Movie Review

Proof

Reviewed by Daniel Ullman

Proof
Miramax Films
Starring Gwyneth Paltrow and Anthony Hopkins
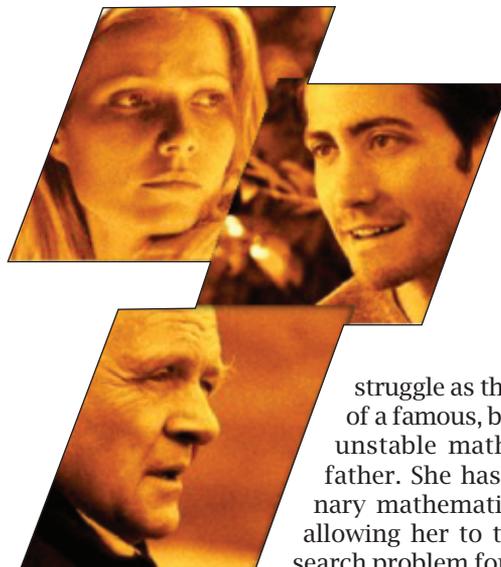*Screenplay by David Auburn and Rebecca Miller*
Directed by John Madden

David Auburn's brilliant play *Proof*, which opened on Broadway in 2000, is now a Miramax film starring Anthony Hopkins and Gwyneth Paltrow as mathematicians, father and daughter. In recent years, there have been several feature films involving mathematicians, and *Proof* is one of the best. Hopkins plays Robert, a man who at one time was a brilliant young mathematician but is beset by mental illness. The diagnosis is not specified, but one can infer that it is schizophrenia. Paltrow plays Catherine, who seems to have inherited her father's brilliance as well as his instability.

It is easy to draw parallels between Robert and John Nash Jr., whose story is now widely known even outside of mathematical and economic circles, owing to the success of Sylvia Nasar's book *A Beautiful Mind* and the Ron Howard film of the same name. Like Nash, Robert suffers from an illness that causes him to believe that there are encrypted messages being communicated to him via newspapers and magazines. Like Nash, Robert attempts to do mathematics during his illness but his efforts reveal not brilliance but the tragic depths of his mental illness. Robert is said to have done revolutionary work in three areas before the age of thirty: game theory, algebraic geometry, and nonlinear operator theory. That seems close enough to Nash that the reference there is impossible to miss.

But *Proof* is more about the daughter Catherine than it is about the father Robert. Auburn must have been fascinated by the story from Nasar's book of John Nash III, who like his father suffered from mental illness. The setting for *Proof* is Catherine's struggle as the offspring of a famous, brilliant, but unstable mathematician father. She has extraordinary mathematical ability, allowing her to tackle a research problem for which her limited formal education ought not have prepared her. In this respect, she is something like the title character in the film *Good Will Hunting*. Both Will and Catherine are unusually brilliant but need psychiatric help, he for childhood trauma, she for depression (or is it schizophrenia, like her father?). Both resist treatment. Will gets his and, predictably, weeps and is cured. Catherine, on the other hand, refuses to submit to the psychiatric plans that have been arranged for her. Still, her need for psychiatric help is apparent.

But Auburn adds an interesting twist: Catherine is female. When Catherine asks Hal, the young protégé of Robert at the University of Chicago, if he knows any female mathematicians, he stammers, and eventually says "There's a woman at Stanford. I don't remember her name." "Sophie Germain?" tests Catherine, in response. "Yeah. I think I've met her at meetings," says Hal, revealing his ignorance. The absence of women in mathematics is a theme in the film, but it is also an inconsistency between the film and reality. Nowadays it would be unthinkable for someone in the field not to know of any women mathematicians. The era of Sophie Germain is past.

Of these three films, *Proof* is the one that most realistically illustrates the world of mathematics and mathematicians. Matt Damon and Ben Affleck co-wrote and co-star in *Good Will Hunting*. It is clear that they are fascinated by the story of Srinivasa Ramanujan. Their brilliant yet unschooled character Will, like Ramanujan, emerges from the wrong side of the tracks and clashes culturally with the mathematicians with whom he collaborates. Perhaps *Good Will Hunting* was envisioned doing for the legend of Ramanujan what *West Side*

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Story did for the legend of Romeo and Juliet. It is a kind of fictionalization of the Ramanujan story set among modern street toughs. But it is not clear that Damon and Affleck have much of an understanding of how and why mathematicians do what they do. The combinatorial problems on the blackboard that Will solves are not the type to baffle professional mathematicians. And there are phrases that ring hollow to a mathematician's ear. For example, the fictional Fields medalist from MIT describes Ramanujan as having created "some of the most exciting math theory ever done". A mathematician wouldn't use that phrasing. This Fields medalist also uses the phrases "solve a theorem" and "prove a problem". These mix-ups broke the spell for me. The feeling of actual mathematicians doing actual mathematics is not present in this film.

Neither is it in the film *A Beautiful Mind*. The film gives the impression that Nash's deep insight into game theory was motivated by the dilemma of how to compete for the attention of pretty girls in a bar. I can imagine that Nash used the competition for female attention as a way of explaining his work to non-mathematicians. But the notion that he discovered his ideas in a flash of insight upon the sight of a beautiful woman is a romanticized version of the story suitable for a movie audience. It is not how mathematicians work. The movie does not show us a mathematical seminar or a mathematical conversation. We see no mention of mathematics as enjoyable or as beautiful (despite the title of the film).

In *Proof*, though, the mathematical life is more realistically rendered. Three of the four characters (Robert, Catherine, and Hal) work in mathematics. We see them read and study and write. Robert describes the pleasure he feels when mathematical ideas are flowing. Catherine describes to Hal how she felt when doing mathematics, speaking of "beautiful, elegant proofs, like music". Hal describes his fear that his own mathematics research doesn't pass muster when compared with Robert's. David Auburn did his homework and is really able to convey how mathematicians work. The script reveals that he knows about John Nash, yes, but also about Ramanujan, Germain, Paul Erdős, and Andrew Wiles. His characters seem very much like they belong in the world of real mathematicians.

There are a couple of breaks from realism in *Proof* where characters speak in a way that is for the benefit of the audience rather than the way mathematicians would actually talk among themselves. When Hal remembers what a Germain prime is, he speaks to Catherine in a way that would be patronizing to another mathematician. After giving the definition, he offers: "Like two. Double plus one is five, another prime." When Robert and Catherine recall the fact, made famous by a story of Ramanujan and G. H. Hardy, about the number

1729—the smallest integer that can be expressed as the sum of two cubes in two different ways—they similarly explain too much to each other. Mathematicians, particularly father and daughter, would have a silent rapport on this.

Auburn and Rebecca Miller, daughter of playwright Arthur Miller, wrote the screenplay for the film, which was directed by John Madden (of *Shakespeare in Love* fame). Madden should be credited with capturing the feeling of the mathematical world; he consulted effectively with Fields medalist Timothy Gowers of Cambridge University in preparation for the film. It is richer and deeper, simultaneously both funnier and more serious, than either *A Beautiful Mind* or *Good Will Hunting*. David Auburn has more to say to mathematicians than do Damon, Affleck, or Akiva Goldsman, the screenwriter for *A Beautiful Mind*. *Proof* is a multifaceted story about sibling rivalry, about gender ability (did Lawrence Summers catch it?), about mental illness, about trust in relationships, all set within the world of mathematics. It is also a mystery about the authorship of a discovered manuscript.

Great stage plays rarely make blockbuster movies, and this may be another example. The play won a Pulitzer Prize, a Tony Award, and played for an astonishing 917 performances on Broadway. Not bad for a first effort from Auburn! It seems unlikely that the film will be as successful, even with its all-star cast. To me, *Proof* works slightly better on the live stage, with its stark setting and small cast, and with intermission to punctuate the shocking last line of the first act. In particular, part of the fascination of the play is the way the action revolves neatly around the one setting, the back porch of Robert's and Catherine's home. The activities in other parts of the house and other parts of town are implied, cleverly woven into the action back on the porch. One of my favorite lines from the play is absent from the film: When Catherine asks what it means about her mental health that she is having a drink with (an image of?) her deceased father, her father replies sadly and poignantly "It could be a bad sign." The film reveals the last name of Robert and Catherine: Llewellyn. There are far more flashbacks in the film, some even momentary. Catherine is 27 and 24 years old in the film where she was 25 and 21 years old in the play. This is presumably to accommodate Paltrow, who was 32 years old when *Proof* was filmed.

The film medium adds a new dimension to any stage play. A camera can follow a character in a way that an audience cannot. The film *Proof* gets around Robert's and Catherine's house. So we have the requisite bedroom scene in the film, a scene that is delicately left to our imagination when we see the play. We visit the party that is merely heard offstage in the play. The film gets out into Chicago, Auburn's home town, as well. There are crowds of

people at a funeral and on the campuses of the University of Chicago and Northwestern University. The play, by contrast, has but four characters in it. Jake Gyllenhaal, who plays Hal in the film, gives a warm and sympathetic portrayal of a young mathematician, more charming than Hals I've seen on stage. And Hope Davis gives a fine performance in the film as the sane, normal, but ultimately annoying Claire, sister to Catherine. The film adds a couple of minor characters, university mathematicians, with spoken lines.

Still, on the whole the movie is similar to the play. To say "based on the play by David Auburn" is to understate the connection. Entire scenes are taken verbatim from the play. Roughly 80 percent of the lines in the film are straight from the play. By contrast, the film version of *A Beautiful Mind* is only loosely based on Nasar's book.

The real significance of the film is that it brings the story to a wider audience, just as *A Beautiful Mind* was seen in film version much more than it was read in book version. (Nasar's book was a *New York Times* best seller, but far fewer Americans read books than see movies.) The vast majority of Americans may well have an image of a mathematical genius that is shaped primarily by the case studies of Will Hunting, John Nash, and Robert and Catherine Llewellyn in *Proof*. I suspect that they will draw the following conclusions about mathematicians from these feature films:

First, mathematicians are disturbed and need psychiatry. Will is emotionally disturbed, John is paranoid and schizophrenic, and Catherine suffers from depression (at least). A reasonable inference is that mathematical talent is itself a psychiatric illness, that madness is a natural result of a mind that can reason mathematically. Or perhaps it is the converse, that madness induces a state in which the ability to reason mathematically is heightened. It is never easy to infer causation from correlation.

Second, mathematicians are arrogant and rude. Will carries his intellect like a weapon, brandishing it on psychiatrists, on irritating Harvard students, and even on the Fields medalist to demonstrate his superiority. John is portrayed as obnoxious, such as when he cuts down a colleague by telling him that his ideas have not an ounce of originality in them. Catherine, too, seems rarely to be nice to anyone but her father.

Third, mathematicians are antisocial. Neither John nor Catherine seems to have any friends. Will does have friends, but his behavior lands him repeatedly in jail. Hal actually describes mathematicians as wild party animals, but that characterization seems to be mostly for laughs, since the stereotype is opposite.

Fourth, mathematicians are competitive and self-promoting. They are more interested in advancing themselves, in being recognized as brilliant, than they are in advancing mathematics. Will's only interest in mathematics seems to be as a tool to demonstrate intellect. The young John searches mightily for some big idea that will make others notice him. Catherine accuses Hal of stealing mathematical results for his own advancement.

Last, mathematicians are young. Will certainly is young, while the aging Fields medalist seems uncertain whether he can understand Will's work. John does all his work when young, certainly. And Hal worries that he too is over the mathematical hill at age twenty-six. Once mathematicians reach a certain age, Hal in the play suggests that they "might as well teach high school". In the film, Hal quips "I'm twenty-six. You know, the downward slope." About the assumption that mathematical ability is the province of the young, Robert in the play says "this is a stereotype that happens to be true."

These impressions are of course all stereotypes. I have not heard any evidence that mental illness is positively correlated with mathematical talent or mathematical interest. Regarding arrogance, my own experience is that mathematicians tend to find mathematics humbling and that they share with other mathematicians a certain fellowship. I suspect that mathematicians may be less social on average than the general population, but probably the same can be said for others who work in cerebral disciplines, where extended solitary concentration is required. I doubt self-promotion is a trait that is attributable to mathematicians more than to any other professionals. And while many young mathematicians accomplish great things, I doubt very much that mathematical ability must necessarily wane with age. Much more likely, it is mental energy that wanes with age. These films associate mathematicians with brilliance rather than diligence, which in turn suggests that mathematical work is easy rather than hard. It makes sense then that a failure to be productive is seen as a loss of intellect rather than a loss of drive or stamina.

Although Robert is mentally ill and Catherine is antisocial, *Proof* does as much to dispel stereotypes as it does to reinforce them. It presents mathematicians who seem passionate about their work for its own sake, and in this respect it presents a more realistic picture of the mathematicians I know. Hal is musical, athletic, energetic, and funny, but also vulnerable, in many ways exactly like a regular person who is a mathematician. *Proof* shows us mathematicians, young and old, working together with fervor to examine an exciting new manuscript. One imagines it might be a proof of the Riemann hypothesis or perhaps the twin primes conjecture. Perhaps in the sequel we'll find out which result it is and get the details of the proof.