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The Eye of Horus and the synodic month

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In ancient Egyptian sources it is well attested both textually and iconographically that the Eye of Horus – an eye made up of six parts, each representing a fraction in a geometric progression – had a strong lunar character. Drawing on one form of cryptic notation found in Graeco-Roman temples, which in turn relies on the fixed length of the Egyptian civil month, a simple mathematical operation is executed to show that the fractions of the Eye signified a lunar period comparable to our notion of the mean synodic month.

The human eye with falcon-like attributes, either as the Eye of Horus or as the Eye of Ra, was unquestionably one of the most frequent symbols in ancient Egypt. It was often depicted in iconographic representations in varying contexts from the 3rd Dynasty onwards,[1] had a preeminent importance already in the Pyramid Texts[2] (5th Dynasty) and was widely used in the cult of the dead as a protective amulet.[3] Clearly, the Eye of Horus was an immensely complex symbol carrying a wide variety of significations for the ancient Egyptians, and it was identified with such diverse phenomena as the royal crown, the land of Egypt or the ritual offering to the dead. Corollary to this, modern interpretations are just as wide-ranging and fraught with uncertainties.[4] However, Ptolemaic texts leave little doubt about the ultimate origins of this symbol: it had sprung up from the belief that the sky was a falcon whose eyes represented the two brightest bodies in the heavens.[5] Thus the right eye was identified with the sun, while the left one stood for the moon.

One well-known mythological event concerning the Eye was the fight between Horus and Seth, during which the latter tore out the left eye of Horus. This story may have belonged to the original Horus-legend, in which the two protagonists very probably represented some opposite forces of nature, though the duel between the two gods may also have been prompted by the acceptance of Horus into the Osirian pantheon.[6] In any case, allusions to the fight in ancient Egyptian texts present the story in its later form, when it was adapted to the Osirian cycle of mythological tales. Horus is now the son of Osiris avenging his father's death, and his Eye is wrenched out by his opponent while the two are being engaged in battle over the succession to the throne of Egypt. Spell 17 of the Book of the Dead[7] (New Kingdom) informs us that the Eye is then magically restored to Horus by Thoth, the god of all sciences, who – using his fingers (perhaps a reference to counting)[8] – makes the Eye full or complete, or in Egyptian, *wḏꜣt*, 'the sound eye'.

Even in antiquity Plutarch suspected – probably basing his views on the testimony of native informants – that this mythological episode allegorically referred to the monthly waning of the moon, or perhaps more particularly to the disappearances of this celestial body either at new moons or during eclipses.[9] From his explanation it is clear that Plutarch meant lunar eclipses, but judged from some passages in the Pyramid Texts (§§ 594-596, 976, 1742), in which Thoth – and in one version, Seth – carries back the torn eye to its owner on his wings, solar eclipses may also have been alluded to, because this imagery forcefully evokes the sun's corona, visible only when the solar disc is totally eclipsed. Solar eclipses only occur at new moon, so here the Eye of Horus seems to be associated with that aspect of the moon.

A number of texts from the New Kingdom and later, however, unambiguously identify the Eye with the full moon when they say that the Eye becomes intact on the 15th day of the month.[10] This identification is expressed particularly strongly on the walls of the Graeco-Roman temples, not only by plenty of inscriptions, but also by numerous images. Two obvious examples from Dendera are the picture strip of the lunar cycle on the ceiling of the pronaos, where the Eye of Horus is drawn within the disc of the full moon (Fig. 1),[11] and the famous zodiac in one of the Osirian chapels, where the lunar disc containing the Eye is depicted between the constellations of Pisces and Aries. Eric Aubourg has determined that this situation refers to a total eclipse of the moon taking place on 25 September 52 BC.[12] Certainly, a lunar eclipse is only possible at opposition, so here the Eye must again refer to the full moon.

It was also believed that Seth tore the Eye into six pieces, and each piece in turn represented a fraction of the whole Eye (Fig. 2). These fractions form the terms of a geometric progression: the first term is 1/2 and the common ratio is 2 (1/2, 1/4, 1/8, 1/16, 1/32, 1/64). The hieratic signs representing the elements of this series were utilized as early as the beginning of the dynastic period to designate the fractions of the *ḥḳꜣt* measure,[13] a measure of volume for corn, though the fully-fledged system with the hieroglyphic representations is attested only from the New Kingdom onwards.[14] While we do not know why the Horus-eye fractions became associated so closely with the *ḥḳꜣt* measure (is a moon = Osiris = corn lineage to blame?), we must nonetheless notice that the multiples of the *ḥḳꜣt* continued the geometric progression inscribed in the Eye naturally, because besides the single *ḥḳꜣt* there was also a double *ḥḳꜣt*, and a quadruple *ḥḳꜣt* (thus we have the series 1/64, 1/32, 1/16, 1/8, 1/4, 1/2, 1, 2, 4).

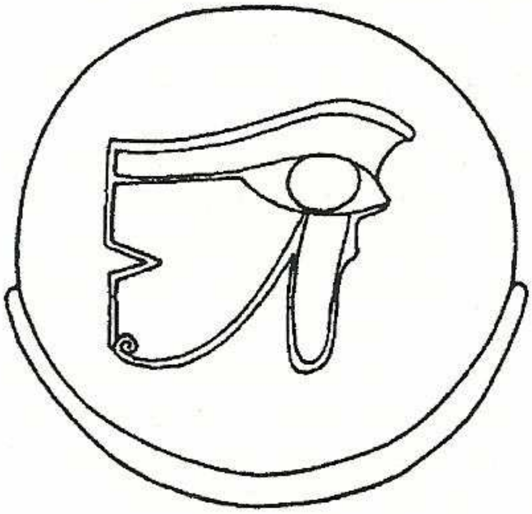

Figure 1 Eye of Horus depicted in full moon at Dendera[15]

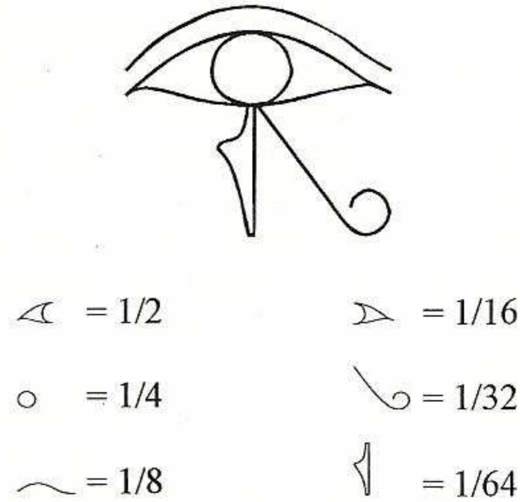

Figure 2 Fractions corresponding to parts of Eye of Horus

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We owe to Georg Möller the discovery in modern times that the signs for the fractions of the *ḥḳ3t* measure can be arranged into the Eye of Horus.[16] He also realized that these fractions added up to 63/64 and believed – together with Alan Gardiner[17]– that the 1/64 that would complete this sum to unity had been supplied magically by Thoth when he healed the Eye. Adriana Belluccio conjectured that the fractions should refer to a number characteristic of the moon, and she claimed to have found this number as the days (384) in a full lunar year of thirteen months.[18] All these observations were steps in the right direction insofar as they highlighted the fact that whenever the Eye of Horus was depicted, by force of the above analogy the fraction of 63/64 was noted down, too. However, researchers have so far disregarded some further important clues coming from the Ptolemaic temples that help to fully understand the symbolism of the Horus-eye fractions.

That there was some rationale behind this series of dimidiated fractions is hinted at by the Egyptians themselves. For example, in one of the eastern crypts of the Dendera temple the title of the scene describing the presentation of the *wnšb*, an object connected in all probability with time, or the measurement of time (if its doubtful identity with an inflow water clock is accepted),[19] goes as follows:

> To be recited: Your ka is healthy, provided with its fractions, its pupil is complete in its place, the *wḏ3t*-eye is hale as Iseden (= Thot) [has made] it so, all its forms are according to right reckoning (*tp-ḥsb*). (*D*. V, 20, 10-12)[20]

Since the ceremonial offering of the *wnšb* had become a stock image of Egyptian temple decoration by the Ptolemaic period, numerous similar passages may be pointed out from Dendera itself or from all over Egypt.[21] What is remarkable at Dendera is the mention of *tp-ḥsb*, right reckoning, an expression that in the same temple also occurs in the texts specifying the ground-plans of the sanctuaries. In an earlier article I showed that *tp-ḥsb* was not used as an empty formula there, but referred to a decipherable rule of simple geometry.[22] Consequently, we must now suppose that this phrase is meaningful in the context of the Eye of Horus as well, and stands for some basic mathematical operation.

We have seen that in the Egyptians' imagination the Eye of Horus frequently referred to the moon, and was perhaps particularly associated with the two most conspicuous barriers of the lunar cycle, the new and the full moon. The number we are looking for therefore must be connected with the monthly cycle of the moon. In ancient Egypt there *was* a number closely tied to the month, and that was 30, the number of days in the civil month. Although it is true that the civil month, being always equal to 30 days, was a highly schematized construction, its connection with the real lunar month has never been forgotten entirely, as shown by the use of the hieroglyph of the crescent moon (𓇹) in civil dates to signify this calendrical unit.[23]

Now, fortunately for us, most of the numbers in the inscriptions of the Graeco-Roman temples are not written with the ordinary Egyptian notation, but involve some kind of play with hieroglyphs, or are in some way cryptic.[24] Thus in the sentences reporting the 25-year (!) period of building for the naos at Edfu (*E.* IV, 7, 1-7; VII, 5, 7 and 6, 3-4; also *E.* IV, 14, 4)[25] the number seven, day 7 of the month, is expressed this way: 1/5 1/30, meaning that these fractions must be applied to 30, so that 1/5 × 30 = 6 and 1/30 × 30 = 1, and 6 + 1 = 7. We of course get the same result if we first add up the fractions and then multiply with 30: (1/5 + 1/30) × 30 = 7. Some other examples of this notation are: 1/30 = 1 (*E.* IV, 8,4; VII, 7, 1); 1/10 = 3 (*DCB.* 131); 1/10 1/30 = 4 (*DCB.* 137 and 164); 1/5 1/10 = 9 (*DCB.* 164); 1/3 1/10 = 13 (*DCB.* 133); 1/2 1/30 = 16 (*DCB.* 164); 1/2 1/10 = 18 (*E.* IV, 9, 1; VII, 7, 6 and 9, 1); 1/2 1/10 1/30 = 19 (*DCB.* 164); 1/2 1/3 1/10 = 28 (*DCB.* 137).[26] Perhaps understandably, this use of the fractions of the number 30 – albeit without any explicit instructions to its required application as a point of reference – only occurs in connection with days. We cannot fail to notice the analogy between this notation and the Horus-eye numbers: in both cases we have a series of fractions. Accordingly, the Horus-eye fractions must also be made use of by calling in the number 30:

$$(1/2 + 1/4 + 1/8 + 1/16 + 1/32 + 1/64) \times 30 = 29\ 1/2\ 1/32 \qquad (= 29.53125)$$

Indeed, we now have arrived at perhaps the most significant number of the moon in astronomical terms, because 29.53125 days is a very good approximation of the mean length of the synodic month (29.53059 days in contemporary reference books,[27] the difference being 0.00066 day, i.e. 57 seconds). The synodic month is by definition the period between two identical phases of the moon, for example between two full moons (oppositions). Since the length of 29.53125 days comes from a neat mathematical exercise, we are made to think that it has been derived from some previously empirically obtained value. Nonetheless, no great margin of error has been allowed for by the Egyptians, and the correspondence with the data computed in modern times suggests that in spite of all the complexities of the earth-moon system,[28] the mean length of the lunations has not changed significantly in the past 5000 years.

So the question arises now as to how the ancient Egyptians could have determined the length of the synodic month with such precision. Since consecutive synodic months vary in length considerably (±0.27 day from mean at maximum), a precise mean value for the astronomical month could only have been obtained from averaging observational months – either 29 or 30 days in length – over sufficiently long periods. Once the civil calendar was adopted, however, such a long period had automatically been developed, as twenty-five 365-day years comprised almost exactly 309 synodic months (25 × 365 days = 9125 days ≅ 309 × 29.53059; this equation, as I argued elsewhere,[29] must have been the source of the Egyptians' fascination with the golden ratio). Also, the civil calendar purported the idea of months of equal length, and the concept of the mean synodic month can be said to be based on the same principle. As soon as the 25-year lunar cycle was discovered – most likely by noticing that after such an interval the lunar phases systematically fell on the same civil dates (in my opinion, this happened very early in Egyptian history, despite the fact that direct evidence for the recognition of the cycle only dates from the 2nd century AD)[30] – the task was fairly simple: the number of days elapsed had to be divided by the number of months elapsed (9125 ÷ 309 = 29.53074 ≅ 29 1/2 1/33). Then it was sheer ingenuity on the part of the Egyptians that they approximated this value by collating a geometrical series of fractions and the length of the civil month.

That such a feat was well within the capabilities of an Egyptian scribe is proved beyond doubt by the Rhind Mathematical Papyrus which is thought to reflect the state of mathematics in the Middle Kingdom. A large number of the problems in the papyrus inevitably require the manipulation of lengthy series of fractions. From these problems no. 36 is one of those that show the most affinities with the arithmetical background of the Eye of Horus.[31] First of all, and perhaps not accidentally, the enunciation of the problem involves the *ḥḳ3t* measure (reconstructed from the problems belonging to the same group): "I go three times [into the *ḥḳ3t* measure], my third and my fifth are then added to me. I return fully satisfied. What is the quantity that says this?" Translated into modern mathematics, this is the statement of a first-degree equation where the multiplicand and the product are given: (3 1/3 1/5)x = 1. By the method of false position, the scribe gets the correct answer of x = 1/4 1/53 1/106 1/212. It is a noteworthy detail that in his calculations he uses the common denominator 30 in place of a more obvious 15.[32] Now, supposing that the Egyptians' intention was to express the length of the lunations in relation to the 30-day civil month, the problem behind the Eye of Horus may be stated like this: 30x = 29 1/2 1/33. Then it followed from the limitations of Egyptian mathematics that the result would be given as a series of unit fractions. A touch of imagination somewhere in the process meant a slight adjustment of the product to 29 1/2 1/32, and thus the result came out as a sleek progression of fractions, each being half the preceding one.

In conclusion, I must now retract my earlier tentative remark that the ancient Egyptians were not familiar with the exact length of the lunations. Their knowledge of it has been clearly preserved in the symbolic language of the Eye of Horus. It is a commonplace of Egyptology that the Eye had lunar, or in broader terms, astronomical symbolism. Now we must acknowledge, however, that this symbolism went beyond being merely astronomical, and was rather mathematical astronomical, as the fractions of the Eye denoted the average length of the synodic month – a mathematical astronomical concept par excellence. As for the development of this concept in ancient Egypt, the idea of a month of constant duration must have set in very early – some time in the third millenium BC – because the use of the civil calendar with its uniform 30-day months is well attested from that early period. It is also a remarkable circumstance that the civil calendar and the Horus-eye fractions are interdependent upon each other to signify the mean length of the actual lunations. This, however, again underlines my conviction that astronomy and mathematics were far more advanced at the dawn of Egyptian history than hitherto believed.

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