (5): new GPU programming models



GPUs originally designed for tasks in computer graphics (e.g. rendering).

GPUs nowadays frequently used for OpenMP-parallelizable scientific computations.

Hardware connection via PCI bus (overhead from data transfer before/after computation).

```
void transform_10000by10000grid(float in[10000][10000], float *out[10000][10000]){
   for(int x=0; x<10000; x++){
      for(int y=0; y<10000; y++){
         *out[x][y] = do_something(in[x][y]); // local operation !!!
      }
   }
}
```

Popular programming languages: CUDA, OpenCL, ...

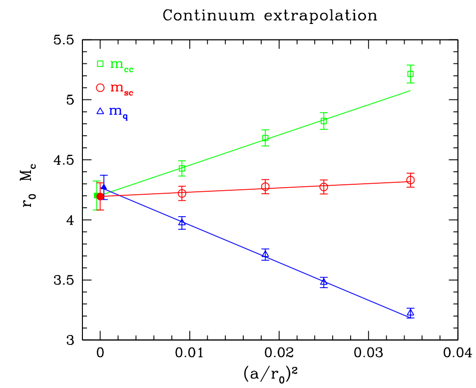
Issues of single (32bit) versus double (64bit) precision ...

Excellent price/performance ratio paid for by human work ...

Lattice techniques (6): theory for $a \to 0$, $L \to \infty$, $m_q \to m_q^{\rm phys}$

Lattice breaks Lorentz symmetry (softly, i.e. recovered in observables under $a \to 0$, $V \to \infty$) but maintains gauge-invariance.

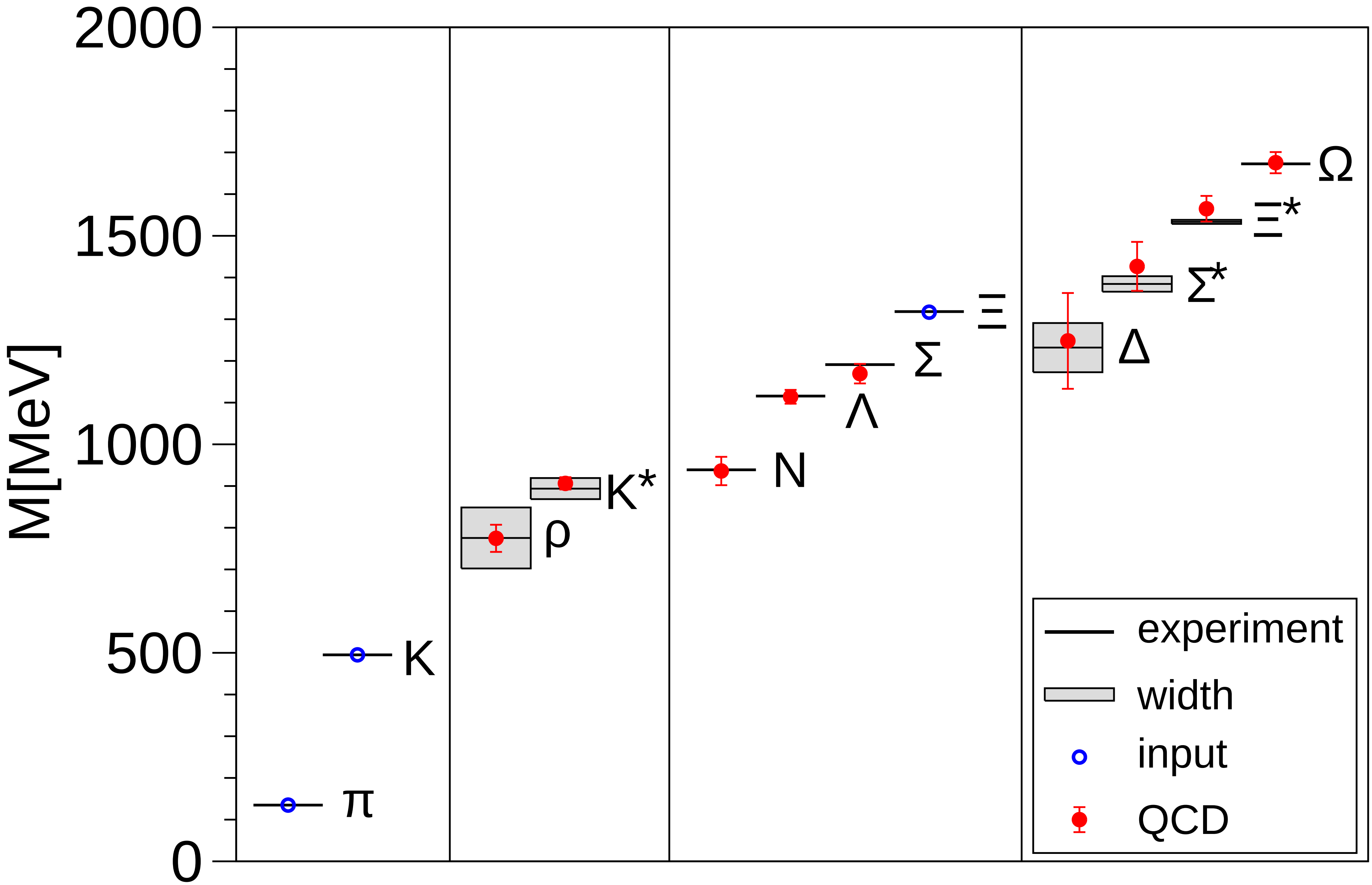
Lattice spacing a and quark masses $m_{ud,s,..}^{\rm scheme}$ are quantities that emerge from the parameters β and $1/\kappa_{ud,s,...}$ of the simulations; hence a suitable number of observables must be "sacrificed" to set the lattice spacing and to adjust the quark masses.



$a \to 0$	Symanzik effective theory of cut-off effects has simple consequence: plot data versus correct power of a (e.g. αa, depends on action used) and extrapolate linearly.
$V \to \infty$	Chiral perturbation theory predicts that every quantity has asymptotic finite-volume effects which scale exponentially in $M_\pi L$; in relative shift $[f_B(L) - f_B(\infty)]/f_B(\infty) = \mathrm{const}\, e^{-M_\pi L}$ often "const" from ChPT.
$m_q \to m_q^{\rm phys}$	Traditionally extrapolation $M_\pi^2 \to (134.8\,\mathrm{MeV})^2$ via ChPT, modern simulations often bracket $m_{ud}^{\rm phys}$ by those in the simulation (in such case linear interpolation seems sufficient).

Almost all lattice computations concern quantities (masses, decay constants, form factors) for which *no backrotation* to Minkowski spacetime is required.

Final result: S. Dürr et al, Science 322, 1224 (2008)



Lattice Phenomenology

- light quark masses from spectroscopy
- decay-constants and form-factors for CKM physics
- light flavor (d, u, s) physics: $f_\pi, f_K, ...$
- heavy flavor (c, b) physics: $f_D, f_{D_s}, f_B, f_{B_s}, ...$
- indirect CP violation: $B_K, B_{\mathrm{BSM}}, B_D, B_B, ...$
- $K \to 2\pi$ amplitudes and $\Delta I = 1/2, \epsilon'/\epsilon$

Quark masses (1): anatomy of $N_f = 2+1$ computation

1. Choose observables to be "sacrificed", e.g. M_π, M_K, M_Ω in $N_f\!=\!2\!+\!1$ QCD, and get "polished" experimental values, e.g. $M_\pi = 134.8(3)\,\mathrm{MeV}$, $M_K = 494.2(5)\,\mathrm{MeV}$ in a world without isospin splitting and without electromagnetism [arXiv:1011.4408].

2. For a given bare coupling β (yields a) tune bare masses $1/\kappa_{ud,s}$ such that the ratios M_π/M_Ω, M_K/M_Ω assume their physical values (in practice: inter-/extrapolation).
$$M_{\pi,K,\Omega} \longleftrightarrow m_q^{\mathrm{bare}}$$

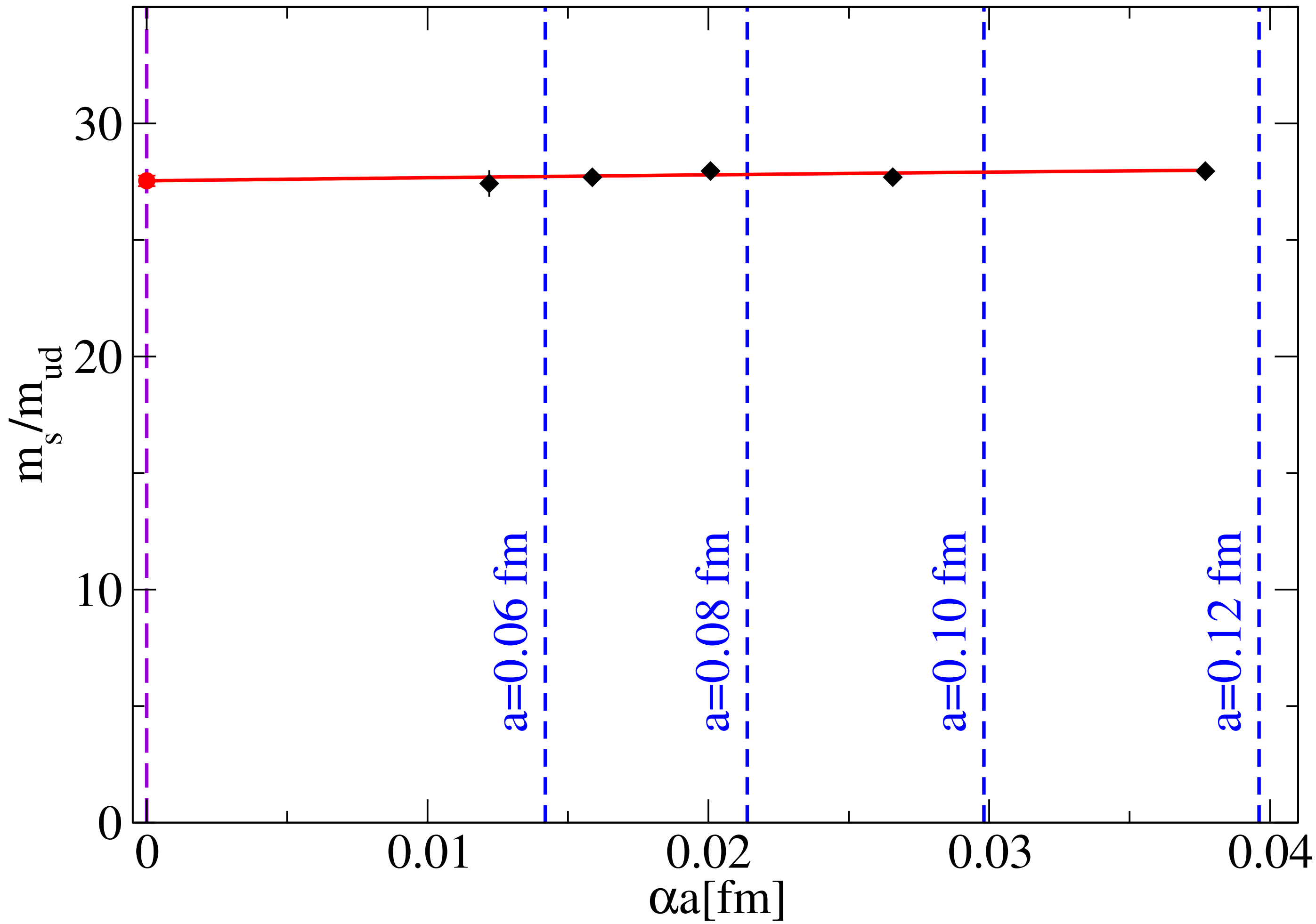
3. Read off $1/\kappa_{ud,s}$ or determine bare $am_{ud,s}$ via AWI and convert them (perturbatively or non-perturbatively) to the scheme of your choice (e.g. $\overline{\mathrm{MS}}$ at $\mu\!=\!3\,\mathrm{GeV}$).
$$m_q^{\mathrm{bare}} \longleftrightarrow m_q^{\mathrm{SF/RI}} \longleftrightarrow m_q^{\overline{\mathrm{MS}}}$$

4. Repeat steps 2 and 3 for at least 3 different lattice spacings and extrapolate the (finite-volume corrected) result to the continuum via Symanzik scaling.

Depending on details, step 3 can be rather demanding [RI/MOM, SF renormalization]. Below, guided tour using plots from BMW-collaboration [arXiv:1011.2403,1011.2711].

Quark masses (2): Final result for ratio m_s/m_{ud}

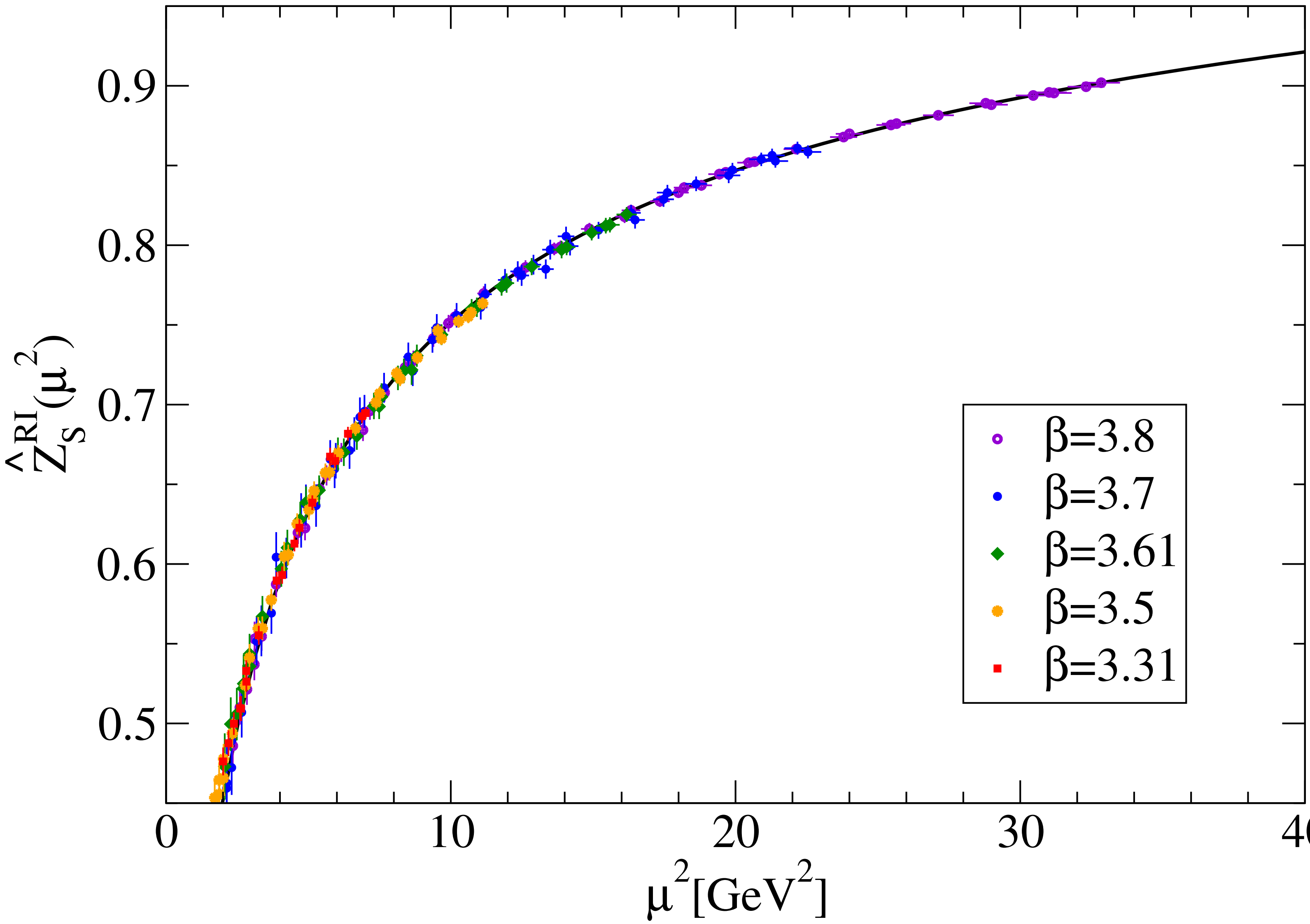
In QCD ratios like m_s/m_{ud} are renormalization group invariant (RGI), hence step 3 in this list is skipped (detail: we invoke αa and a^2 scaling).



Final result $m_s/m_{ud} = 27.53(20)(08)$ amounts to 0.78% precision.

Quark masses (3): $N_f = 3$ RI-running extrapolation for Z_S

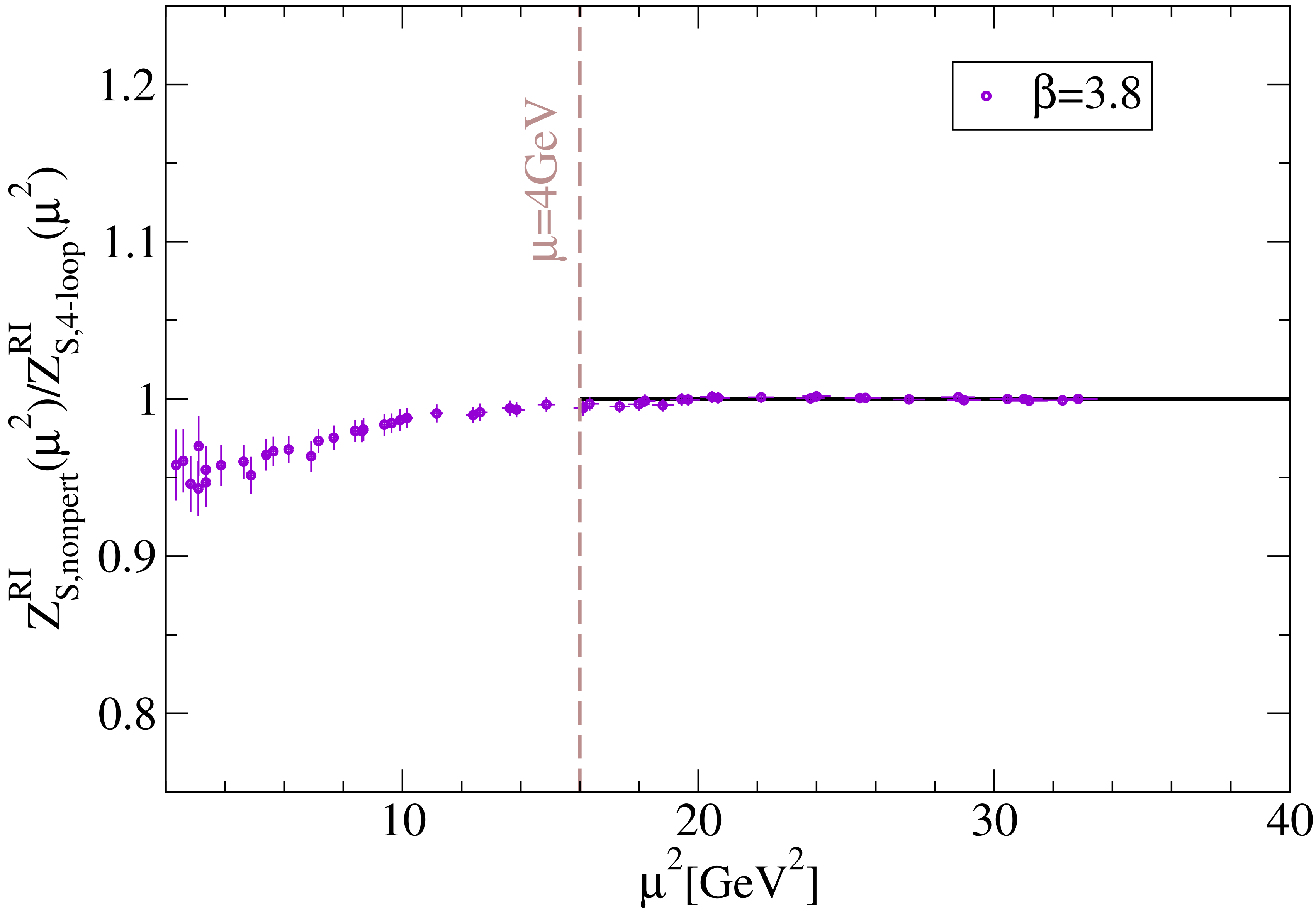
Evolution $Z_S^{\rm RI}(\mu)/Z_S^{\rm RI}(4\,{\rm GeV})$ has no visible cut-off effects among three finest lattices:



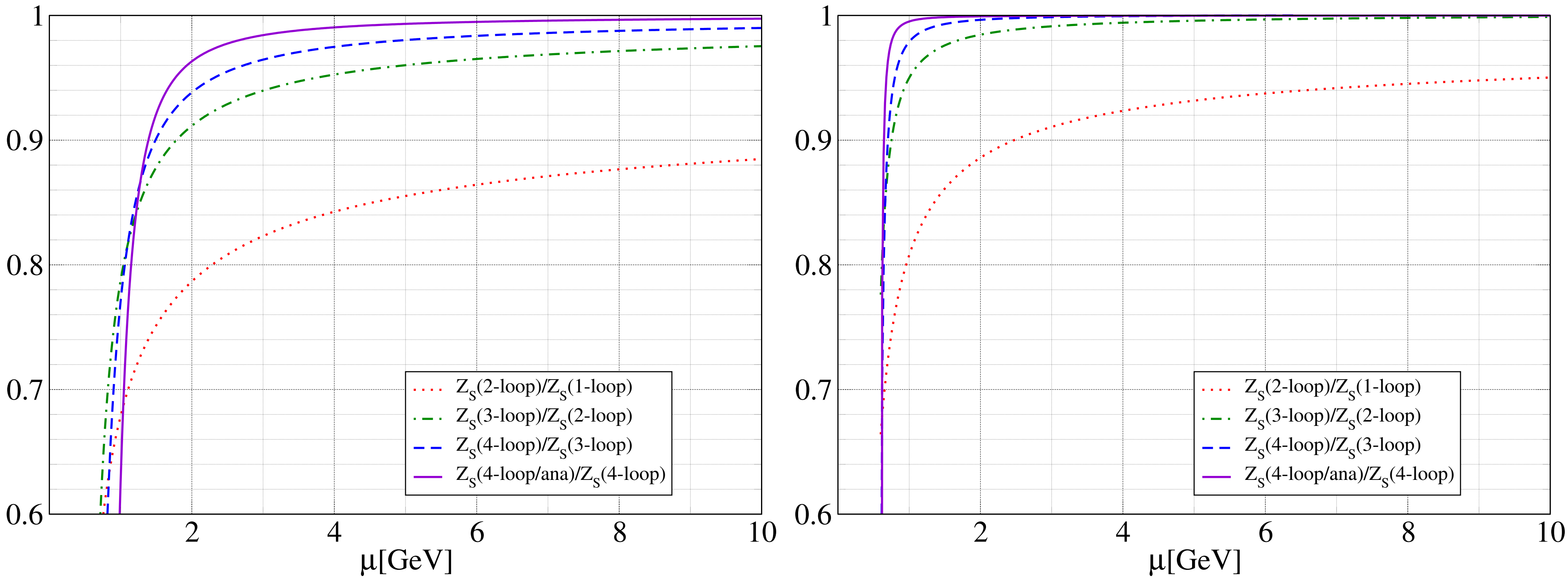
⟶ separate continuum limit with $R_S^{\rm RI}(\mu, 4\,{\rm GeV}) = \lim_{\beta\to\infty} Z_{S,\beta}^{\rm RI}(4\,{\rm GeV})/Z_{S,\beta}^{\rm RI}(\mu)$

Quark masses (4): $N_f = 3$ RI-scheme-running ratio for Z_S

On the finest lattice we make contact within errors to 4-loop PT for $\mu \geq 4\,\mathrm{GeV}$:



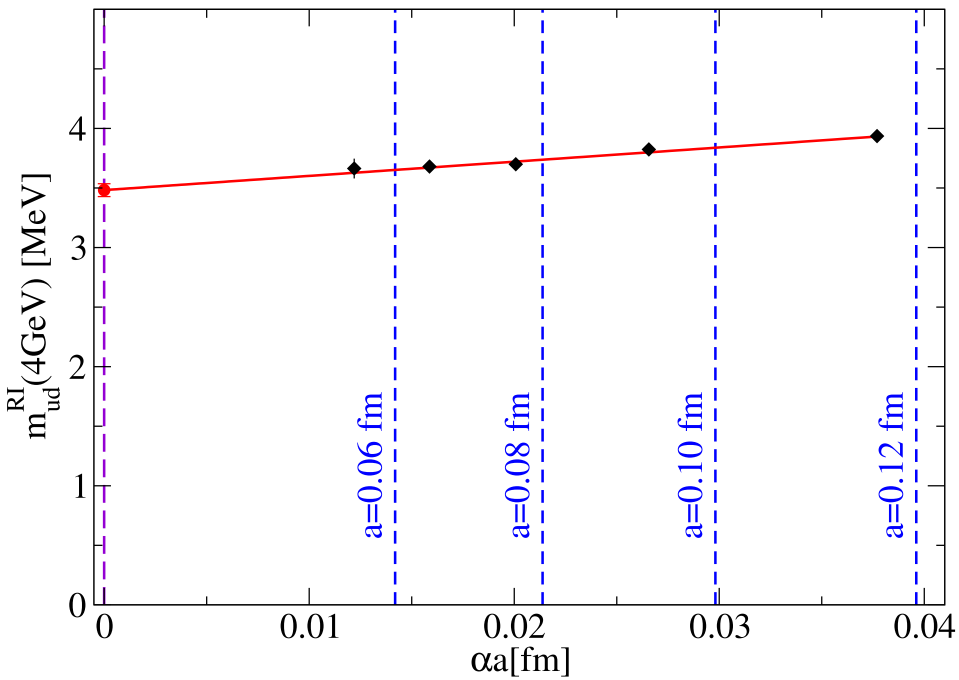
Quark masses (5): $N_f = 3$ RI and $\overline{\mathrm{MS}}$ perturbative series for Z_S

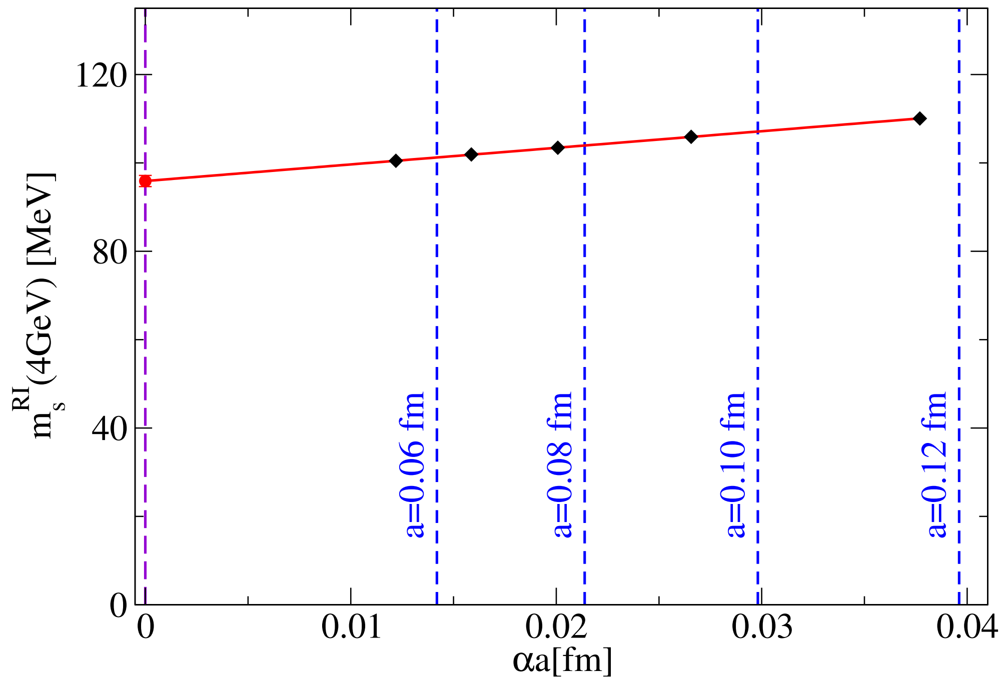


- RI series (left) converges less convincingly than $\overline{\mathrm{MS}}$ series (right)
- difference "4-loop" to "4-loop/ana" indicates size of 5-loop effects
- ratio suggests that higher-loop effects in RI are $< 1\%$ at $\mu = 4\,\mathrm{GeV}$
- ratio suggests that higher-loop effects in $\overline{\mathrm{MS}}$ are negligible down to $\mu = 2\,\mathrm{GeV}$

Quark masses (6): Final results for m_s and m_{ud}

Good scaling of $m^{\rm RI}_{ud,s}(4\,{\rm GeV})$ out to the coarsest lattice ($a\!\sim\!0.116\,{\rm fm}$):





Conversion with analytical 4-loop formula at $4\,{\rm GeV}$ and downwards running in $\overline{\rm MS}$:

	m_{ud}	m_s
RI$(4\,{\rm GeV})$	3.503(48)(49)	96.4(1.1)(1.5)
RGI	4.624(63)(64)	127.3(1.5)(1.9)
$\overline{\rm MS}(2\,{\rm GeV})$	3.469(47)(48)	95.5(1.1)(1.5)

RGI/$\overline{\rm MS}$ results (table 1.9% prec.) need to be augmented by a $\sim\!1\%$ conversion error.

Quark masses (7): splitting m_{ud} with information from $\eta \to 3\pi$

The process $\eta \to 3\pi$ is highly sensitive to QCD isospin breaking (from $m_u \neq m_d$) but rather insensitive to QED isospin breaking (from $q_u \neq q_d$), and this is captured in Q.

Rewrite the Leutwyler ellipse in the form

$$\frac{1}{Q^2} = 4 \left(\frac{m_{ud}}{m_s}\right)^2 \frac{m_d - m_u}{m_d + m_u}$$

and use the conservative estimate $Q = 22.3(8)$ of [Leutwyler, Chiral Dynamics 09] together with our result $m_s/m_{ud} = 27.53(20)(08)$ to get the asymmetry parameter

$$\frac{m_d - m_u}{m_d + m_u} = 0.381(05)(27) \quad \longleftrightarrow \quad m_u/m_d = 0.448(06)(29)$$

from which we then obtain individual m_u, m_d values (note: $m_u = 0$ strongly disfavored)

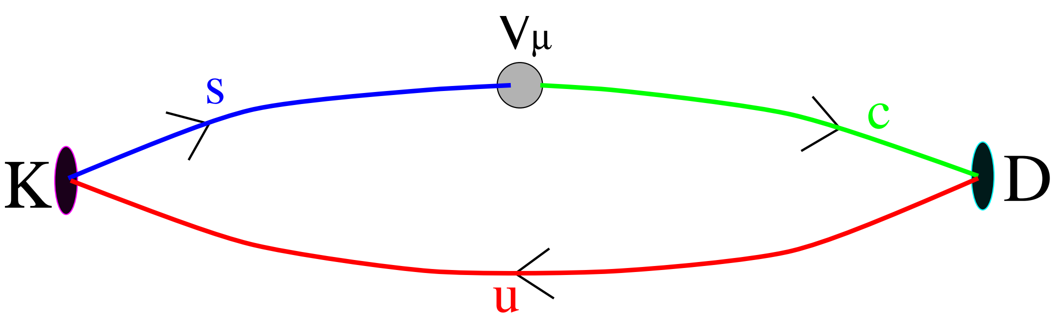
	m_u	m_d	m_s
RI$(4\,\mathrm{GeV})$	2.17(04)(10)	4.84(07)(12)	96.4(1.1)(1.5)
RGI	2.86(05)(13)	6.39(09)(15)	127.3(1.5)(1.9)
$\overline{\mathrm{MS}}(2\,\mathrm{GeV})$	2.15(03)(10)	4.79(07)(12)	95.5(1.1)(1.5)

Lattice phenomenology (1): CKM physics ...

$D \to K l \nu$ $\quad D \to K^* l \nu$ $\qquad K \to \pi l \nu$ $\qquad B \to \pi l \nu$ $\quad B \to \rho l \nu$

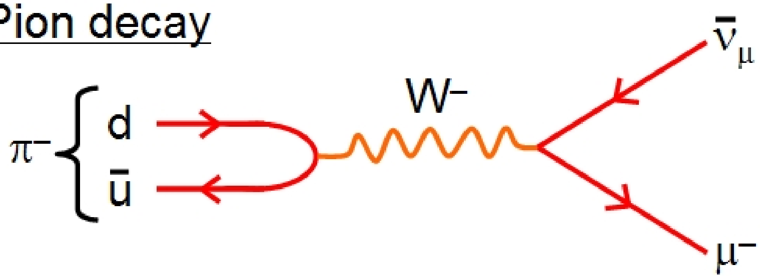
$$V_{CKM} \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$B^0_{\rm d}$–$\overline{B}^0_{\rm d}$ Mixing (B_B) $\qquad B^0_{\rm s}$–$\overline{B}^0_{\rm s}$ Mixing (B_{B_s}) $\qquad B \to K^*\gamma$ (Rare Decays) $\qquad B \to D l \nu$ $\quad B \to D^* l \nu$



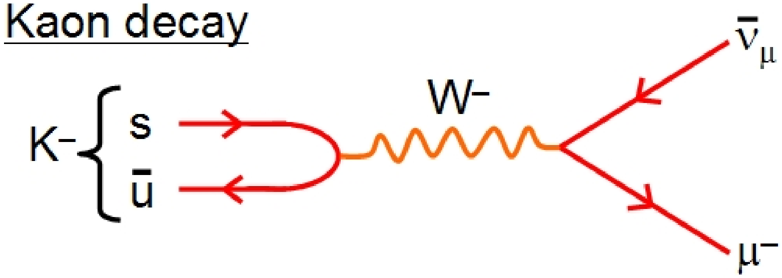
Lattice phenomenology (2): ... via external currents

Pion decay



$\pi^-(d\bar{u}) \to \mu^- + \bar{\nu}_\mu$

Kaon decay

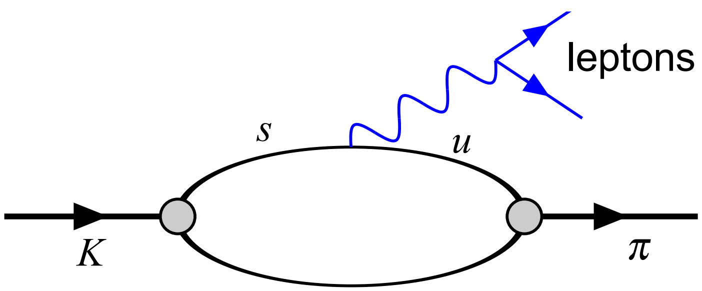


$K^-(s\bar{u}) \to \mu^- + \bar{\nu}_\mu$

$$J_\mu^{\mathrm{CC}} = (\bar{u}, \bar{c}, \bar{t})\gamma_\mu \tfrac{1}{2}[1-\gamma_5] V_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\langle 0|(\bar{u}\gamma_\mu\gamma_5 d)(x)|\pi^-(p)\rangle = \mathrm{i} f_\pi p_\mu e^{ipx}$$
$$\langle 0|(\bar{u}\gamma_\mu\gamma_5 s)(x)|K^-(p)\rangle = \mathrm{i} f_K p_\mu e^{ipx}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{V_{\mathrm{CKM}}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



$\Rightarrow \quad V_{us}$

$\Longrightarrow$ strong dynamics restricted to matrix elements $\langle 0|A_\mu|\pi\rangle$, $\langle 0|A_\mu|K\rangle$ and form factors $\langle \pi|V_\mu|K\rangle$ etc.

f_K/f_π calculation (1): Marciano's observation

- $|V_{ud}|$ is known, from super-allowed nuclear β-decays, with 0.03% precision [HT].

- $|V_{us}|$ is much less precisely known, but can be linked to $|V_{ud}|$ via a relation involving f_K/f_π, with everything else known rather accurately:

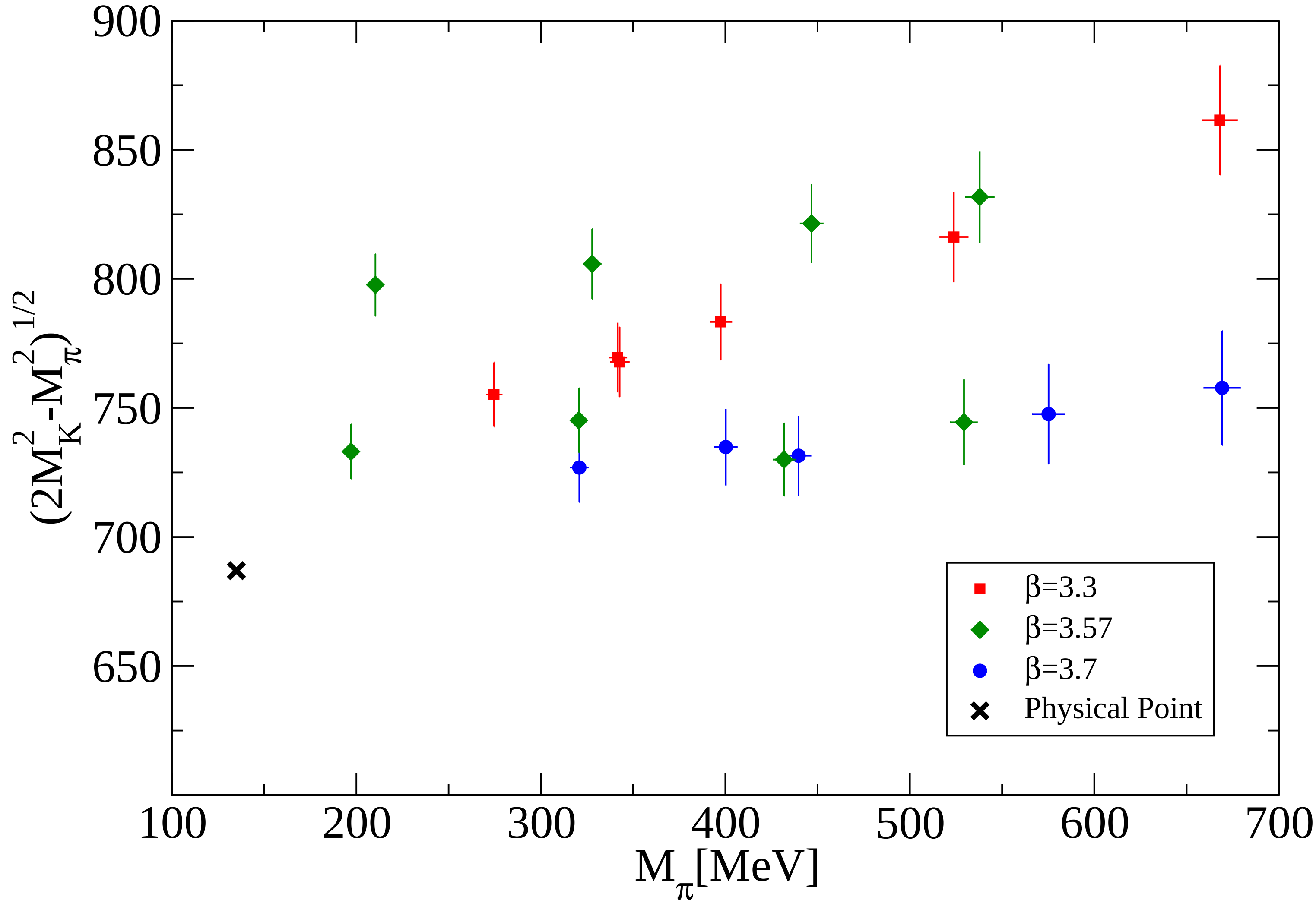
$$\frac{\Gamma(K \to l\bar{\nu}_l)}{\Gamma(\pi \to l\bar{\nu}_l)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{M_K(1-m_l^2/M_K^2)^2}{M_\pi(1-m_l^2/M_\pi^2)^2} \left\{1+\frac{\alpha}{\pi}(C_K - C_\pi)\right\}$$

- CKM unitarity $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ (with $|V_{ub}|$ being negligibly small) is genuine to the SM; any deviation is a *unambiguous* signal of BSM physics.

$\Longrightarrow$ calculate f_K/f_π in $N_f = 2+1$ QCD (with quark masses extrapolated to the physical point) on the lattice; the precision attained gives the precision of $|V_{us}|$.

f_K/f_π calculation (2): adjusting quark masses

$N_f = 2+1$ lattice QCD: set m_{ud}, m_s by adjusting M_π, M_K to their physical values



$\longrightarrow$ extract f_K/f_π on unitary ensembles and extrapolate to the physical mass point

$\longrightarrow$ $f_K/f_\pi = 1$ at $m_{ud} = m_s$ means that $f_K/f_\pi - 1$ is calculated with $\sim 5\%$ accuracy

f_K/f_π calculation (3): chiral extrapolation

- chiral $SU(3)$ formula:

$$\frac{F_K}{F_\pi} = 1+\frac{1}{32\pi^2F_0^2}\Big\{\frac{5}{4}M_\pi^2\log(\frac{M_\pi^2}{\mu^2})-\frac{1}{2}M_K^2\log(\frac{M_K^2}{\mu^2}) -[M_K^2-\frac{1}{4}M_\pi^2]\log(\frac{4M_K^2-M_\pi^2}{3\mu^2})\Big\}+\frac{4}{F_0^2}[M_K^2-M_\pi^2]L_5$$

- chiral $SU(2)$_plus_strange formula [RBC/UKQCD 08], simplified form:

$$\frac{F_K}{F_\pi}=\frac{F_K}{F_\pi}\Big|_{m_{ud}=0}\left\{1+\frac{5}{8}\frac{M_\pi^2}{(4\pi F)^2}\log\Big(\frac{M_\pi^2}{\Lambda^2}\Big)\right\}$$

- polynomial expansion $F_\pi/F_K = d_0+d_1(M_\pi-M_\pi^{\mathrm{ref}})+d_2(M_\pi-M_\pi^{\mathrm{ref}})^2$, e.g. around $M_\pi^{\mathrm{ref}}=300\,\mathrm{MeV}$, at fixed physical m_s, with $\Delta_{\pi,K}\equiv(M_{\pi,K}^2-M_{\pi,K}^{\mathrm{ref}\,2})/M_\Omega^2$ suggests:

$$\frac{F_K}{F_\pi}=c_0+c_1\Delta_\pi+c_2\Delta_\pi^2+c_3\Delta_K$$

$\longrightarrow$ use all of them and count spread towards systematic uncertainty

f_K/f_π calculation (4): infinite volume extrapolation

- finite volume effects on F_K, F_π are known at the 2-loop level [CDH 05]

$$\frac{F_\pi(L)}{F_\pi} = 1 + \sum_{n=1}^{\infty} \frac{m(n)}{\sqrt{n}} \frac{1}{M_\pi L} \; 1 \; \frac{M_\pi^2}{(4\pi F_\pi)^2} \left[I_{F_\pi}^{(2)} + \frac{M_\pi^2}{(4\pi F_\pi)^2} I_{F_\pi}^{(4)} + \ldots \right]$$

$$\frac{F_K(L)}{F_K} = 1 + \sum_{n=1}^{\infty} \frac{m(n)}{\sqrt{n}} \frac{1}{M_\pi L} \frac{F_\pi}{F_K} \frac{M_\pi^2}{(4\pi F_\pi)^2} \left[I_{F_K}^{(2)} + \frac{M_K^2}{(4\pi F_\pi)^2} I_{F_K}^{(4)} + \ldots \right]$$

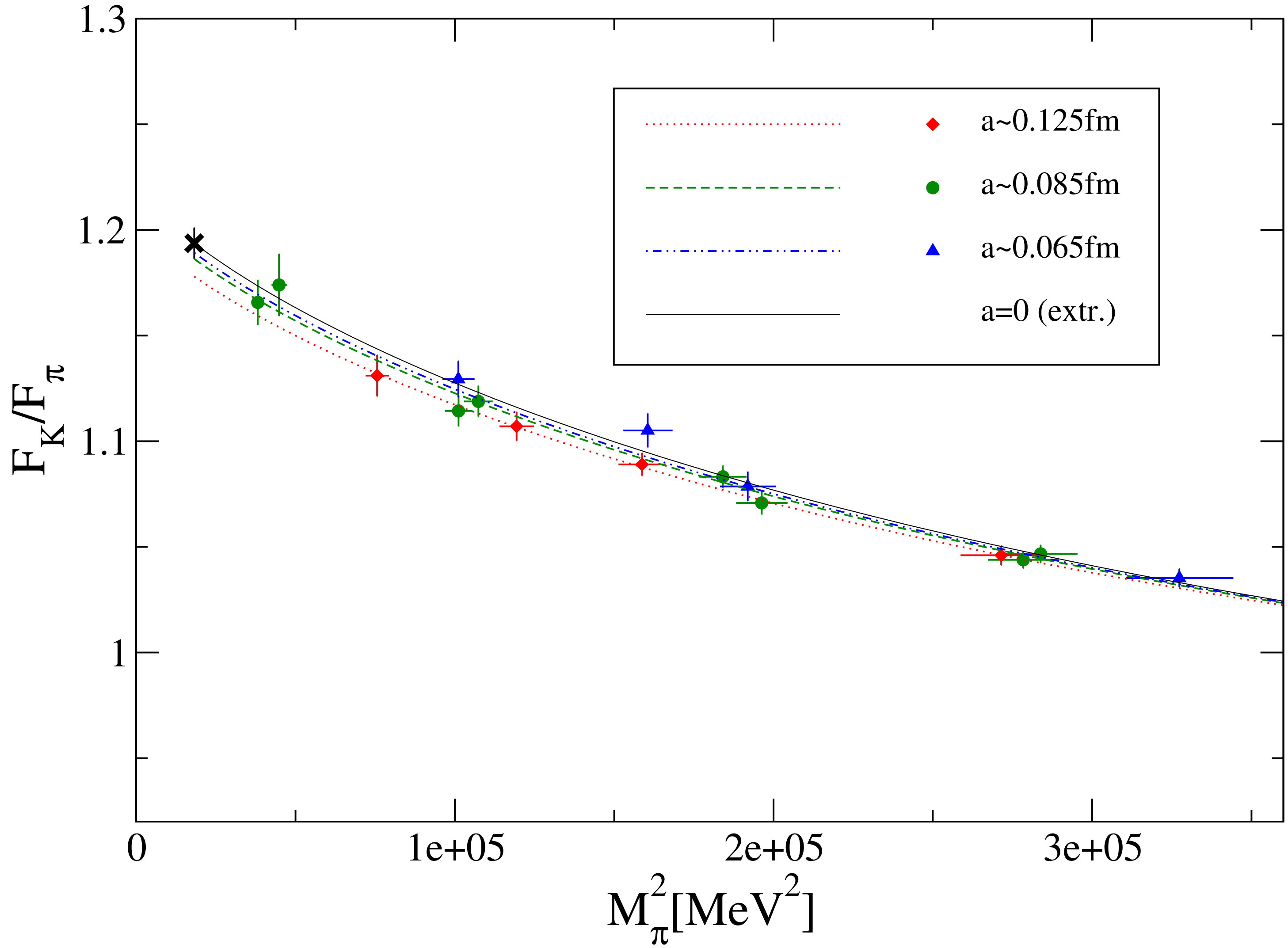
with $I_{F_\pi}^{(2)} = -4K_1(\sqrt{n}\, M_\pi L)$ and $I_{F_K}^{(2)} = -\frac{3}{2}K_1(\sqrt{n}\, M_\pi L)$, where $K_1(.)$ is a Bessel function of the second kind, and lengthy expressions for $I_{F_\pi}^{(4)}, I_{F_K}^{(4)}$

- finite volume effects cancel partly in the ratio, as evident from the 1-loop formula

$$\frac{F_K(L)}{F_\pi(L)} = \frac{F_K}{F_\pi} \left\{ 1 + \sum_{n=1}^{\infty} \frac{m(n)}{\sqrt{n}} \frac{1}{M_\pi L} \frac{M_\pi^2}{(4\pi F_\pi)^2} \left[\frac{F_\pi}{F_K} I_{F_K}^{(2)} - I_{F_\pi}^{(2)} \right] \right\}$$

- BMW uses $\frac{F_K(L)}{F_\pi(L)} / \frac{F_K}{F_\pi}$ at 1-loop and 2-loop level, and $F_\pi(L)/F_\pi$ at 2-loop level

f_K/f_π calculation (5): combined fits



$\longrightarrow$ plot shows $\mathrm{data}(M_\pi^2, 2M_K^2{-}M_\pi^2) - \mathrm{fit}(M_\pi^2, 2M_K^2{-}M_\pi^2) + \mathrm{fit}(M_\pi^2, [2M_K^2{-}M_\pi^2]_{\mathrm{phys}})$

$\longrightarrow$ f_K/f_π scales rather nicely [note $a^2/\mathrm{fm}^2 = 0.0042, 0.0072, 0.0156$]

$\Longrightarrow$ $f_K/f_\pi = 1.192(7)(6)$ at physical m_{ud} and m_s, in continuum, in infinite volume

f_K/f_π calculation (6): update on $|V_{us}|$ and CKM unitarity

- Latest nuclear structure calculations [Hardy Towner'09] give

$$|V_{ud}| = 0.97425(22) \; .$$

- Plug experimental information $\Gamma(K\to\mu\bar{\nu})/\Gamma(\pi\to\mu\bar{\nu}) = 1.3363(37)$ [PDG'08] and $C_K - C_\pi = -3.0 \pm 1.5$ [Marciano] into Marciano's equation; this yields

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.27599(59) \; .$$

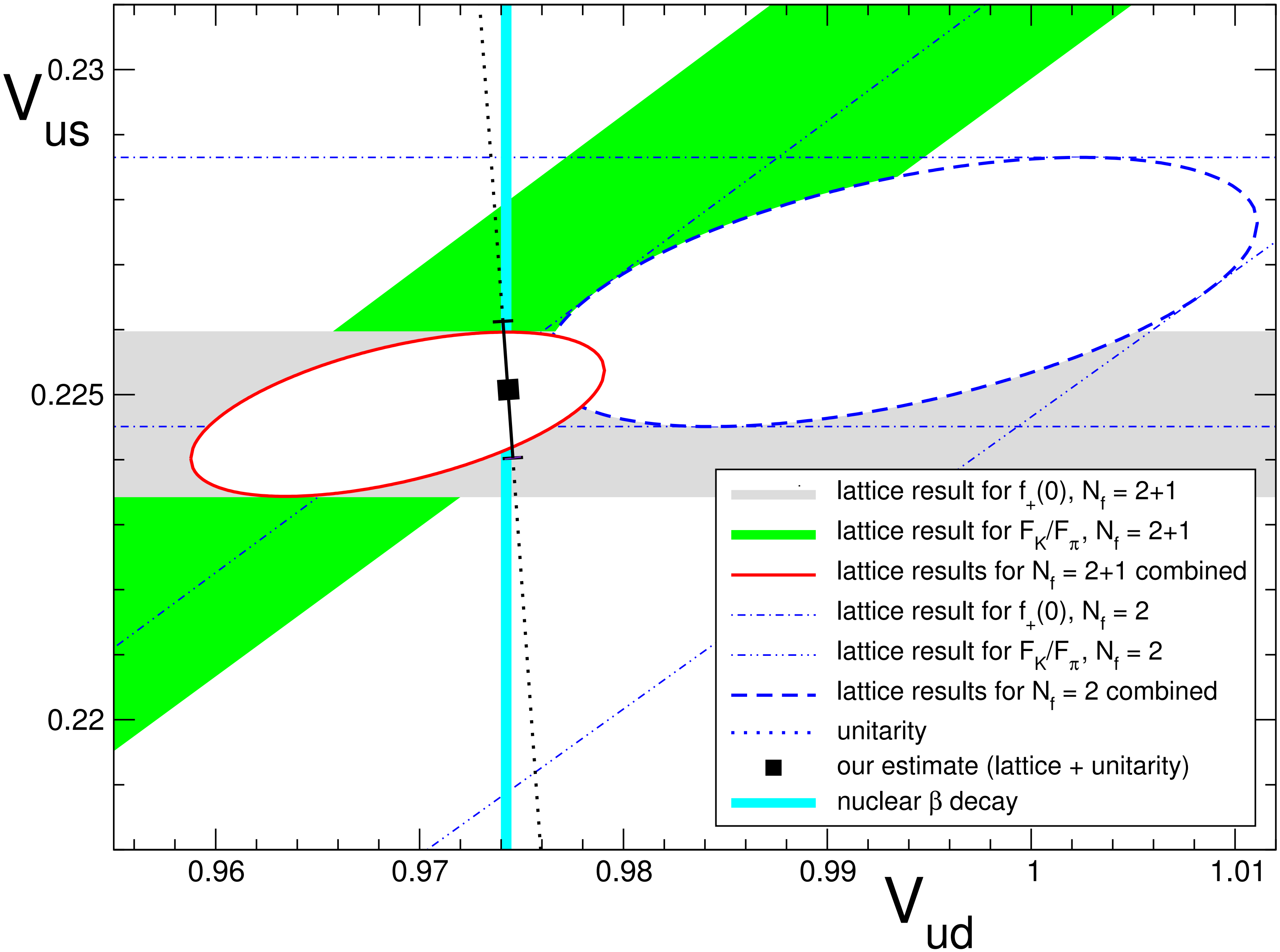
- Upon combining the previous one/two points and our value for f_K/f_π we obtain

$$\frac{|V_{us}|}{|V_{ud}|} = 0.2315(19) \quad \text{and} \quad |V_{us}| = 0.2256(17) \; .$$

- Upon including $|V_{ub}|=3.39(36)10^{-3}$ [PDG'08] we end up with [BMW, 1001.4692]

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0001(9) \; .$$

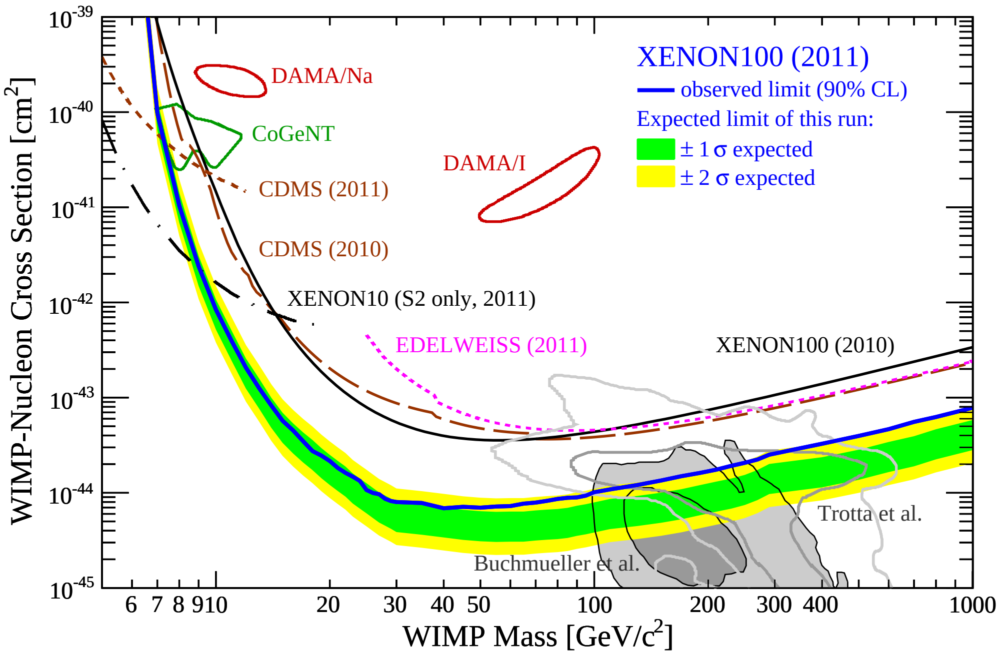
f_K/f_π calculation (7): FLAG summary



Lattice Outreach

- baryon sigma terms and dark matter
- nuclear physics from first principles
- QCD thermodynamics at $\mu=0$
- QCD thermodynamics at $\mu>0$
- hadronic contributions to muon $g-2$
- isospin splitting and electromagnetism
- large N_c, larger N_f, different representations

Lattice outreach (1): WIMPS via nucleon sigma terms



XENON100, PRL 107 (2011) 131302

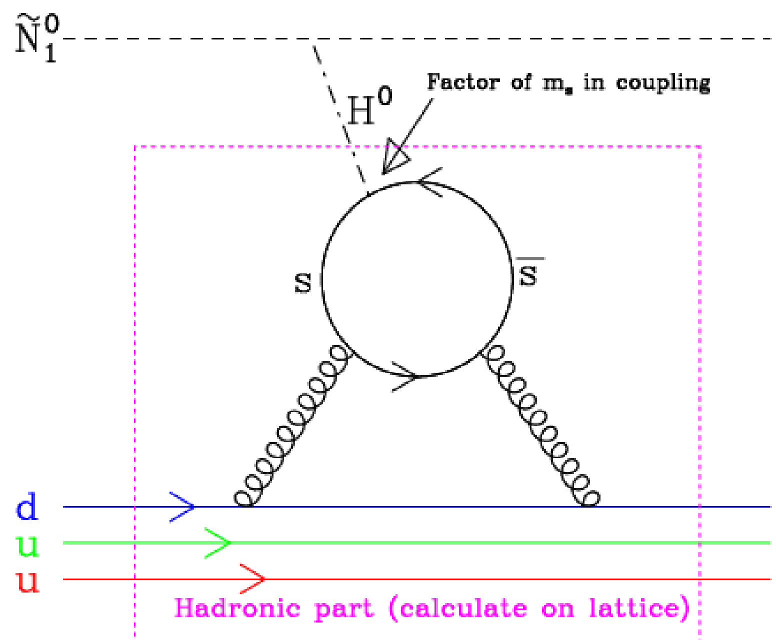
Universe: 73% dark energy
23% dark matter
4% baryons

Dark matter stays dark, unless WIMP-Nucleon scattering can be probed down to tiny cross-sections.

Traditionally large uncertainty from matrix elements

$$\sigma_{ud} = m_{ud} \langle N|\bar{u}u+\bar{d}d|N\rangle$$
$$\sigma_s = m_s \langle N|\bar{s}s|N\rangle$$

(RGI, dimension of mass)

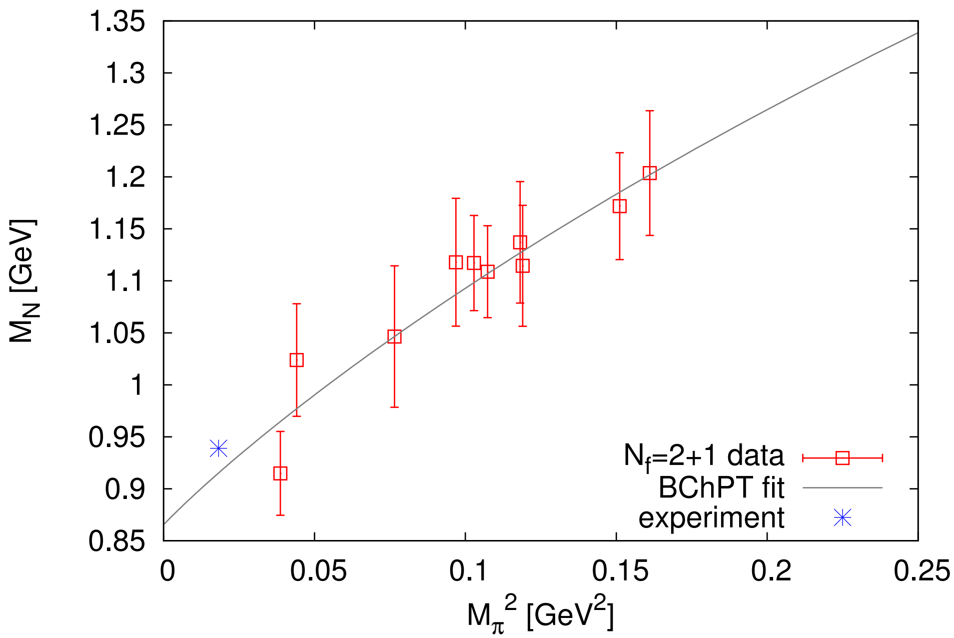


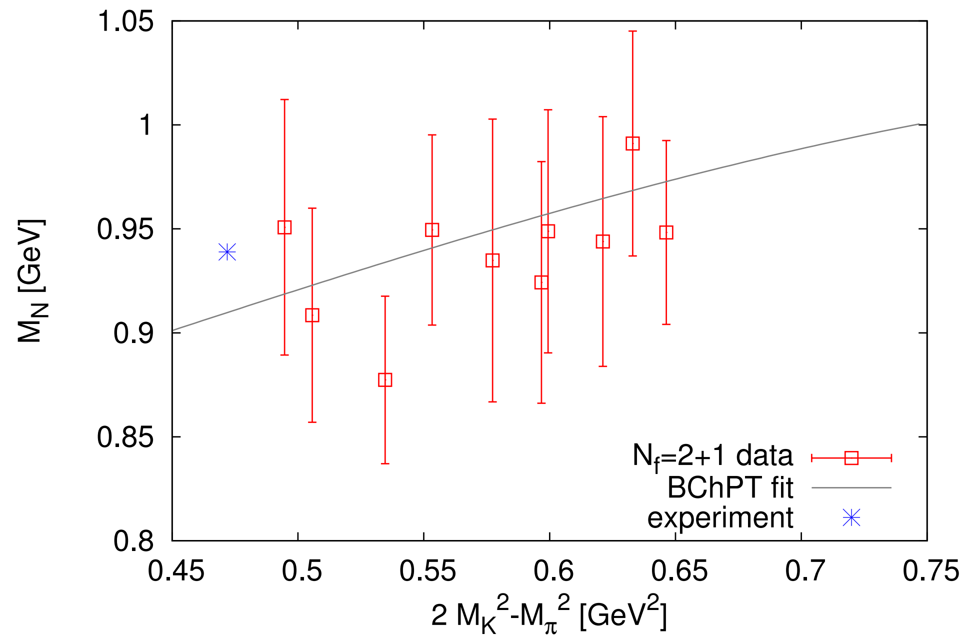
W. Freeman et al, arXiv:1204.3866

Lattice outreach (2): sigma terms via Feynman-Hellmann

Lattice can compute σ_{ud} and σ_s directly or via Feynman-Hellmann theorem:

$\sigma_{ud} = m_{ud}\frac{\partial M_N}{\partial m_{ud}} = M_\pi^2 \frac{\partial M_N}{\partial M_\pi^2}$ and $\sigma_s = m_s \frac{\partial M_N}{\partial m_s} = (2M_K^2 - M_\pi^2)\frac{\partial M_N}{\partial(2M_K^2 - M_\pi^2)}$

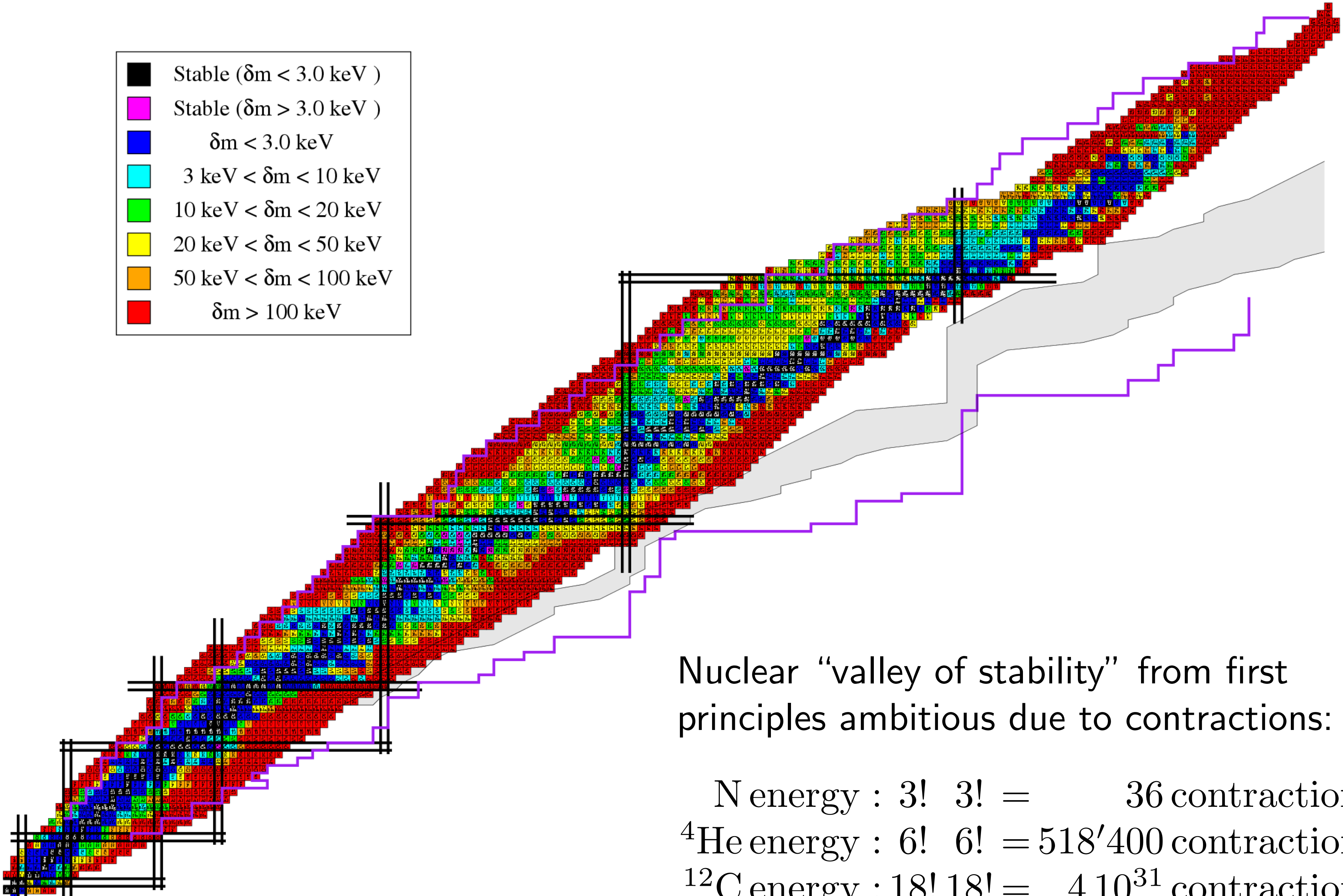




$\Longrightarrow$ we find $\sigma_{ud} = 39(4)(^{+18}_{-7})\,\mathrm{MeV}$ and $\sigma_s = 34(14)(^{+28}_{-24})\,\mathrm{MeV}$

$\longrightarrow$ in consequence $m_{ud}\langle N|\bar{u}u+\bar{d}d|N\rangle \simeq m_s\langle N|\bar{s}s|N\rangle$

Lattice outreach (3): nuclear physics from first principles



Nuclear “valley of stability” from first principles ambitious due to contractions:

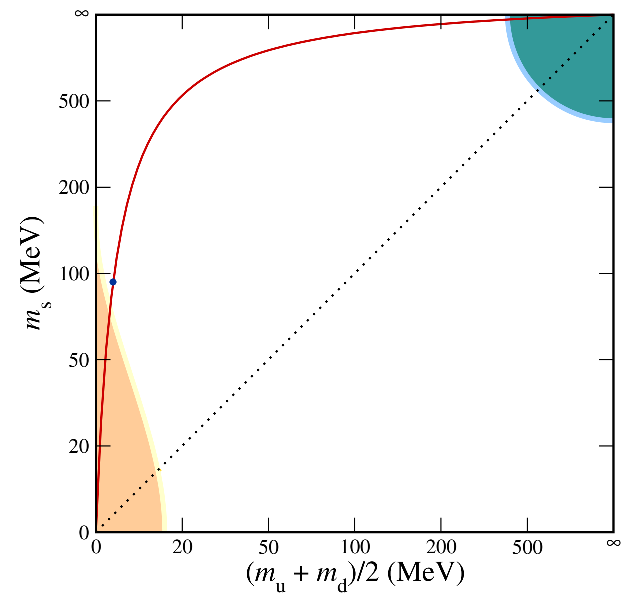
$$
\begin{aligned}
\mathrm{N\,energy} &: 3!\ \ 3! = 36 \text{ contractions}\\
{}^{4}\mathrm{He\,energy} &: 6!\ \ 6! = 518'400 \text{ contractions}\\
{}^{12}\mathrm{C\,energy} &: 18!\,18! = 4\,10^{31} \text{ contractions}
\end{aligned}
$$

Lattice outreach (4): QCD thermodynamics at $\mu = 0$

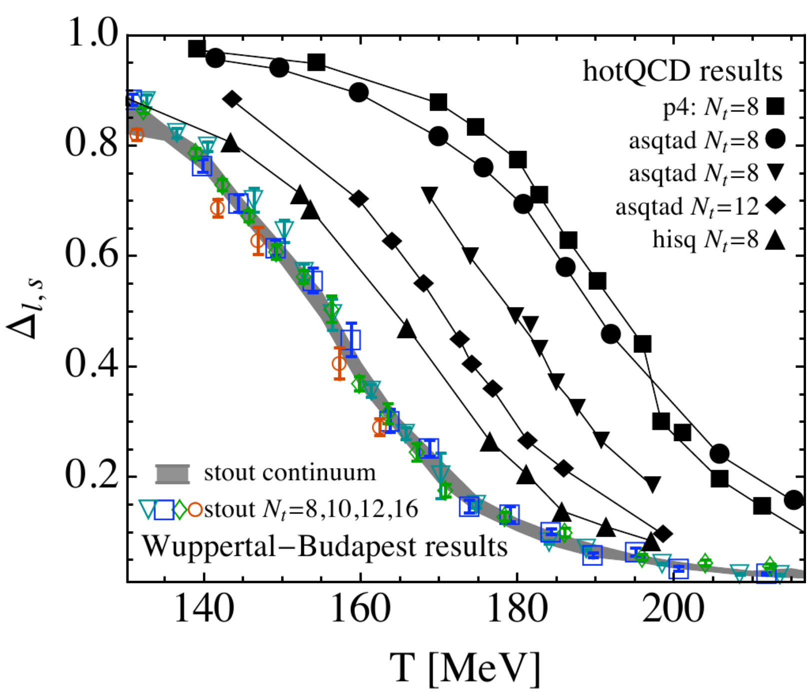
Established: QCD with physical m_{ud}, m_s at zero chemical potential (as relevant in early universe) shows *crossover*.

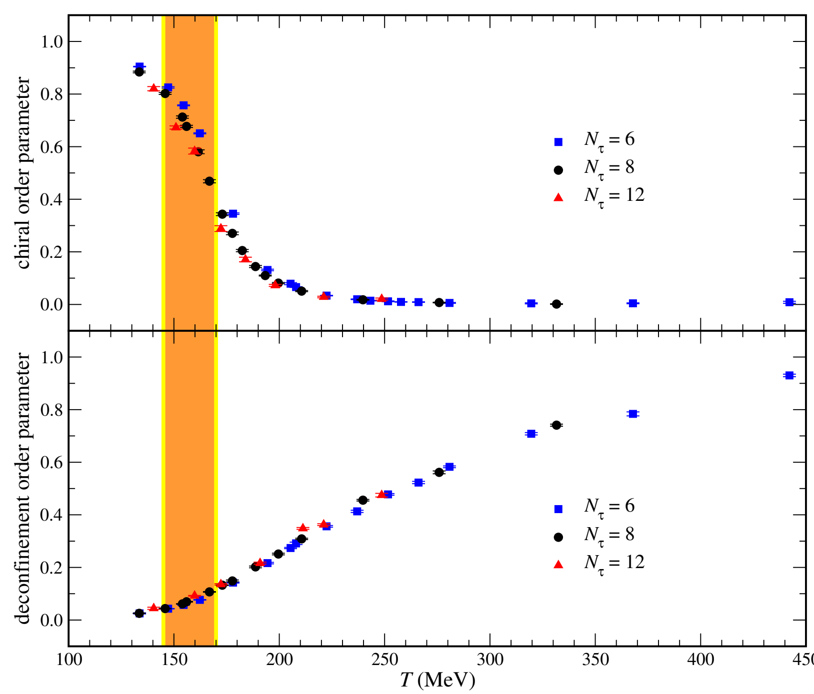
Different definitions of "transition temperature" T_c yield different values [P, $\langle\bar{\psi}\psi\rangle$, ...], but for one definition everyone should agree in the continuum.

Long standing discrepancy between Wuppertal-Budapest (left) and HotQCD (right) now resolved.



plot by A. Kronfeld

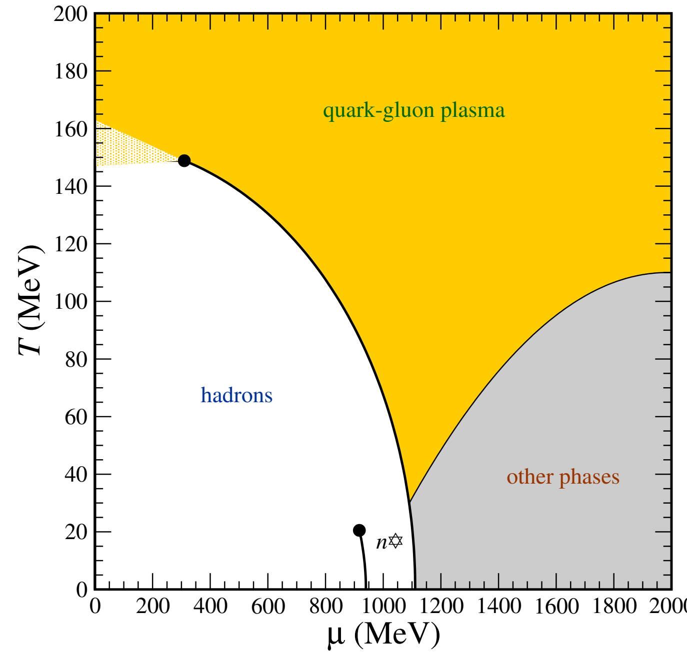




Lattice outreach (5): QCD thermodynamics at $\mu > 0$

At non-zero baryon density (equivalent: chemical potential $\mu \neq 0$) the fermion determinant becomes complex, which creates a major difficulty to the concept of importance sampling.

A clear establishment of a second-order endpoint would be a major leap forward.



plot by A. Kronfeld

In QCD many approaches to solve the sign problem have been tried:

- absorb phase in observable [ancient]
- two-parameter reweighting from $\mu = 0$ [Fodor Katz]
- work at imaginary μ and continue [Philipsen deForcrand]
- compute Taylor coefficients at $\mu = 0$

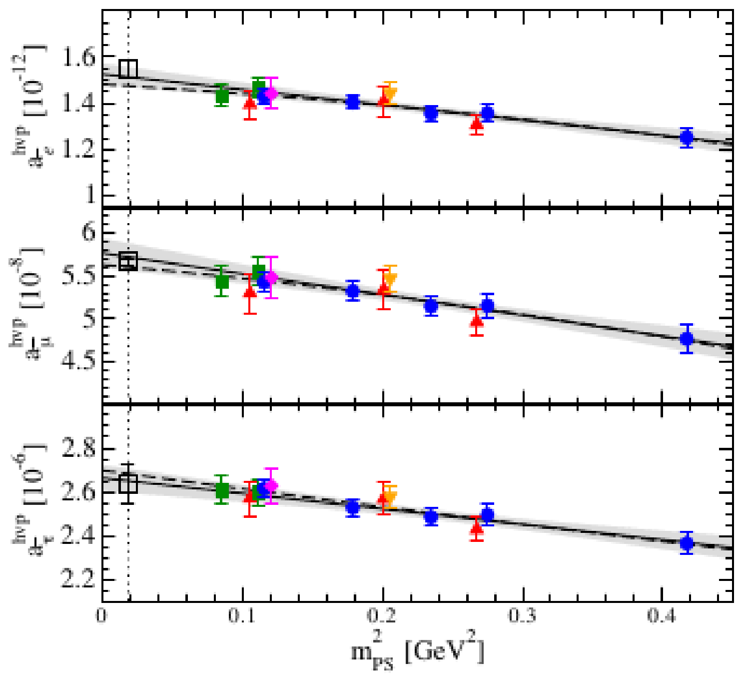
In QCD-inspired models many tricks/reformulations become possible.

Lattice outreach (6): hadronic contributions to muon $g-2$

Hadronic contributions to vacuum polarization provide one of the major sources of systematic uncertainty in the computation of $a_\mu = (g_\mu - 2)/2$. Can the lattice help ?

$$a_\ell^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, f(Q^2)\, \bar{\Pi}(Q^2)$$

with known f and $\bar{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$ and $\Pi_{\mu\nu}(q) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$ can be computed as the Fourier transformed 2-point function of the electromagnetic current.



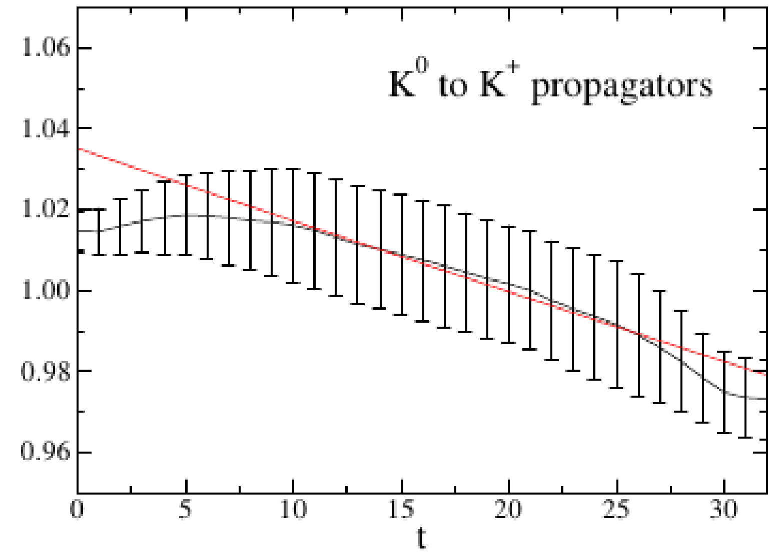
Recent computations include:
Feng et al, Phys.Rev.Lett. 107 (2011) 081802 [arXiv:1103.4818]
Della Morte et al, JHEP 1203 (2012) 055 [arXiv:1112.2894]
Kerrane et al, Phys.Rev. D85 (2012) 074504 [arXiv:1107.1497]

Lattice outreach (7): isospin splittings and electromagnetism

In standard $N_f = 2+1$ lattice studies two sources of isospin breaking are ignored (up-down mass difference, electromagnetic). Since they are both small, it would appear reasonable to include both of them a posteriori, by reweighting the configurations.

PACS-CS has long experience with reweighting in the quark mass; they used reweighting in m_{ud} to shift M_π from $156\,\mathrm{MeV}$ to $135\,\mathrm{MeV}$.

In arXiv:1205.2961 they extend this approach to account for QED effects and the up-down quark mass difference. They find $M_{K^0} > M_K^\pm$.



Pioneering publication for QCD+QED on the lattice is Duncan et al, Phys. Rev. Lett. 76 (1996) 3894-3897 [hep-lat/9602005].
Continuation by RBC/UKQCD Phys.Rev. D76 (2007), Phys.Rev. D82 (2010) 094508.

Still, there remain issues relating to finite-volume corrections, see e.g. Hayakawa Uno, Prog.Theor.Phys. 120 (2008) 413 and Portelli et al, PoS LATTICE2011 (2011) 136.

Lattice outreach (8): $N_f = 1{+}1{+}1{+}1$ plus QED simulations

- 2002-20??:

  $N_f = 2{+}1$ QCD requires 3 polished input values [e.g. M_π, M_K, M_Ω in theory with $m_u, m_d \to (m_u{+}m_d)/2$ and $e \to 0$]

  $\longrightarrow$ analysis suggests $M_\pi{=}134.8(3)\mathrm{MeV}, M_K{=}494.2(5)\mathrm{MeV}$ [see FLAG report]

- 2010-????:

  $N_f = 2{+}1{+}1$ QCD requires 4 polished input values [ditto and M_{D_s} in theory with $m_u, m_d \to (m_u{+}m_d)/2$ and $e \to 0$]

  $\longrightarrow$ charm unquenched, but no conceptual change on isospin issue

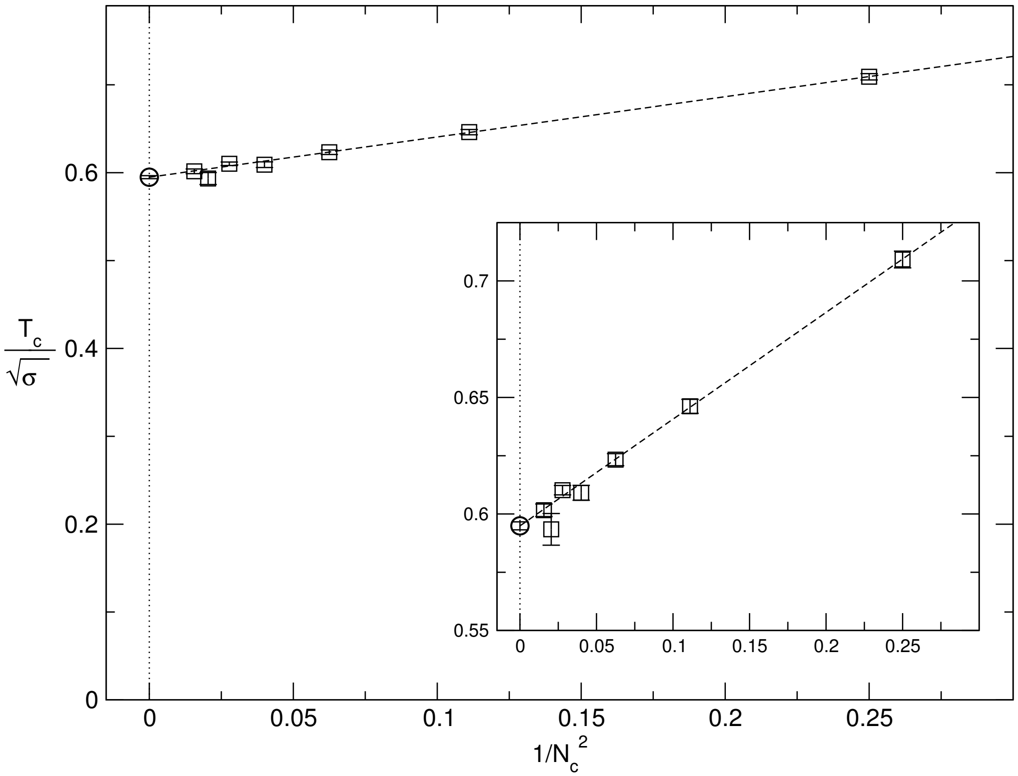
- 2014-????:

  $N_f = 1{+}1{+}1{+}1$ QCD requires 5 input variables [e.g. $M_{\pi^\pm}, M_{K^\pm}, M_{K^0}, M_{D_s}, M_\Omega$]

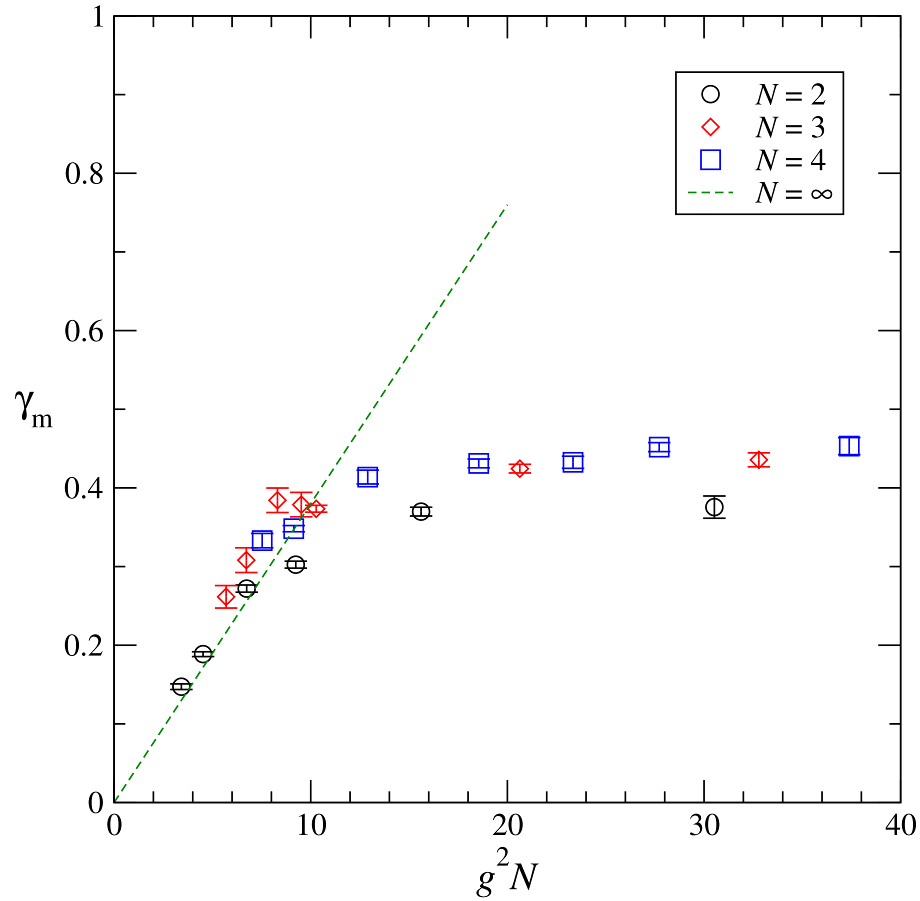
  $\longrightarrow$ requires disconnected contribution to flavor-singlet quantities
  $\longrightarrow$ analysis of π^0-η-η'-γ mixing mandatory to extract physical masses
  $\longrightarrow$ QED and QCD renormalization intertwined (m_s/m_d is RGI, m_u/m_d is not)
  $\longrightarrow$ final word on $m_u \stackrel{?}{=} 0$ [in QCD+QED] will be possible

Lattice outreach (9): Large N_c, larger N_f, higher representations

QCD with $N_c \to \infty$ and fixed $\lambda = g^2 N_c$ gets much simpler [weakly coupled hadrons, OZI exact, chiral loops $\sim 1/N$, axial anomaly $\sim 1/N$]; lattice is almost unnecessary ;-)

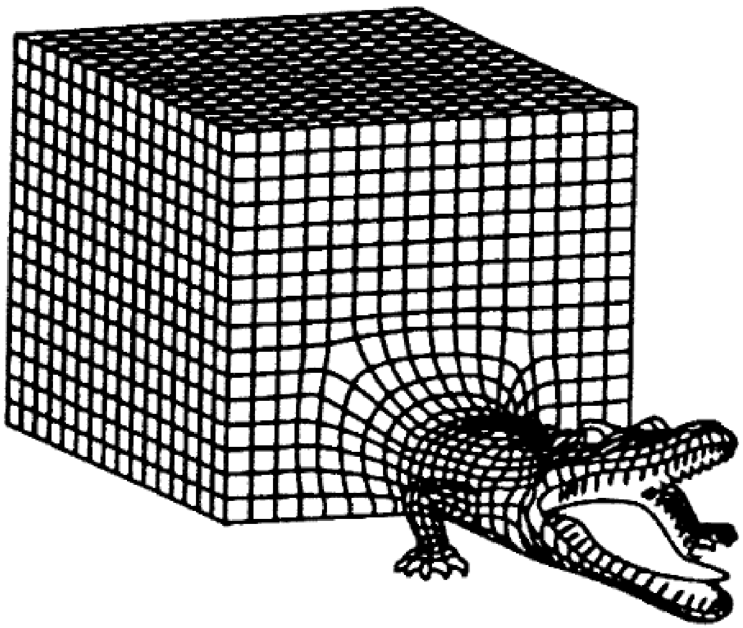


$T_c/\sqrt{\sigma}$ for $N_c \to \infty$
Lucini, Rago, Rinaldi, 1202.6684



Anomalous mass dimension
2-index symm-representation, $N_f = 2$
DeGrand, Shamir, Svetitsky, 1202.2675

Summary



Lattice'90, Tallahassee

- Lattice solves QCD from first principles: euclidean QFT, analytical and numerical methods
- Remnants of lattice formulation to be removed:
  - ◇ continuum extrapolation: $a \to 0$
  - ◇ infinite volume extrapolation: $L \to \infty$
  - ◇ chiral inter/extrapolation: $m_q \to m_q^{\mathrm{phys}}$

- Hadron spectroscopy with one stable particle on in and out side is simple
- Hadron spectroscopy with multiparticle states on in or our side is challenging
- Wealth of applications in flavor physics, nuclear physics, (perhaps) BSM physics
- Formulation useful for addressing conceptual issues in euclidean QFT

Epilogue: lattice literature

- G. Colangelo et al. [FLAG], Eur. Phys. Jour. C 71, 1695 (2011) [arXiv:1011.4408].
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- T. DeGrand, C. DeTar, Lattice Methods for Quantum Chromodynamics, World Scientific, 2006.
- J. Smit, Introduction to Quantum Fields on a Lattice, Cambridge University Press, 2002.
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