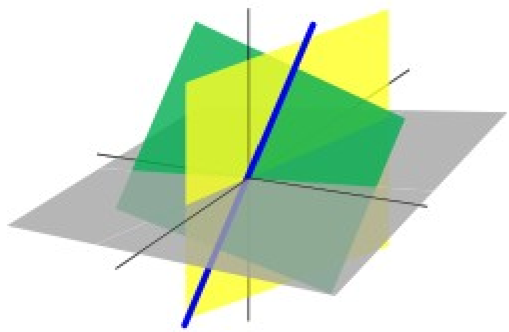
A Brief History of Linear Algebra



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In order to unfold the history of linear algebra, it is important that we first determine what Linear Algebra is. As such, this definition is not a complete and comprehensive answer, but rather a broad definition loosely wrapping itself around the subject. I will use several different answers so that we can see these perspectives. First, linear algebra is the study of a certain algebraic structure called a vector space (BYU). Second, linear algebra is the study of linear sets of equations and their transformation properties. Finally, it is the branch of mathematics charged with investigating the properties of finite dimensional vector spaces and linear mappings between such spaces (wiki). This project will discuss the history of linear algebra as it relates linear sets of equations and their transformations and vector spaces. The project seeks to give a brief overview of the history of linear algebra and its practical applications touching on the various topics used in concordance with it.

Around 4000 years ago, the people of Babylon knew how to solve a simple 2X2 system of linear equations with two unknowns. Around 200 BC, the Chinese published that "Nine Chapters of the Mathematical Art," they displayed the ability to solve a 3X3 system of equations (Perotti). The simple equation of ax+b=0 is an ancient question worked on by people from all walks of life. The power and progress in linear algebra did not come to fruition until the late 17th century.

The emergence of the subject came from determinants, values connected to a square matrix, studied by the founder of calculus, Leibnitz, in the late 17th century. Lagrange came out with his work regarding Lagrange multipliers, a way to "characterize the maxima and minima multivariate functions." (Darkwing) More than fifty years later, Cramer presented his ideas of solving systems of linear equations based on determinants more than 50 years after Leibnitz (Darkwing). Interestingly enough, Cramer provided no proof for solving an nxn system.   As we see, linear algebra has become more relevant since the emergence of calculus even though it's foundational equation of ax+b=0 dates back centuries.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Euler brought to light the idea that a system of equations doesn't necessarily have to have a solution (Perotti). He recognized the need for conditions to be placed upon unknown variables in order to find a solution. The initial work up until this period mainly dealt with the concept of unique solutions and square matrices where the number of equations matched the number of unknowns.

With the turn into the 19^{th} century Gauss introduced a procedure to be used for solving a system of linear equations. His work dealt mainly with the linear equations and had yet to bring in the idea of matrices or their notations. His efforts dealt with equations of differing numbers and variables as well as the traditional pre-19^{th} century works of Euler, Leibnitz, and Cramer. Gauss' work is now summed up in the term Gaussian elimination. This method uses the concepts of combining, swapping, or multiplying rows with each other in order to eliminate variables from certain equations. After variables are determined, the student is then to use back substitution to help find the remaining unknown variables.

As mentioned before, Gauss work dealt much with solving linear equations themselves initially, but did not have as much to do with matrices. In order for matrix algebra to develop, a proper notation or method of describing the process was necessary. Also vital to this process was a definition of matrix multiplication and the facets involving it. "The introduction of matrix notation and the invention of the word matrix were motivated by attempts to develop the right algebraic language for studying determinants. In 1848, J.J. Sylvester introduced the term "matrix," the Latin word for womb, as a name for an array of numbers. He used womb, because he viewed a matrix as a generator of determinants (Tucker, 1993). The other part, matrix multiplication or matrix algebra came from the work of Arthur Cayley in 1855.

Cayley's defined matrix multiplication as, "the matrix of coefficients for the composite transformation T_2T_1 is the product of the matrix for T_2 times the matrix of T_1" (Tucker, 1993). His work dealing with Matrix multiplication culminated in his theorem, the Cayley-Hamilton Theorem. Simply stated, a square matrix satisfies its characteristic equation. Cayley's efforts were published in two papers, one in 1850 and the other in 1858. His works introduced the idea of the identity matrix as well as the inverse of a square matrix. He also did much to further the ongoing transformation of the use of matrices and symbolic algebra. He used the letter "A" to represent a matrix, something that had been very little before his works. His efforts were little recognized outside of England until the 1880s.

Matrices at the end of the 19^{th} century were heavily connected with Physics issues and for mathematicians, more attention was given to vectors as they proved to be basic mathematical elements. For a time, however, interest in a lot of linear algebra slowed until the end of World War II brought on the development of computers. Now instead of having to break down an enormous nxn matrix, computers could quickly and accurately solve these systems of linear algebra. With the advancement of technology using the methods of Cayley, Gauss, Leibnitz, Euler, and others determinants and linear algebra moved forward more quickly and more effective. Regardless of the technology though Gaussian elimination still proves to be the best way known to solve a system of linear equations (Tucker, 1993).

The influence of Linear Algebra in the mathematical world is spread wide because it provides an important base to many of the principles and practices. Some of the things Linear Algebra is used for are to solve systems of linear format, to find least-square best fit lines to predict future outcomes or find trends, and the use of the Fourier series expansion as a means to solving partial differential equations. Other more broad topics that it is used for are to solve questions of energy in Quantum mechanics. It is also used to create simple every day household games like Sudoku. It is because of these practical applications that Linear Algebra has spread so far and advanced. The key, however, is to understand that the history of linear algebra provides the basis for these applications.

Although linear algebra is a fairly new subject when compared to other mathematical practices, it's uses are widespread. With the efforts of calculus savvy Leibnitz the concept of using systems of linear equations to solve unknowns was formalized. Other efforts from scholars like Cayley, Euler, Sylvester, and others changed linear systems into the use of matrices to represent them. Gauss brought his theory to solve systems of equations proving to be the most effective basis for solving unknowns. Technology continues to push the use further and further, but the history of Linear Algebra continues to provide the foundation. Even though every few years companies update their textbooks, the fundamentals stay the same.

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