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EARTH-MOON NEAR RECTILINEAR HALO AND BUTTERFLY ORBITS FOR LUNAR SURFACE EXPLORATION

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NASA is planning the next phase of crewed spaceflight with a set of missions designed to establish a human-tended presence beyond low Earth orbit. The current investigation focuses on the staging post potential of cislunar infrastructure by seeking to maximize performance capabilities by optimizing transits in the lunar vicinity for both crewed and robotic spacecraft. In this paper, the current reference orbit for the cislunar architecture, known as a Near Rectilinear Halo Orbit (NRHO), is examined alongside the bifurcated L_2 butterfly family for their use as a staging location to the lunar surface.

INTRODUCTION

NASA has proposed the Gateway[1] concept as a testing ground for deep-space technologies, a staging point for missions beyond cislunar space, and a platform for Earth, lunar and space science. Currently, assembly of the Gateway components is expected to occur on orbit; tended by crew transported to the Gateway via the Orion spacecraft. Careful selection of orbit architecture is required to ensure that constraints are met while fulfilling the goals of the Gateway mission.[2] One objective of the Gateway is to serve as a staging post for lunar surface access, both for crewed and robotic applications. With the capability to stage crewed missions, the Gateway will support the return of humans to the surface of the Moon, a NASA objective outlined recently in Space Policy Directive 1.[3]

The reference orbit for the Gateway is a Near Rectilinear Halo Orbit (NRHO).[4] NRHOs comprise a subset of the halo families of orbits in the Earth-Moon system, characterized by close passages of the Moon and nearly stable behavior. More specifically under consideration is an L_2 southern NRHO in a 9:2 synodic resonance with the Moon's orbit around the Earth that completes an orbital period about every 6.5 days. As a destination for the Gateway, the NRHO offers distinct advantages. These include relatively low transfer costs from Earth that fit within the capabilities of the Orion spacecraft, low orbit maintenance costs, and favorable communications opportunities to both Earth and the lunar south pole.[5] NRHOs also offer opportunities for transfers to other orbits within cislunar space, including other members of the halo families, butterfly orbits, and DROs. In addition, a spacecraft in an NRHO can serve as a potential staging point for transfers to the lunar surface.

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Current architecture designs for crew landings on the lunar surface envision a multiple launch scenario. It is this fact that makes NRHO attractive as a staging location to the surface. For example, a dual launch scenario in which a crewed lunar lander is delivered months in advance on a ballistic lunar transfer could save significant total propellant prior to rendezvous with a crewed vehicle when it arrives at the NRHO on a separate launch. The lunar surface mission cost is then primarily a function of the cost to transfer from NRHO to different landing sites and the capability to perform aborts back to NRHO. Previous work provided limited analysis on the relative cost for these transfers, focusing on a single reference NRHO.[2] In this current investigation, multiple sizes of NRHO and L_2 butterfly orbits, which bifurcate from the halo family, are considered.

Future missions to the lunar surface will feature both crewed and robotic spacecraft. The two categories of missions involve different sets of constraints and challenges. Crewed missions from Earth to the NRHO and from the NRHO to the Lunar surface require short transfer times and must allow for abort opportunities throughout the transfer sequence. Crewed missions are round trip with relatively heavy spacecraft, generally powered by chemical propulsion to allow for short times of flight. Uncrewed transfers are less constrained by flight time and abort planning, and they may be one-way voyages of lightweight spacecraft. They may employ chemical propulsion or low-thrust solar electric propulsion (SEP). Short crewed transfer options can differ significantly from longer, uncrewed transfers in flight time, cost, and appearance.

The current investigation first explores transfers from the Earth to the NRHO. Examples of crewed transfer options[6] are compared to a new exploration of ballistic lunar transfers for cargo delivery to the NRHO and to low thrust cargo delivery results.[7] Then, transfers from the NRHO to the surface are explored to enable robotic science missions as well as human access to the Moon. Finally, butterfly orbits are investigated as an alternative starting location for transfers to the Lunar surface.

THE L_2 NRHOS AND BUTTERFLY ORBITS FOR LUNAR EXPLORATION

Dynamical Models

The current plans for Gateway orbit architecture exploit the advantages of cislunar space, where the influences of the Sun, Earth, and Moon can be simultaneously significant. Two dynamical models are employed to investigate the design space. The Circular Restricted Three-Body Problem (CR3BP) provides the framework for generation of periodic orbits and initial solutions for transfer trajectories, which are then converged in an n-body ephemeris model for higher-fidelity mission analysis.

In the cislunar regime, the CR3BP is an effective model for exploring the complex orbital dynamics. The CR3BP assumes that two primary bodies, such as the Earth and Moon, orbit in circular paths about their barycenter, and that the mass of a much smaller third body, in this case a spacecraft, does not affect their orbits. No closed form solution exists to the CR3BP equations of motion, but five equilibrium points, the libration points, are stationary in the rotating frame. Families of stable and unstable periodic orbits exist in the vicinity of the libration points. A single integral of the motion exists in the CR3BP. The Jacobi integral, or Jacobi constant, J, is written:

$$J = 2U^* - v^2 \tag{1}$$

where v is the velocity magnitude of the spacecraft relative to the rotating frame. The pseudo-potential U^* is a function of the spacecraft position,

$$U^* = \frac{1}{2}(x^2 + y^2) + \frac{\mu}{r} + \frac{(1-\mu)}{d} \tag{2}$$

where x and y are components of spacecraft position in the nondimensional barycentered rotating frame. The values d and r are the distances between the spacecraft and the larger and smaller primary body respectively, and $\mu = m_2/(m_1 + m_2)$ is the mass parameter of the system where m_1 and m_2 are the masses of the two bodies. An energy-like quantity, the Jacobi constant limits the motion of the spacecraft to regions in space where $v^2 > 0$, with zero velocity curves (ZVCs) bounding the regions. The ZVCs form boundaries within which the spacecraft can move freely. For values of the Jacobi constant greater than that associated with the L_1 libration point, the ZVCs form closed regions around each of the two primaries. As the energy of the spacecraft's trajectory increases, the value of Jacobi constant decreases until, at the L_1 value, the ZVCs open at the L_1 libration point and the spacecraft is able to move freely between the two primaries. Similarly, when the value of the Jacobi constant decreases to the value associated with L_2, the ZVCs open at L_2 and the spacecraft is able to escape the vicinity of the primaries entirely.

The CR3BP is an autonomous system, and without epoch dependence, it is an effective model for characterizing behavior in both the Sun-Earth and the Earth-Moon regimes. The CR3BP provides a framework for the generation of families of orbits of many kinds that can then be transitioned to higher-fidelity models at various epochs. The higher-fidelity ephemeris model employed in the current investigation incorporates N-body equations of motion and NAIF planetary ephemerides. Integration is performed in a Moon-centered J2000 inertial coordinate frame with point-mass perturbations by the Sun and Earth included. For transfers in the Sun-Earth regime, the Moon is modeled as a point mass. For NRHO and butterfly propagations and transfers to LLO, the Moon's gravity is modeled using the GRAIL (GRGM660PRIM) model truncated to degree and order 8.

Periodic Orbits in the Earth-Moon System

The current investigation focuses on three types of periodic orbits in the vicinity of the Moon: Near Rectilinear Halo Orbits (NRHOs), which are periodic in the Earth-Moon rotating frame in the CR3BP; butterfly orbits, a periodic family that bifurcates off the NRHOs; and circular Low Lunar Orbits (LLOs), which are periodic in a Keplerian sense.

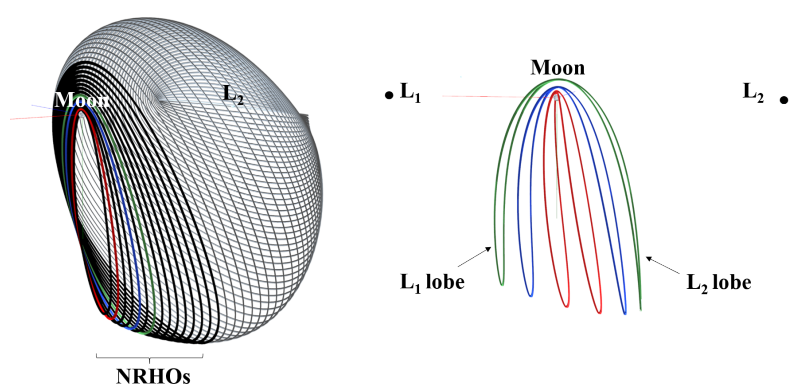


Figure 1: L_2 Family of NRHOs and Butterflies

The primary Gateway orbit is assumed to be an NRHO. The NRHOs form a subset of the larger halo families of orbits, well-known families near the collinear libration points L_1, L_2, and L3. In the Earth-Moon system, the L_1 and L_2 families bifurcate from planar Lyapunov orbits around the libration points and evolve out of plane until they approach the Moon. The NRHO portion of each halo family is defined by its stability properties: the NRHOs are those members of the halo family with bounded stability. That is, the NRHOs are marginally stable, or nearly so, from a linear analysis in the CR3BP.[4] The southern L_2 halo orbits appear in Figure 1. As the NRHO family approaches the Moon, the energy of the orbits decreases as the Jacobi constant increases. The Jacobi constant J is plotted as a function of perilune radius, r_p, in orange for low-r_p NRHOs in Figure 2a. The orbital periods of the NRHOs also decrease in length as the family approaches the Moon. Orbital period as a function of perilune radius for a portion of the L_2 NRHO family appears in orange in Figure 2b.

The current investigation focuses on three members of the southern L_2 NRHO family. The first is an NRHO in a 9:2 resonance with the lunar synodic period. The lunar synodic resonance is attractive for eclipse

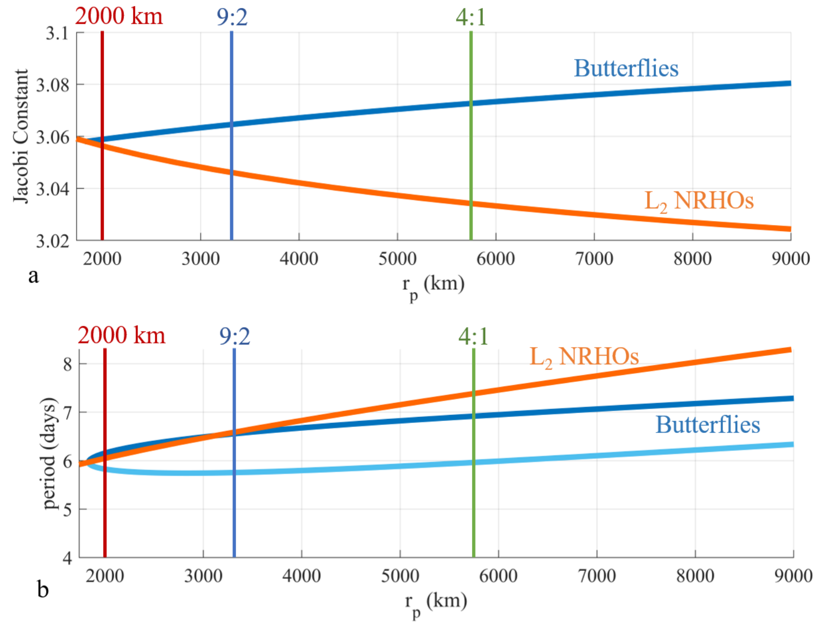


Figure 2: Orbital Periods of NRHOs and Butterflies

avoidance applications. By phasing the spacecraft within the NRHO such that perilune passages avoid alignment of the Sun and Earth, long eclipses by the Earth's shadow are avoided. The 9:2 lunar synodic resonant NRHO is characterized by a period of 6.5 days, a perilune radius of about 3,250 km, and an apolune radius of approximately 71,000 km. The 9:2 NRHO is the lowest-altitude NRHO that demonstrates a useful resonance; it is the current baseline orbit for the Gateway investigation. The 9:2 NRHO is marked with a blue line in Figure 2. A second, higher NRHO is in a 4:1 resonance with the lunar synodic period. With appropriate phasing in the 4:1 NRHO, eclipses by the Moon's shadow as well as the Earth's shadow are avoided. The 4:1 NRHO has a longer period of about 7.3 days, and it passes farther from the Moon with a perilune radius of approximately 5,750 km and an apolune radius of about 75,000 km. The 4:1 NRHO is marked with a green line in Figure 2. The third NRHO considered in the current investigation is the smallest, with a radius at closest approach of 2,000 km. The 2,000 km NRHO reaches a radius of about 68,000 km radius at apolune and has shorter period of 5.9 days. The 2,000 km NRHO is marked with a red line in Figure 2.

Like the NRHOs, the butterfly orbits exhibit nearly-stable behavior. The butterfly family bifurcates off of the L_2 NRHO with $r_p \approx$ 1,830 km. The tangent bifurcation doubles the period of the orbits, and the butterfly orbits thus have two lobes wrapping around the Moon, one lobe on the L_1 side of the Moon (towards the Earth) and the other on the L_2 side (away from the Earth). Three members of the southern L_2 butterfly family appear in Figure 1b. As the family grows, perilune radius increases and the lobes grow in width and duration. The lobe on the L_2 side of each orbit is somewhat larger, and the spacecraft spends more time on the L_2 side of the butterfly than in the L_1 lobe. The two half-periods of the butterfly, that is, the period of the L_1 lobe and that of the L_2 lobe, appear as a function of perilune radius in Figure 2b in blue.

Three southern butterfly orbits are considered in the current analysis, selected to match the perilune radius values of the three NRHOs under investigation. The butterfly corresponding to the 9:2 NRHO has a perilune radius equal to 3,250 km and half-periods of 5.75 days on the L_1 side and 6.54 days on the L_2 side. It is marked with a blue line in Figure 2. The larger butterfly analogous to the 4:1 NRHO is characterized by r_p = 5750 km. Its L_1 lobe and L_2 lobe half-periods are 5.96 days and 6.92 days respectively; it is designated by a green line in Figure 2. The smallest butterfly orbit considered in this study has a perilune radius of 2,000 km and half-periods of 5.83 days and 6.15 days; it is marked by a red line in Figure 2.

Finally, a low lunar orbit is investigated as a destination for lunar surface access. A 100 km circular, polar

LLO is selected to allow for access to landing sites at the poles. With a period of about 100 minutes and a semi-major axis of 1837 km, the LLO is significantly smaller than the NRHO and Butterfly orbits. Unlike the larger multi-body periodic orbits, the LLO is affected by the Earth's gravity only slightly; it is a small perturbation on the near-Keplerian circular orbit.

TRANSFERS FROM EARTH TO NRHO

Previous studies have compared the relative cost of transfer trajectories from Earth to various types of multi-body cislunar orbits.[2,6,8,9] As missions to the lunar surface have recently received more interest within the human spaceflight community, it has become vital to seek staging orbits that enable the cheapest possible transfers to the lunar surface. All phases from Earth to lunar surface must be optimized. To start, both short and long transfers from Earth to the three NRHOs of interest are explored.

Fast Lunar Transfers from Earth to NRHO

It has been shown that the total cost to enter into and return from low perilune NRHOs is relatively constant across r_p with ΔV variations driven primarily by the cyclical behavior of the Moon's orbit around the Earth.[6] Dropping the perilune radius by 1000 km and exploring costs to enter the 2000 km NRHO appears to validate the trend. In Figure 3 the transfer trajectories for a common epoch, December 18, 2019, are shown.

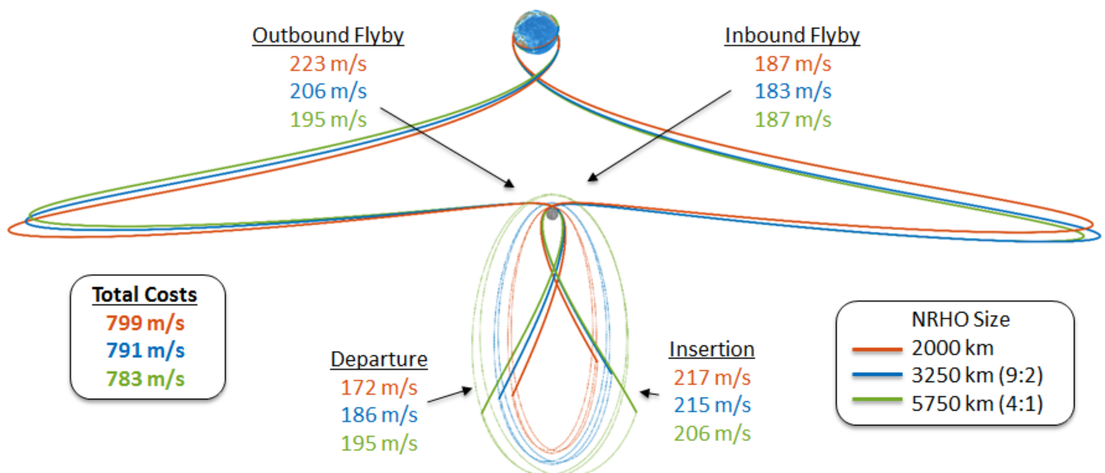


Figure 3: Comparison of Earth to NRHO transfers

While the cost is higher for the 2000 km compared to the 9:2 and 4:1 resonant NRHOs, the increase is negligible overall. The period of the orbit is 5.9 days compared to 6.4 and 7.3 for the 9:2 and 4:1 NRHOs respectively, likely improving abort capability back to Earth, for which future analysis will be needed to confirm. Thus, it appears that the 2000 km NRHO, as shown in Figure 4, is equally feasible from a performance standpoint as compared to the other NRHOs previously examined. For the launch epoch of December 18, 2019, the cost for each transfer is around 800 m/s roundtrip. This epoch represents a minimum value in the

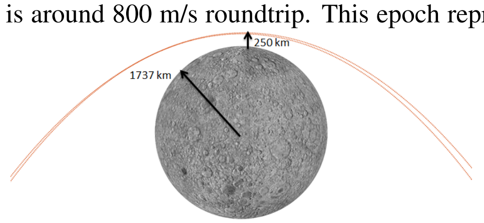


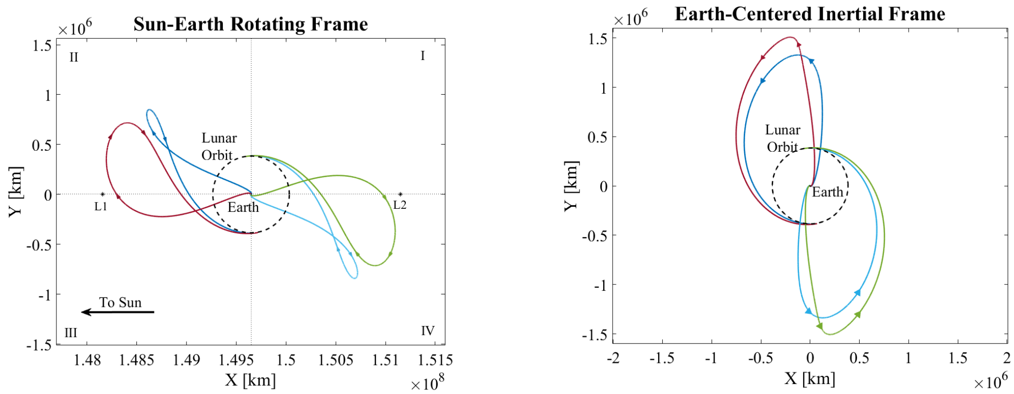
Figure 4: Perilune Passage for 2000 km NRHO

lunar cycle as the moon is closest to Earth during the mission. The total cost varies by up to 100 m/s and can thus increase to as much as 900 m/s.

Ballistic Lunar Transfers from Earth to NRHO

For uncrewed missions to the Gateway, time of flight and abort constraints are relaxed, and the primary design consideration is minimization of maneuver costs. Low-energy ballistic lunar transfer (BLT) geometries are examined with the goal of maximizing payload mass for resupply vehicles to the Gateway. Extensive work by Parker, et al.[10,11,12,13] has been performed to understand the dynamics behind BLTs. NASA's GRAIL mission used BLTs to capture into low lunar orbit in 2011.[14]

Low-energy BLTs seek to reduce maneuver costs by exploiting solar gravity to raise the perigee radius of an Earth-to-NRHO transfer trajectory arc to match the lunar orbit radius. In Figure 5a, four BLT geometries with $J = 3.0008$ appear in the Sun-Earth CR3BP; each could be a viable candidate for transfer to an NRHO. The region around the Earth is divided into four quandrants, labeled in Figure 5a. By departing Earth such that the apogee of the transfer arc is in quadrants II or IV, the Sun tends to circularize the trajectory, raising perigee. Conversely, the Sun stretches the orbit when apogee is in quadrants I or III, thus lowering perigee. In the inertial view in Figure 5b, the candidate trajectories depart Earth along highly eccentric paths. As the trajectories pass the radius of the lunar orbit and approach apogee, they become perturbed by solar gravity to raise the orbit perigee. Upon return to perigee, the trajectories match the lunar orbit radius, where rendezvous and capture into the target NRHO can occur.



(a) (b)

Figure 5: Candidate ballistic lunar transfer arcs in the Sun-Earth rotating frame (a) and Earth-centered inertial frame (b). Quadrants I through IV are marked in the Sun-Earth rotating frame.

The problem of constructing end to end Earth to NRHO BLTs is split into three pieces. The first is performed in the Sun-Earth CR3BP, by selecting a ballistic arc similar to those in Figure 5. The second piece is obtained in the Earth-Moon CR3BP. Currently, the reference orbit for the Gateway is a 9:2 synodic L_2 southern NRHO. By sampling points along the reference and imparting a ΔV to increase energy at each point, candidate arrival trajectories are generated by propagating each point in reverse time. Once the trajectories reach the lunar sphere of influence, the arc is chosen based on the minimum magnitude of the out-of-plane motion, z, calculated using the non-dimensional z-component of position and velocity: $\sqrt{z^2 + v_z^2}$. The final segment of the trajectory is comprised of seven revolutions of the 9:2 NRHO reference orbit.

Discretizing the aforementioned Sun-Earth CR3BP arc, the Earth-Moon CR3BP NRHO arrival arc, and seven revolutions of the target 9:2 synodic L_2 southern NRHO in the Earth-Moon CR3BP, the Copernicus[15] optimization software is used to correct the arcs into a continuous trajectory in the Sun-Earth-Moon ephemeris model. Figure 6 shows three transfer geometries converged in the ephemeris model. The insertion maneuver ΔV and time of flight is 6.3 m/s and 136 days for the geometry in Figure 6a, and 26 m/s and 104 days in

Figure 6b.

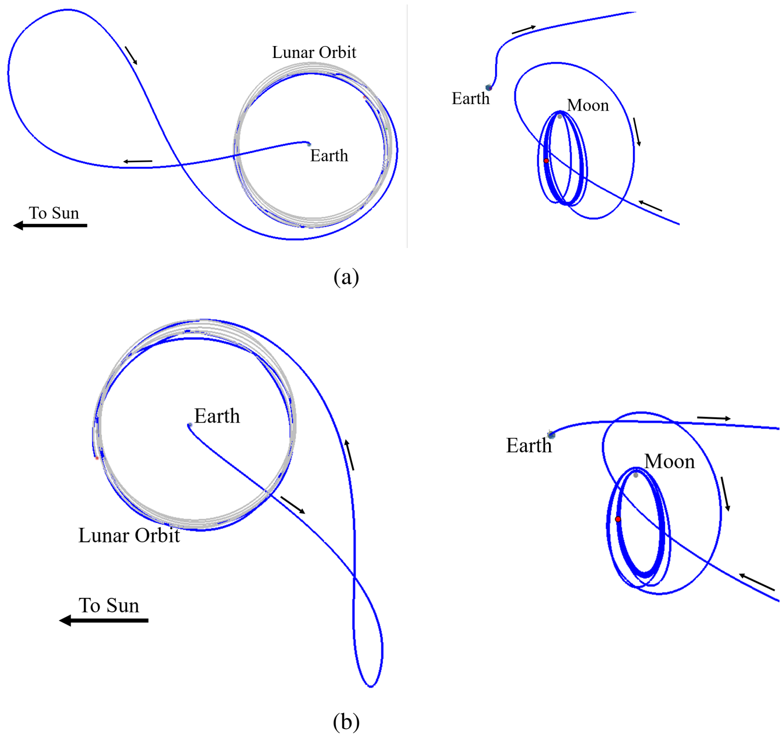


Figure 6: Ballistic lunar transfers to the 9:2 synodic L_2 southern NRHO converged in the ephemeris model.

To further assess the feasibility of these low-energy transfers in the ephemeris model, it is important to consider the variations in maneuver cost and time of flight over a range of possible mission epochs.

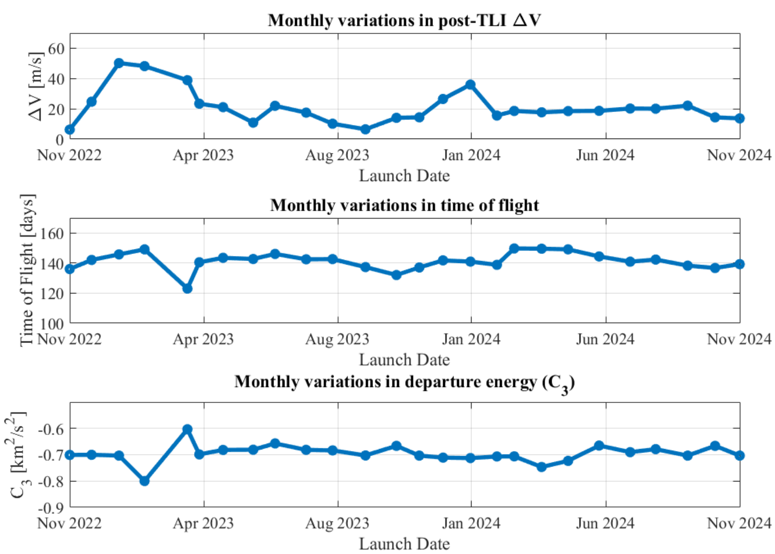


Figure 7: Monthly variations in maneuver cost (top), time of flight (middle), and departure energy (bottom) for transfer geometry represented in Figure 6a

The variations are studied by reconverging the transfer geometry in Figure 6a for 26 launch epochs between November 2022 and November 2024. The launch epochs are roughly 29.5 days apart, or approximately equal to a lunar synodic cycle to ensure the Moon is in a favorable location when the spacecraft arrives in the lunar vicinity. Over the two-year scan, the ΔV ranges from 6.3 and 50.2 m/s for this geometry, while the times of flight vary from 123 and 150 days. Results appear in Figure 7. Departure energy remains consistent near -0.7 km^2/s^2, indicating monthly variations do not have a significant impact on launch vehicle sizing.

In the first quarter of 2023, the maneuver costs reach a maximum within the two year scan. The increased costs are related to a disadvantageous orientation of the lunar orbit plane relative to the ecliptic plane. Deterministic trajectory correction maneuvers (TCMs) are performed near apogee for launch dates of Jan 30, 2023 and Mar 18, 2023 to reduce the post-TLI ΔV costs to those shown in Figure 7. Based on the significant ΔV savings that the TCMs provide, it is likely that the addition of TCMs to each of the trajectories throughout the scan will further reduce ΔV costs.

NRHO TO LUNAR SURFACE TRANSFERS

To date, the only vehicles to ascend into orbit from a planetary body other than Earth are NASA's six Apollo Lunar Modules and Russia's three Luna sample return landers. If the NRHO is to be a staging node for two-way lunar surface access for crew and robotic spacecraft, minimizing the total transfer ΔV to and from the surface is vital. For crewed missions it is also necessary to minimize transfer time, as crew require keep-alive resources that increase lander dry mass, causing propellant mass to also increase. The gear ratio of propellant mass to dry mass for a roundtrip to the lunar surface from a given orbit can be as much as 10:1 depending on the total ΔV, number of lander stages, and lander performance assumed.

One of the major findings from NASA's LLO staged Constellation lunar program was the ΔV costs associated with changing the orbit plane to achieve a non-equatorial LLO. In fact, a non-equatorial sortie can require up to 50% more ΔV than the no-plane change minimal one way insertion cost of 900 m/s.[16] The plane change costs are created by misalignment between the departure plane at Earth and the orbit right ascension at the Moon, and can only be solved by loitering in orbit between 7 and 14 days. In contrast, the transfer to NRHO from Earth is independent of plane change but requires a stay time in LLO commensurate with the orbit period of around 6 to 7 days. Thus, both NRHO and LLO staged architectures require similar loiter times to eliminate expensive plane change maneuvers.

To determine total transfer costs, the costs of the actual ascent and descent from the lunar surface are assumed to be constant. This is a good assumption as an ascent from a given location on the lunar surface to a staging orbit that passes directly overhead is driven almost entirely by the thrust-to-weight ratio of the lander. To a circular LLO of 100 km the total required ΔV is approximately 4000 m/s. The descent ΔV is larger than ascent to ensure a soft touchdown, so while the exact energy change for each leg is identical, the actual split is approximately 2100 m/s down and 1900 m/s up.[17] With ascent and descent costs assumed to be constant, only the transfers to LLOs associated with a given latitude and longitude on the surface are calculated.

Fast impulsive transfers for crewed applications

A previous study provided costs for half day transfers from a 4500 km r_p NRHO to any location on the lunar surface.[2] That study showed that the cheapest costs to polar (either North or South) locations is around 730 m/s each way while most expensive transits were just under 900 m/s each way to the equatorial faces of the near and far sides the Moon, that is, latitude/longitude pairs of (0,0) and (0,180) respectively.

In this paper, the polar and equatorial landing sites are examined as representative for best and worst cases respectively to characterize the global surface access extremes. Figure 8 and Table 1 show the trajectories and transit costs to access these two locations on the surface to/from each NRHO. For polar regions, the half day transit cost is cheaper for NRHOs with lower r_p values. Unsurprisingly, accessing the equator is more challenging so the transfer times are lengthened from .5 day to 1 day each way to reduce the costs. As a result, and following the trend associated with polar sites, the lower perilune NRHOs are then able to achieve transits under 800 m/s. Note that since the 2000 km NRHO is near the minimum feasible NRHO with an altitude of about 250 km, the 684 m/s transfer cost to the polar LLO is probably the minimum cost possible

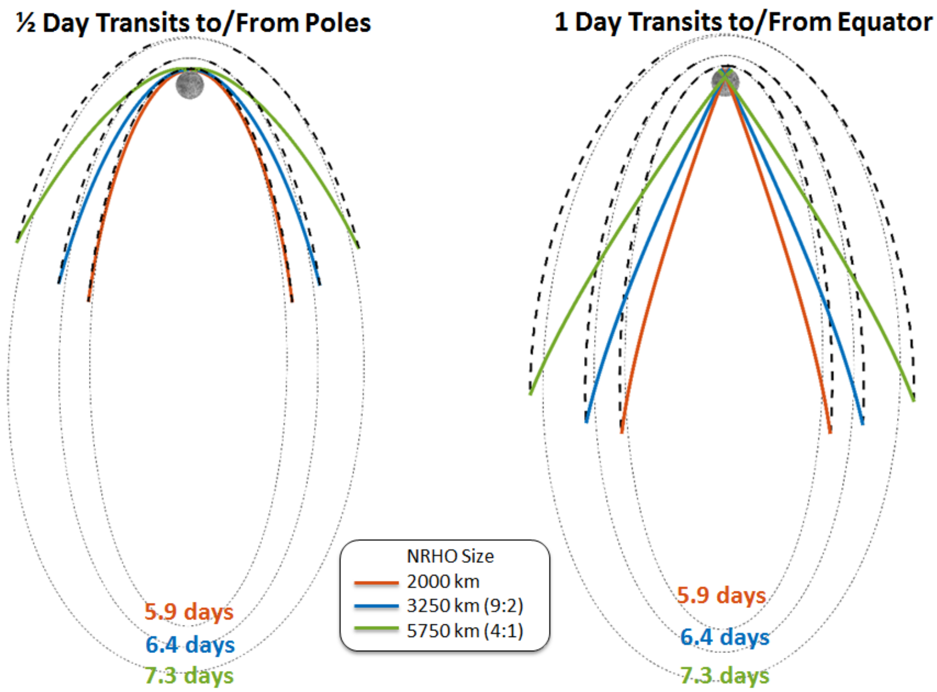


Figure 8: Nominal NRHO to Poles and Equator

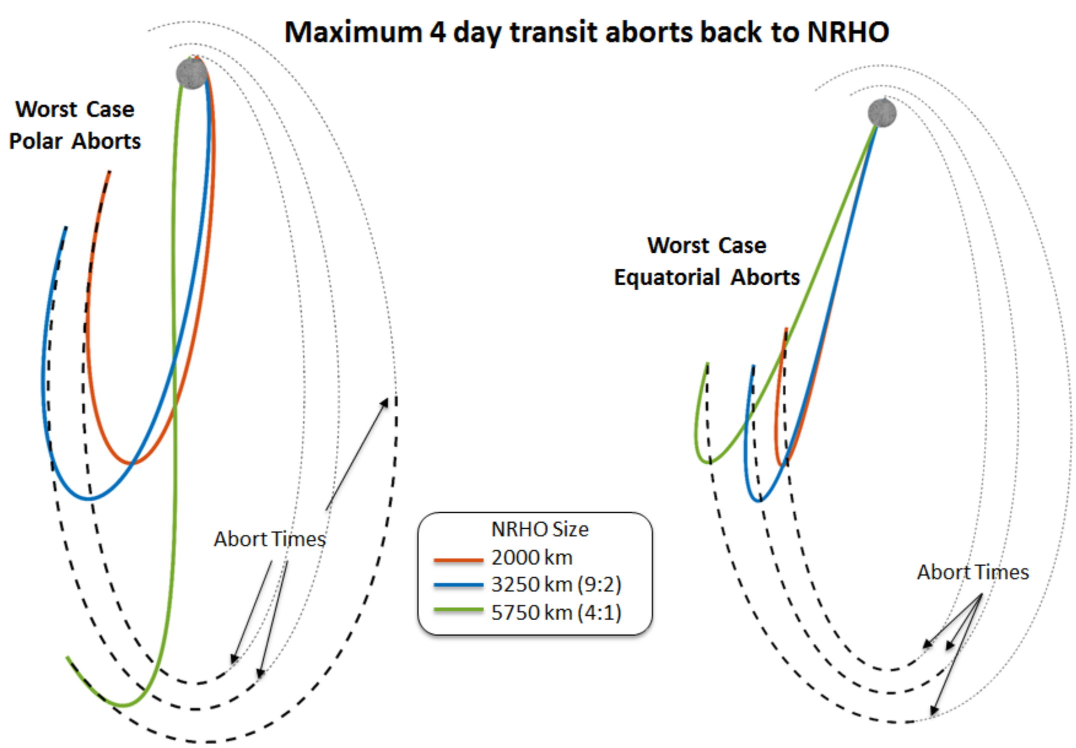


Figure 9: Most challenging abort to NRHO trajectories

Table 1: NRHO to LLO Round Trip Transfers

NRHO Orbit	Polar Site (90/-90 Latitude) To ΔV m/s	To ΔT $days$	From ΔV m/s	From ΔT $days$	Stay Time ΔT $days$	Equator (0,0 or 0,180) To ΔV m/s	To ΔT $days$	From ΔV m/s	From ΔT $days$
r_p = 2000 km	684	0.5	705	0.5	5.9	778	1.0	777	1.0
9:2 r_p = 3250 km	707	0.5	717	0.5	6.4	798	1.0	795	1.0
4:1 r_p = 5750 km	758	0.5	762	0.5	7.3	831	1.0	825	1.0

for a polar or near lunar rim mission. In fact, this is approaching the minimum found for long transits at 665 and 672 m/s respectively.

The difference between 'transfer to' and 'transfer from' LLO transit costs is due to the fact that the perilune pass of the NRHO is not centered exactly over the pole. Additionally each perilune pass has a slightly different geometry due to perturbations by the Sun, oblateness of the Earth and moon as well as the ellipticity of their respective orbits.

Short transits are possible when the NRHO return vehicle is close to the moon. However, if the crew needs to abort back to the NRHO during the lunar surface stay, the transit takes longer, generally around 3- 4 days. If abort transits in the 1-2 day range are desired, the total ΔV capability quickly escalates significantly above the baseline cost. Thus, the mass trade favors longer aborts. In this instance, a maximum permitted abort time of 4 days appears the most reasonable constraint to balance propellant to the dry mass of the lander. For all transfers and aborts, the actual ascent time to LLO is a few hours with an option for a few hour loiter in LLO if desired.

Continuous aborts from the surface are examined and the most expensive from a ΔV perspective are shown in Figure 9 and Table 2. For both the polar and equatorial regions the abort ΔVs exceed the minimum nominal return costs as given in Table 1. Thus, an abort capable lander must have propellant and lifetime capabilities equivalent to or exceeding the nominal values. Fortunately, such a constraint only applies to the return transit; the shorter transits and ΔV costs can be maintained on the way down to the lunar surface.

Table 2: LLO to NRHO Largest Abort Cost

NRHO Orbit	Maximum Anytime Abort From Pole T_{abort} $days$	From Pole ΔV m/s	From Pole ΔT $days$	From Equator T_{abort} $days$	From Equator ΔV m/s	From Equator ΔT $days$
r_p = 2000 km	2.5	732	3.2	2.5	792	2.9
9:2 r_p = 3250 km	2.5	739	3.6	2.5	789	3.3
4:1 r_p = 5750 km	1	763	4	3.5	831	3.0

An important observation to note is that the optimal trajectories in this investigation are free to choose the LLO that best connects a given landing site and the NRHO. Thus, the LLO that provides the cheapest overall transfer from NRHO to LLO is not necessarily related to the LLO that is selected for the return trip or for abort analysis. That is, rendezvous with an asset left in LLO is not considered. If it were desired to break up the ΔV between elements and leave a tug or equivalent in LLO orbit, the total ΔVs would change based on the constraints associated with rendezvous back in LLO. A quick look at the optimal return and abort geometries suggest this penalty is severe and likely untenable. At this point the only solution for the NRHOs for additional elements is as drop stages or one way tugs. One option would be to tug the lander to LLO and then return it to the NRHO for refueling on its own, preventing the need for a rendezvous but saving the lander the descent cost into LLO.

Longer impulsive transfer options for robotic spacecraft

While crewed missions into and out of the NRHO require short transfer times, robotic spacecraft are free to leverage longer times of flight in an effort to achieve lower total transfer costs. The current investigation explores the existence and feasibility of longer duration transfers from the reference NRHOs into LLO. Analysis begins with the bounding of theoretical minimal transfer costs in the CR3BP, then continues with the employment of periapse maps to find potential long duration transfers and ends with convergence of these transfers in the higher-fidelity ephemeris force model, all targeting an inclination of 90 degrees which maximizes landing site flexibility.

The theoretical minimum transfer cost is simply the Δv required to change the energy of the spacecraft from the value associated with the NRHO to the value along the LLO, without considering the transfer path. The minimum theoretical transfer cost is evaluated in the CR3BP for the 9:2 and 4:1 lunar synodic resonant NRHOs. The basis of this evaluation is the Jacobi constant, the energy-like integral of the motion in the CR3BP given in Equation 1. The value of J is constant along a ballistic arc. The instantaneous Δv required to move from an initial value, J_0, to a second value, J_1, is expressed as

$$\Delta v = \sqrt{2U^* - J_1} - \sqrt{2U^* - J_0} \tag{3}$$

Equation 3 is a function of the initial and final values of J as well as the pseudopotential, U^*, a function only of position. For a given J_0 and J_1, the Δv magnitude is strictly decreasing as U^* increases. Therefore, the required Δv is minimized for a given J_0 and J_1 when the value of U^* is maximized. For an NRHO to LLO transfer that remains near the Moon, the largest value of U^* occurs on the LLO itself. Thus, to bound the theoretical minimal cost, the required Δv to move from J_{NRHO} to J_{LLO} is evaluated at the highest U^* along the LLO. Theoretical minima are computed for four cases: the 4:1 NRHO, the 9:2 NRHO, a 100 km altitude NRHO that is tangent to the LLO, and the L_1 point itself. The computed theoretical minimum Δv values appear in Table 3. As the r_p of the NRHOs decrease, the minimum Δv values decrease as well. The 100 km altitude NRHO represents a limiting case with a Δv of 637 m/s.

For transfers that depart the vicinity of the Moon before returning to the LLO, the greater value of U^* around the Earth may be leveraged for cost savings as well. However, within the CR3BP, the maximum possible Jacobi that can be reached at a burn near Earth is limited by the ZVCs. To re-enter the vicinity of the Moon, the ZVCs must be open at the L_1 libration point. Thus, J must remain below the value at L_1 itself. Assuming the spacecraft has reached $J = J_{L_1}$, the minimum theoretical transfer cost is 622.2 m/s. Note that this theoretical minimum is based solely on the Jacobi constant analysis and does not include the cost of the maneuver near Earth.

Table 3: Minimal Energy Change Costs to 100 km LLO

Starting Condition	Jacobi Constant	Minimum Energy Change m/s
4:1, $r_p = 5750$ km	3.03	657.9
9:2, $r_p = 3250$ km	3.05	654.8
100 km altitude NRHO, $r_p = 1837$ km	3.06	637.0
L_1	3.19	622.2

With baseline theoretical minimum costs established, long-duration transfers between the NRHOs and the LLO are generated. The driving constraints for the design are altitude and inclination of the final LLO: a polar orbit with a 100 km altitude is the desired destination. To design transfers off of the NRHO, periapse maps are employed.[18] First, Δv maneuvers are applied at multiple points around the NRHO in a range from 1 m/s to 200 m/s to generate a large set of potential transfer trajectories. The potential transfer orbits are propagated forward, and periapses relative to the Moon are mapped. The resulting periapse maps are visualized in altitude-inclination space in Figure 10 for the 9:2 NRHO (left) and the 4:1 NRHO (right). Points are colored according to periapse passage. Noticeably, the 4:1 map is much sparser at the lower altitudes due to its higher r_p. Potential transfers correspond to periapses with altitudes near 100 km and inclinations near

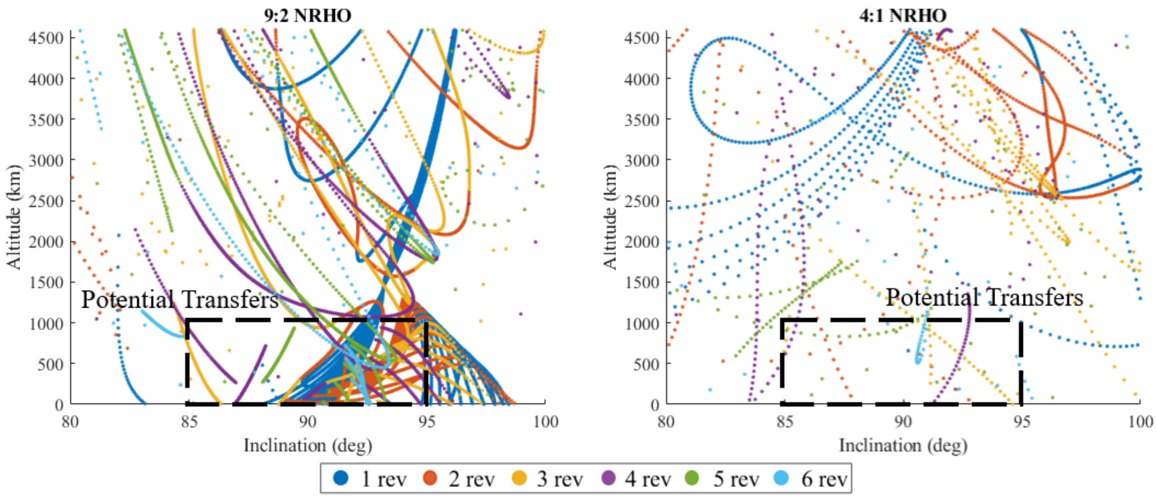


Figure 10: Periapse maps for Δv steps off of reference NRHOs

90 deg. Selecting such points as initial guesses, the transfers are corrected into continuous NRHO to LLO transfers in the CR3BP.

Looking first at transfers from the 9:2 NRHO to the LLO, continuous trajectories in the CR3BP are re-converged and optimized in the higher-fidelity ephemeris force model using Copernicus.[15] Comparing the transfer cost of the initial guesses taken from the CR3BP to the converged cost in the ephemeris force model, the Δv requirements for transfers converged in the ephemeris model from the 9:2 reference NRHO to the LLO are substantial. The primary reason for the change in cost between the two models is the application of the inclination constraint. While the initial guess inclination may sit around 87°, costs climb rapidly as this inclination is forced toward 90°. After converging the initial guess transfer at 87°, a cost of around 30 m/s per degree is required to move the inclination to 90°. This effect grows as the number of revolutions is increased, resulting in low convergence rates.

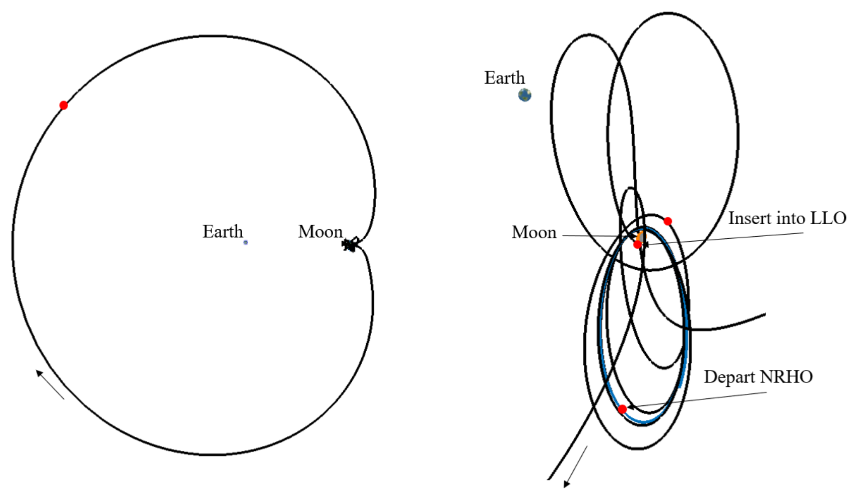


Figure 11: 100.1 day transfer from the 9:2 NRHO to 100 km altitude LLO requiring 664.9 m/s with maneuvers marked in red

Two main mitigation strategies are employed to lower the cost of the inclination change for transfers from the 9:2 NRHO. First, intermediate burns are added to multi-revolution transfers that do not initially meet inclination constraints. Over the course of the transfer, the burns slowly change the inclination to meet the

90° constraint. For example, a two-rev transfer trajectory is found that departs the NRHO with a ΔV of 13 m/s prior to perilune. Four small intermediate maneuvers adjust the orbit until the large LLO insertion maneuver with a ΔV of 651 m/s completes the transfer. The minimal-cost transfer has an 11.8 day time of flight that requires a total ΔV of 673.5 m/s.

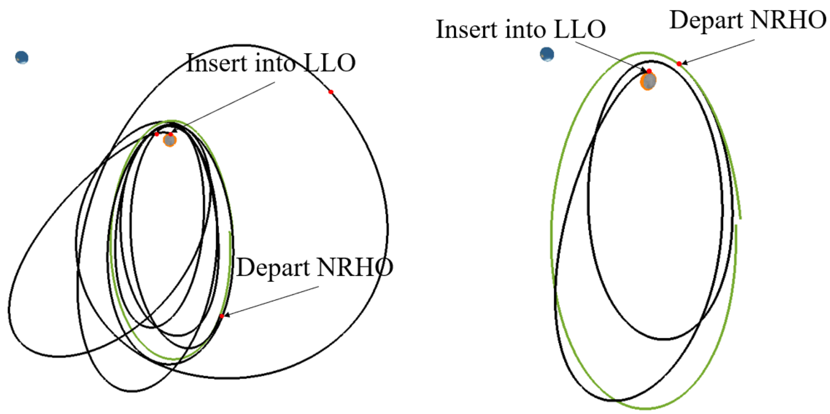


Figure 12: Long (left) and short (right) duration transfers from the 4:1 NRHO to 100 km altitude LLO requiring 700.2 m/s and 672.2 m/s, respectively, with maneuvers marked in red

The second method employed to reduced the cost leverages the solar perturbation to assist with the inclination change. Ideally, solar effects raise *J* and allow easier targeting of the 90° inclination requirement. Such transfers are found using the periapse maps by extending propagation time to locate arcs that exit out of the vicinity of the Moon near L_2 and return to make later periapse passages near 100 km at the Moon. After correction into continuous orbits in the CR3BP, candidate transfers are re-converged and optimized in Copernicus. One sample appears in Figure 11; it requires 664.9 m/s and a time of flight of 100 days.

Initial guesses for transfers from the 4:1 NRHO to the LLO are again taken from the periapse map. After correction into continuous transfers in the CR3BP, the trajectories are again moved into the higher fidelity ephemeris model in Copernicus, converged, and optimized to minimize total Δv. For transfers from the 4:1 NRHO, the departure location along the NRHO is correlated to transfer cost. Departures from the sensitive region around perilune are observed to generate lower-cost, faster transfers than similar departures near apolune. Two examples appear in Figure 12. On the left, a locally optimal 52 day transfer departs near apolune with a total transfer cost of 700.2 m/s. On the right, a shorter transfer of 12 days leaves the NRHO near perilune for a total cost of 672.2 m/s. The departure near apolune results in longer time of flight as well as the addition of intermediate burns to approach the lower cost of the transfer that departs near perilune.

The cost associated with four sample long-duration transfers are summarized in Table 2. The locally optimal transfers remain above the theoretical lower bound set by the Jacobi analysis. Transfers from lower-altitude NRHOs are aided by including maneuvers near the apses and leveraging solar perturbations. Additionally, shorter transfers can be completed at lower costs if departure from the NRHO occurs closer to perilune.

Table 4: Long Duration NRHO to LLO Transfer Costs

Starting NRHO	Time of Flight	Total Δv	No. of Maneuvers
	days	m/s	
9:2 r_p = 3250 km	11.8	673.5	6
9:2 r_p = 3250 km	100.1	664.9	5
4:1 r_p = 5750 km	12.1	672.2	2
4:1 r_p = 5750 km	51.5	700.2	5

Low-thrust transfer options for robotic spacecraft

If a SEP engine is available such as that planned on the Gateway, robotic spacecraft could also be delivered via low thrust spiral to LLO. Preliminary analysis on accessing LLO from NRHO reveals that there is a minimum acceleration a vehicle must be able to produce to stay within the immediate vicinity of the Moon during the transfer. For vehicles that have propulsion systems incapable of generating at least this minimum acceleration for the vehicle's mass, the required ΔV to stay near the Moon is not performed quickly enough after escaping from the NRHO. When this minimum acceleration is not achieved, a a 1-rev transfer is required, similar to Figure 11. The transfer time increases significantly since the vehicle traverses the far side of the Earth before entering a spiral-in period.

The analysis is performed to identify the minimum acceleration necessary for transfer in the lunar vicinity as well as generate data to provide an estimate of the amount of propellant required to perform NRHO to polar LLOs transfers with altitudes ranging from 50 km to 10,000 km. To generalize this dataset, it is assumed that the vehicle's propulsion system provides constant acceleration during the transfer. This assumption is not the case in an actual transfer scenario where the consumption of propellant continually increases the vehicle's acceleration. However, the assumption adds conservatism into propellant estimates generated with this dataset.

Because of the high computational time required to optimize transfers to very low LLOs within Copernicus,[15] a 2-step approach is used to generate the full NRHO to LLO datasets: Copernicus is used to optimize the trajectory from the 9:2 NRHO to a 10,000 km altitude polar LLO and then an Edelbaum approximation from 10,000 km down to the lower LLOs.[19] For the spiral, the burn times are calculated based on the ΔV values calculated using the Edelbaum assumption and the same vehicle accelerations with which the Copernicus trajectories are optimized. An example low thrust transfer from NRHO (black) to a 1,000 km circular LLO is shown in Figure 13.

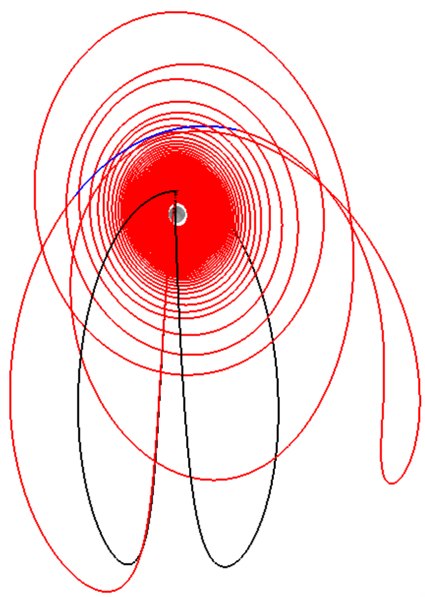


Figure 13: Transfer from 9:2 NRHO to 1,000 km Polar LLO

The costs required to lower LLO-altitude orbits is added to the curve generated from the Copernicus data to define a relationship between the required burn time and the vehicle acceleration; results appear in Figure 14. If the mass and propulsion system characteristics are known for a low thrust vehicle, the data in Figure 14 can be interpolated to obtain the burn time required to transfer the vehicle to the desired LLO. The curves in Figuree 14 are combined with the knowledge of a thruster's mass flow rate to estimate required transfer

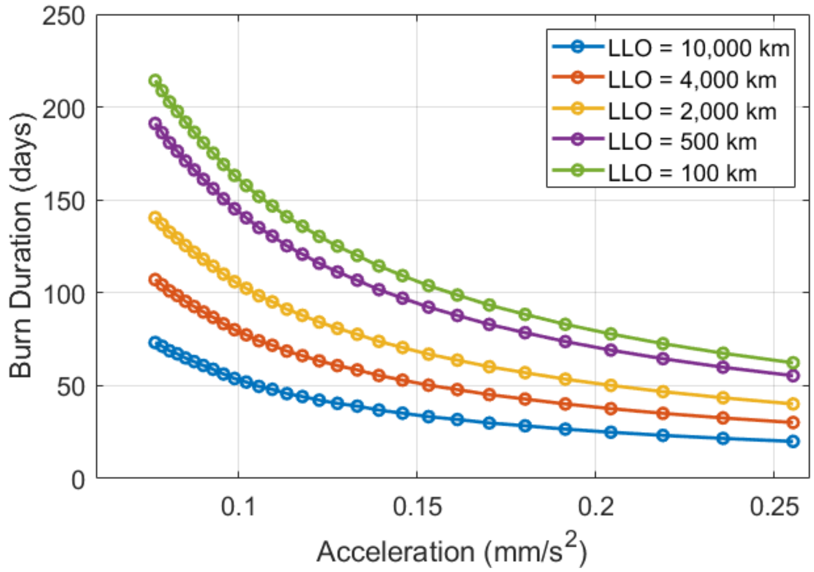


Figure 14: NRHO to 50 - 10,000 km Polar LLOs: Vehicle Acceleration vs. Burn Time Required for Transfer

propellant. The actual time of flight required to perform the transfer is slightly longer than the times of flight in this analysis due to shadowing and coasting during the trajectory following NRHO departure and before entering a high lunar orbit. Coasting durations range between zero depends on the orientation of the LLO and season.

Several SEP cases of interest are listed in Table 5. Using this data, the initial acceleration for a range of initial stack masses in the NRHO is calculated. After generating the transfer burn times from Figure 14, the mass flow rate is used to calculate the estimated transfer propellant required for each vehicle to perform the transfer to the full range of LLO altitudes. The resulting propellant versus initial stack mass data appears in Figure 15.

Table 5: Assumed SEP Bus Performance

40 kW	1.53 N	6.2E-05 kg/s
26.6 kW	1.02 N	4.1E-05 kg/s

For the 40 kW bus, stack masses over 19.95 t result in initial accelerations less than 7.65859E-08 km/s^2. For the 26.6 kW bus, stack masses over 13.31 t result in initial accelerations less than 7.65859E-08 km/s^2. In instances of heavier stack masses, the longer 1-rev transfers are necessary.

Actual transfers from NRHO to several polar, circular LLOs are generated using a 40 kW SEP bus. There is close agreement between the results from these fully integrated trajectories and the estimated required amount of transfer propellant. For longer transfer times, the largest differences occur when acceleration is near the minimum threshold and the desired LLO altitude is low. This case corresponds to the largest acceleration change in the realistic model. The maximum observed difference between the actual and estimated 40 kW burn durations is ˜6.5 days.

Although this data can be used to estimate transfer propellant requirements from equivalent energy elliptical orbits, the arugment of periapsis (AOP) of the elliptical LLO may be unpredictable. Larger, more eccentric elliptical orbits are affected differently depending on orientation by Earth, Moon, and Sun's gravity. Capturing into the LLO, depending on AOP, may be faster or slower, or require an infeasible velocity vector direction from the NRHO. Future analysis should be done on transfer requirements to an elliptical LLO that may be of interest.

While a SEP vehicle requires a longer time of flight to reach LLO than a high-thrust vehicle, the high

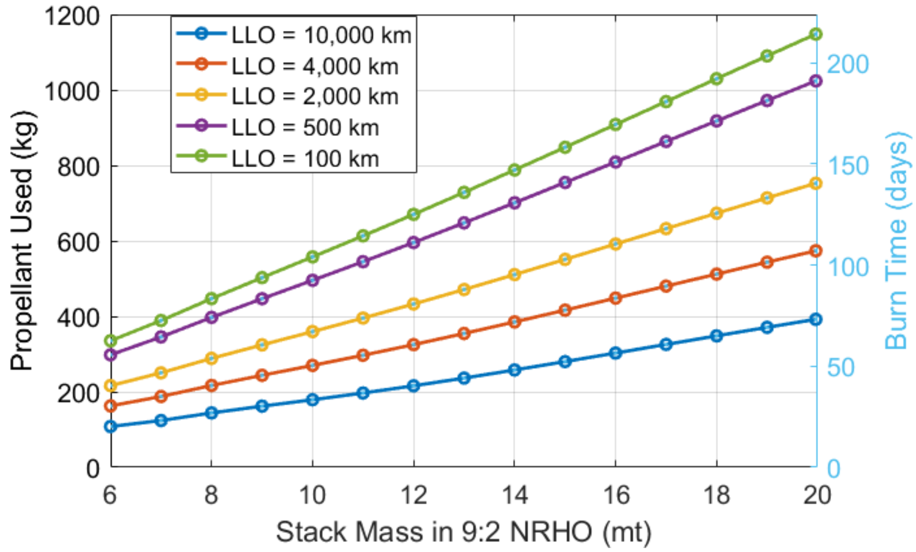


Figure 15: Estimated Transfer Propellant Required for $40kW$ SEP Vehicles

efficiency of a SEP propulsion system results in significant propellant mass savings, particularly for larger payloads. For comparison, if a 40 kW SEP bus and payload having an initial mass of 18 t performs an 9:2 NRHO to 100 km LLO transfer, approximately 1.03t of xenon is required, as seen in Figure 15. A high-thrust vehicle of the same mass performing the 673.5 m/s lunar vicinity transfer from Table 4 requires 3.68 t of chemical propellant.

BUTTERFLY ORBITS FOR LUNAR SURFACE ACCESS

As described above, butterfly orbits share characteristics with NRHOs. However, instead of a 6 to 7 day repeating geometry, butterflies have two lobes that pass over the near and far sides of the Moon, each having 5 to 7 day period ranges. The distance between the lobes increases in size as r_p increases. As with the NRHOs, three sizes of butterfly orbits are examined, defined by the same range of perilune radii: 2000, 3250 and 5750 km. The lobes result in passes not constrained solely to the lunar rim. It is this feature that may allow cheaper costs to a wider variety of lunar surface sites beyond the poles. This paper does not address NRHO to butterfly transfers, as previous investigations have confirmed their feasiblity within 1 or 2 revolutions for low ΔV costs.[18] Future work will address a wider range of transfer opportunities with longer transfer times and lower costs.

Fast Earth to Butterfly transfers

The optimal Earth to butterfly transfer is very similar to the optimal Earth to NRHO transfer. The primary difference is where there is only one entry and exit geometry for the NRHOs, butterfly orbits have two possible geometries due to the two lobes. Shown in Figure 16 are transfers to the lobe on the far side of the moon and the respective costs. The performance is very comparable to the NRHO and only diverges in trend when increased to the 5750 km butterfly. It is possible this is due to the geometry of the butterfly as the lobes further outward away towards L_2, but perhaps the nature of the convergence into the ephemeris model is also a contributor. Future studies can examine the cyclic performance to see if the trend holds. In the meantime, comparable costs suggest the butterfly orbits in the same low perilune family are just as accessible as equivalent NRHOs.

Transfers from Butterfly Orbits to the Lunar Surface

The three butterflies are examined in a similar fashion to the NRHOs in terms of total transit cost to and from landing site specific LLOs, including abort options. As expected the costs to the equator decrease and

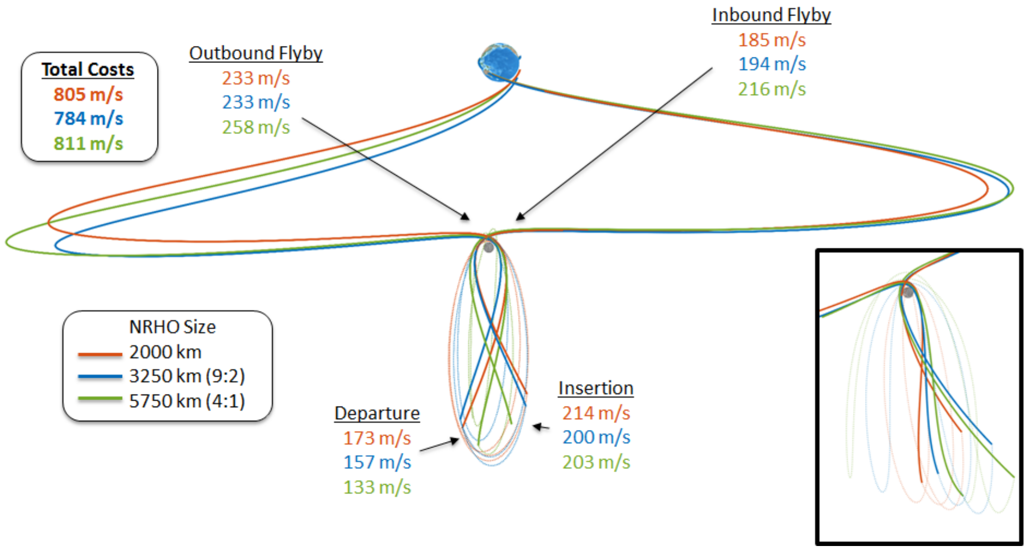


Figure 16: Comparison of Earth to butterfly transfers

costs to the poles remain about the same. In fact, the costs to the equator decrease sufficiently to enable half day transits to the center of the far side for the same cost as one day transits from the NRHOs. Landing sites along the rim are not examined, but likely will require higher ΔV as the minimum cost fast transit landing sites are aligned with the ground track of the butterfly that occurs at other longitudes. Tables 6 and 7 provide comparable performance data for the butterflies compared to the NRHOs previously examined.

Table 6: L_2 Butterfly to LLO Round Trip Transfers

Butterfly Orbit	Polar Site (90/-90 Latitude) To ΔV m/s	To ΔT $days$	From ΔV m/s	From ΔT $days$	Stay Time ΔT $days$	Equator (0,0 or 0,180) To ΔV m/s	To ΔT $days$	From ΔV m/s	From ΔT $days$
r_p = 2000 km	691	0.5	679	0.5	5.9	778	1.0	776	1.0
9:2 r_p = 3250 km	689	0.5	690	0.5	5.7	795	0.5	796	0.5
4:1 r_p = 5750 km	746	0.5	754	0.5	6.1	768	0.5	785	0.5

While the biggest gain shown in Table 7 is for transfers to the equatorial landing site from the 5750 km butterfly, the most interesting gain is in the 3250 km for polar landing sites. Not only are the transfer costs lower by a total of 45 m/s, the time on the surface is lower by a total of 0.7 days permitting shorter missions and smaller landers. The geometry of the transfers and the aborts is shown in Figure 17 from a top down perspective.

Table 7: Difference between NRHO and Butterfly to LLO Round Trip Transfers

NRHO - Butterfly Orbit	Polar Site ΔV m/s	Polar Site ΔT $days$	Stay Time ΔT $days$	Equatorial Site ΔV m/s	Equatorial Site ΔT $days$
r_p = 2000 km	-19	–	–	-1	–
9:2 r_p = 3250 km	-45	–	-.7	-2	-1
4:1 r_p = 5750 km	-20	–	-1.2	-103	-1

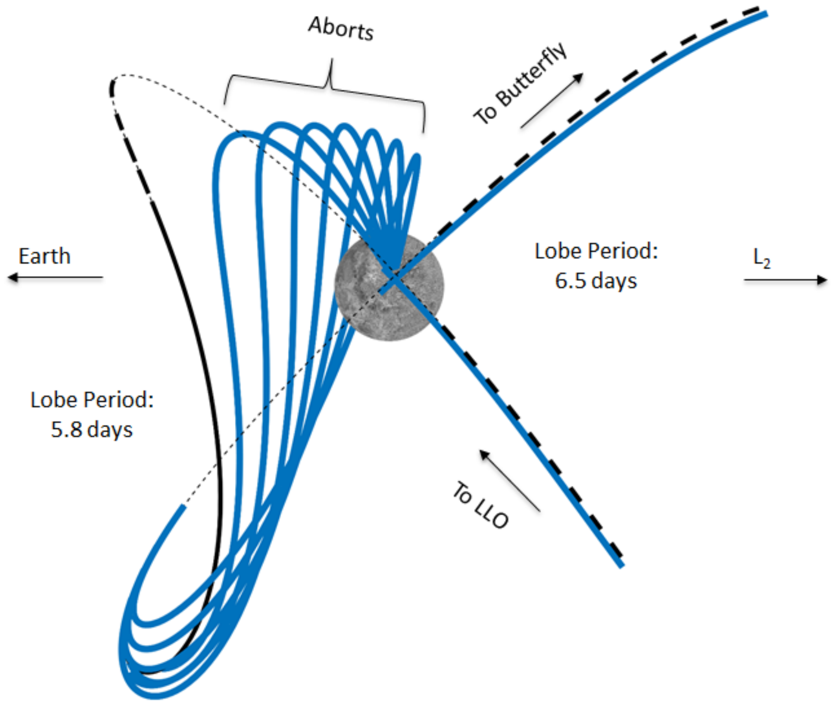


Figure 17: 3250 km Butterfly mominal mission plus aborts from Polar Landing Site

Opportunity for Butterfly to LLO Tug

An unexpected trait associated with transfers from the 3250 km butterfly to LLO is noted by examining Figure 17 more closely. After the transfer to polar LLO, all the optimal aborts progress in right-ascension around the moon until the nominal return leg is right approximately 90 degrees from the original LLO orientation after about 5.8 days. These optimal orientations happen to correspond closely with the natural precession rate of the LLO.

As discussed previously, plane changes in LLO are expensive, and thus without precise alignment, LLO rendezvous can be cost prohibitive. However, if the plane of the LLO aligns with the desired departure trajectory, LLO rendezvous could be enabled. An LLO rendezvous then permits leaving infrastructure, such as a tug, in orbit that then re-docks to the lander after the surface mission, or prior if the crew must abort from the surface.

The ΔV due to plane change makes rendezvous cost prohibitive for all NRHOs. While a lander could launch from the poles and rendezvous with a tug, it would not be aligned properly to return to the NRHO. Recall that:

$$\Delta V_{pc} = 2vsin\left[\frac{\Delta i}{2}\right] \tag{4}$$

Of course this cost can be mitigated in a transfer to a higher orbit so that the majority of plane change occurs at a lower velocity, but the plane change must be reasonable. For instance, the trajectories in Figure 8 designed for equatorial landing site access minimize the plane change by targeting an inclined orbit (60° inclination or higher generally) that has a right ascension of ascending node (RAAN) that passes over the landing site. Transfer to a purely equatorial orbit would be too expensive from the NRHOs, which are oriented nearly perpendicular to the lunar equator.

Thus, recognizing the opportunity of the close alignment of the LLOs with desired departure orientations for the 3250 km butterfly, analysis is run to fix the LLO over the surface stay and determine the resultant costs. As the initial LLO is determined by the first transit, it becomes clear that the target LLO must be

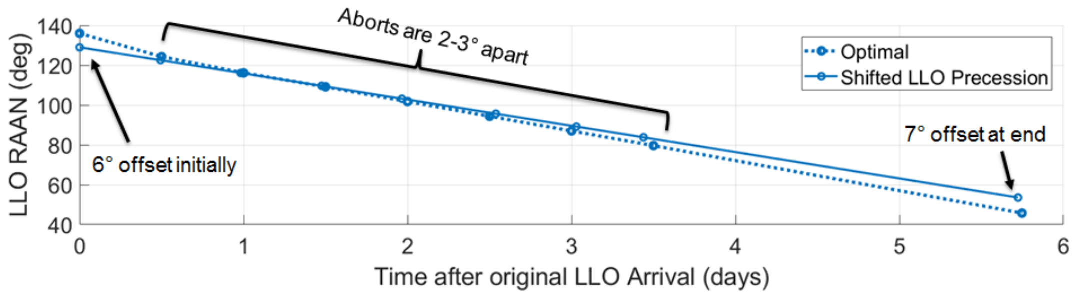


Figure 18: Body-fixed right ascension of ascending node (RAAN) compared to optimal departure RAAN

biased so that the difference between the precession rate and optimal transfers throughout the surface stay is minimized. Figure 18 compares the optimal RAAN of a transfer to and from the 3250 km butterfly with the shifted precessed RAAN. The maximum difference occurs at the beginning and ending transit, balancing the RAAN difference so that it is minimized during the surface stay, enabling anytime abort to LLO. Figure 19 traces the resultant trajectories optimal transits.

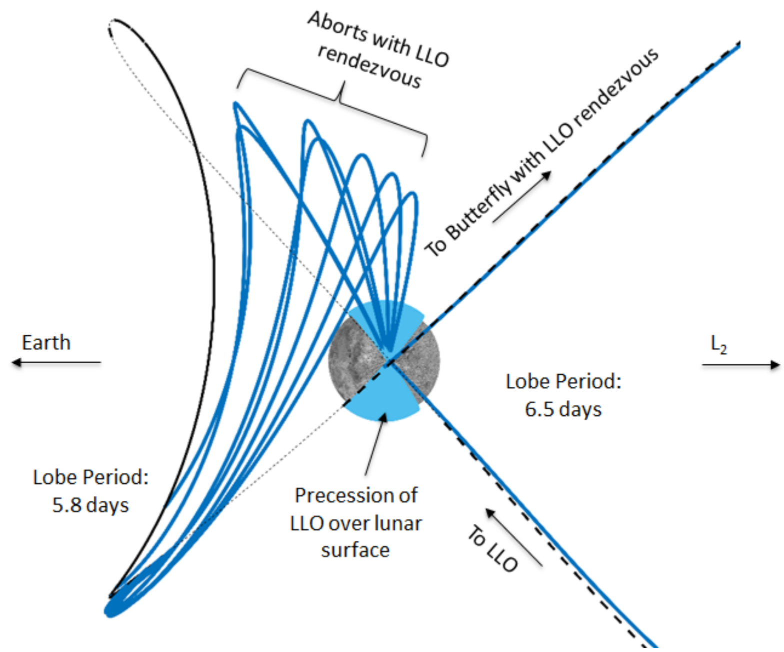


Figure 19: 3250 km Butterfly with fixed LLO orbit

Fixing the LLO for all transits results in higher ΔVs. However, the opportunity to break up the cost between an orbiting asset such as a tug and a lander is enabled. In the end, an architecture trade may favor a small increase in ΔV if less spacecraft must transit all the way to the surface. Comparisons between a tug based transit architecture and one free to choose any LLO back to the butterfly appear in Table 8.

Table 8: 3250 km L_2 Butterfly to Polar LLO Round Trip Comparison

	To		From		Least Favorable Abort	
	ΔV (m/s)	ΔT (days)	ΔV (m/s)	ΔT ()days)	ΔV (m/s)	ΔT (days)
LLO Free	689	0.5	690	0.5	710	4.0
LLO Fixed	761	1.0	760	1.0	727	4.0

CONCLUDING REMARKS

Options for conducting missions to the lunar surface using various sizes of Earth-Moon L_2 Near Rectilinear Halo Orbits (NRHOs) and butterfly orbits as staging locations have been examined. Fast transits to both NRHO and butterfly orbits are available within Orion's propellant budget and are relatively constant, around 400 m/s for the example epoch examined. Long time of flight ballistic lunar transits from a similar Earth departure energy average around 20 m/s for insertion into the reference 9:2 NRHO with opportunities for further optimization. Likewise, fast transits to surface specific LLOs are available for less than 700 m/s to the poles and 800 m/s to the most difficult to access equatorial landing sites, enabling overall global access for less than 800 m/s each way. Slower transits to LLO measured in weeks and months can be as low as 665 m/s. Additionally, high efficiency low thrust engines can save up to 70% of the propellant associated with the cheapest high thrust transits. Butterfly orbits that bifurcate from the NRHOs further reduce costs to both polar and equatorial landing sites. Finally, while most transits require free LLO conditions, the 3250 km butterfly orbit enables the use of tugs or other fixed LLO assets through the unique alignment of the transfer LLO as it precesses around the lunar surface and the optimal lunar surface to orbit return geometry.

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