Aerodynamic Stability of Finned Rockets

LabRat Scientific
© 2018

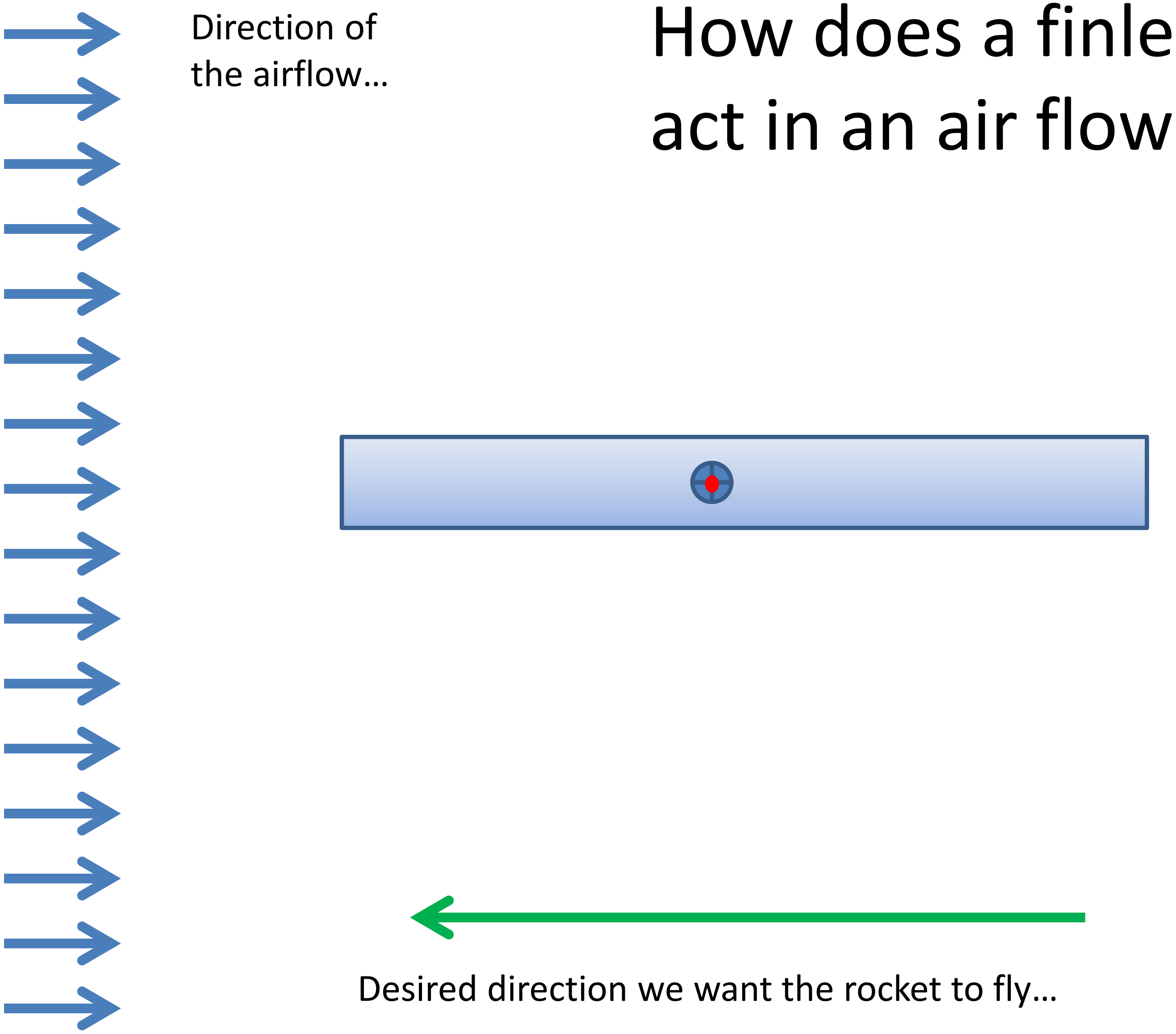
Basic Concepts

- Center of Gravity (CG)
  - Point where the mass loads can be concentrated
  - Physical balancing point

- Center of Pressure (CP)
  - Point where aerodynamic forces can be concentrated
  - Sort of the aerodynamic balancing point

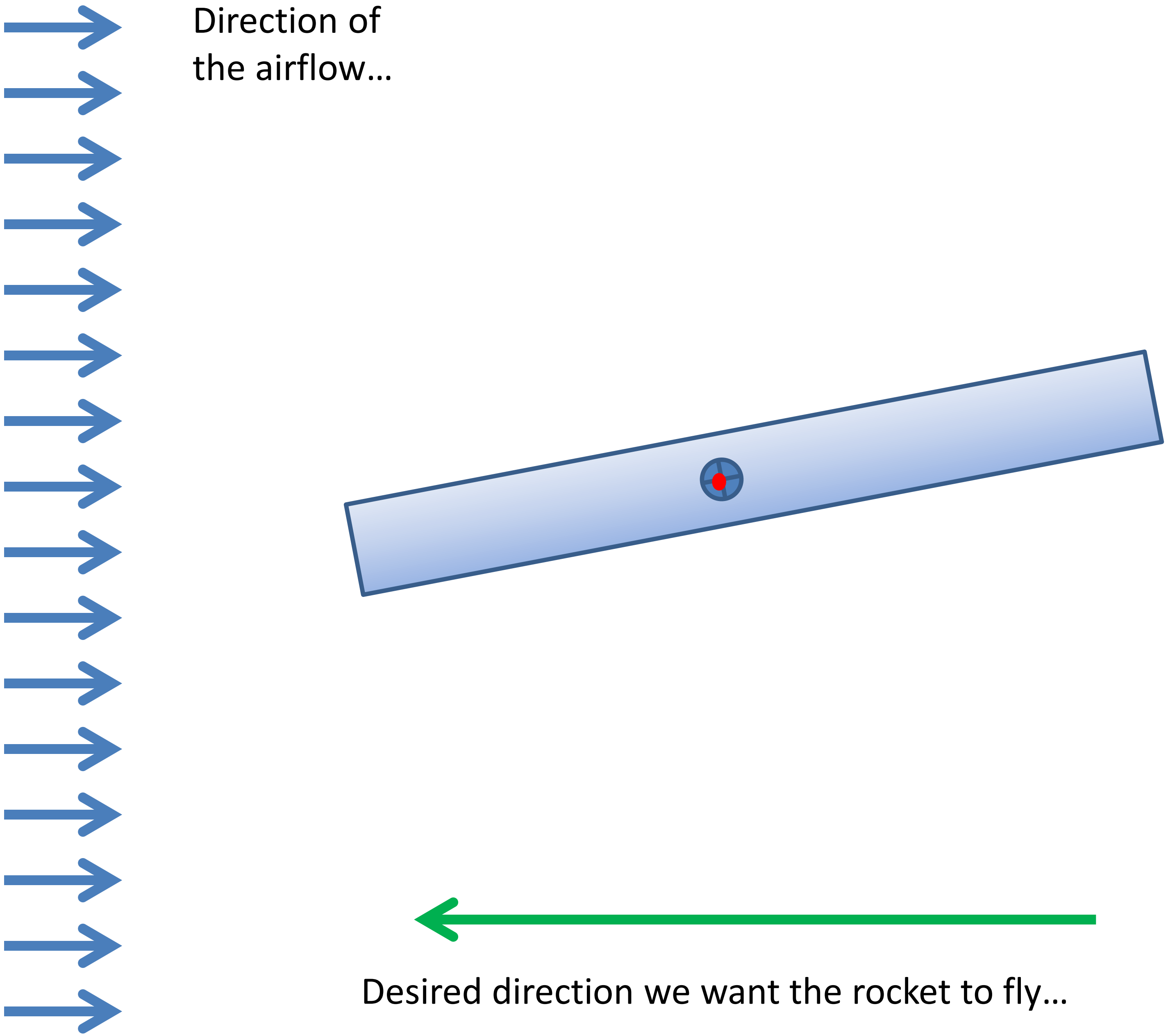
- Static Margin
  - Distance between the CG and CP

How does a finless cylinder act in an air flow?



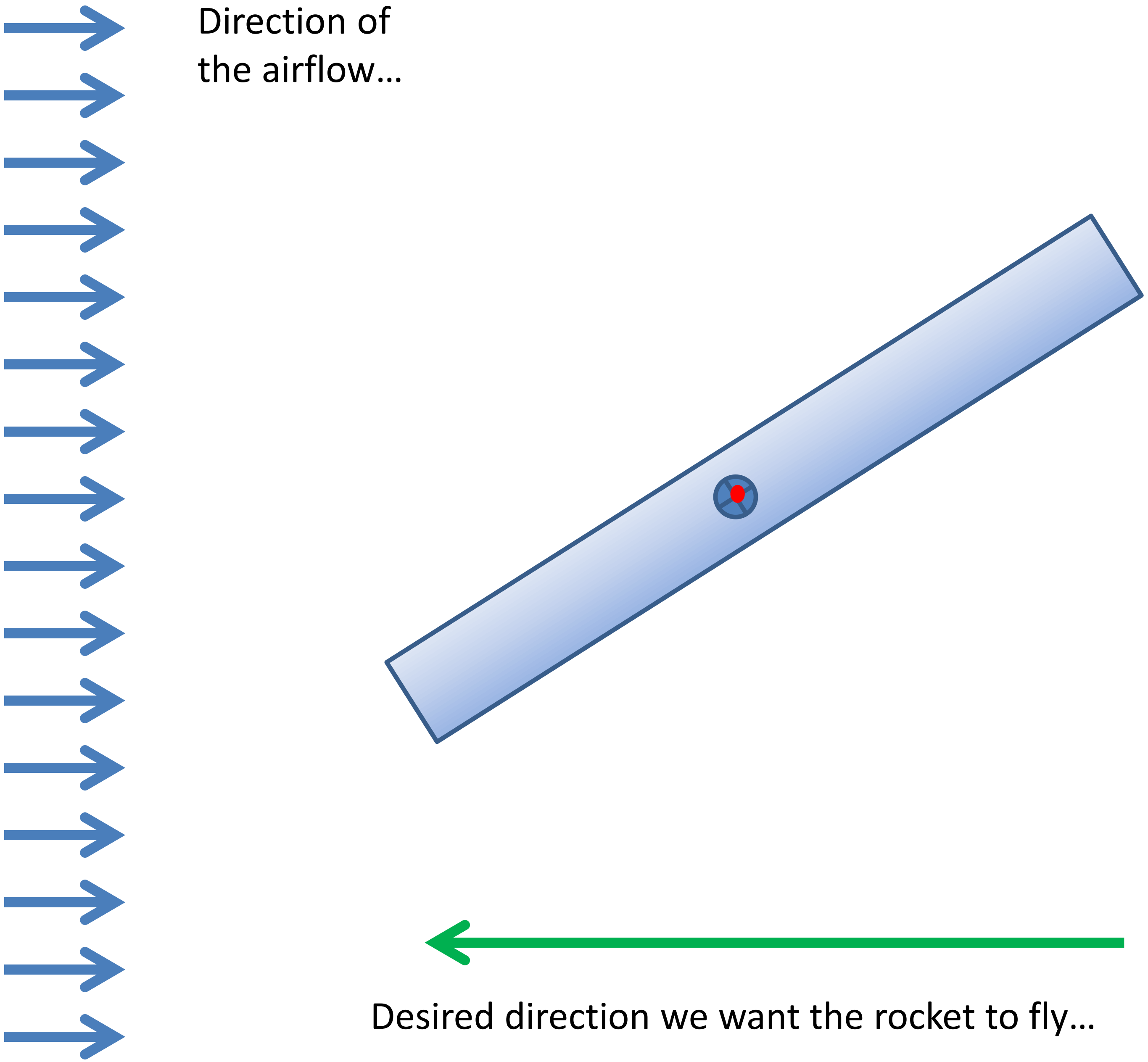
Direction of the airflow…

Desired direction we want the rocket to fly…



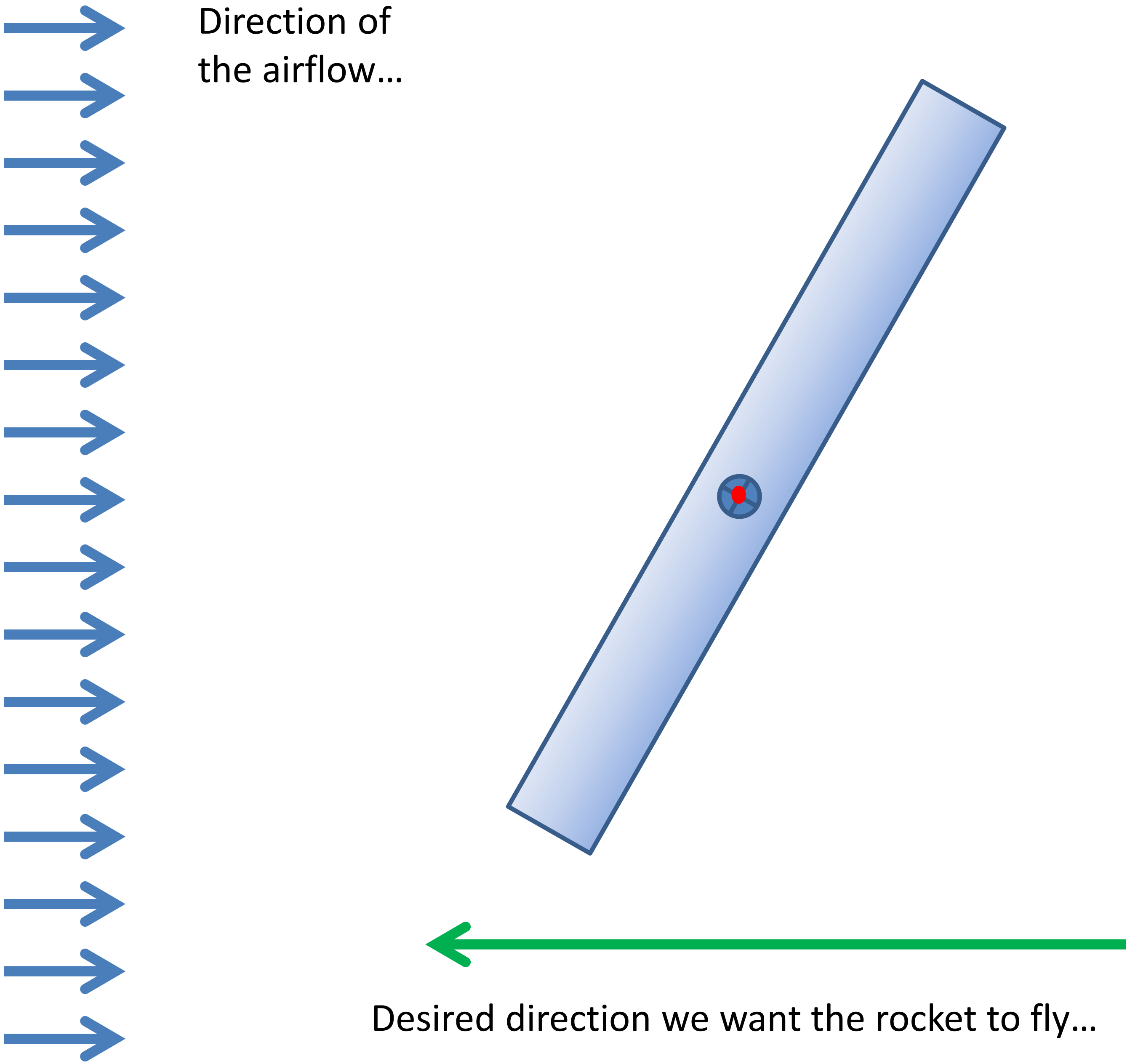
Direction of
the airflow…

Desired direction we want the rocket to fly…



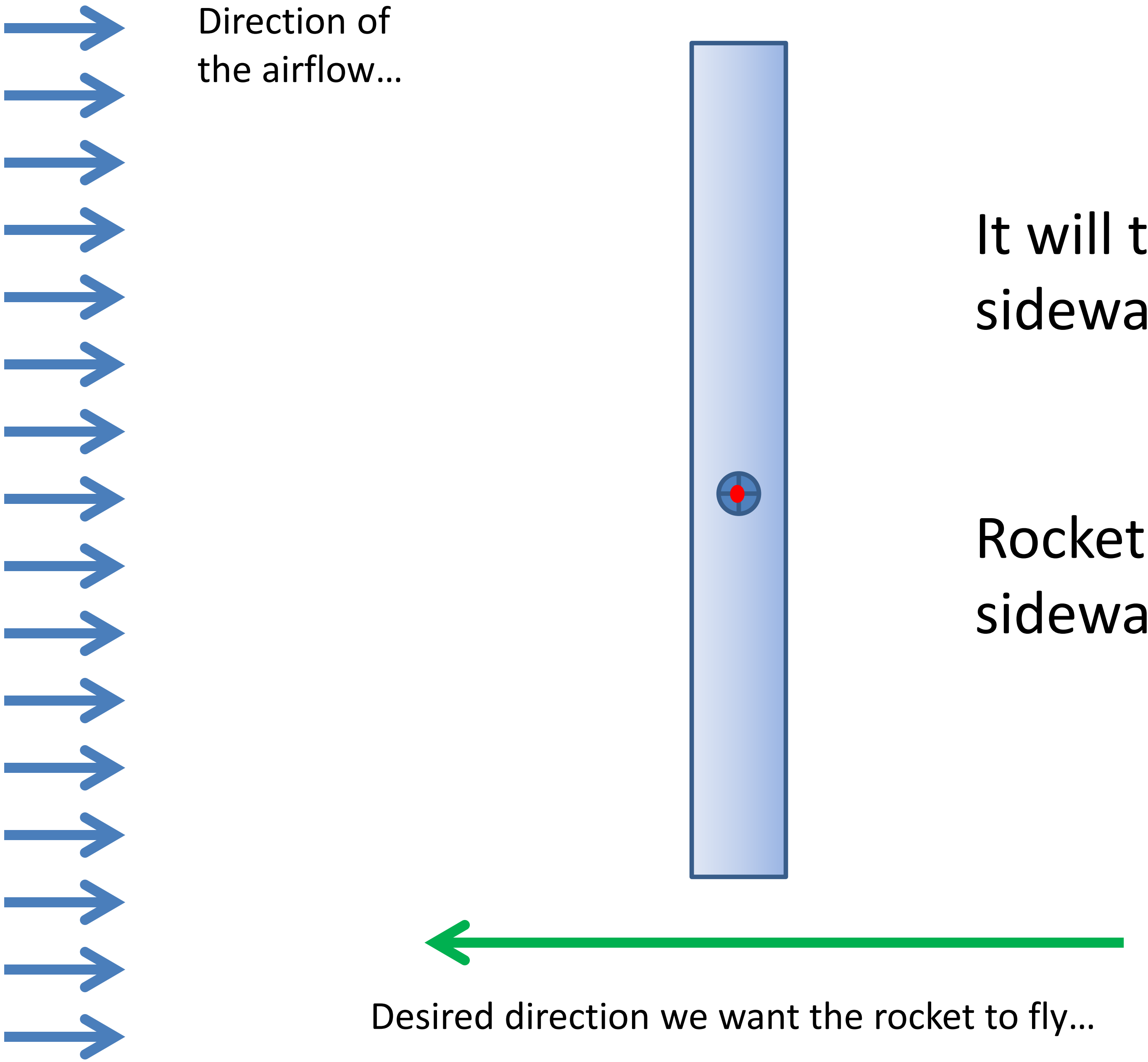
Direction of
the airflow…

Desired direction we want the rocket to fly…



Direction of
the airflow…

Desired direction we want the rocket to fly…

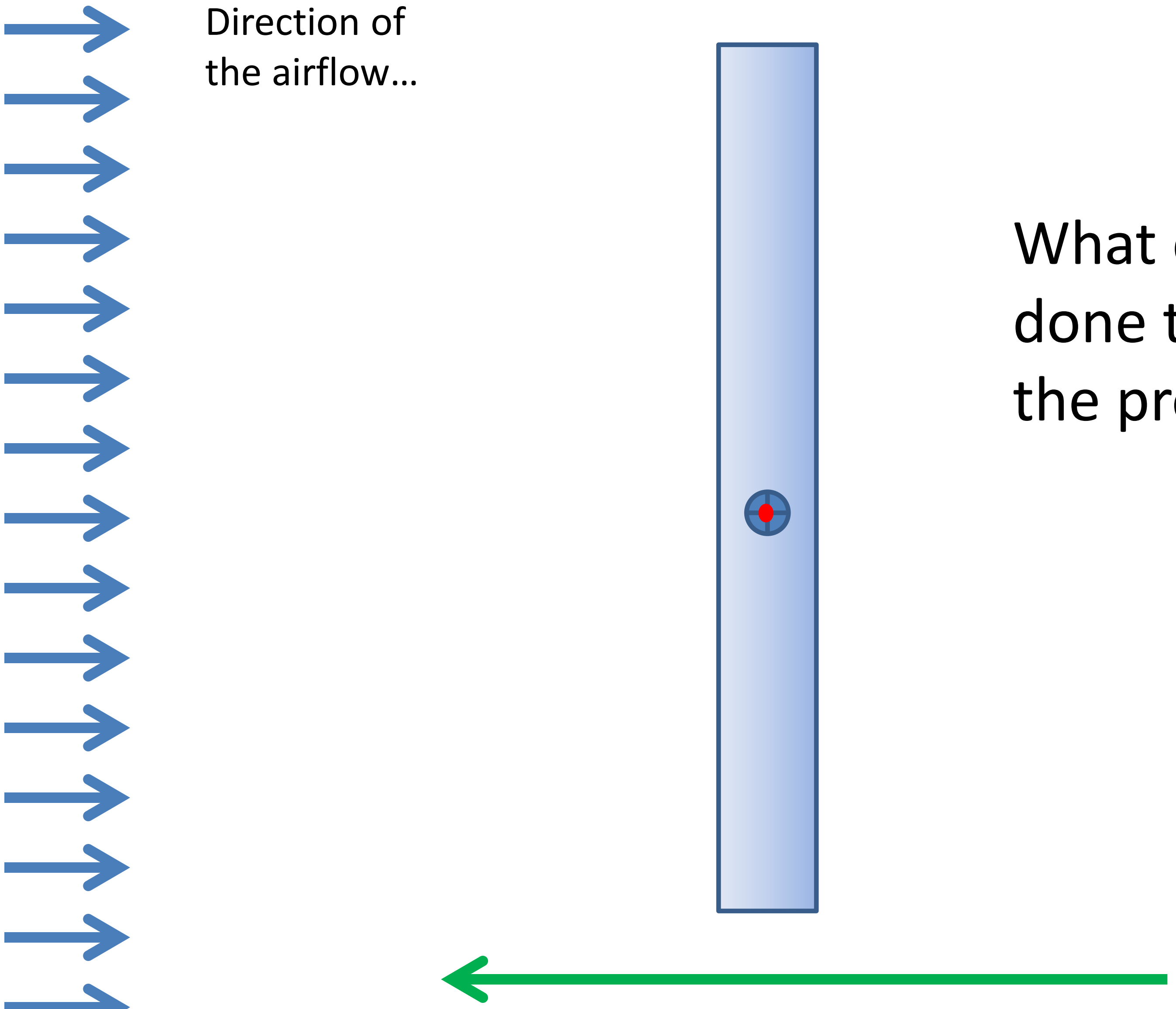


Direction of
the airflow…

Desired direction we want the rocket to fly…

It will tend to orient sideways.

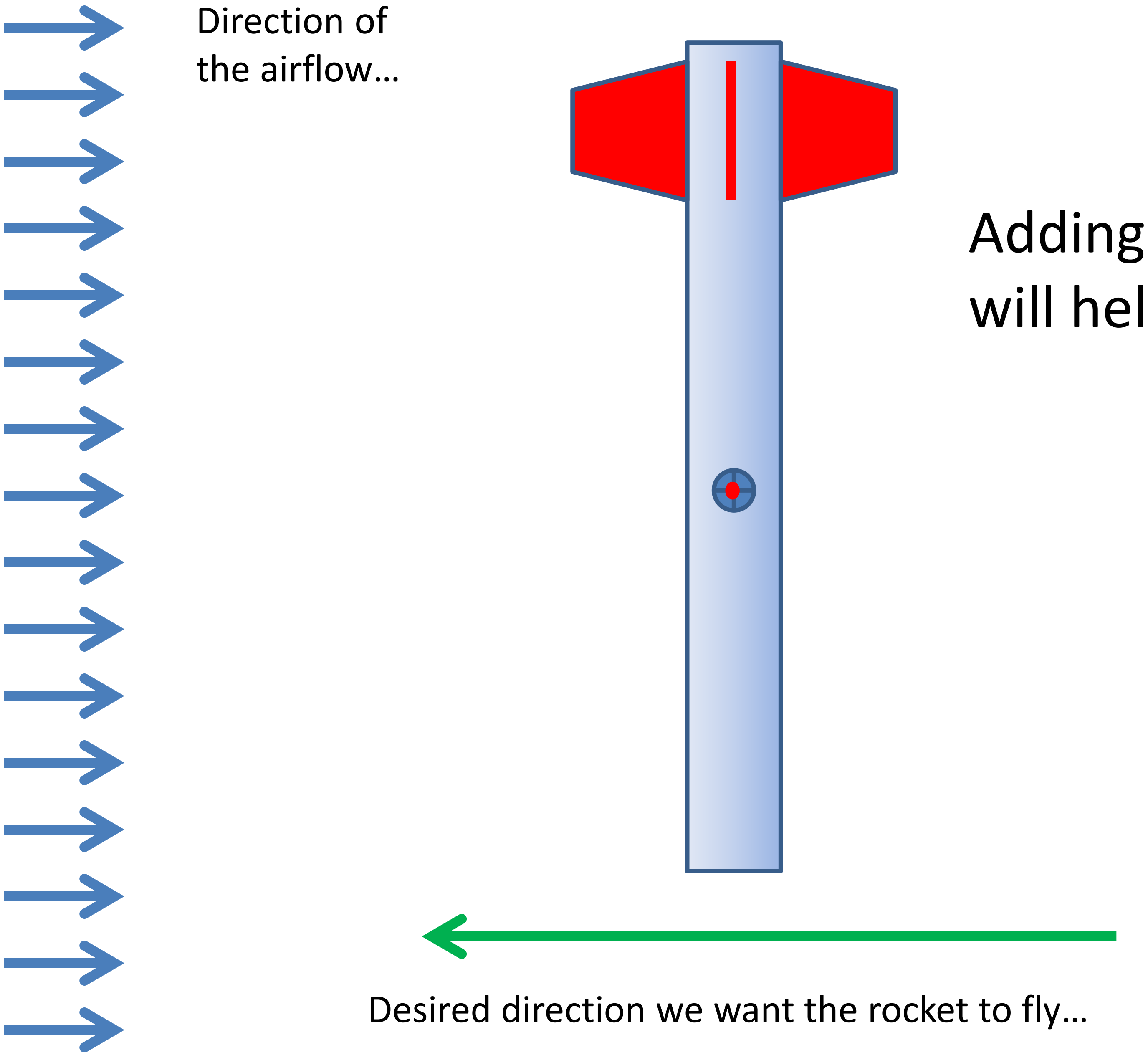
Rockets don’t fly sideways too well…



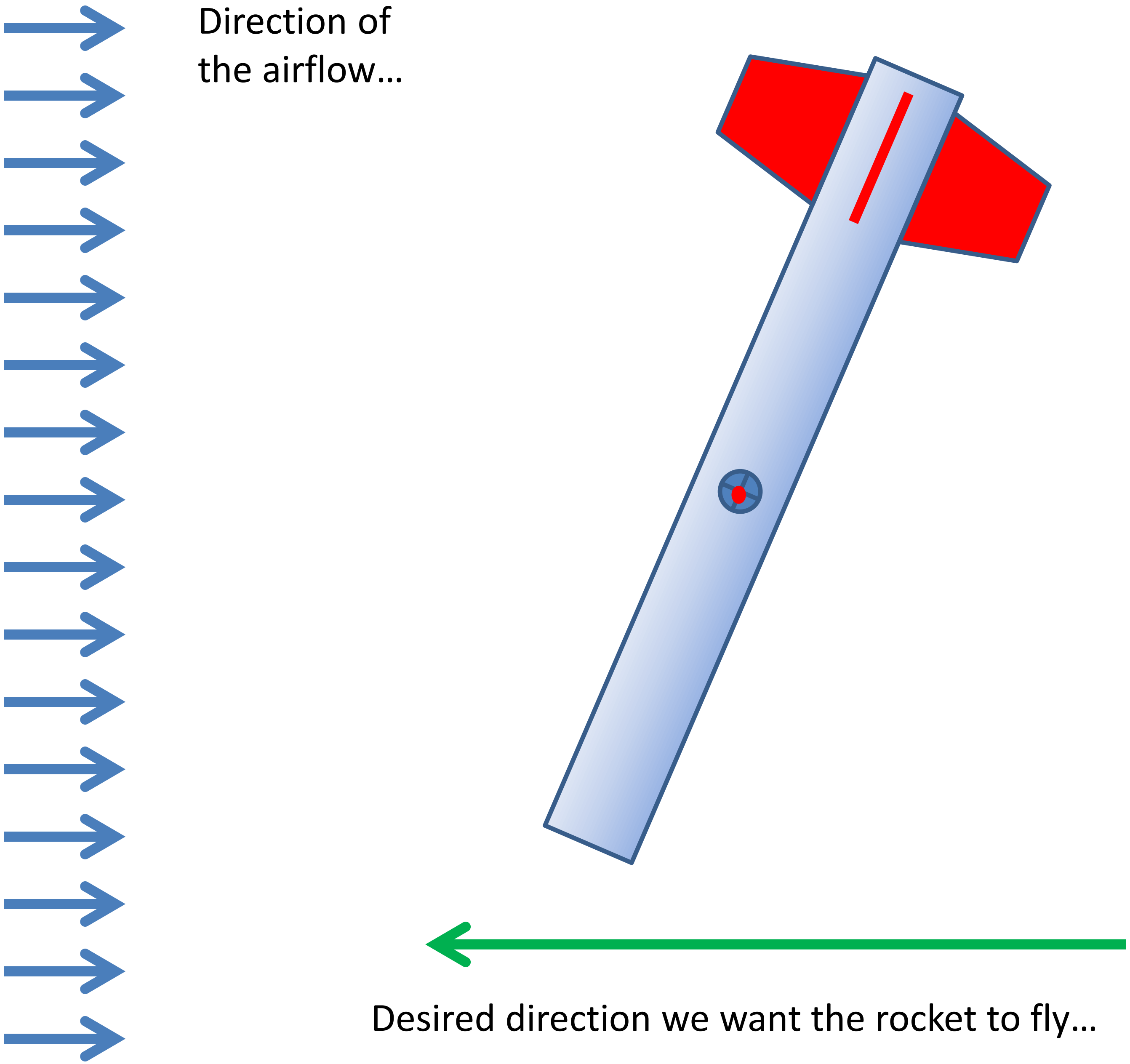
Direction of
the airflow…

What can be
done to rectify
the problem?

Desired direction we want the rocket to fly…

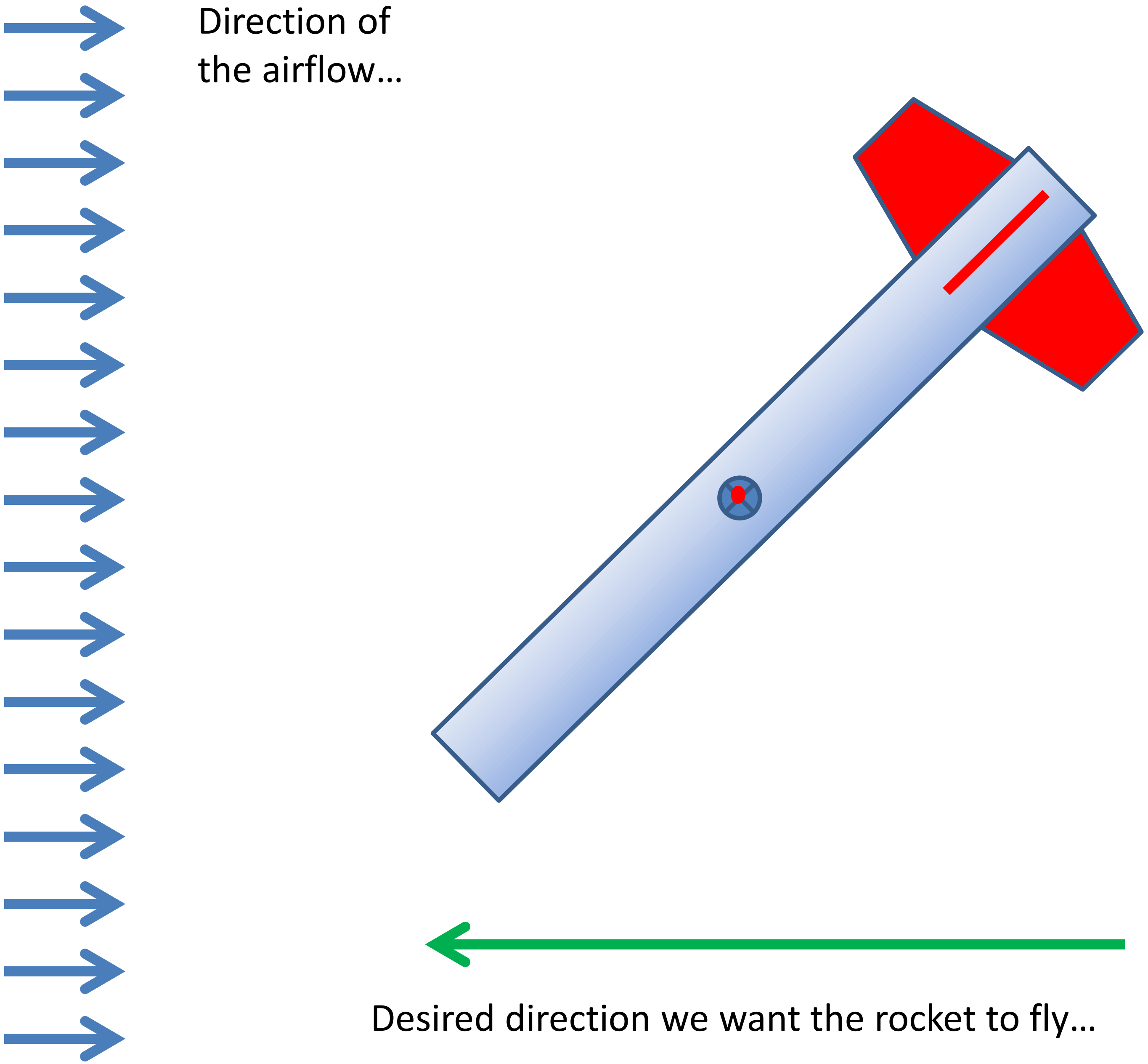


Adding fins (not “wings”) will help the situation.



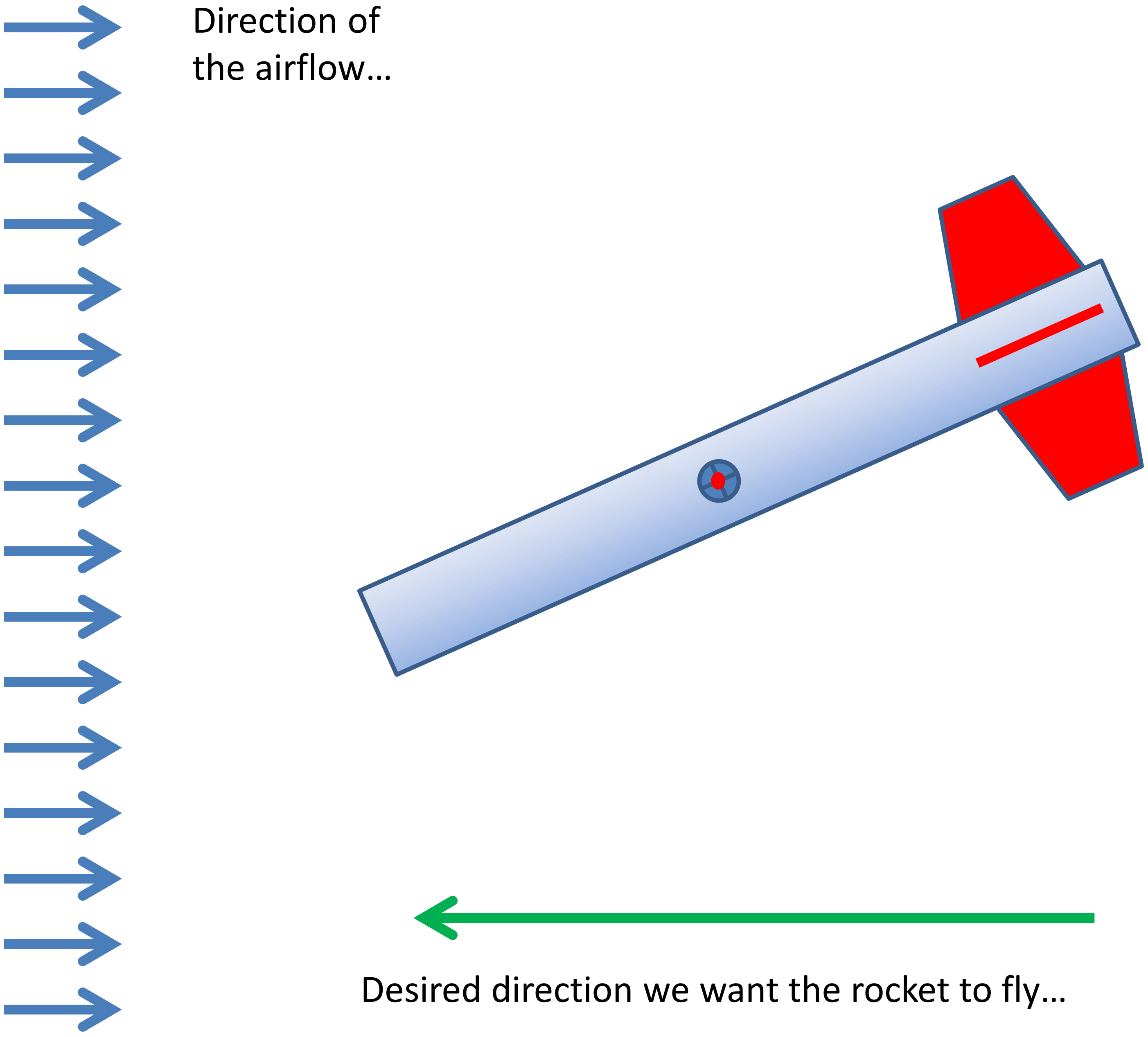
Direction of the airflow…

Desired direction we want the rocket to fly…



Direction of
the airflow…

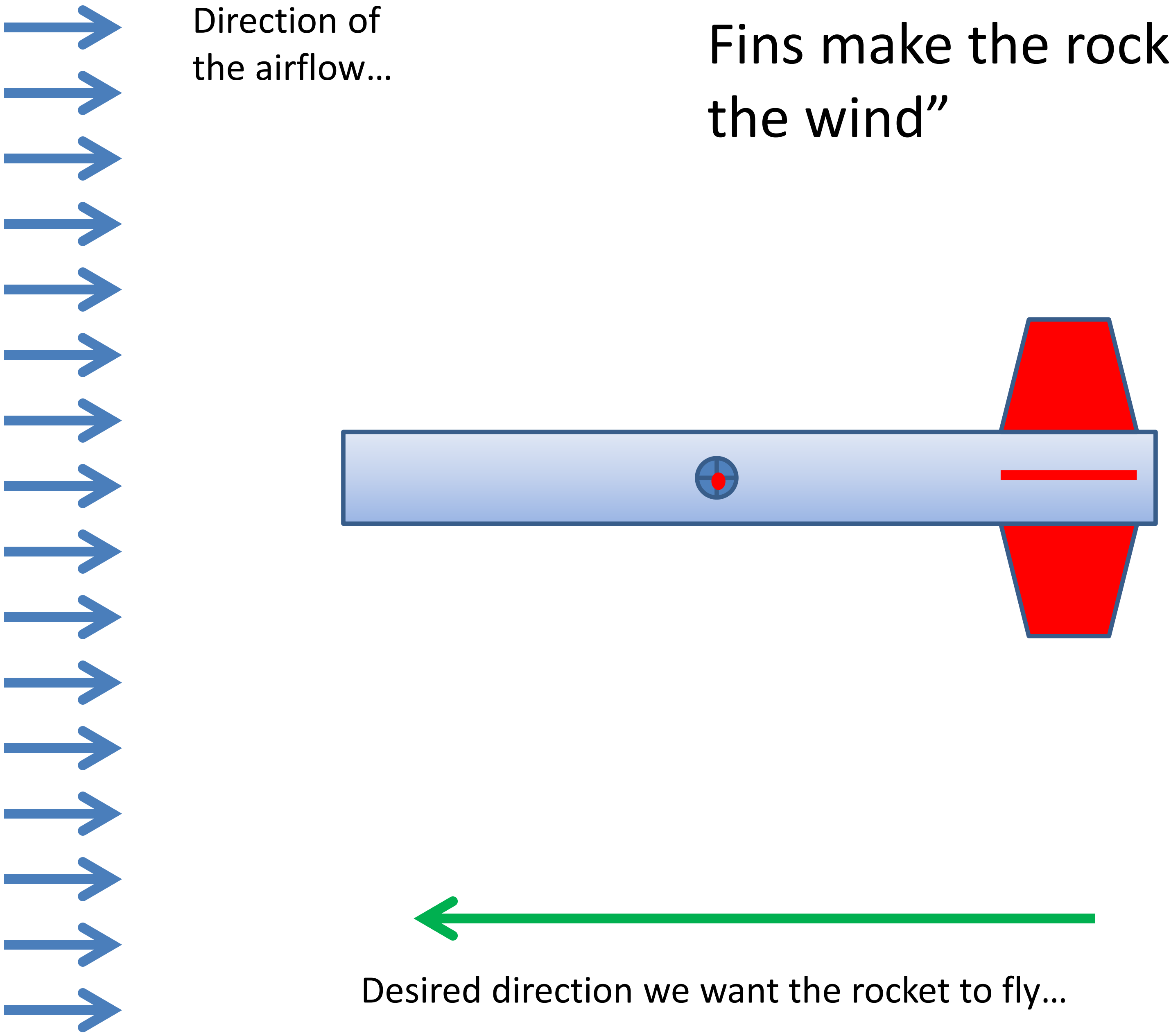
Desired direction we want the rocket to fly…



Direction of
the airflow…

Desired direction we want the rocket to fly…

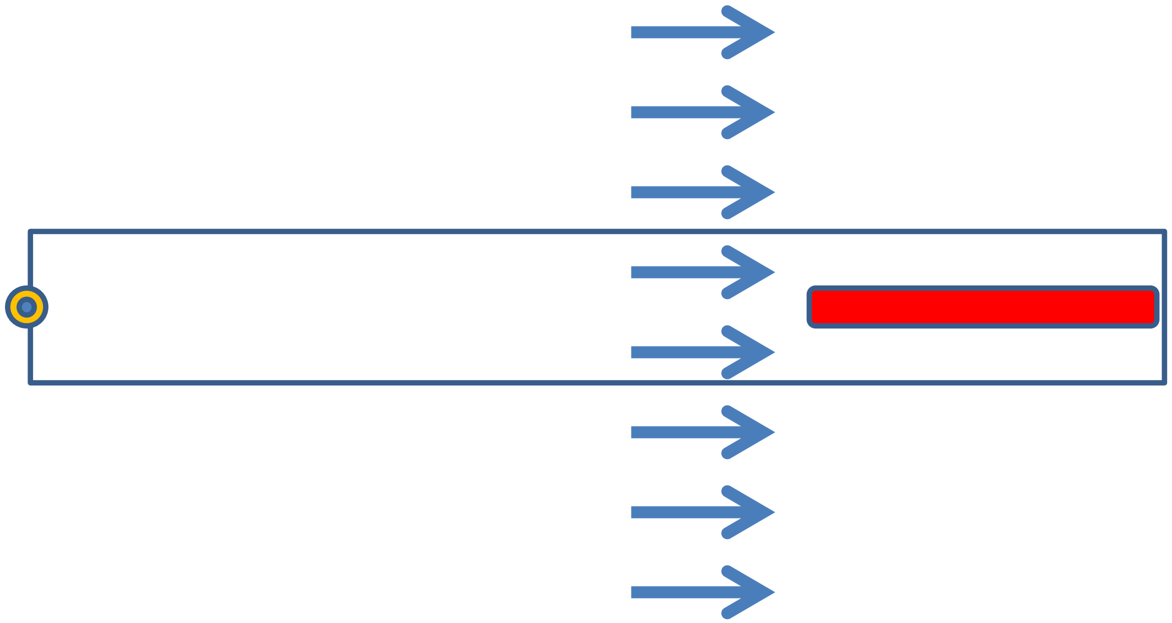
Fins make the rocket “point into the wind”



Direction of the airflow…

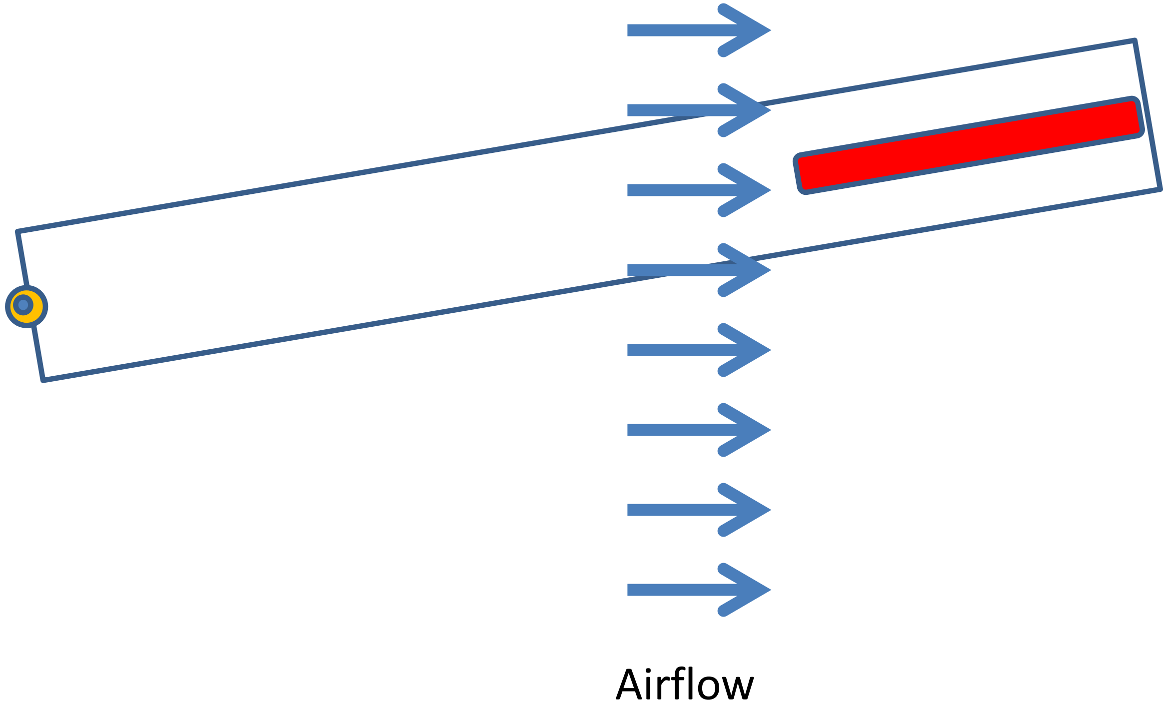
Desired direction we want the rocket to fly…

Fins are very similar to a wing, but they are designed to work equally well with a positive or negative angle of attack.

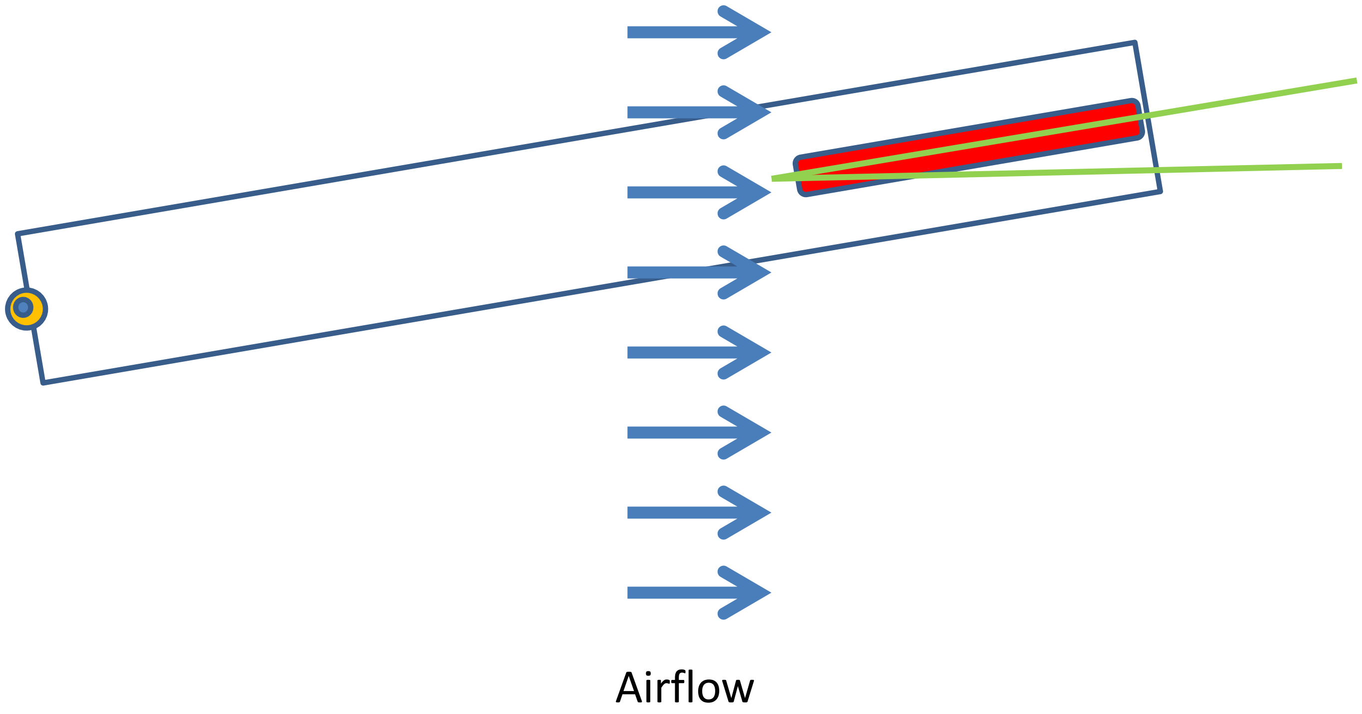


Airflow

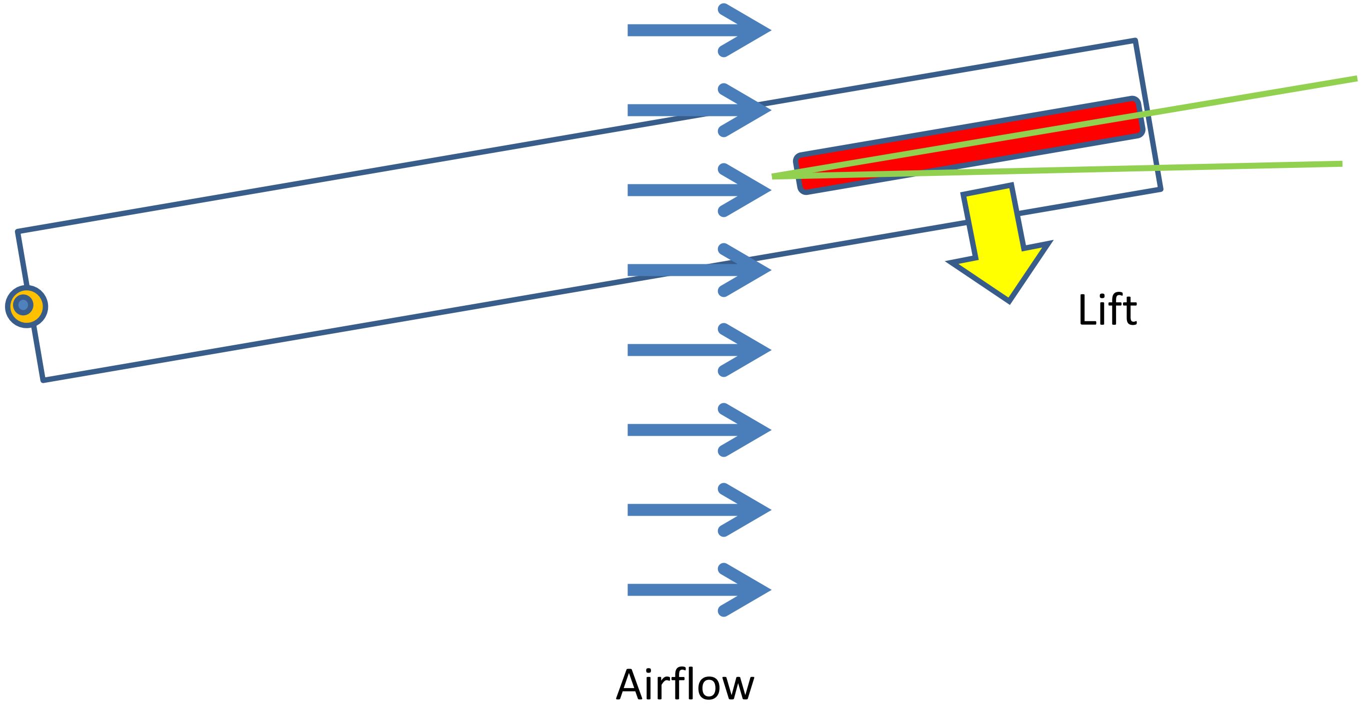
Fins are very similar to a wing, but they are designed to work equally well with a positive or negative angle of attack.



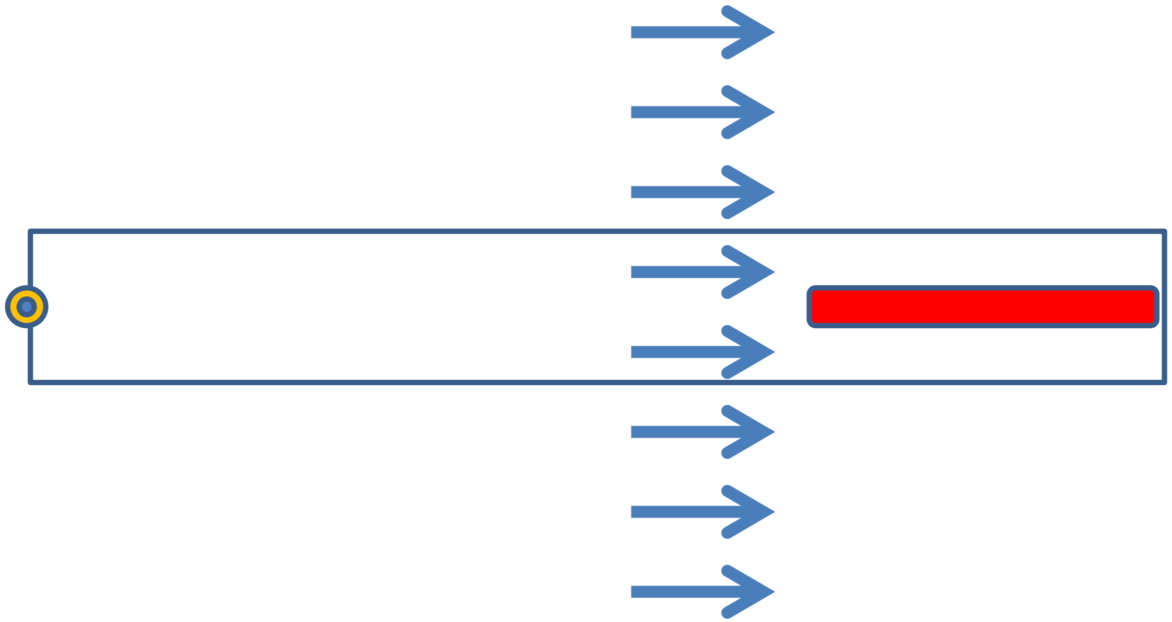
Fins are very similar to a wing, but they are designed to work equally well with a positive or negative angle of attack.



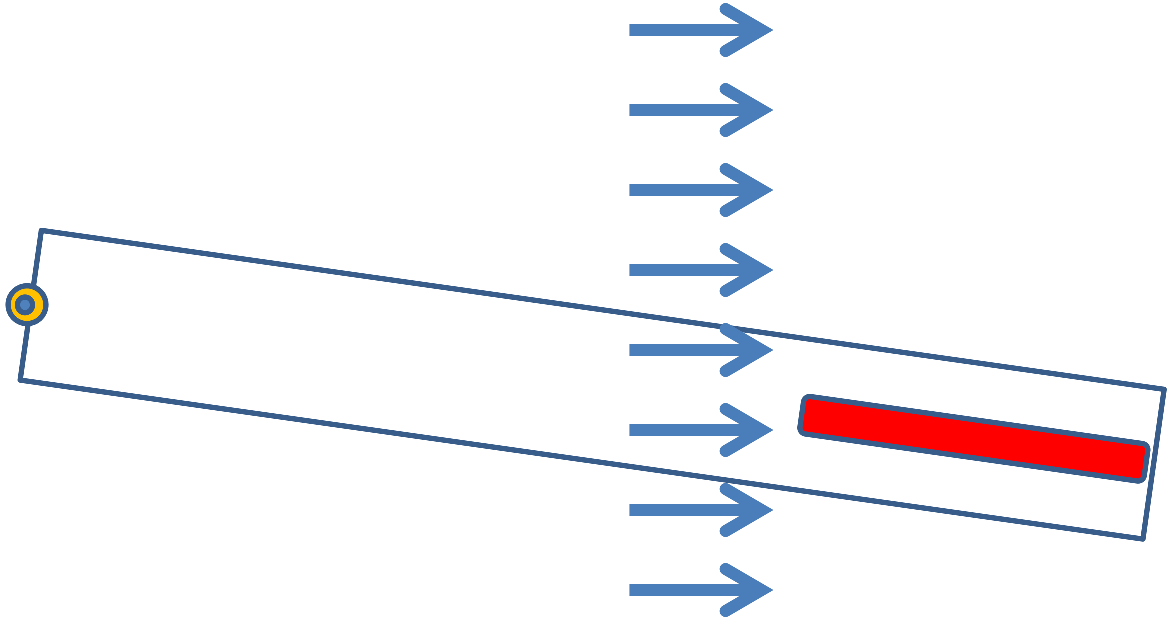
Fins are very similar to a wing, but they are designed to work equally well with a positive or negative angle of attack.



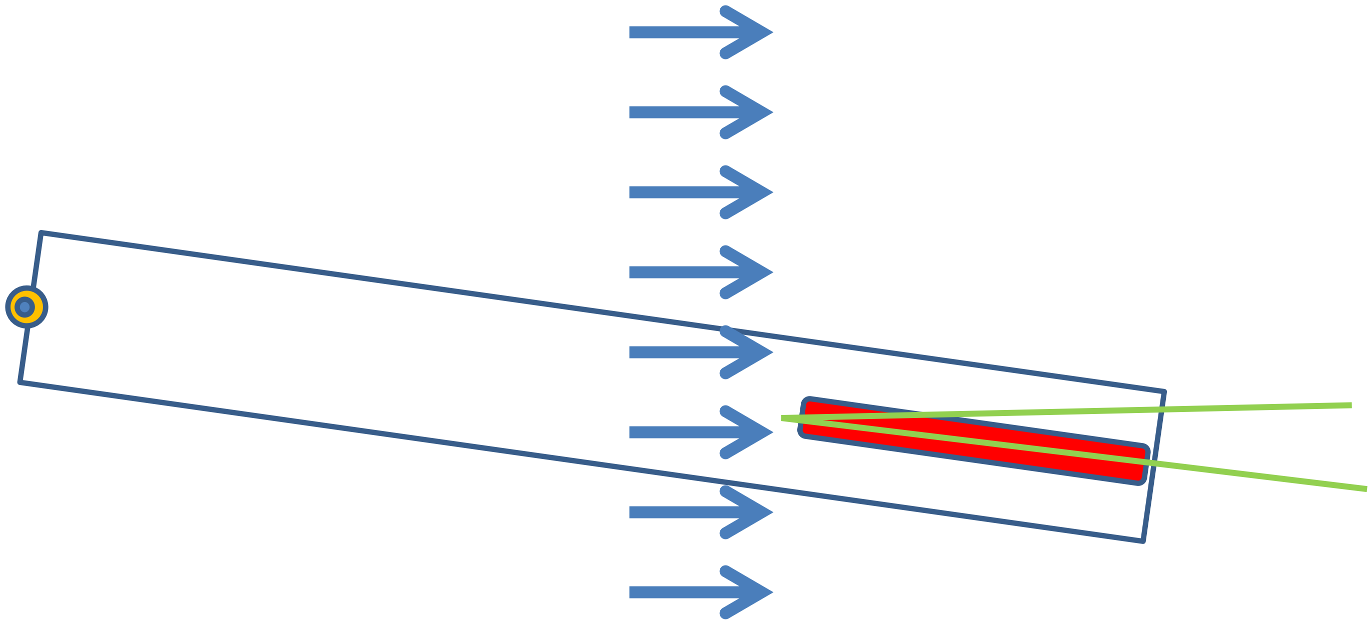
Fins are very similar to a wing, but they are designed to work equally well with a positive or negative angle of attack.



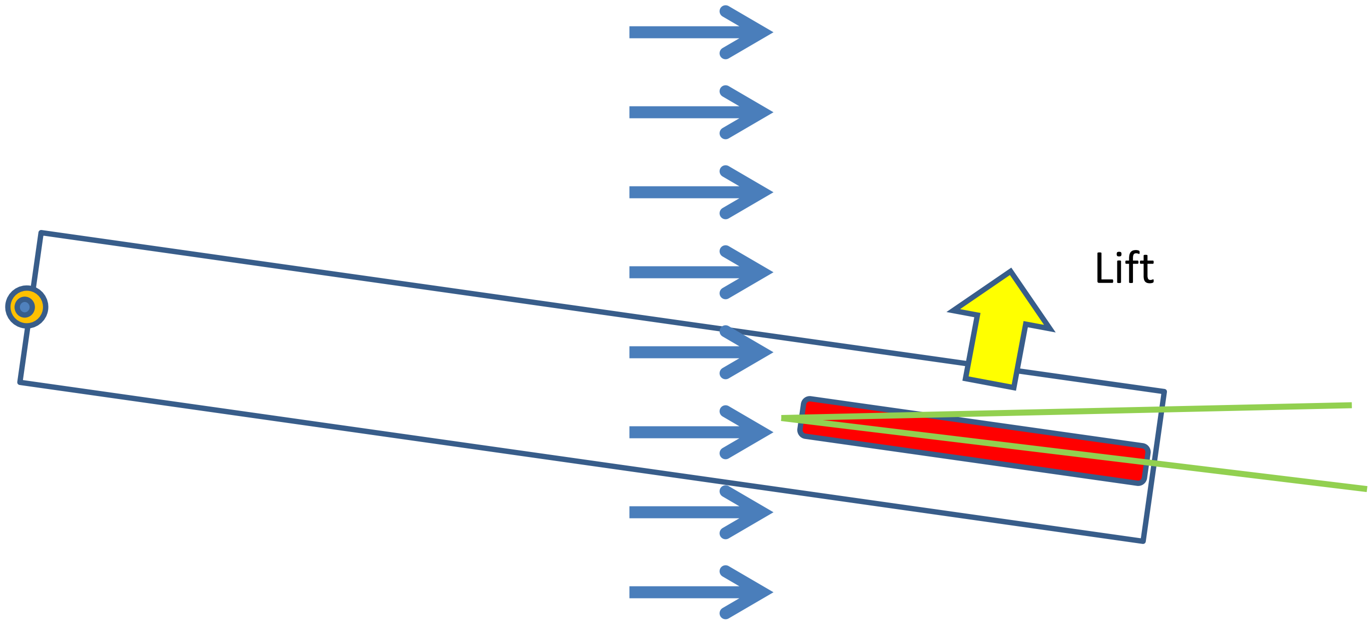
Fins are very similar to a wing, but they are designed to work equally well with a positive or negative angle of attack.



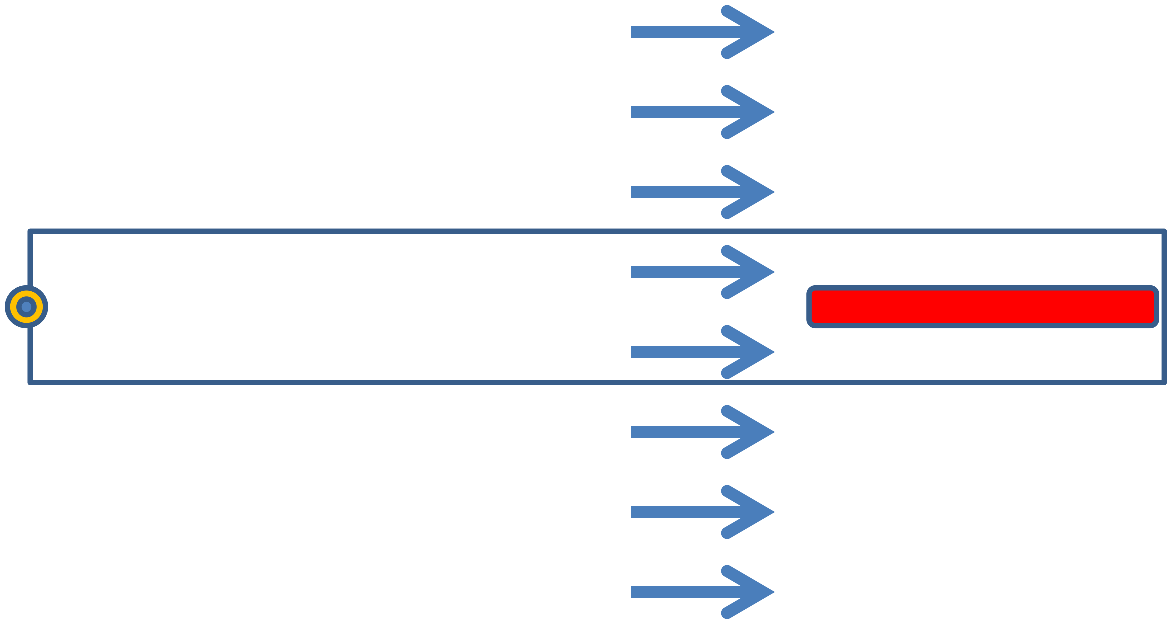
Fins are very similar to a wing, but they are designed to work equally well with a positive or negative angle of attack.



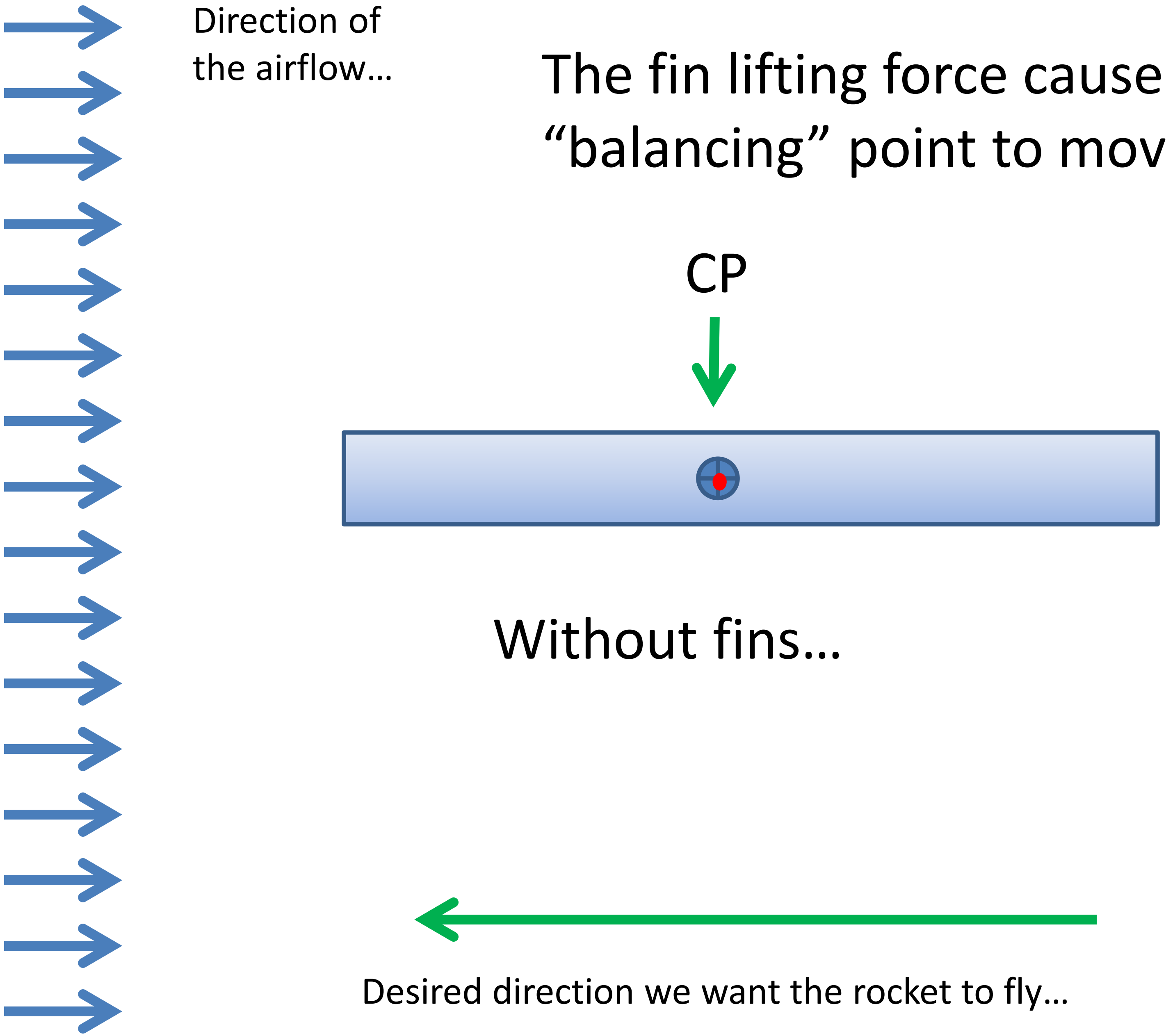
Fins are very similar to a wing, but they are designed to work equally well with a positive or negative angle of attack.



Fins are very similar to a wing, but they are designed to work equally well with a positive or negative angle of attack.



The fin lifting force causes the aerodynamic "balancing" point to move back



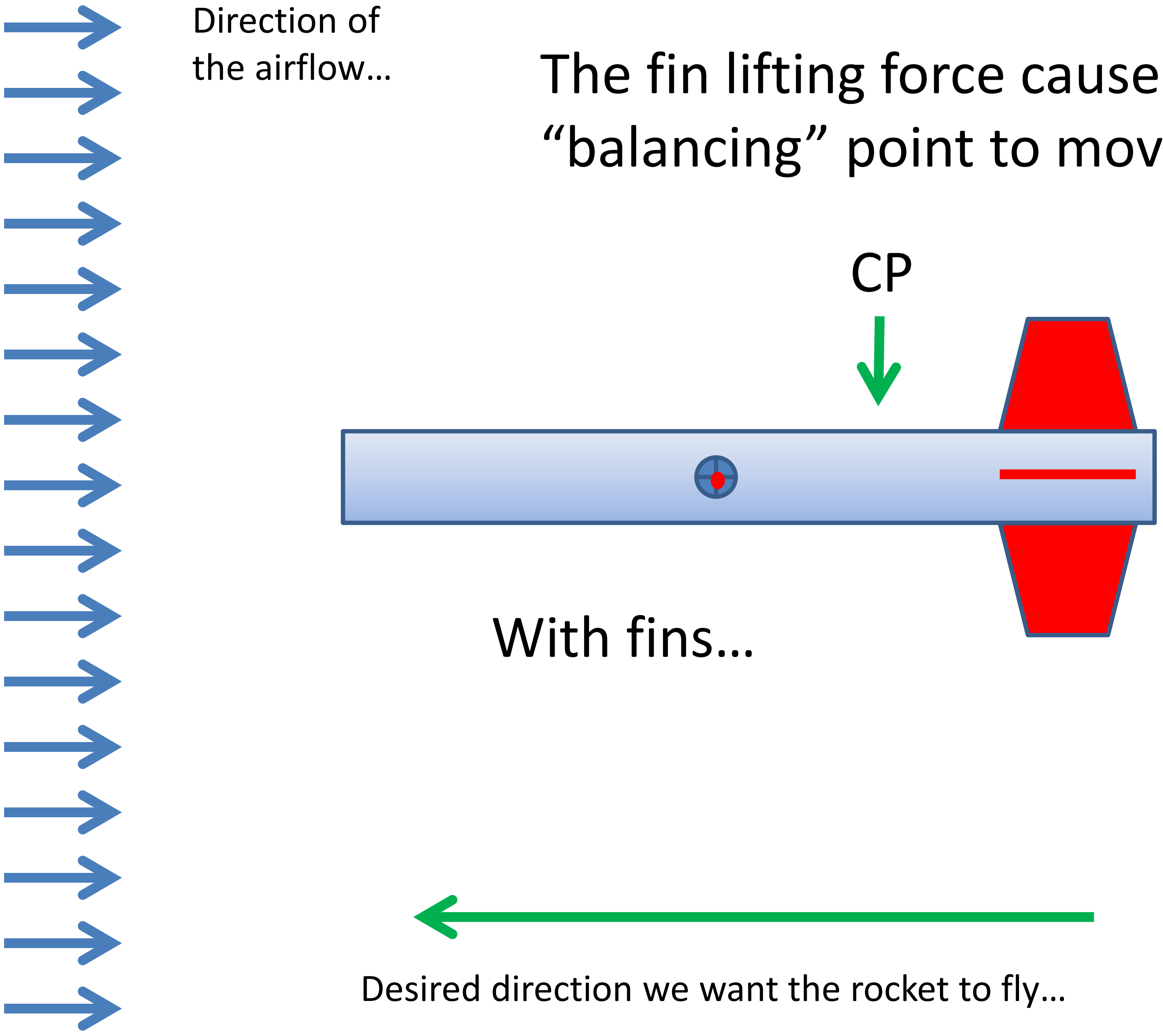
Direction of
the airflow…

CP

Without fins…

Desired direction we want the rocket to fly…

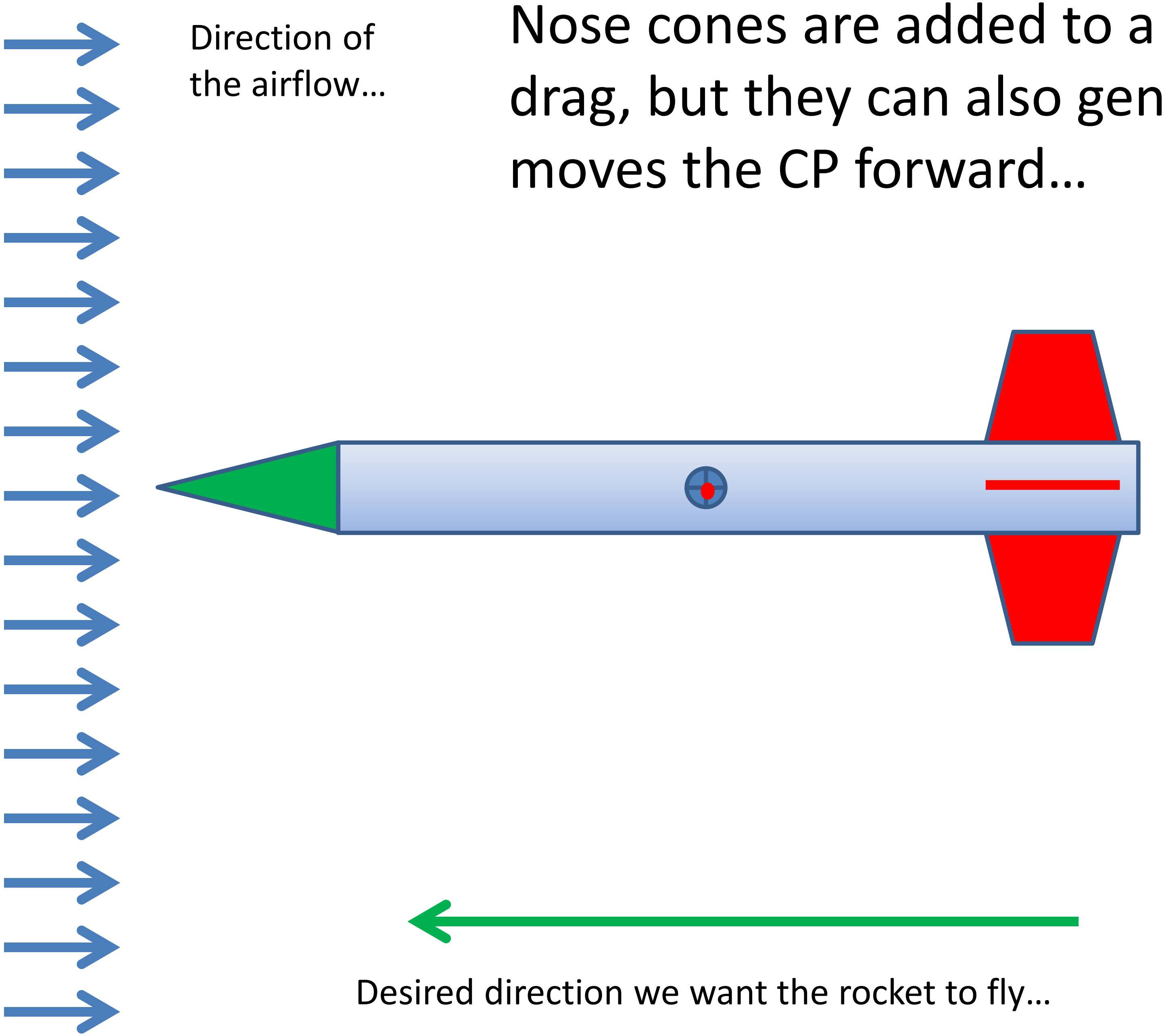
The fin lifting force causes the aerodynamic "balancing" point to move back



With fins…

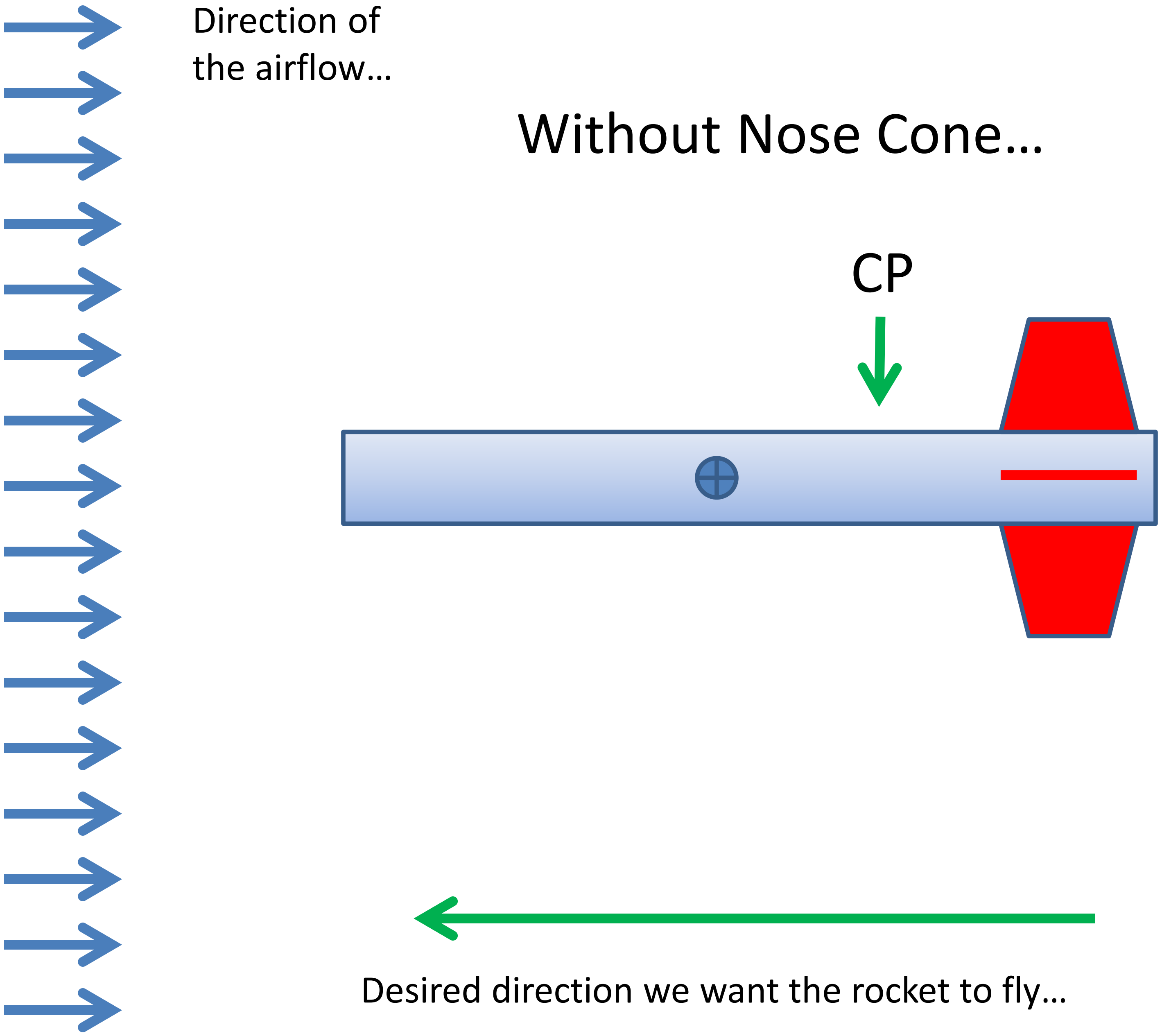
Desired direction we want the rocket to fly…

Nose cones are added to a rocket to reduce drag, but they can also generate lift which moves the CP forward…


Direction of the airflow…

Desired direction we want the rocket to fly…

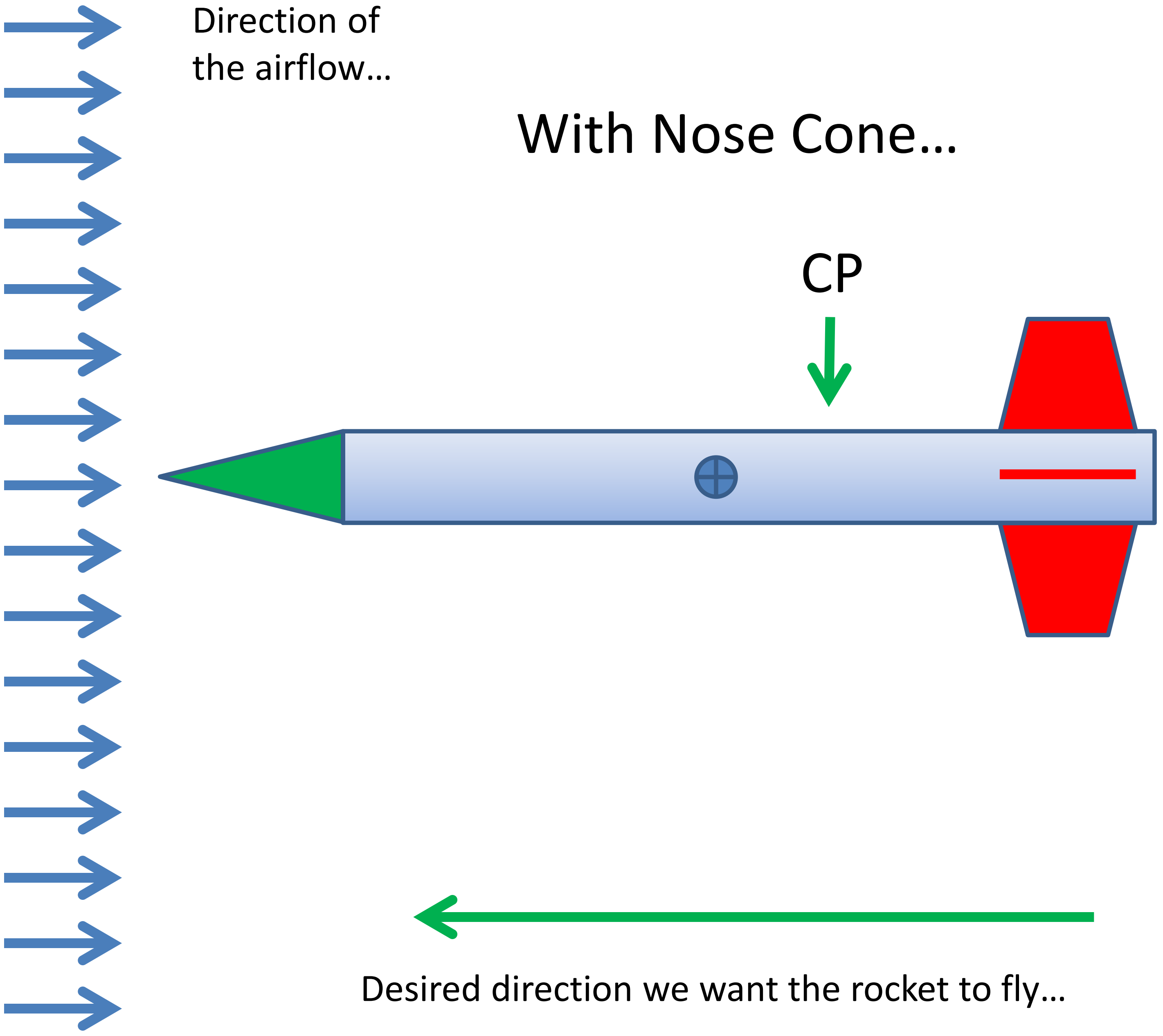
Without Nose Cone…



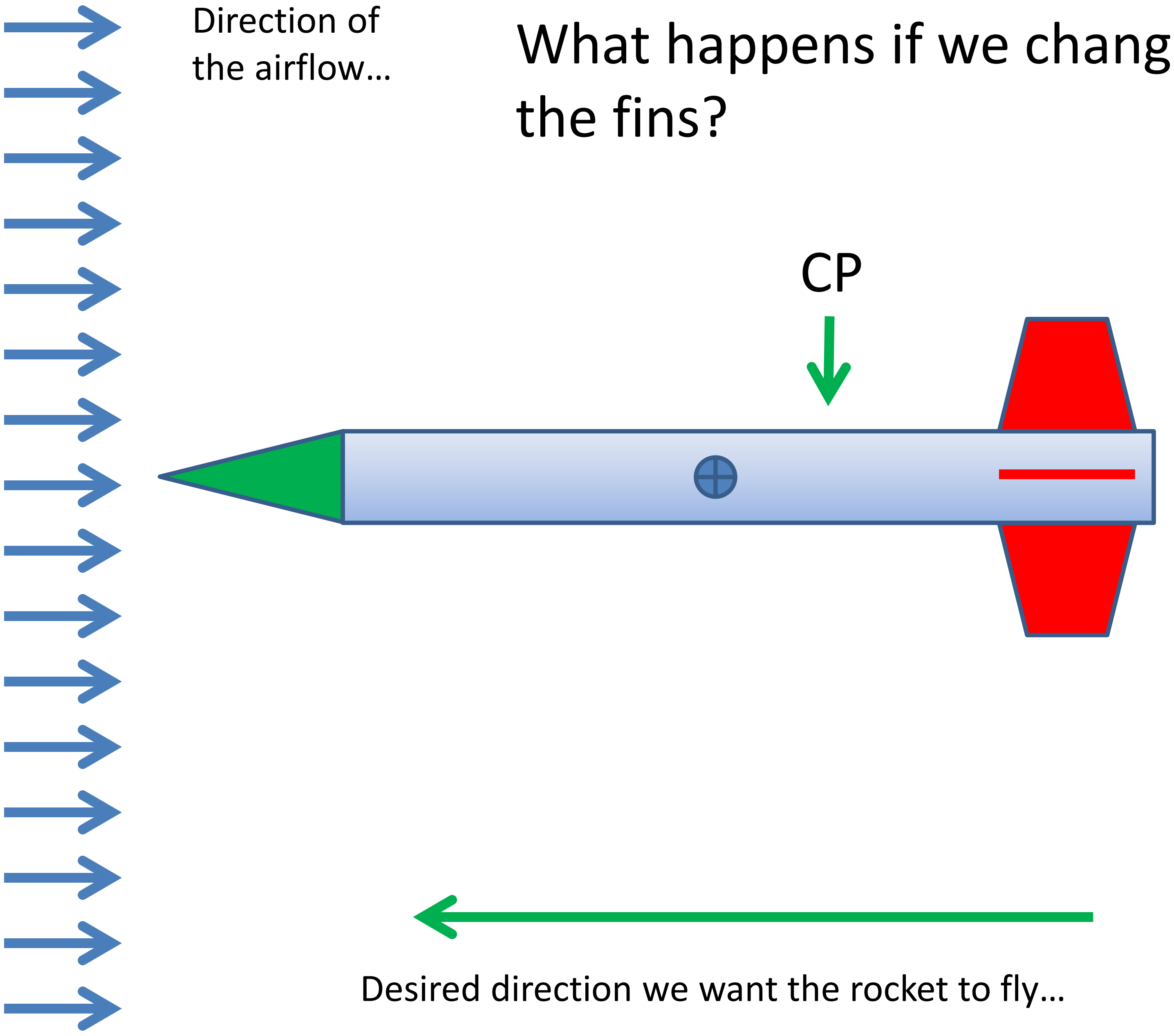
Direction of
the airflow…

Desired direction we want the rocket to fly…

With Nose Cone…



What happens if we change the size of the fins?

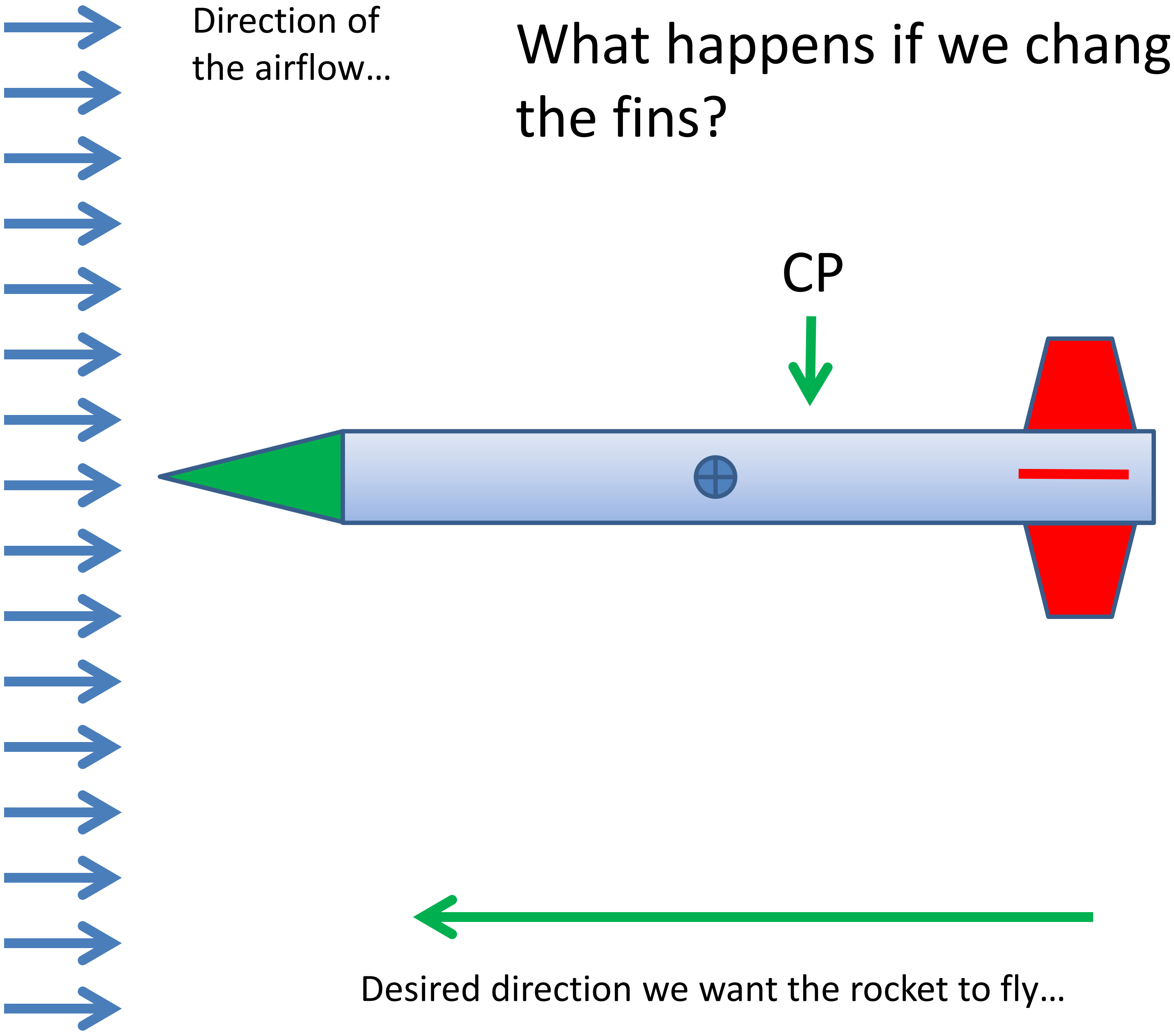


Direction of the airflow…

CP

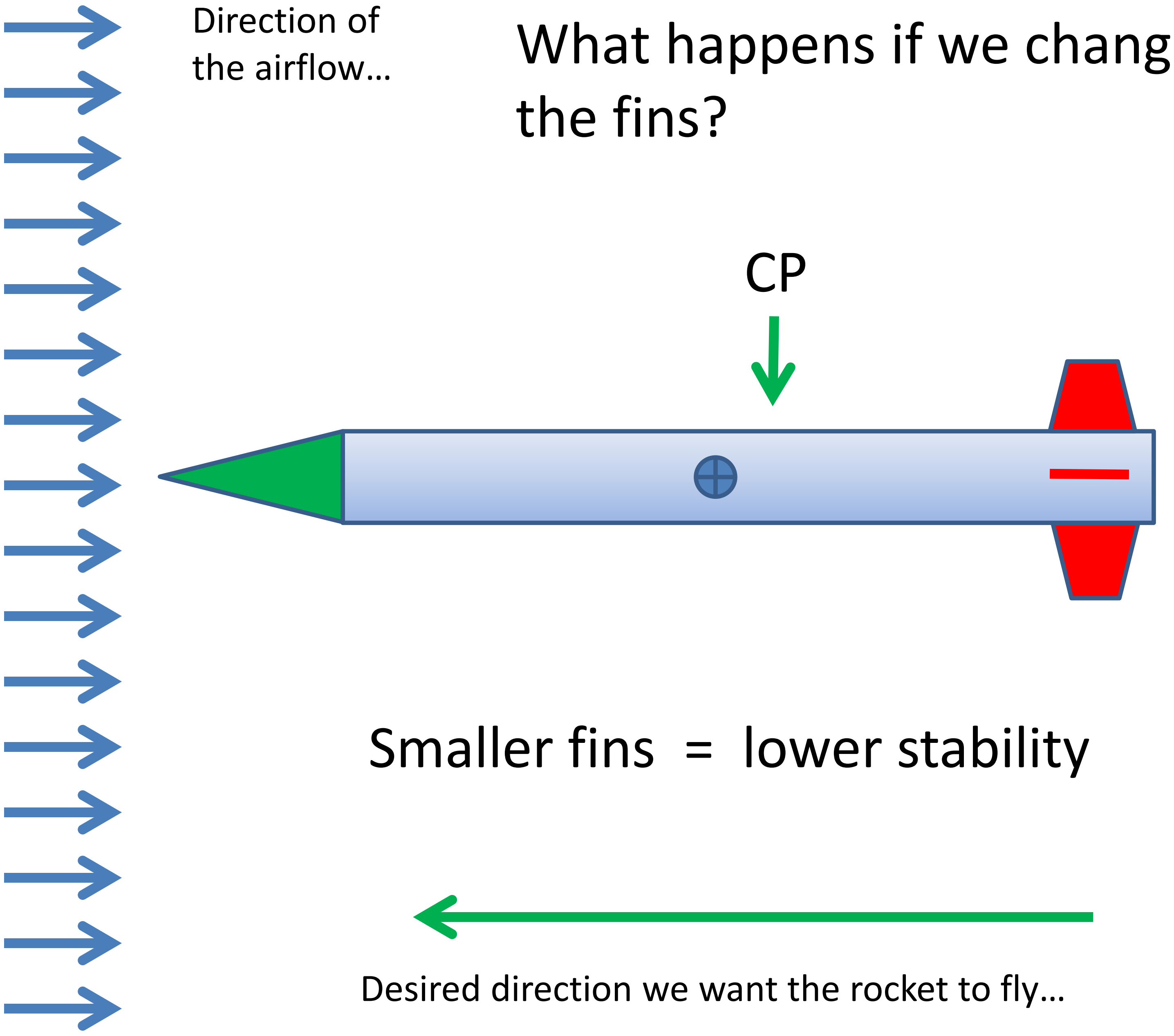
Desired direction we want the rocket to fly…

What happens if we change the size of the fins?



Desired direction we want the rocket to fly…

What happens if we change the size of the fins?

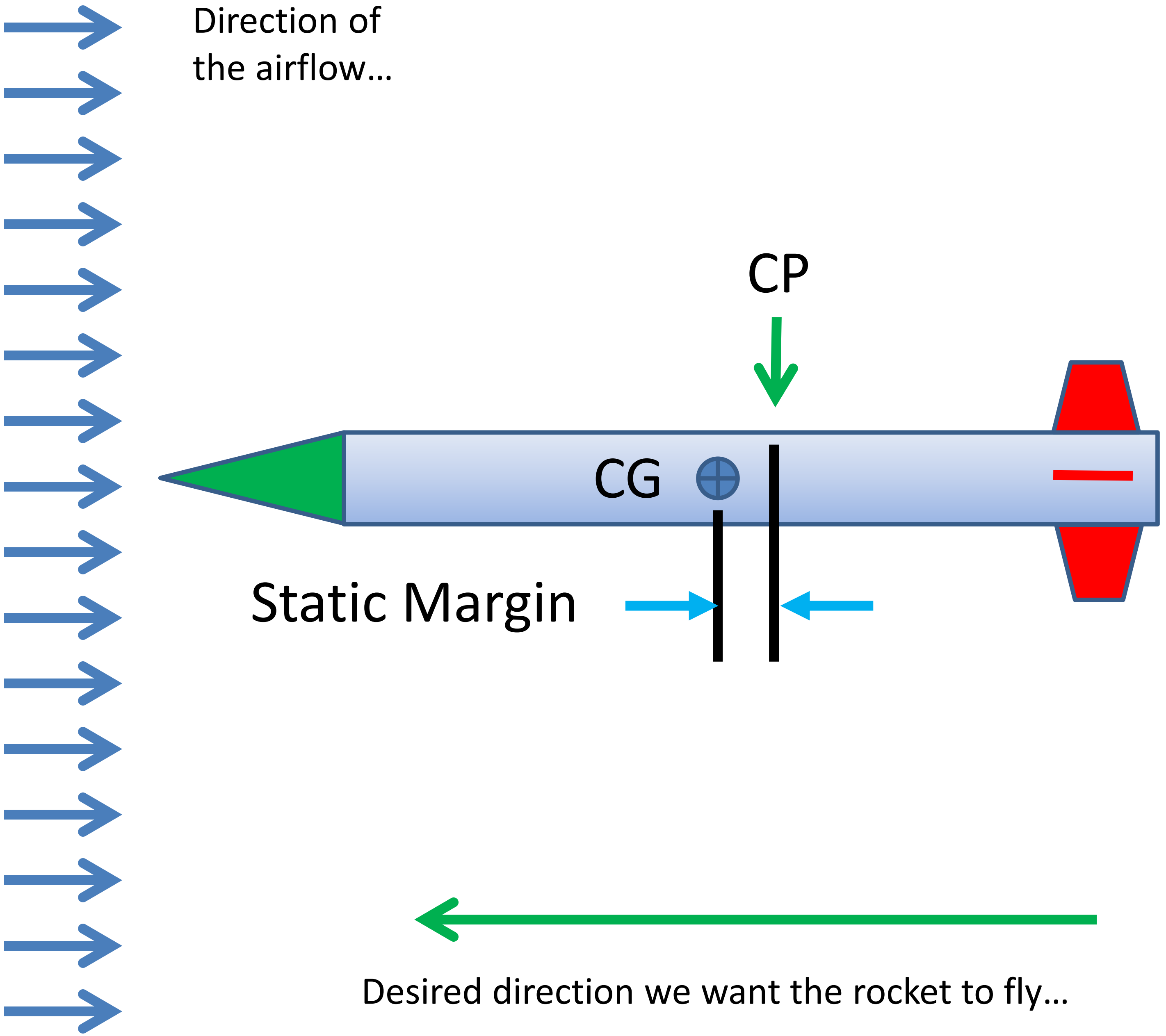


Direction of the airflow…

CP

Smaller fins = lower stability

Desired direction we want the rocket to fly…



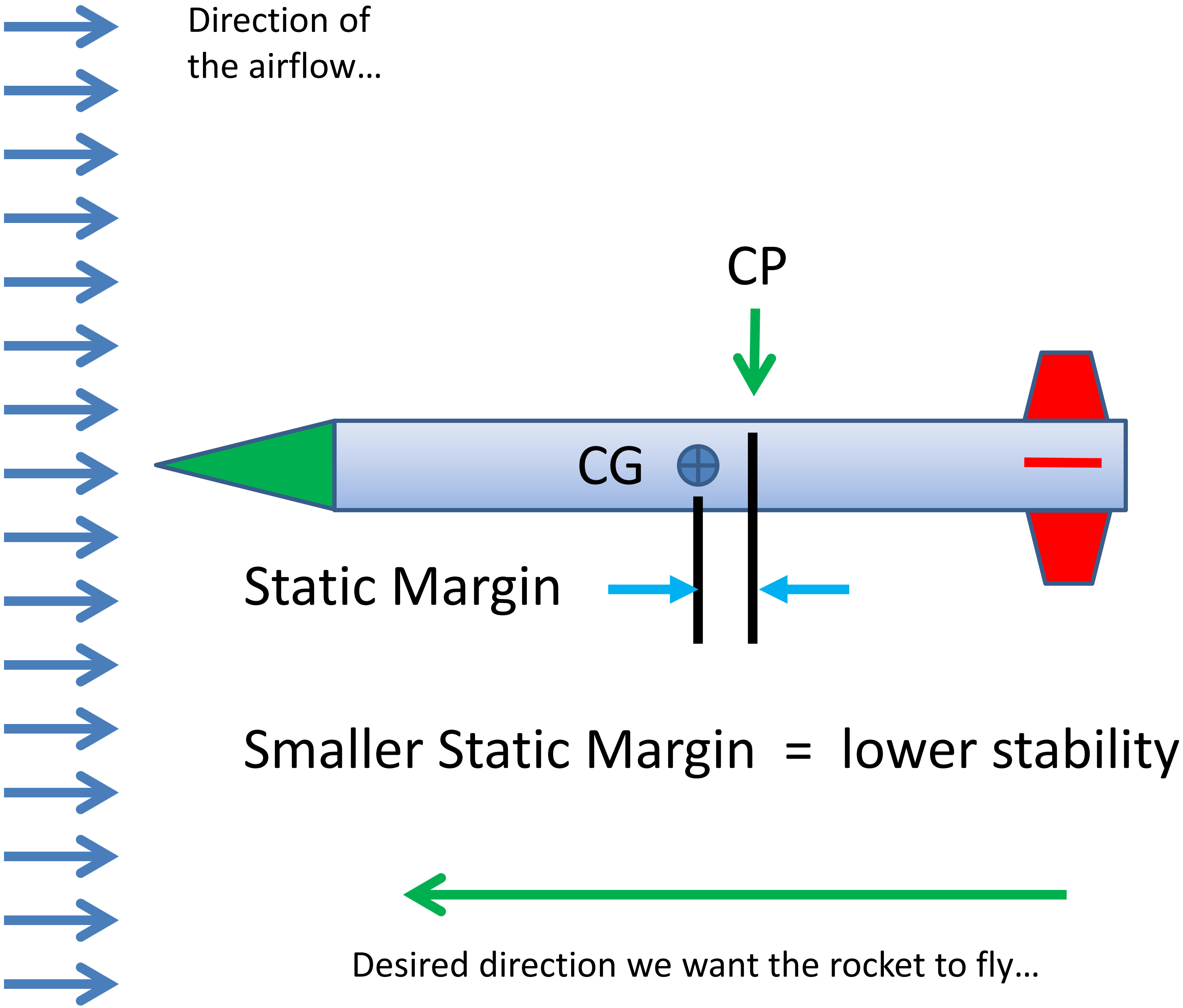
Direction of
the airflow…

CP

CG

Static Margin

Desired direction we want the rocket to fly…



Direction of
the airflow…

CP

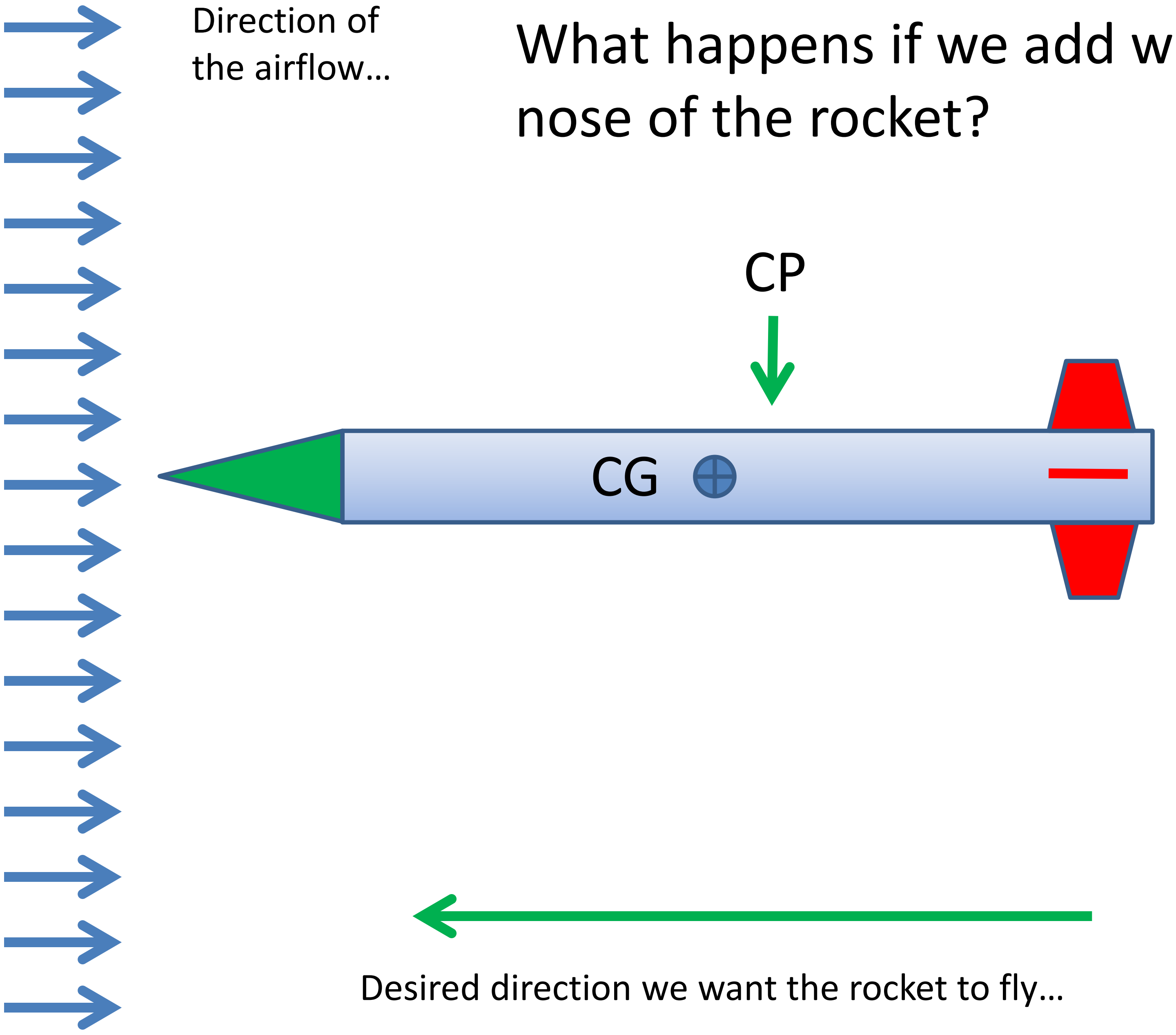
CG

Static Margin

Smaller Static Margin = lower stability

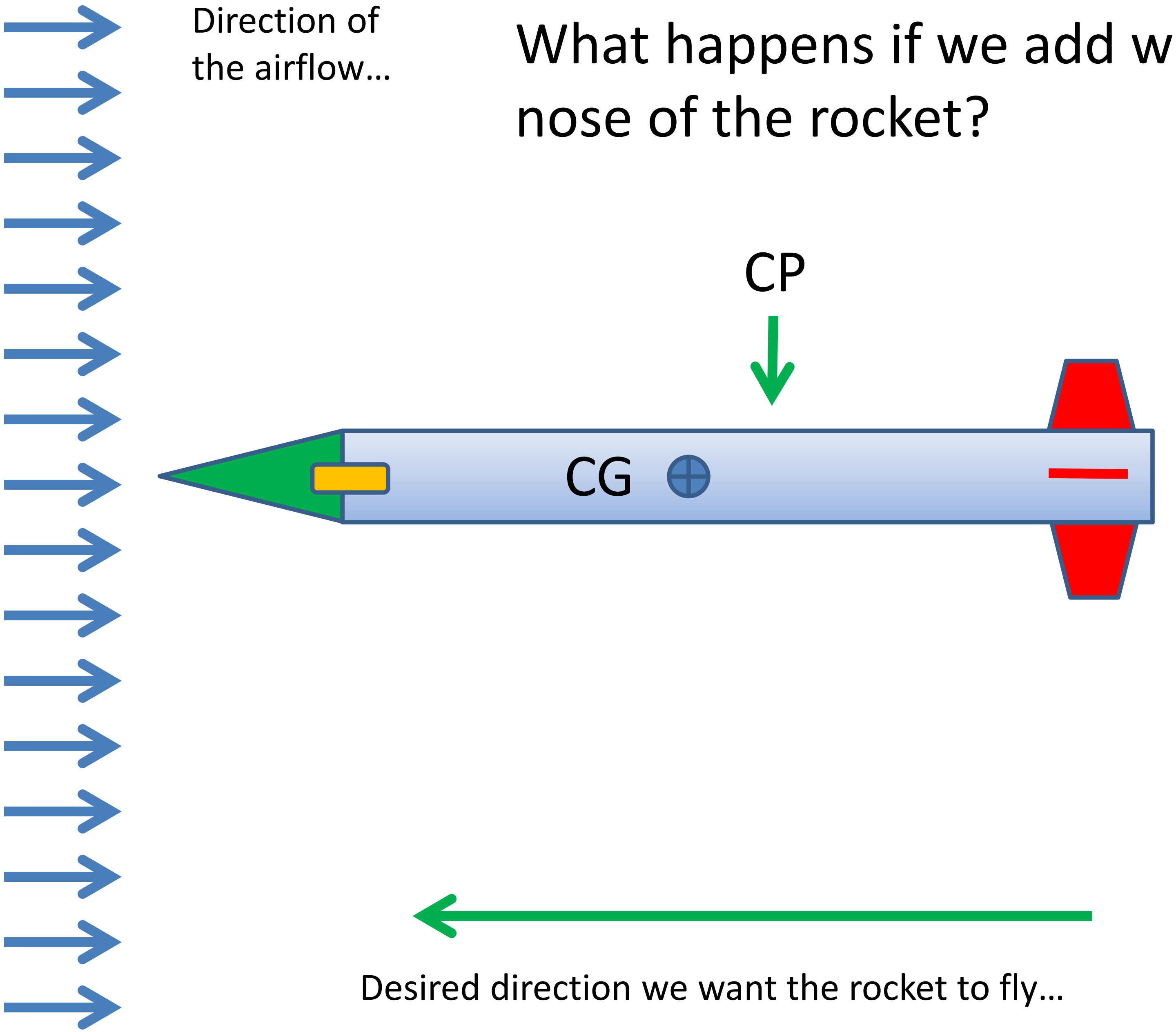
Desired direction we want the rocket to fly…

What happens if we add weight to the nose of the rocket?



Desired direction we want the rocket to fly…

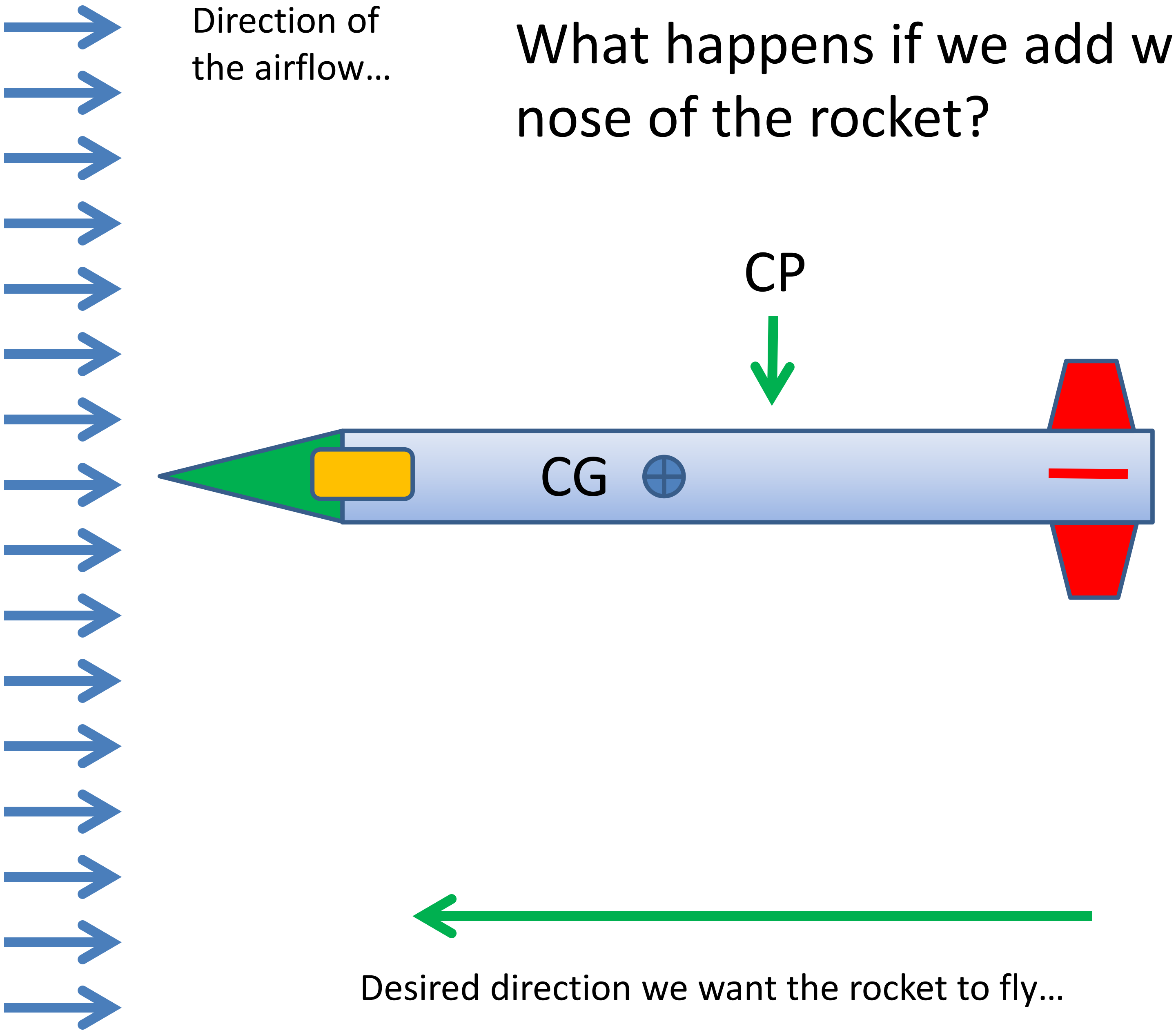
What happens if we add weight to the nose of the rocket?



Direction of
the airflow…

Desired direction we want the rocket to fly…

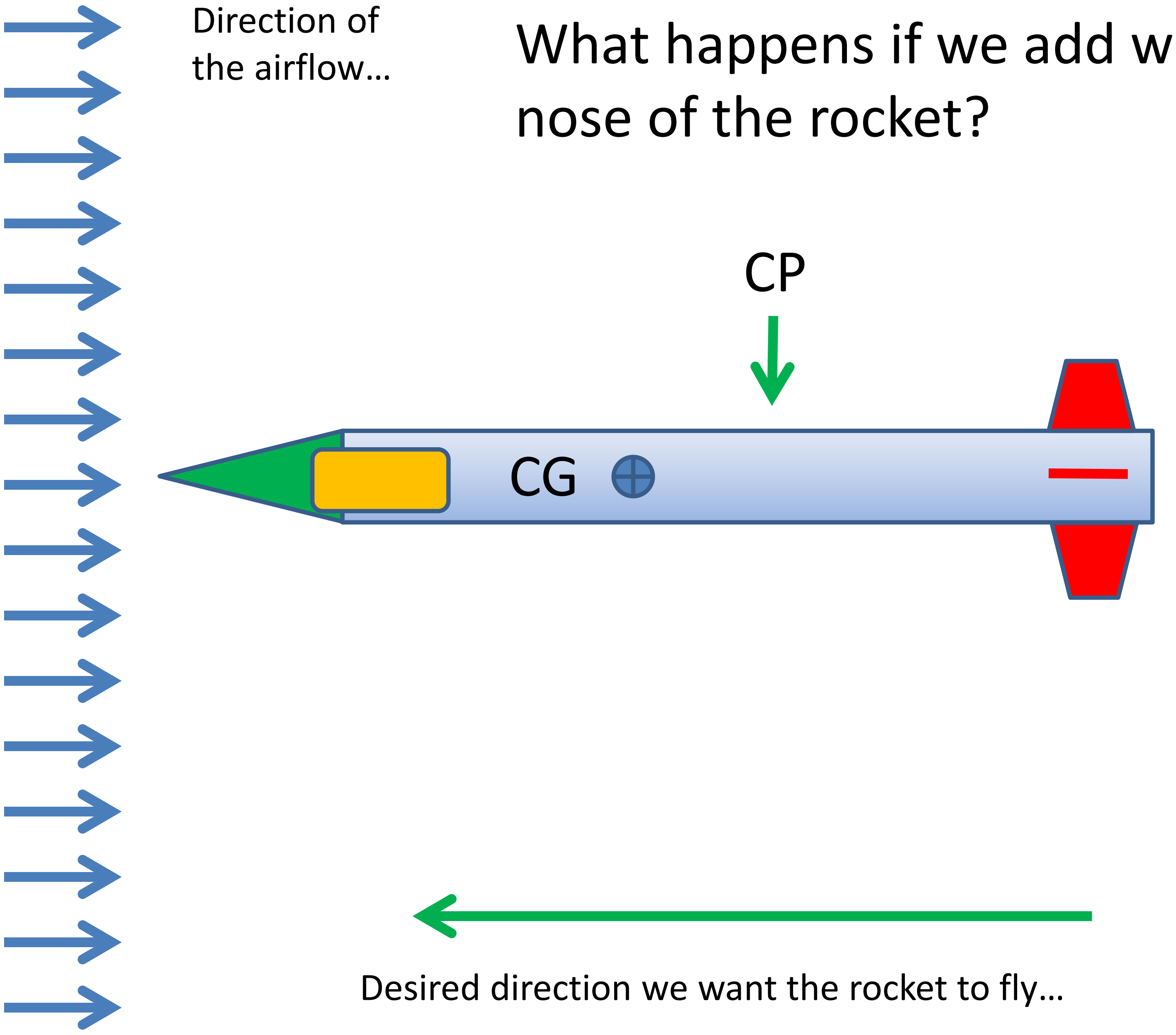
What happens if we add weight to the nose of the rocket?



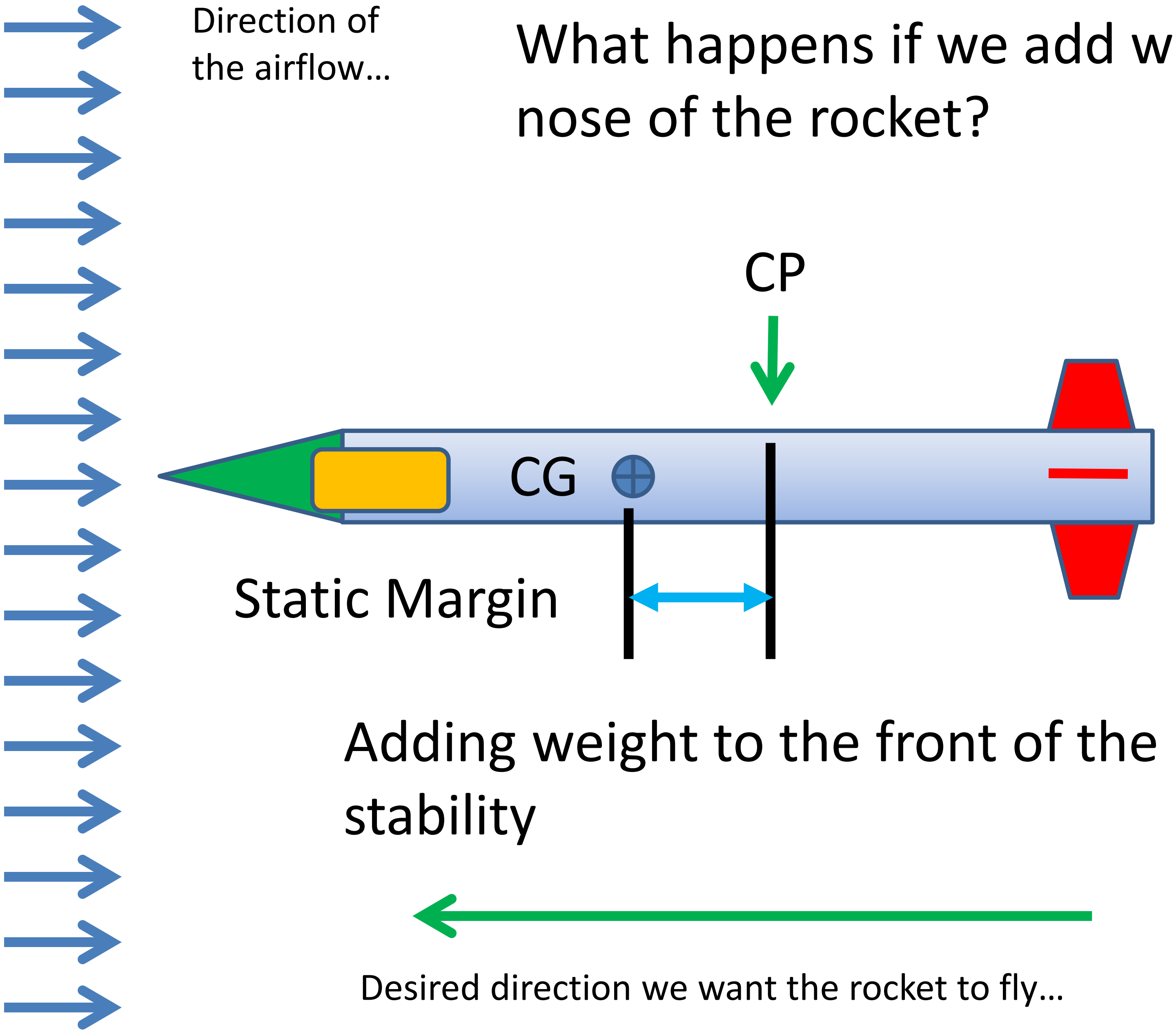
Direction of
the airflow…

Desired direction we want the rocket to fly…

What happens if we add weight to the nose of the rocket?



What happens if we add weight to the nose of the rocket?



Direction of the airflow…

CP

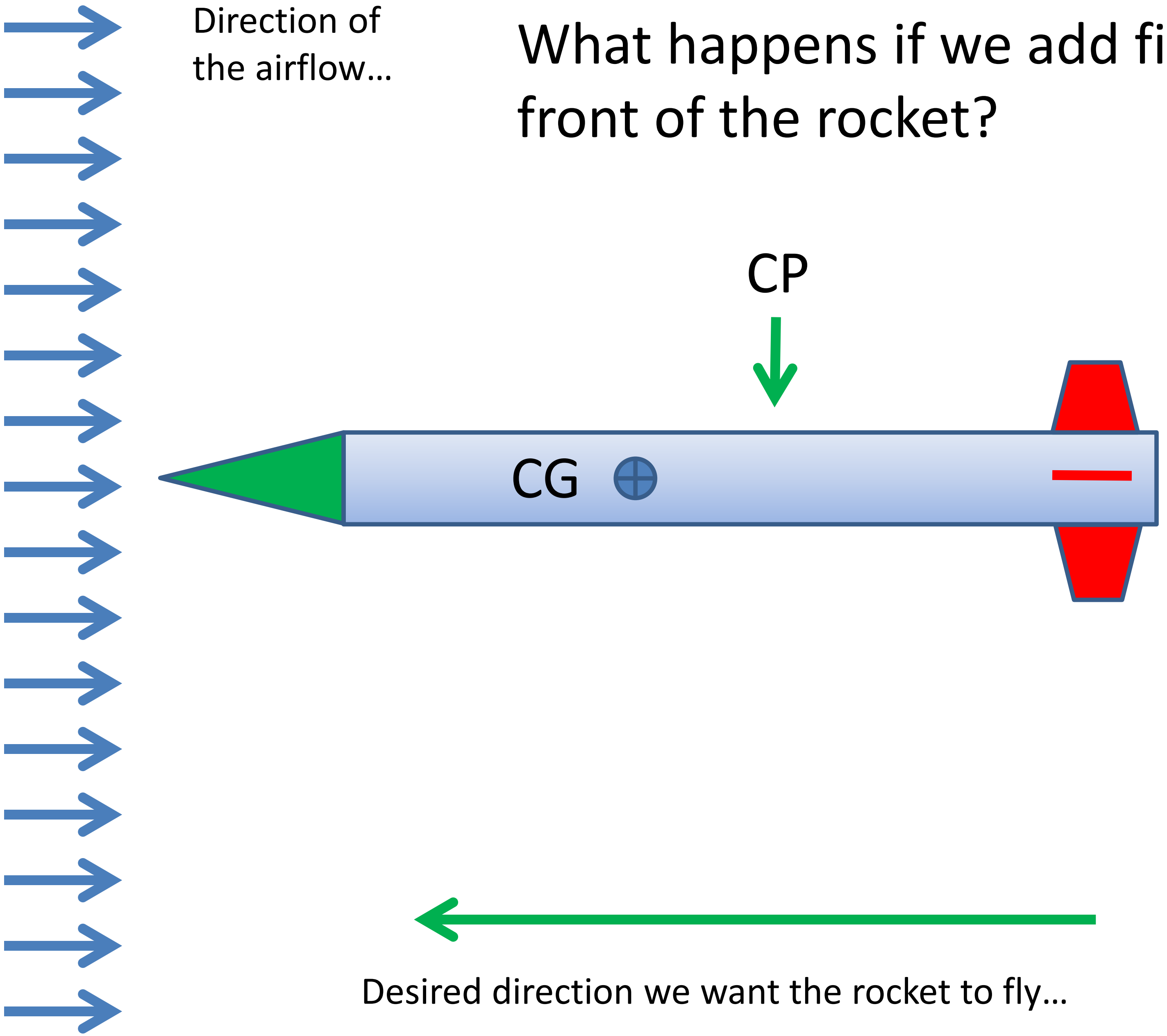
CG

Static Margin

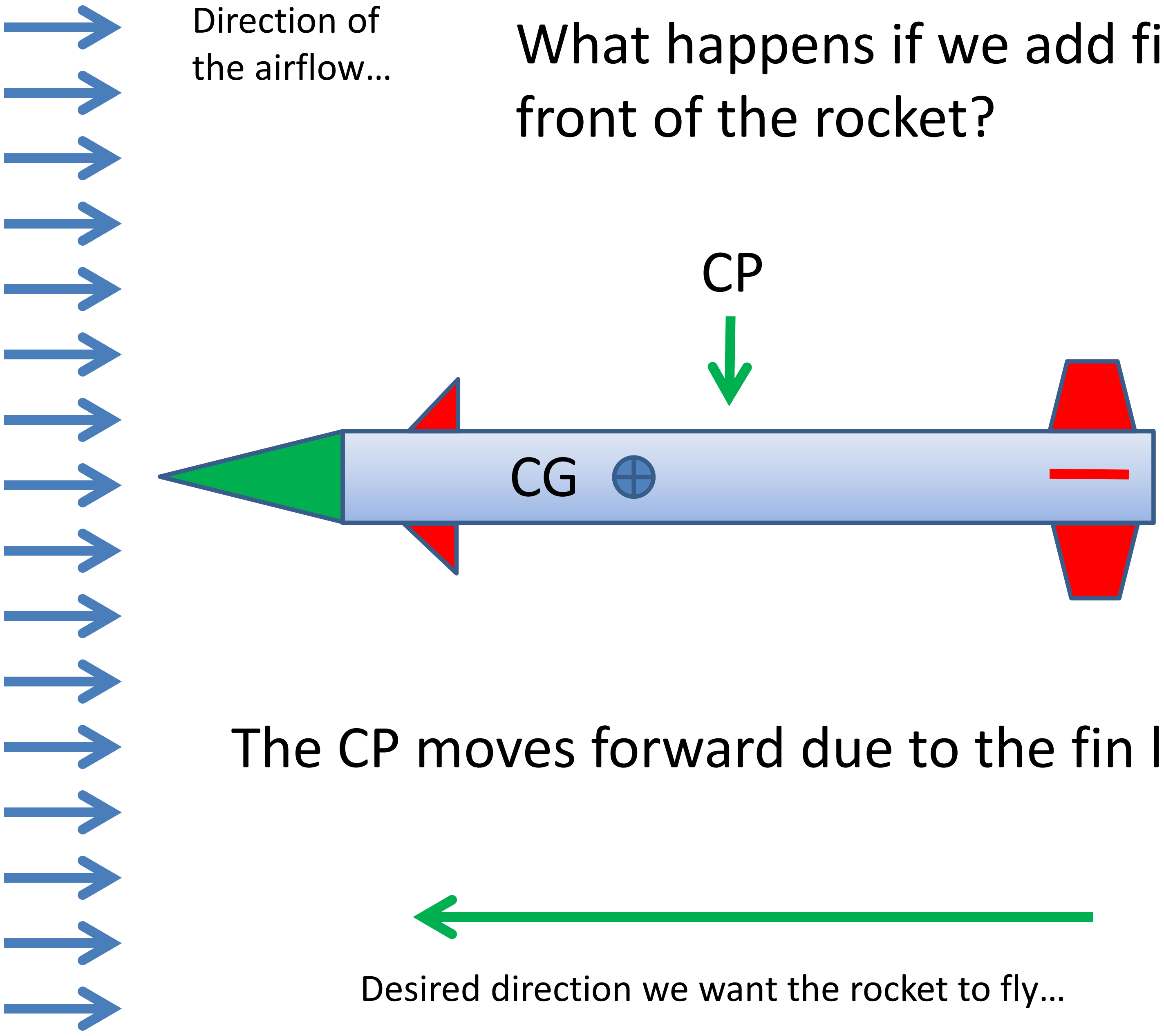
Adding weight to the front of the rocket increases stability

Desired direction we want the rocket to fly…

What happens if we add fins towards the front of the rocket?



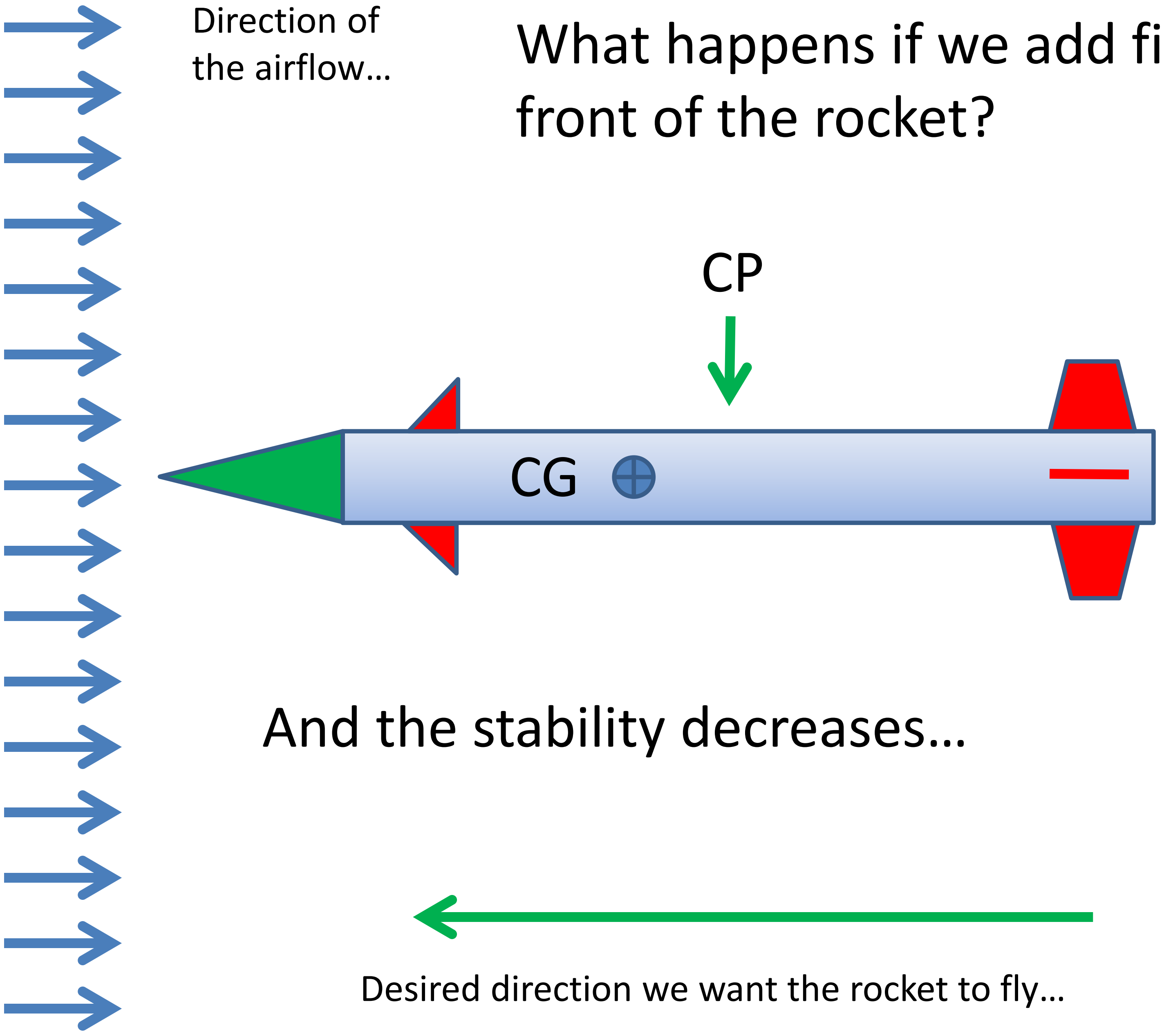
What happens if we add fins towards the front of the rocket?



The CP moves forward due to the fin lift…

Desired direction we want the rocket to fly…

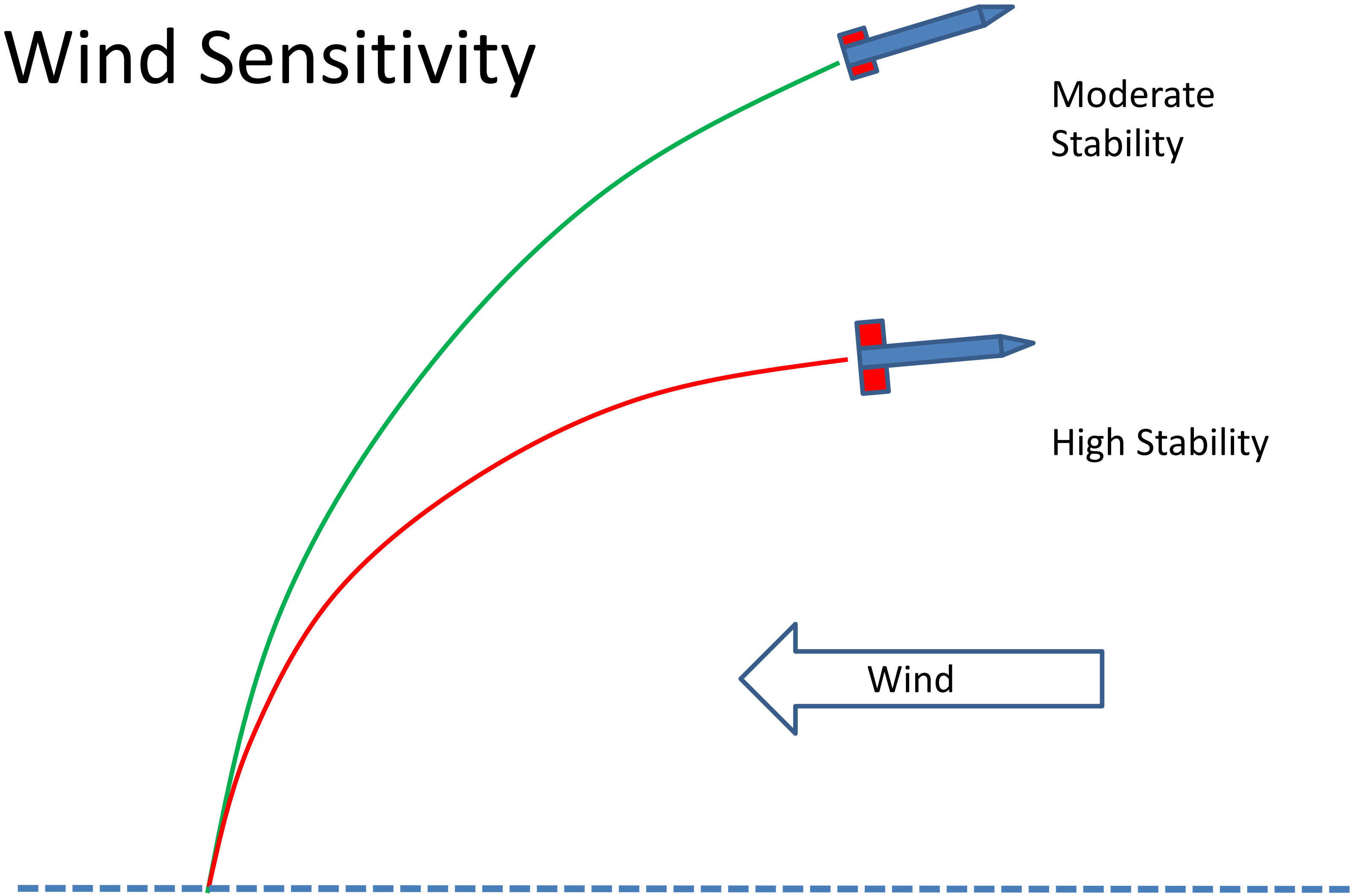
What happens if we add fins towards the front of the rocket?



And the stability decreases…

Desired direction we want the rocket to fly…

Wind Sensitivity



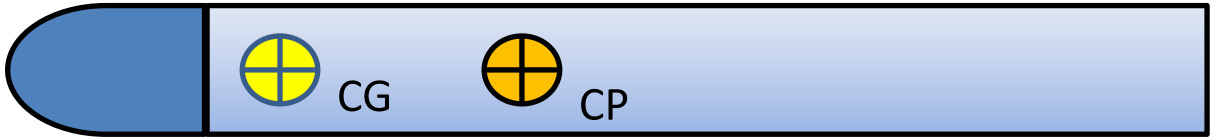
A rocket with higher stability will tend to "weather-cock" more than a rocket with less stability.

Is it possible for a finless rocket to be stable?

What do we need to think about to answer this question?

1. Center of Pressure (CP)
2. Center of Gravity (CG)

What is the key relationship between CP and CG?



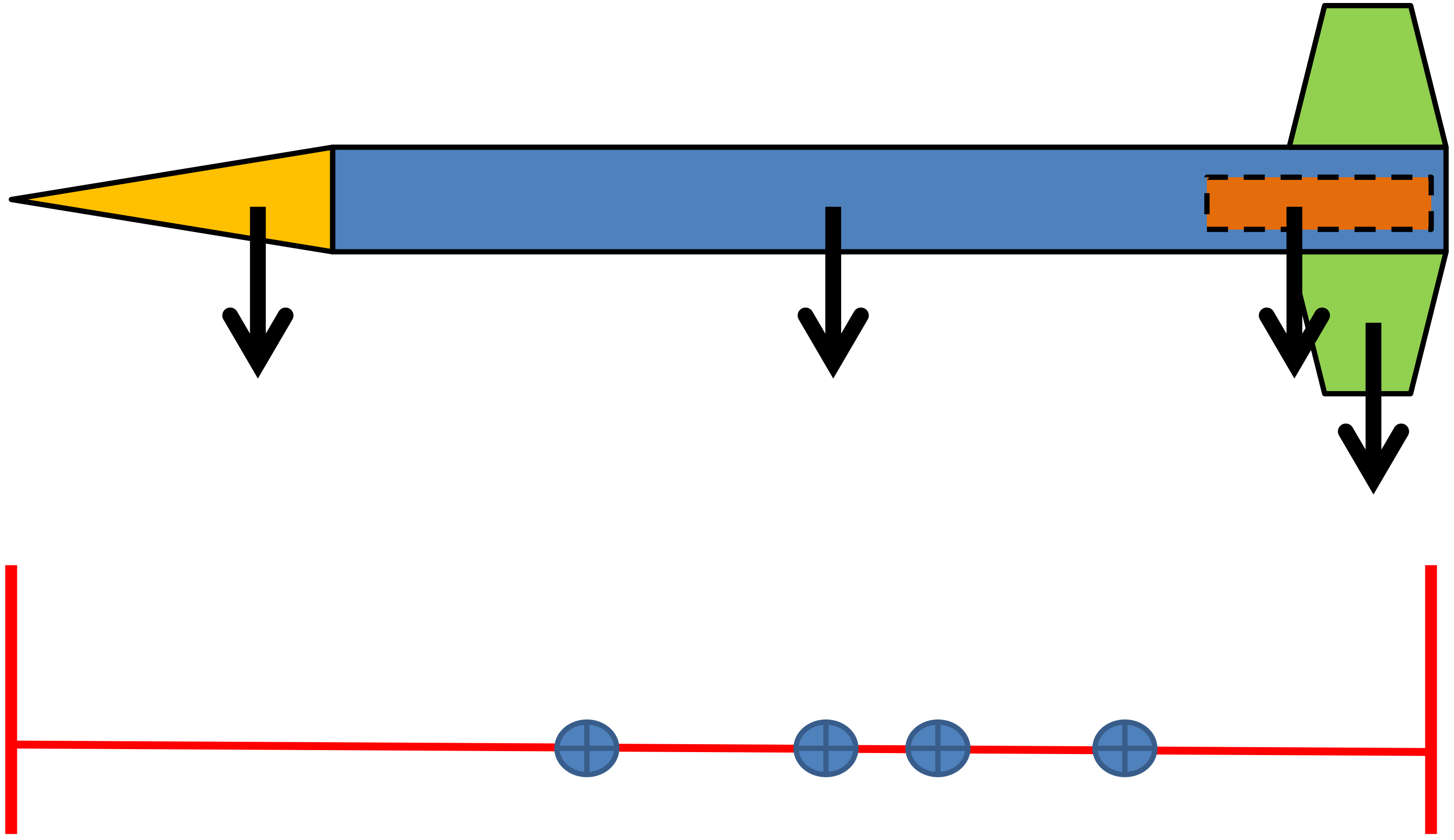
Finless Rocket Experiment

- The rocket flew “stable” for a couple of seconds
  - It takes time of overcome the inertia
  - The rocket must gain speed to build up adverse nose lift
- Once it started to diverge, it deviated from stable flight very quickly
- Unstable flight during coasting isn’t quite so drastic because the rocket isn’t being pushed
  - The rocket tends to tumble rather than fly here and there…
  - Drag is high due to high angle of attack
  - Trajectory tends to “smooth out“ due to the lack of a thrusting force

Calculating the Theoretical Location of the Center of Gravity
and
Center of Pressure

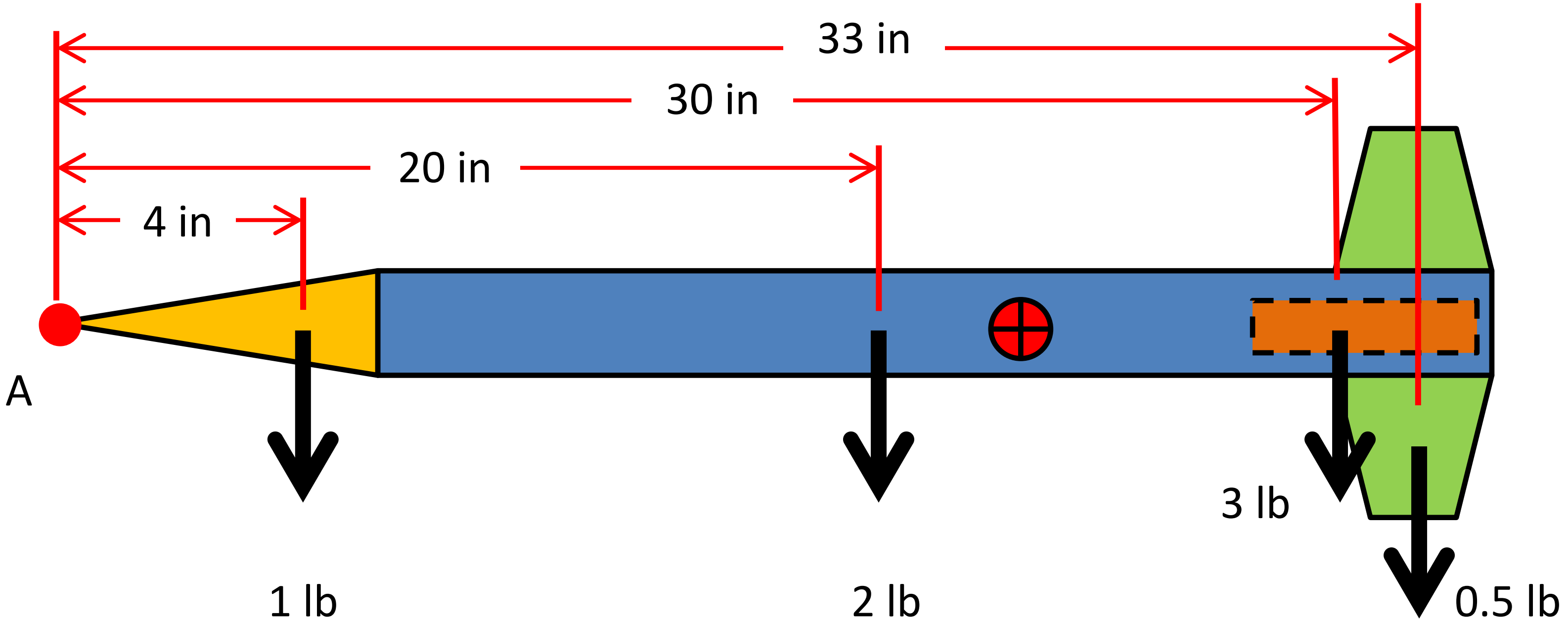
Center of Gravity

- The physical balancing point of the gravity forces



Body Position

Calculating Center of Gravity



Sum the moments of the components about Point A, and set equal to the total moment:

CG (in) x total weight (lb) = (4 in)x(1 lb) + (20 in)x(2 lb) + (30 in)x(3 lb) + (33 in)x(0.5 lb)

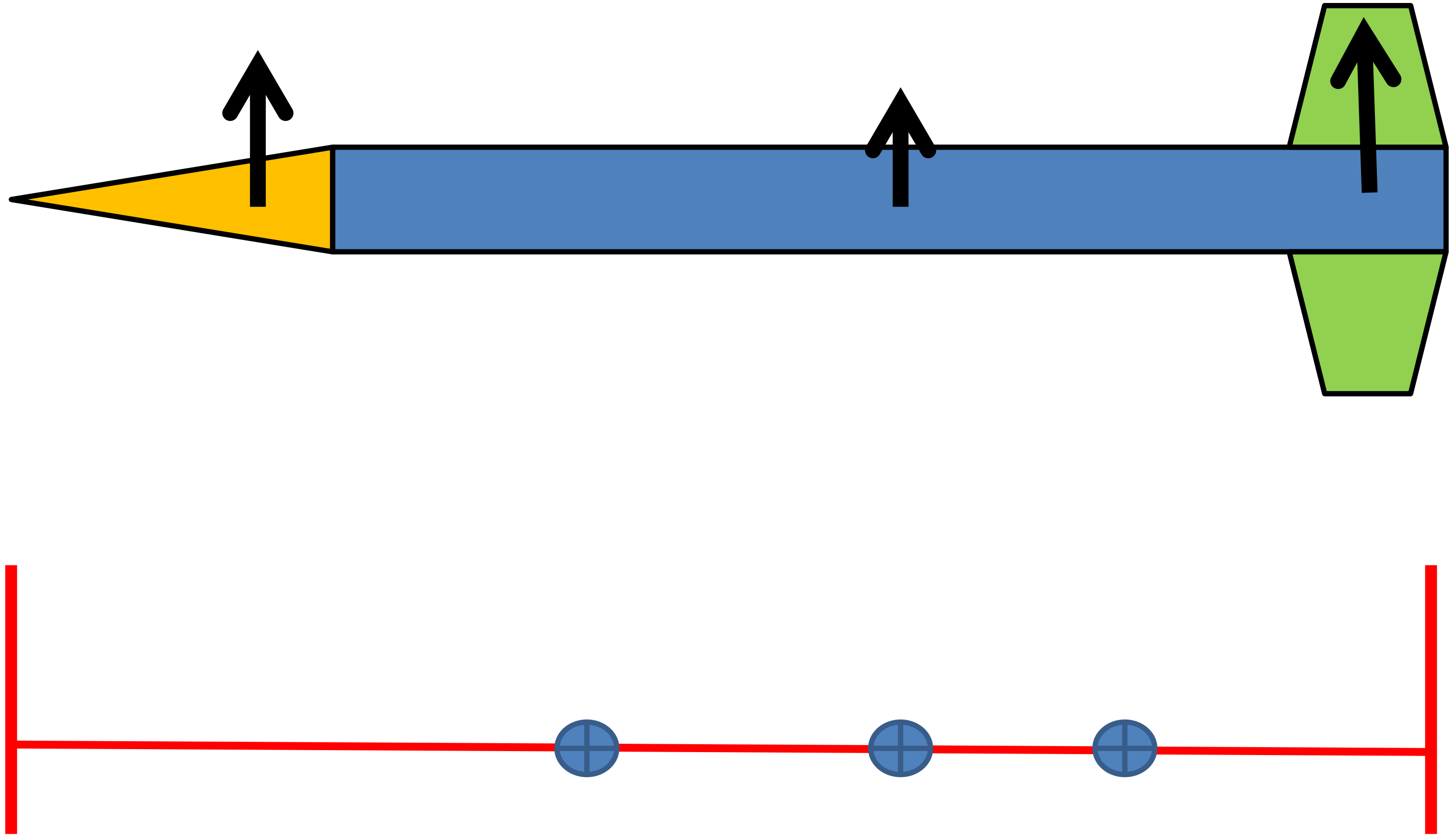
CG (in) x 6.5 lb = 4 in*lb + 40 in*lb + 90 in*lb + 16.5 in*lb

CG (in) x 6.5 lb = 150.5 in*lb

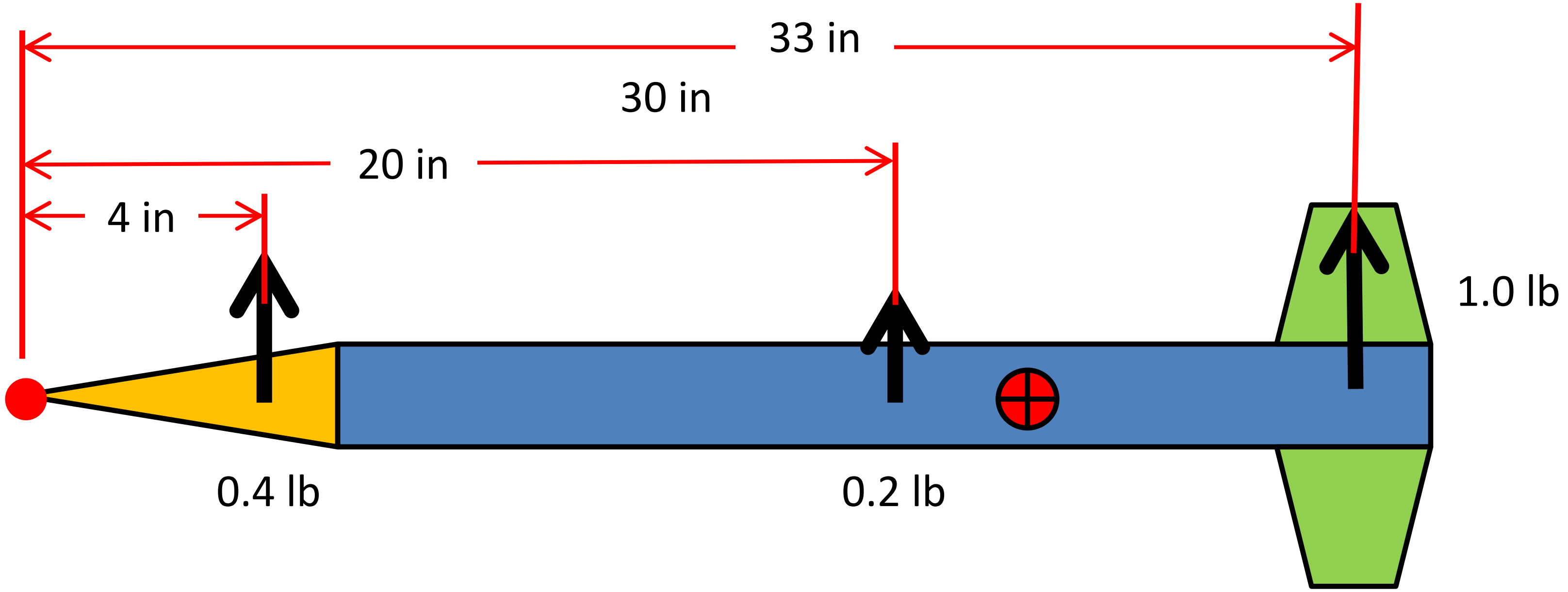
CG (in) = 150.5 in*lb / 6.5 lb = 23.2 in (measured from Point A)

Center of Lift (a.k.a Center of Pressure)

- The “balancing” point of the lifting forces



Center of Lift (a.k.a Center of Pressure)



Sum the lift moments of the components about Point A, and set equal to the total moment:

CP (in) x total weight (lb) = (4 in)x(.4 lb) + (20 in)x(.2 lb) + (30 in)x(3 lb) + (33 in)x(1.0 lb)

CP (in) x 1.6 lb = 1.6 in*lb + 4 in*lb + 33 in*lb

CP (in) x 1.6 lb = 38.6 in*lb

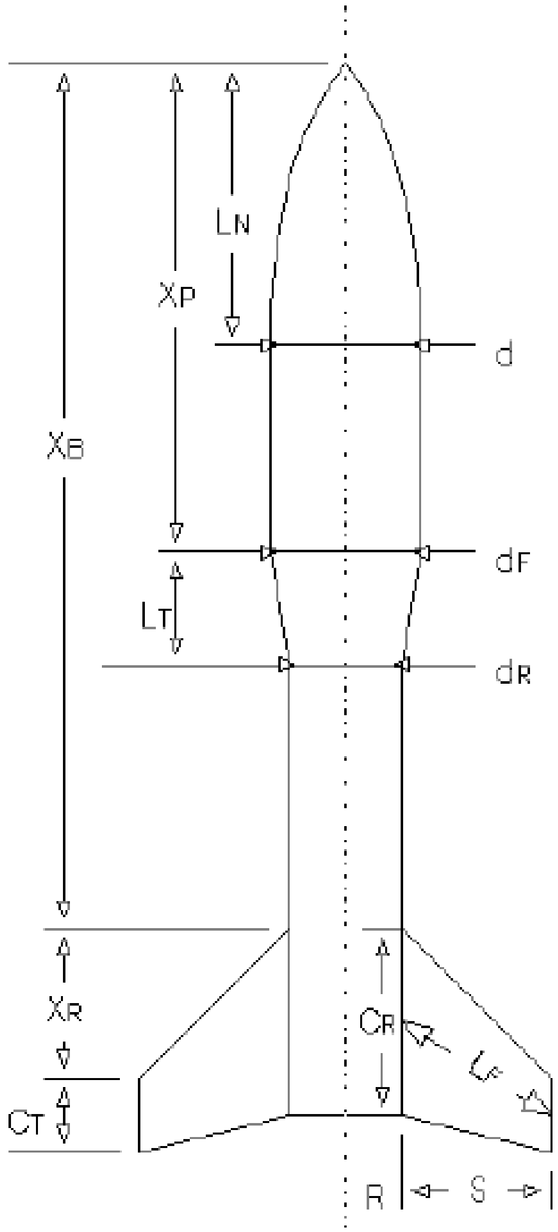
CP (in) = 38.6 in*lb / 1.6 lb = 24.1 in (measured from Point A)

Calculating the lift for the various rocket sections can be a little bit laborious.

In reality, we simplify things by just calculating the Normal Force Coefficients (CN) and skip actually calculating the lift.

Note that the Normal Force Coefficient is the same thing as the Lift Coefficient.

Parameters used in Calculating Model Rocket Aerodynamics



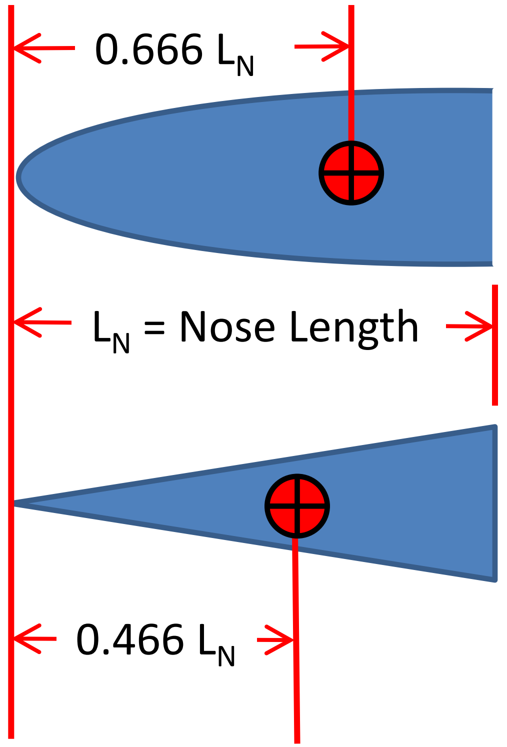
L_N = length of nose
d = diameter at base of nose
d_F = diameter at front of transition
d_R = diameter at rear of transition
L_T = length of transition
X_P = distance from tip of nose to front of transition
C_R = fin root chord
C_T = fin tip chord
S = fin semispan
L_F = length of fin mid-chord line
R = radius of body at aft end
X_R = distance between fin root leading edge and fin tip leading edge parallel to body
X_B = distance from nose tip to fin root chord leading edge
N = number of fins

This approach assumes the straight body tube segmentss do not generate lift. This is a reasonable assumption since the angle of attack of a rocket is generally small.

Nose Normal Force Coefficient and CP Position

Normal Force Coefficient of Nose = $(C_N)_N$

$(C_N)_N = 2.0$ *this value is a constant regardless of shape and size*



$X_N = 0.666\ L_N$ Ogive

$X_N = 0.466\ L_N$ Conical

Transition Section Normal Force Coefficient and CP Position

Normal Force Coefficient of the Body Tube = $(C_N)_T$

$$(C_N)_T = 2 \times \left[\left[\frac{d_R}{d} \right]^2 - \left[\frac{d_F}{d} \right]^2 \right]$$

Note: If your rocket does not have a transition section, then this parameter is not calculated…

Transition Section Normal Force Coefficient and CP Position

Location of Normal Force Coefficient of the Transition = X_T

$$X_T = X_P + \frac{LT}{3}\left[1 + \frac{1 - \left[\frac{d_F}{d_R}\right]}{1 - \left[\frac{d_F}{d_R}\right]^2}\right]$$

Note: If your rocket does not have a transition section, then this parameter is not calculated…

Fin Normal Force Coefficient and CP Position

Normal Force Coefficient of the Body Tube = $(C_N)_F$

$$(C_N)_F = \left[1 + \frac{R}{S + R}\right] \times \frac{4N\left[\frac{S}{d}\right]^2}{1 + \sqrt{1 + \left[\frac{2L_F}{C_R + C_T}\right]^2}}$$

Fin Normal Force Coefficient and CP Position

Location of Normal Force Coefficient of the Transition $= X_F$

$$X_F = X_B + \frac{X_R\,(C_R + 2C_T)}{3\,(C_R + C_T)} + \frac{1}{6}\left[(C_R + C_T) - \frac{(C_R \times C_T)}{(C_R + C_T)}\right]$$

Calculating the total CP Position

Sum the Normal Force Coefficients:

$$(C_N)_R = (C_N)_N + (C_N)_T + (C_N)_F$$

Find the CP distance from the nose tip:

$$X_{CP} = \frac{(C_N)_N X_N + (C_N)_T X_T + (C_N)_F X_F}{(C_N)_R}$$

These are essentially the "moments" resulting from the lift being generated by the three primary rocket components. We don't have to go as far as calculating the lift since the associated terms simply cancel out.

Summary

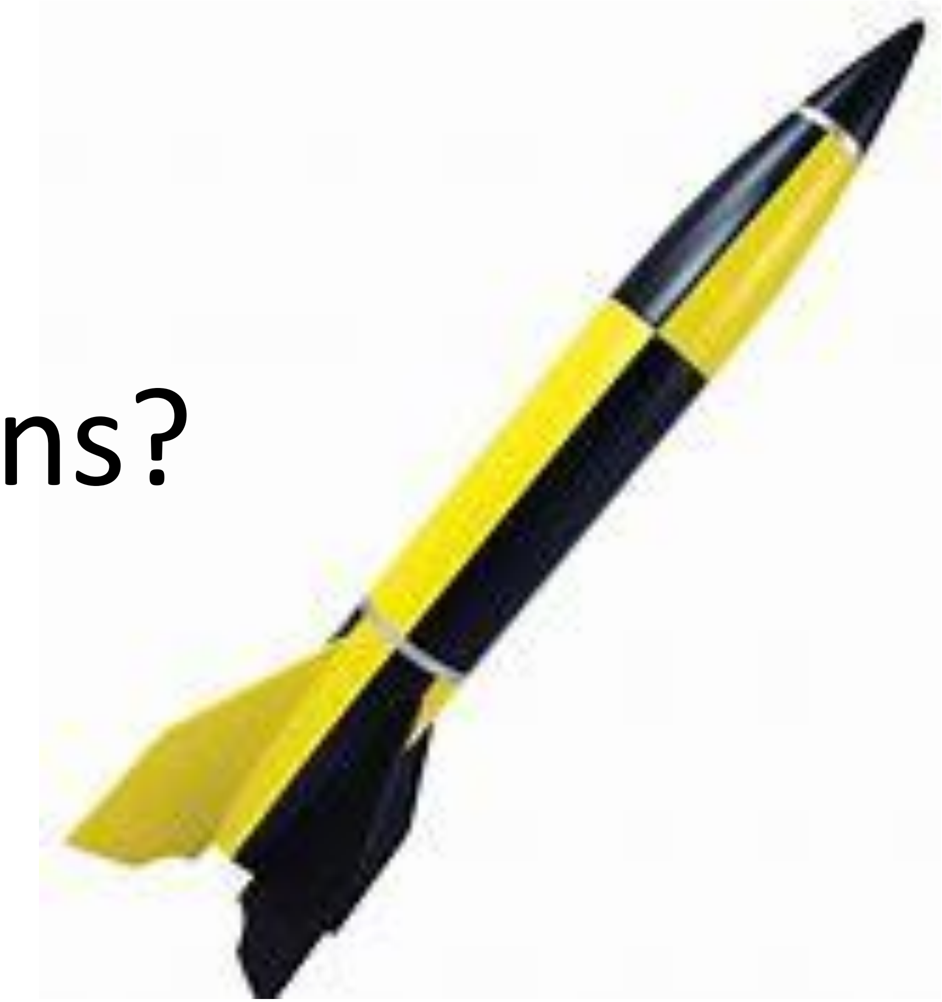
- The static margin is the distance between the CP and CG
- The CG needs to be forward of the CP in order to have a stable rocket
  - Ideally the CG should be 2 body diameters in front of the CP
- Fins at the back of the rocket will move the CP aft
  - Makes the rocket more stable
- Larger fins tend to make the rocket more stable
  - But, larger fins create more drag, which leads to lower acceleration, which leads to lower altitude

Summary

- Adding weight to the front of the rocket will move the CG forward
  - Makes the rocket more stable
  - But Newton says the rocket will not accelerate as well

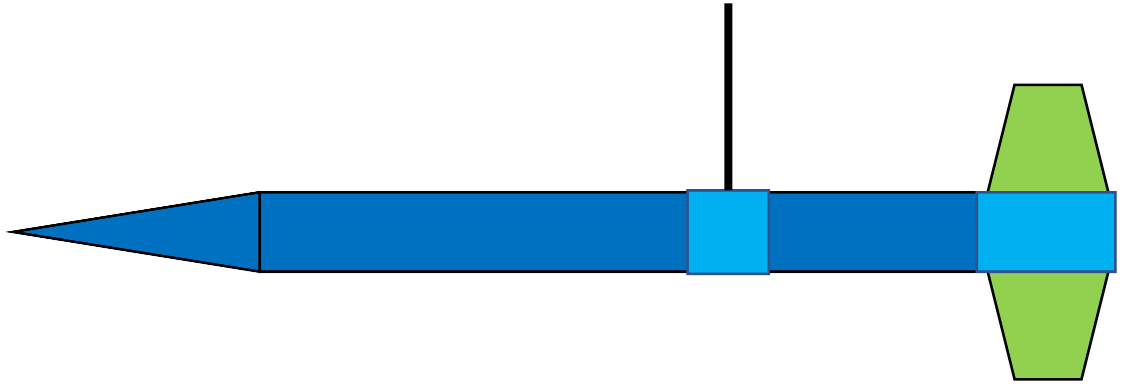
- Higher stability tends to make the rocket more wind sensitive

Questions?



Rocket Stability

Building a Test Rocket



LabRat Scientific

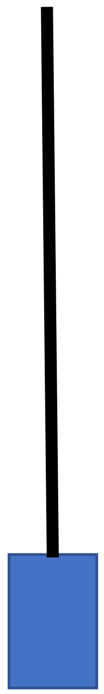
Materials and Supplies:

- Model Rocket Kit
- Balsawood sheet
- Paper Towel Tube
- Sheet of Paper
- Aluminum Foil
- Utility/Hobby Knife
- Masking Tape
- Pencil / Marker
- White Glue
- 5-minute Epoxy

It is suggested that a model rocket kit be purchased for this project. You should find a rocket that has a diameter on the order of 5 cm (~2 in.) and a length around 50 cm (~20 in.). The rocket kit will include tubes, nose cone, and balsawood for fins.

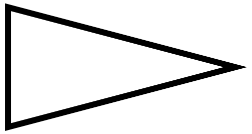
The dimensions of the tubes are not critical, its just easier to work with a larger model rocket.

Items to be Built

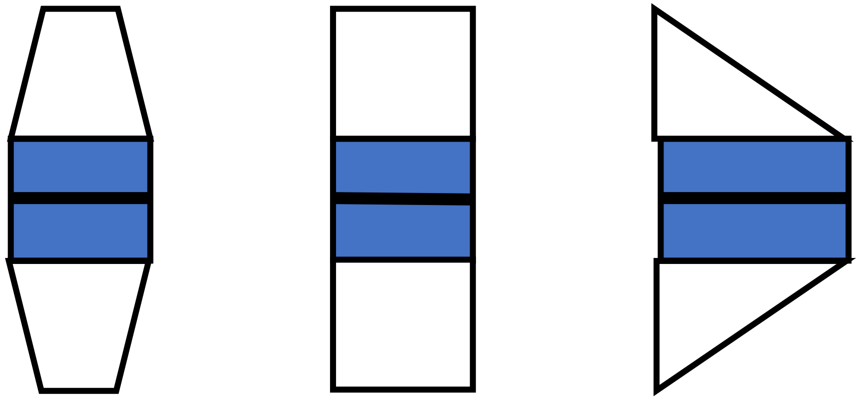


Movable String
Collar

Rocket Body



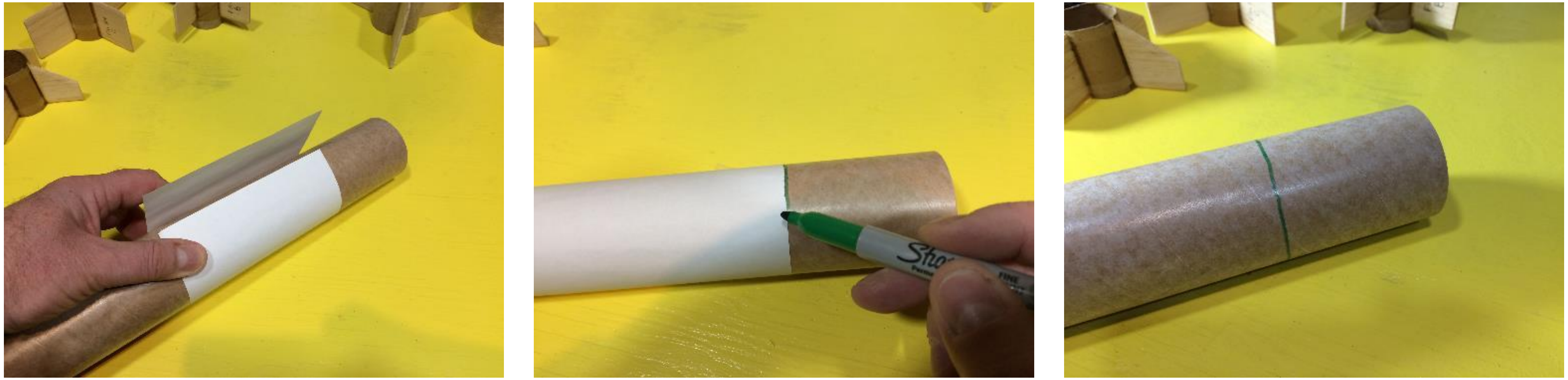
Nose Cone



Fin Sets Mounted to
Fin Collars

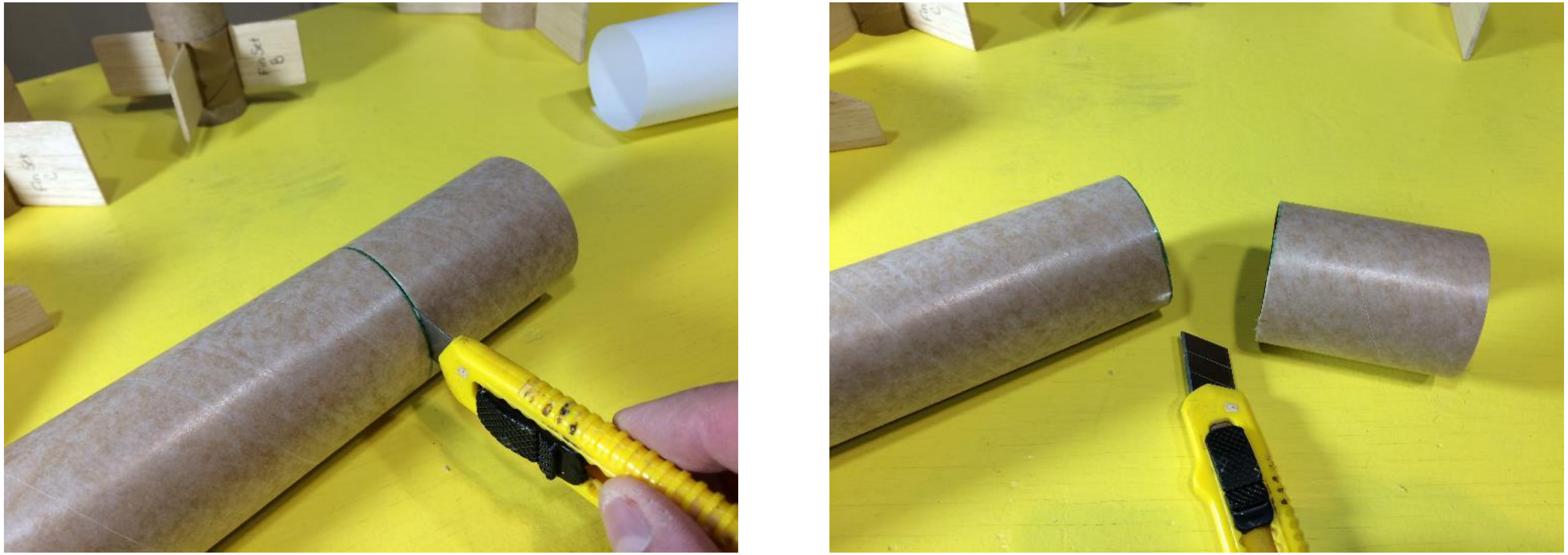
Maintain a constant fin area for each fin set.

Marking Fin Mounting Collars



Use a strip of paper to serve as a marking guide. Align the edges of the paper and tape. If the edges of the paper are aligned prior to marking, the faces of the cut tube will be parallel.

Cutting Fin Mounting Collars



Use a utility knife to cut the tube. Make sure the blade is sharp to ensure a clean cut.

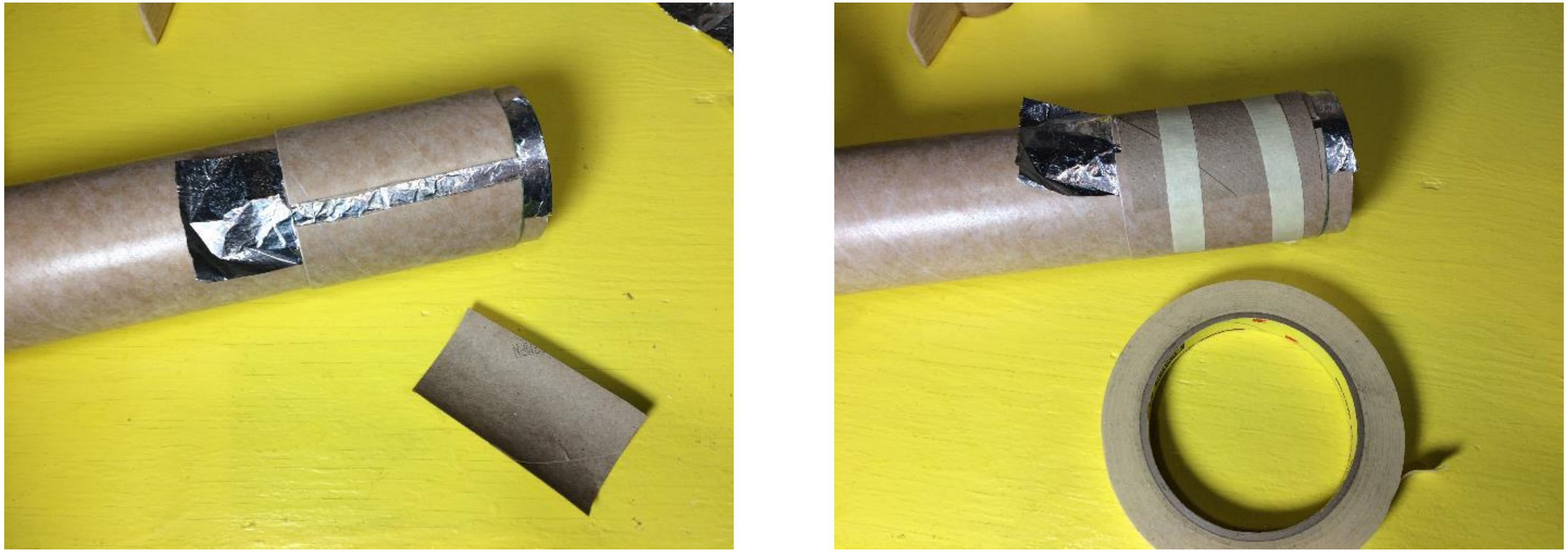
Longitudinal Cut Line

Place the collar tubes in the notch in a door frame and mark a longitudinal cut line on the tubes. This method ensures the cut line is parallel to the tube's sides (or you can free hand it with scissors….)

Making the Collar Patch

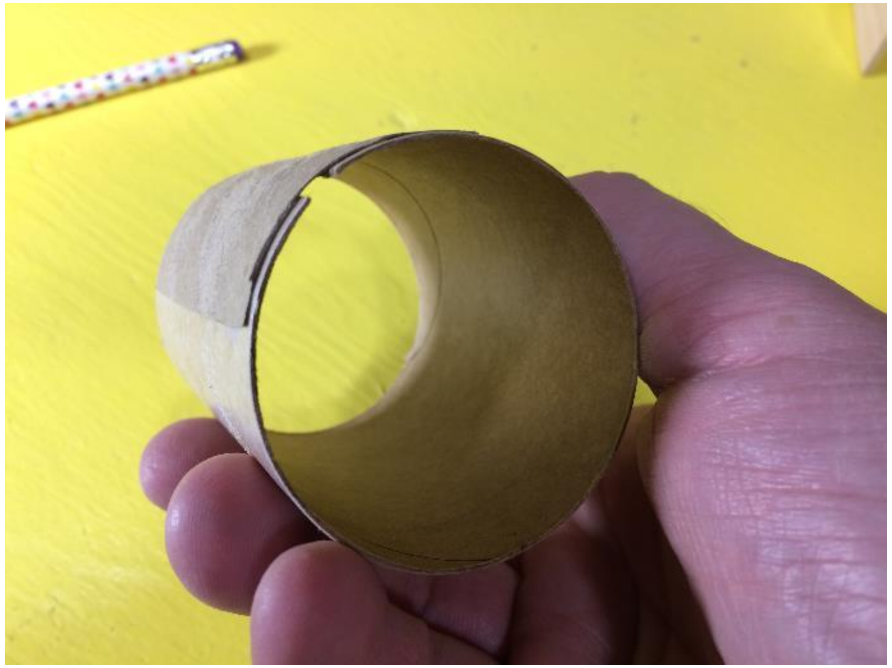
This view shows the cut collar. Cut out a collar patch out of a section of old paper towel tube.

Gluing the Collar Patch

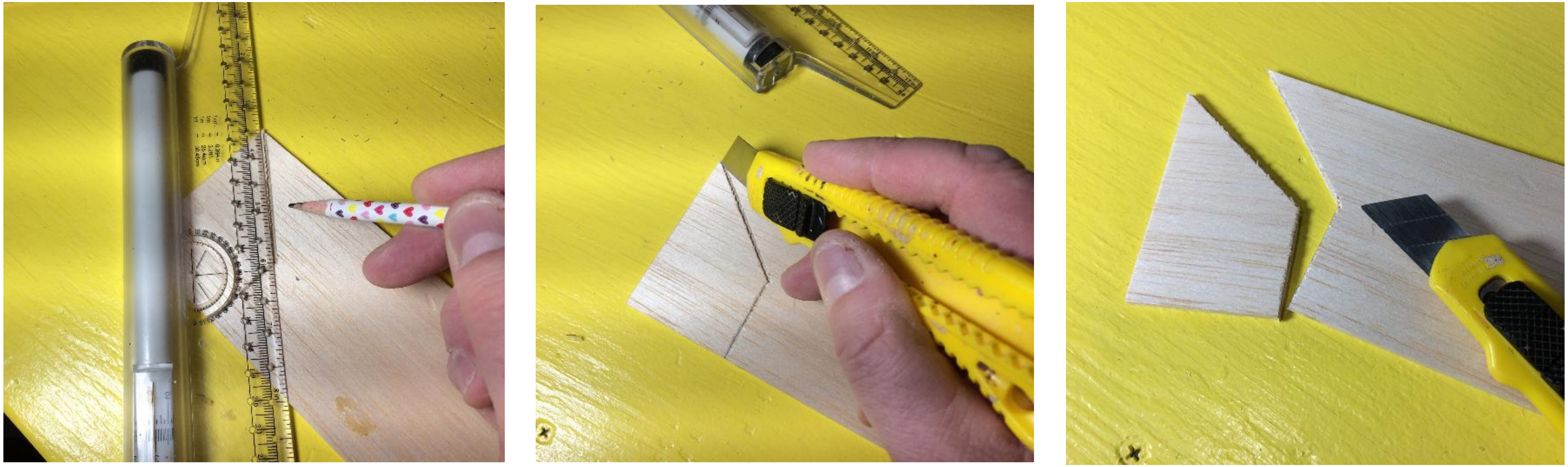


Use the rocket body tube to properly size the collar circumference. Place a piece of aluminum foil under the gap in the collar. This will prevent the collar from being accidentally glued to the body tube. Smear white glue on the back side of the collar patch and place on the collar. Secure with masking tape, making sure the collar fits snuggly on the tube

Completed Fin Collar

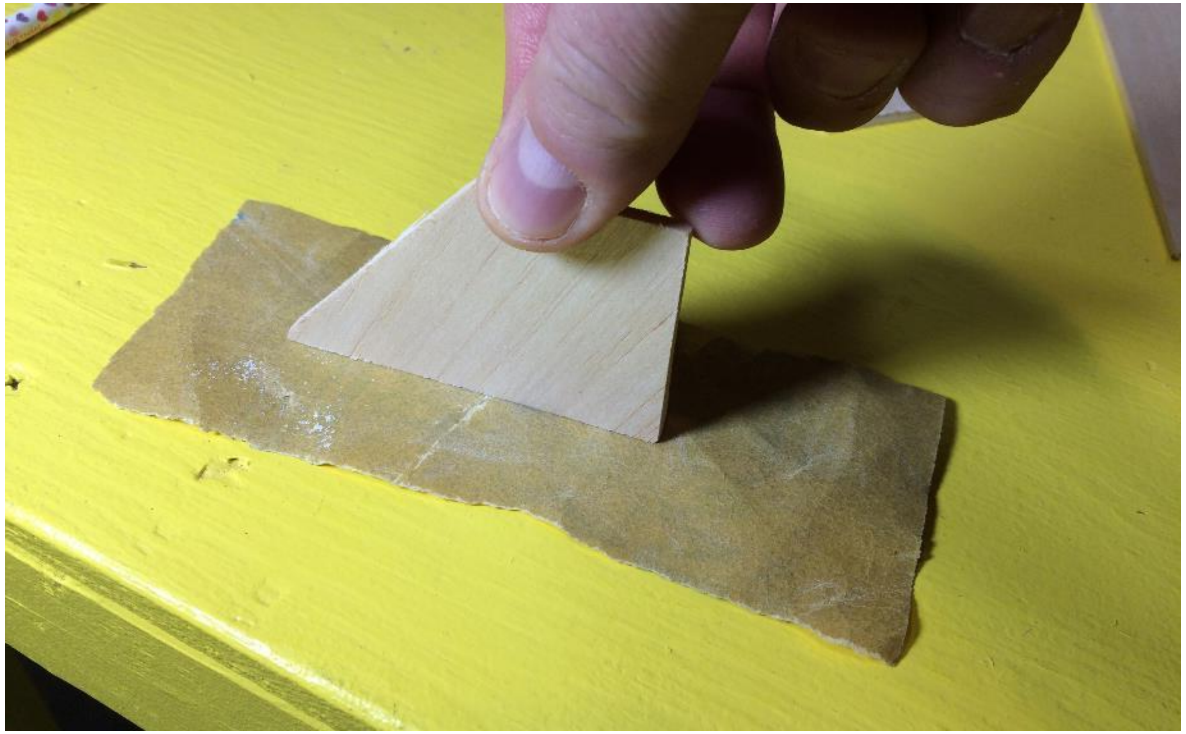


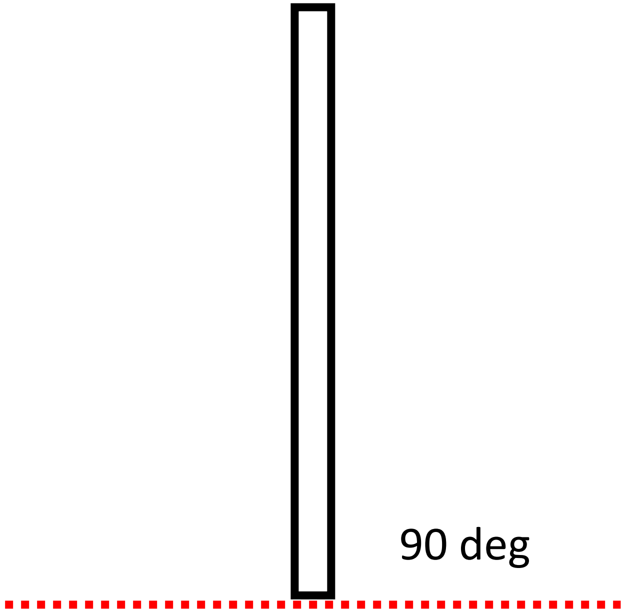
Cutting out Fins



Design your fins, then draw them on the balsa wood sheet. Carefully cut out the fins with a sharp utility knife.

Fin Sanding



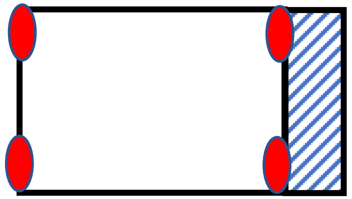


Place a piece of sandpaper on the table top. Smooth each of the edges of the fins. Keep the fin perpendicular to the sand paper.

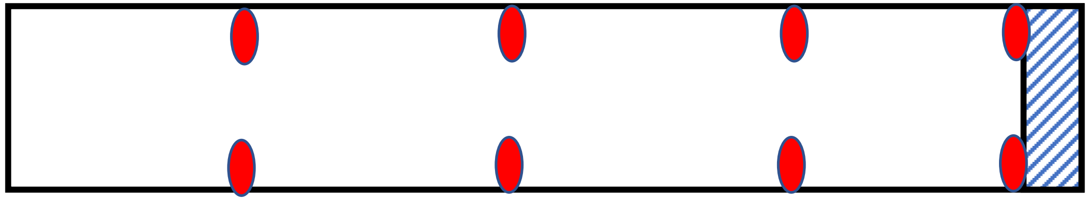
Making a Paper Fin Alignment Guide

Cut a 2” (approx.) wide strip of paper. Make it long enough so it can be wrapped around the body tube. Leave some overlap (cross hatched area)

Fold the paper strip in half, leaving the overlap section sticking out… Make marks on the fold line.



Fold the paper strip in half again, leaving the overlap section sticking out as before… Make marks on the fold line.

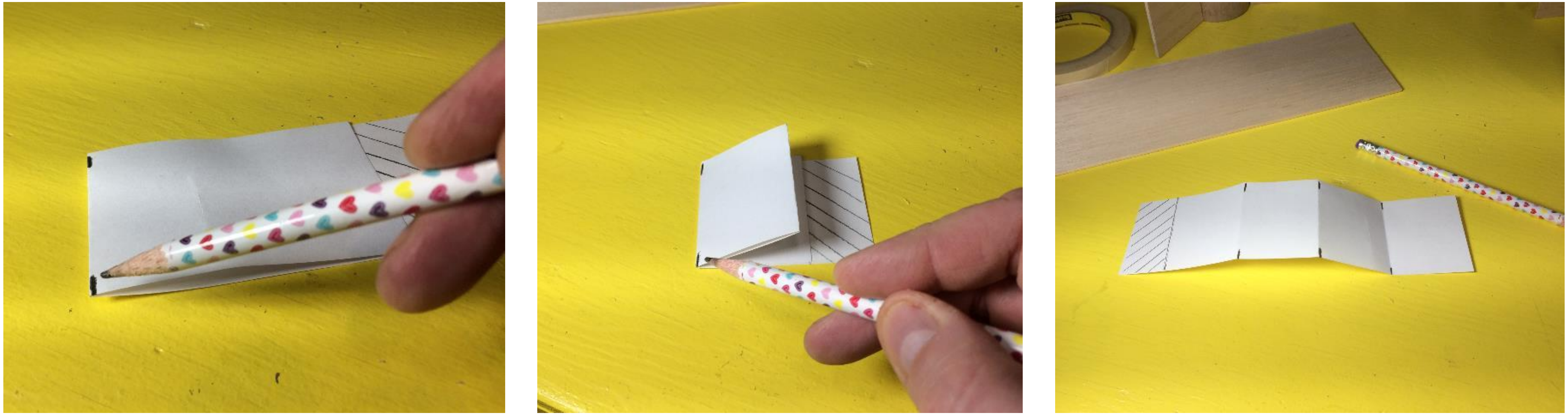


When you unfold the paper strip, you should end up with evenly spaced reference marks.

Making a Paper Fin Alignment Guide

Wrap the paper strip around the fin collar. The cross-hatched area denotes the overlap. Fold the strip in half up to the edge of the overlap area.

Making a Paper Fin Alignment Guide



Fold the strip of paper in half, then mark the fold line. Fold the paper in half a second time and mark the fold lines. Once this is done, the strip will be divided into 4 equal sections.

Making a Paper Fin Alignment Guide

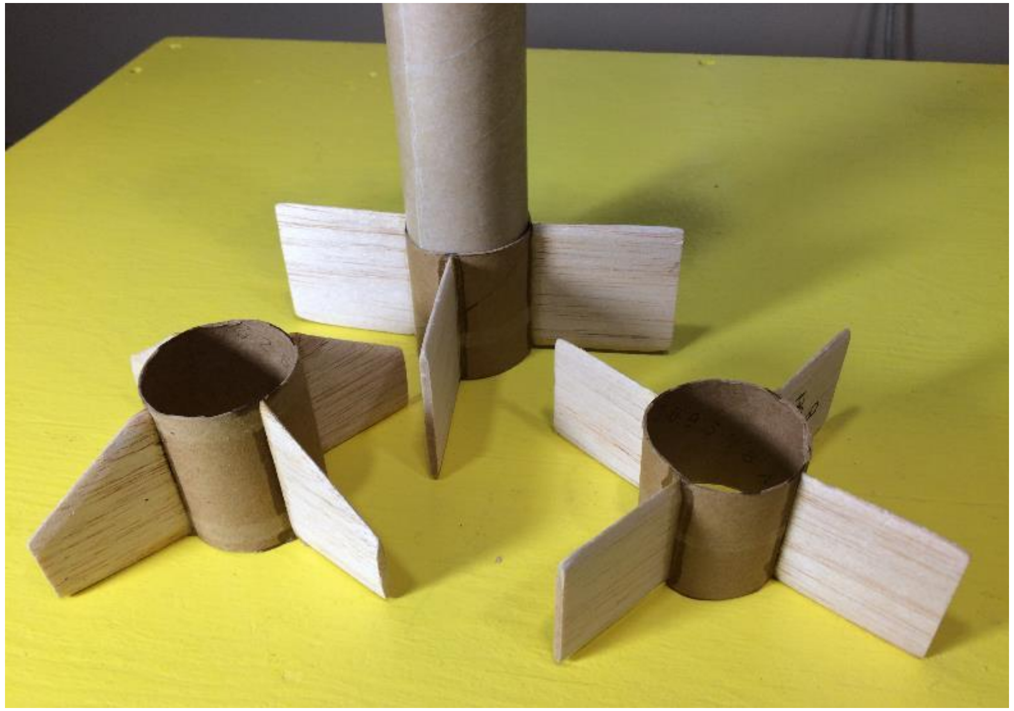
Wrap the completed fin guide around the fin collar and transfer marks to the tube.

Mark the fin alignment lines on the Fin Collar

Fin alignment line (4 total)

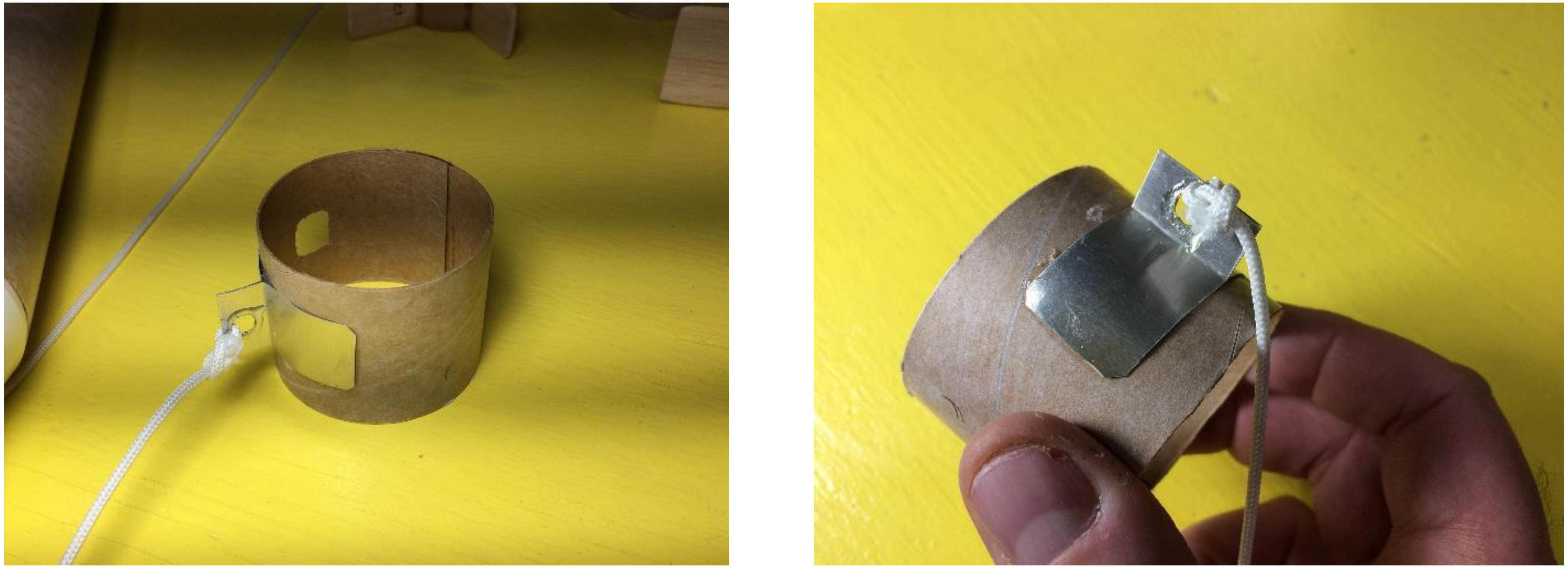
Use a ruler to connect the reference lines, or use the door frame notch technique. The fins will be glued along these lines. If done properly, the fins will be equally spaced and aligned with the rocket body tube

Securing Fins to the Fin Collars



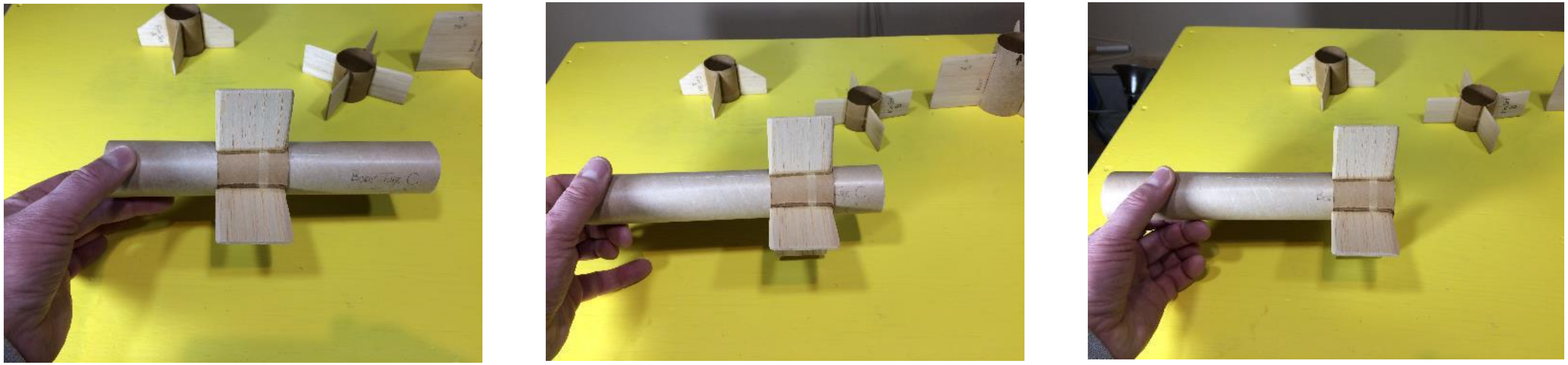
Use 5-minute epoxy to secure the fins to the fin collars. Use of the epoxy greatly speeds up the fin gluing process (white glue takes a long time to dry…).

String Collar



The same collar fabrication techniques can be used to fabricate a movable string collar. A string attachment loop can be made out of a paperclip, poster board or thin metal flashing.

Fin Set on Body Tube

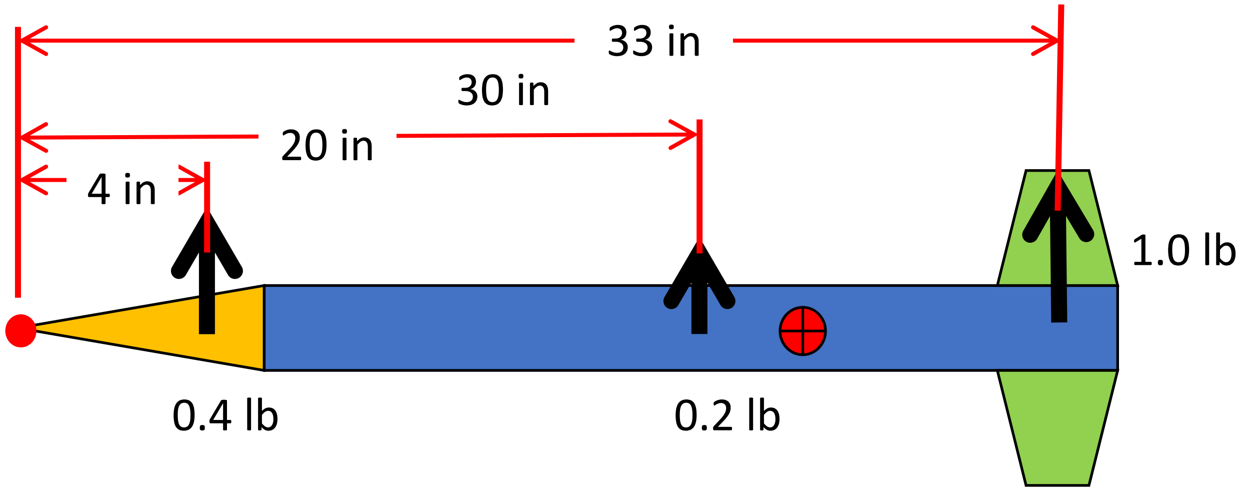


These images show how the fin sets can be positioned along the body tube.

The completed test rocket with large square fins.

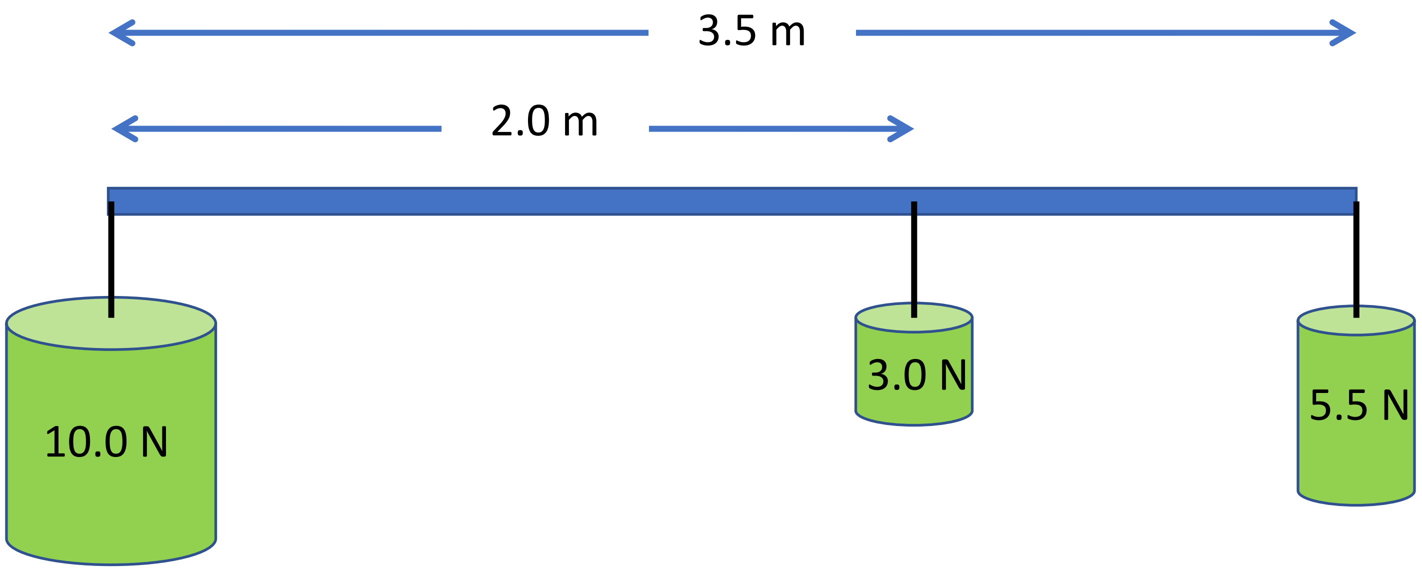
Rocket Stability

Practice Problems



LabRat Scientific

Calculate the Center of Gravity (balance point) of the following system.



The first step is to select a reference point so that the moments can be calculated. Lets pick the left end of the beam.

Next we calculate the moments generated by each weight.

$$\text{Moment}_A = 5.5\ \text{N} * 3.5\ \text{m} = 19.25\ \text{N*m}$$

$$\text{Moment}_B = 3.0\ \text{N} * 2.0\ \text{m} = 6.0\ \text{N*m}$$

$$\text{Moment}_C = 10.0\ \text{N} * 0.0\ \text{m} = 0.0\ \text{N*m}$$

Note: This weight does not generate a moment about the left end

Next we sum the moments:

$$\sum\text{Moments} = 19.25\ \text{N*m} + 6.0\ \text{N*m} + 0.0\ \text{N*m}$$

$$\sum\text{Moments} = 25.25\ \text{N*m}$$

We then sum the masses (or actually the weights):

$$\sum \text{Mass} = 5.5\ \text{N} + 3.0\ \text{N} + 10.0\ \text{N}$$

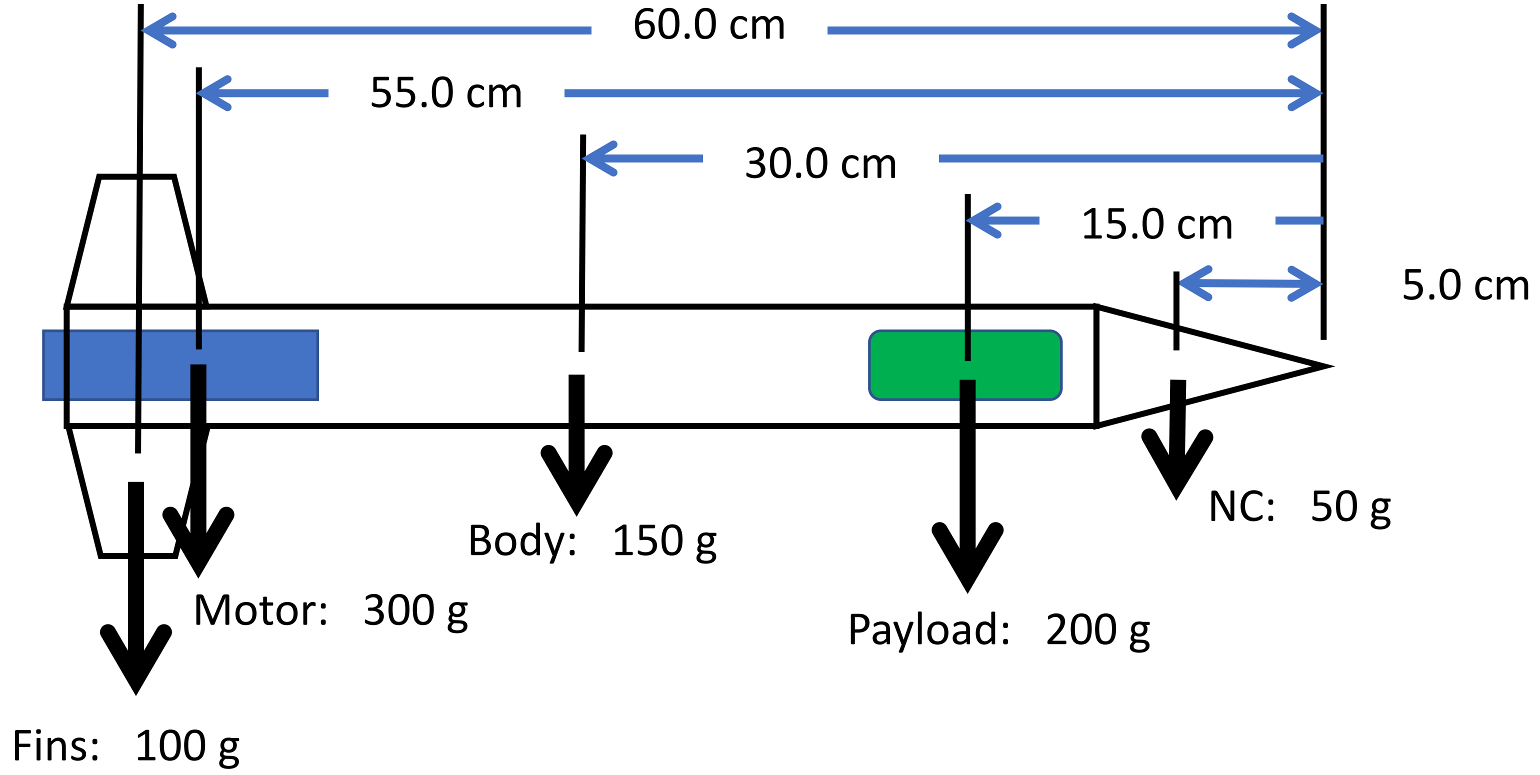
$$\sum \text{Mass} = 18.5\ \text{N}$$

Finally, we divide the sum of the moments by the sum of the masses:

$$\text{XCg} = \frac{25.25\ \text{N*m}}{18.5\ \text{N}} = \mathbf{1.36\ m}$$

The center of gravity of the system is located 1.36 m from the left end of the beam.

Calculate the Center of Gravity of the rocket shown below. Neglect any components that are not defined.



As usual in these types of problems, we need to select a point about which to calculate the moments. This solution uses the nose tip of the rocket.

Next we calculate the aerodynamic moments:

$\text{Moment}_{\text{Nose}} = 50.0\text{ g} * 5.0\text{ cm} = 250.0\text{ g*cm}$

$\text{Moment}_{\text{Payload}} = 200.0\text{ g} * 15.0\text{ cm} = 3{,}000.0\text{ g*cm}$

$\text{Moment}_{\text{Body}} = 150.0\text{ g} * 30.0\text{ cm} = 4{,}500.0\text{ g*cm}$

$\text{Moment}_{\text{Motor}} = 300\text{ g} * 55.0\text{ cm} = 16{,}500.0\text{ g*cm}$

$\text{Moment}_{\text{Fins}} = 100.0\text{ g} * 60.0\text{ cm} = 6{,}000.0\text{ g*cm}$

Because of the math process, it is not necessary to convert the grams to Newtons or cm’s to m’s. The units will ultimately work out out…

Next we calculate the sum of the moments:

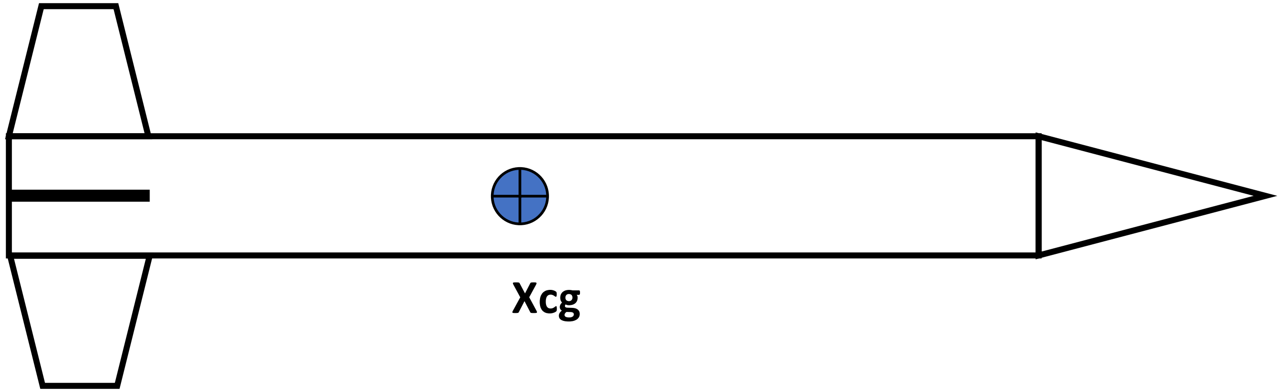
∑Moment = 250.0 g*cm + 3000.0 g*cm + 4,500 g*cm + 16,500 g*cm + 6,000 g/cm
= **30,250 g*cm**

Next we calculate the sum of the component masses:

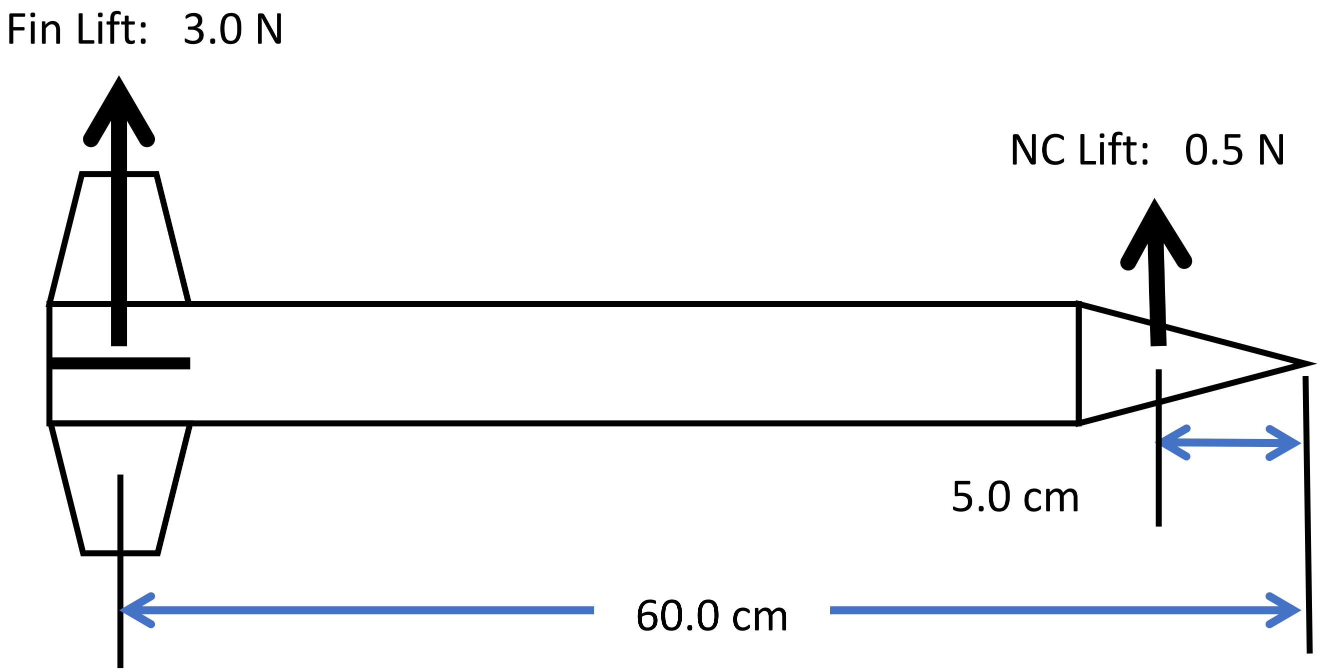
∑Mass = 50.0 g + 200.0 g + 150.0 g + 300.0 g + 100.0 g
= **800 g**

Finally, we divide the sum of the moments by the sum of the masses:

$$XCg = \frac{30{,}250 \text{ g*cm}}{800.0 \text{ g}} = \mathbf{37.8\ cm}$$



Calculate the Center of Pressure location on the rocket.



As usual in these types of problems, we need to select a point about which to calculate the moments. This solution is uses the nose tip of the rocket.

Next we calculate the aerodynamic moments:

$$\text{Moment}_{\text{Fin}} = 3.0\text{ N} * 60.0\text{ cm} = 180.0\text{ N}*\text{m}$$

$$\text{Moment}_{\text{Nose}} = 0.5\text{ N} * 5.0\text{ cm} = 2.5\text{ N}*\text{m}$$

Next we calculate the sum of the aerodynamic moments:

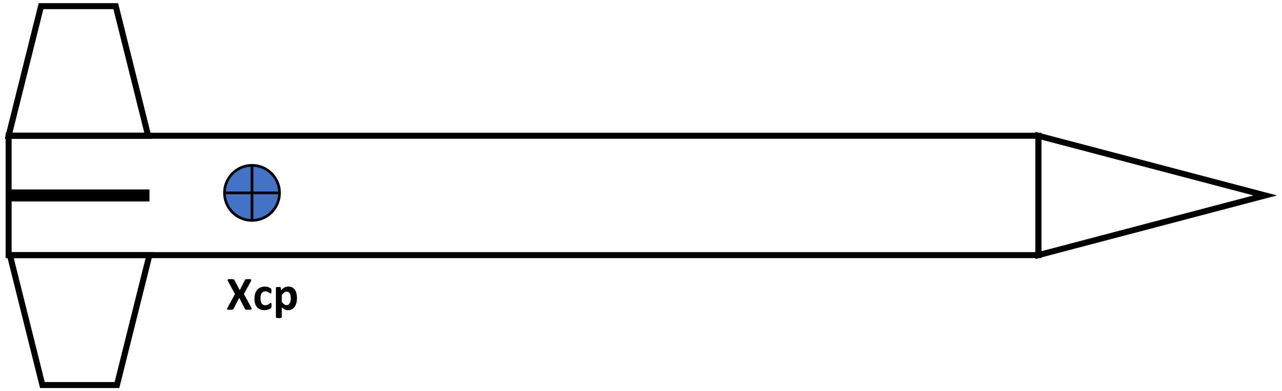
$$\sum\text{Moment} = 180.0\text{ N}*\text{m} + 2.5\text{ N}*\text{m} = 182.5\text{ N}*\text{m}$$

Next we calculate the sum of the aerodynamic forces:

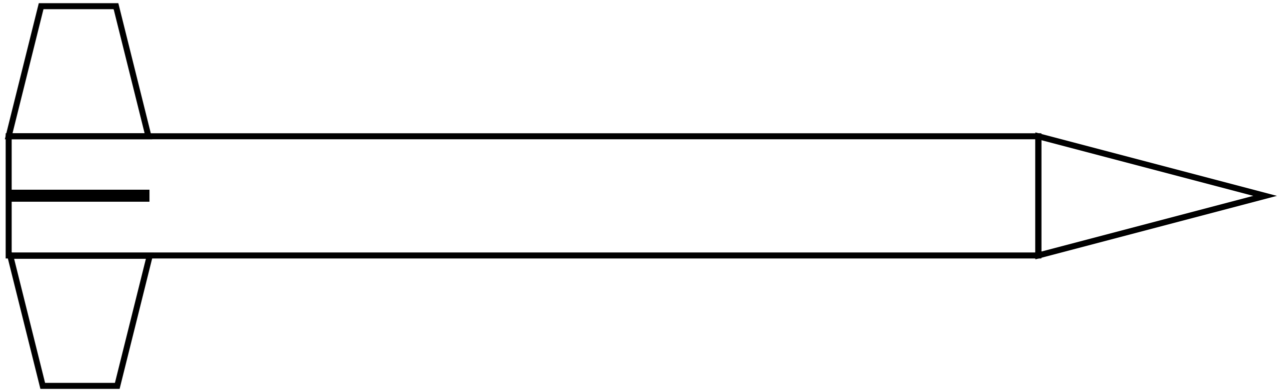
$$\sum \text{Forces} = 3.0\ \text{N} + 0.5\ \text{N} = 3.5\ \text{N}$$

Finally, we divide the sum of the moments by the sum of the forces:

$$\text{XCp} = \frac{182.5\ \text{N*m}}{3.5\ \text{N}} = \mathbf{52.1\ cm}$$



Calculate the Static Margin based on the XCg and XCp results from the previous problems. Is the rocket stable?

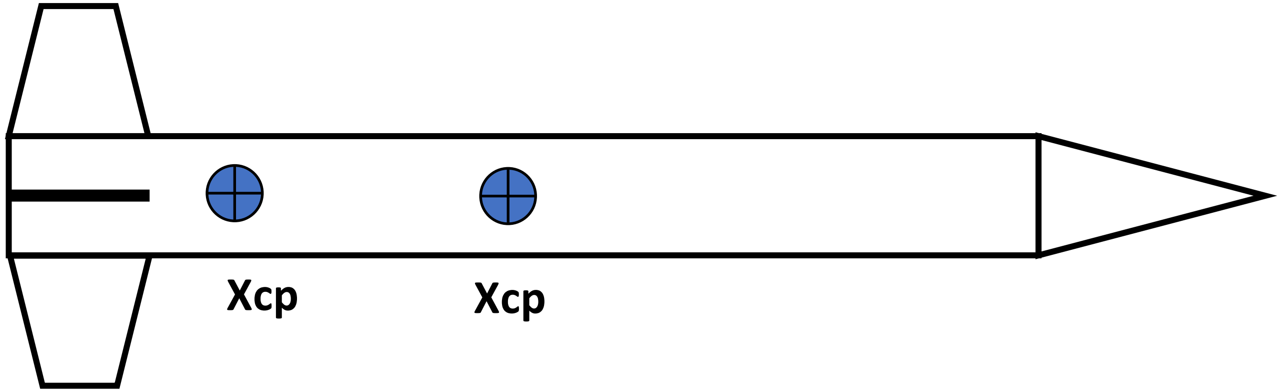


The static margin is the difference between the Xcg and Xcp. The governing equation is as follows:

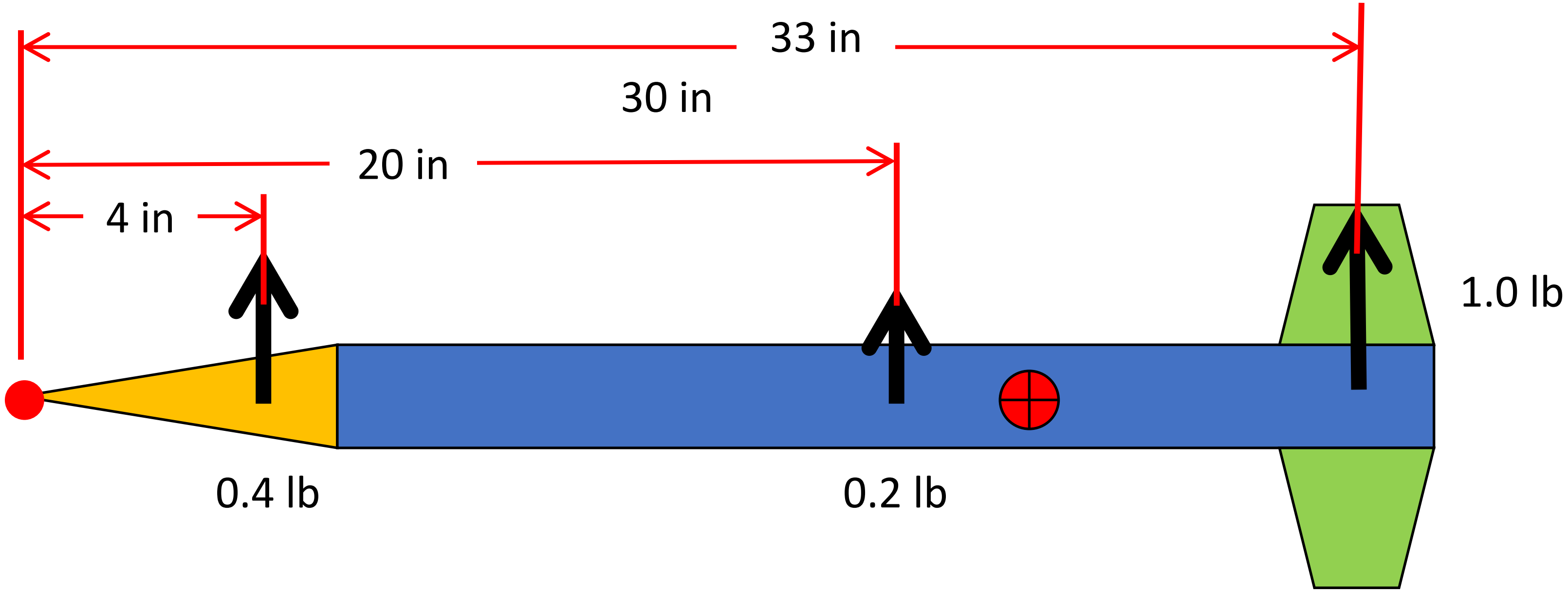
$$\text{Static Margin} = X_{cp} - X_{cg}$$

If the Xcp is behind the Xcg the static margin will be positive (Xcp > Xcg). If the SM is negative, the rocket if unstable. Positive SM means "stable".

$$\begin{aligned}\text{Static Margin} &= 52.1\text{ cm} - 37.8\text{ cm}\\ &= 14.3\text{ cm}\end{aligned}$$



Since the SM is positive the rocket is stable. Actually very stable…



Rocket Stability

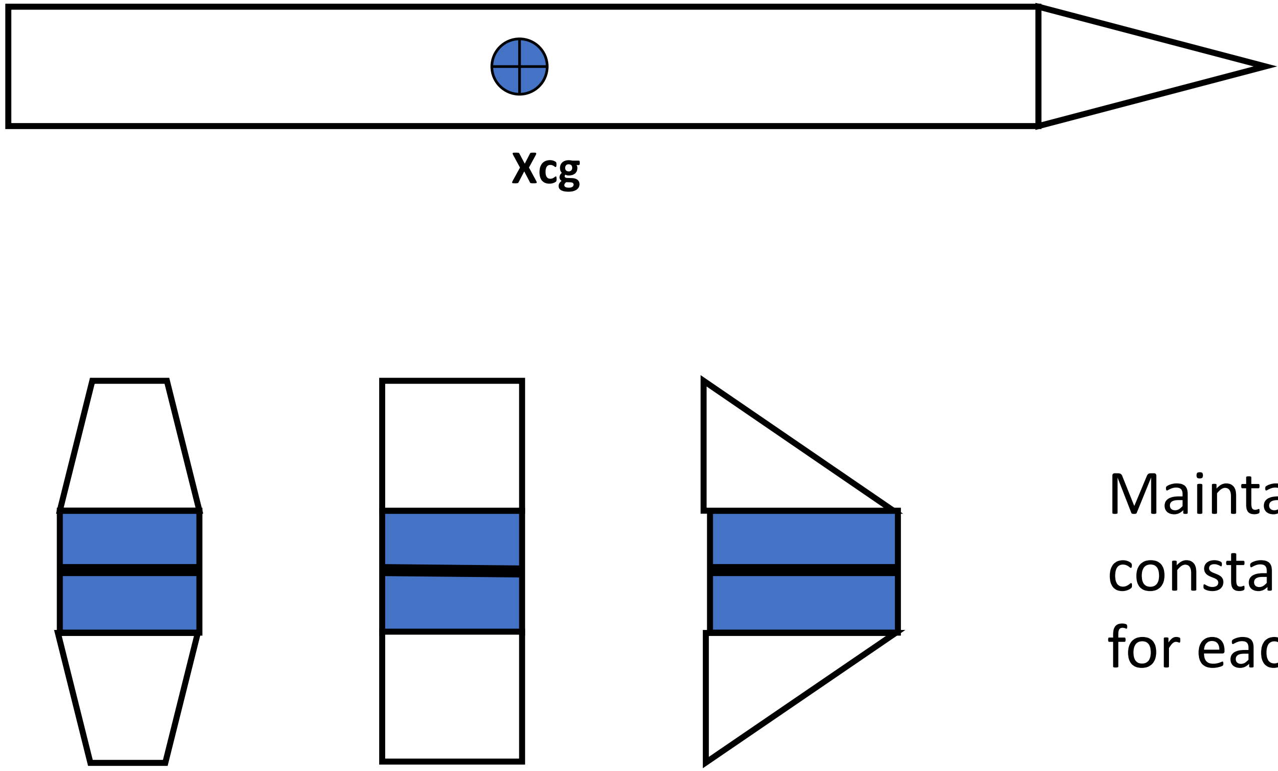
Suggested Experiments

LabRat Scientific

Exp. 1 – Stability as a Function of Fin Location

- Scientific Question
  - How does the location of the fins affect the stability of a rocket?

- Equipment and Materials
  - Experimental rocket with several movable fin sets
  - String



Maintain a constant fin area for each fin set.

Exp. 1 – Stability as a Function of Fin Location

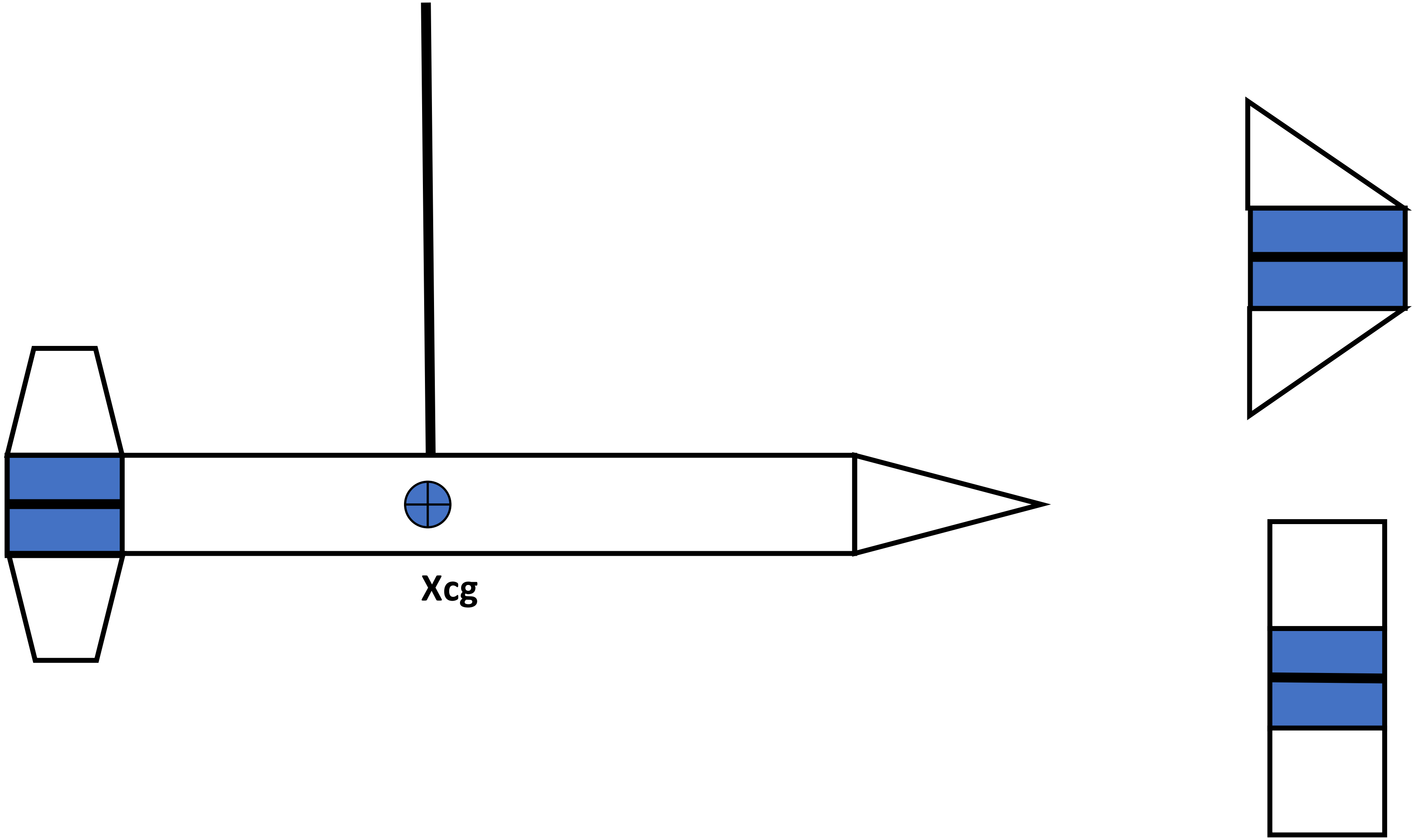
- Approach
    1. Build the test rocket with a movable fin sets (build 3 sets using 3 different fin shapes/sizes)
    2. Place the fins at the rear of the rocket
    3. Hang the rocket from its Center of Gravity
    4. Perform a spin test to see if the rocket is stable
    5. Move the fins 1” forward
    6. Keep the string at the same location by adding weights to the front or back of the rocket as necessary to keep the rocket balanced
    7. Perform a spin test to see if the rocket is stable
    8. Repeat steps 5 – 7
    9. Repeat the experiment until the rocket becomes unstable
    10. Repeat the experiment for the other fin set

Exp. 1 – Stability as a Function of Fin Location

- Data Analysis
    - Determine the fin location where the rocket becomes unstable for each fin set
    - Calculate the rockets static margin for each fin set

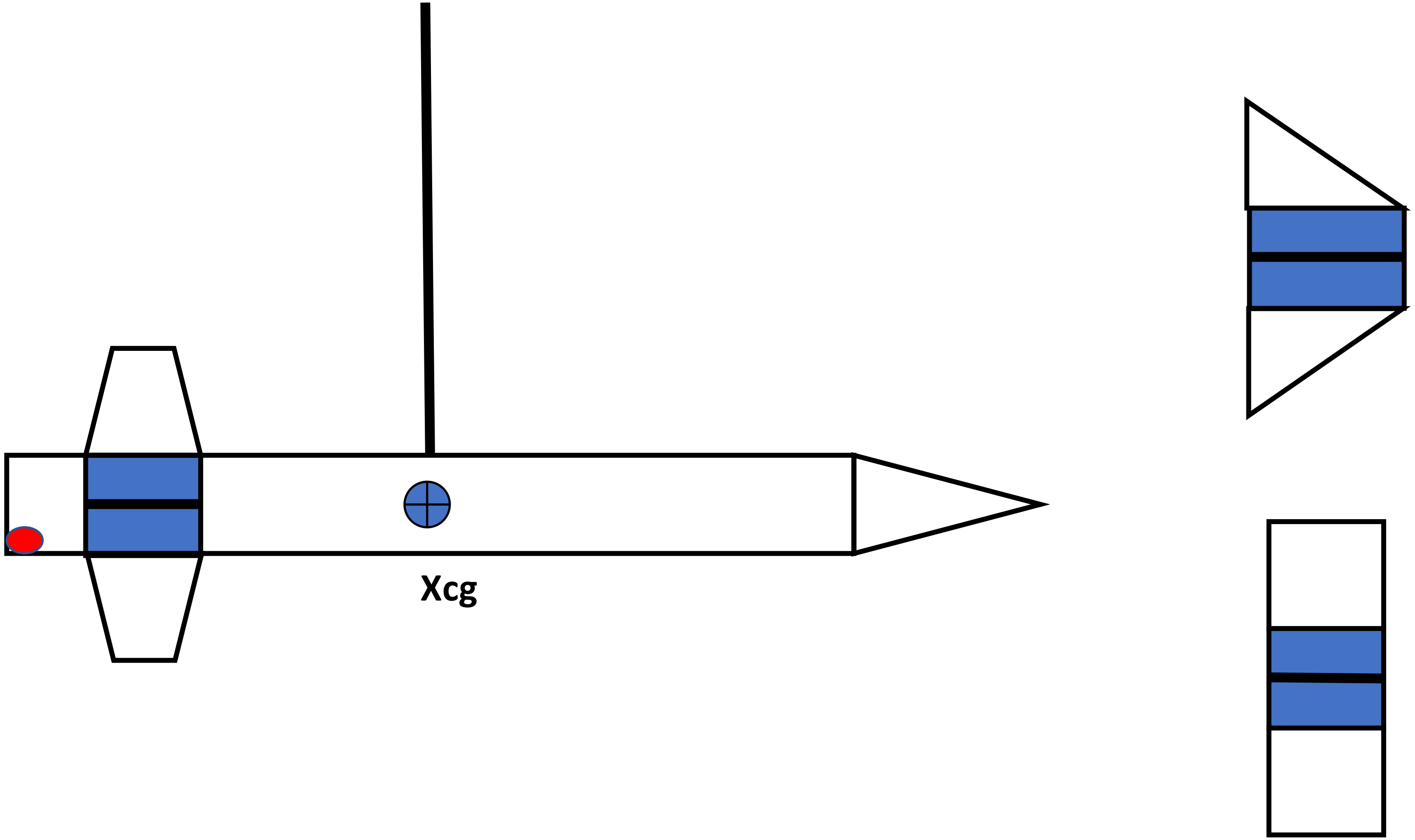
- Drawing Conclusions
    - How does the fin shape affect the static margin of the rocket?

Exp. 1 – Stability as a Function of Fin Location



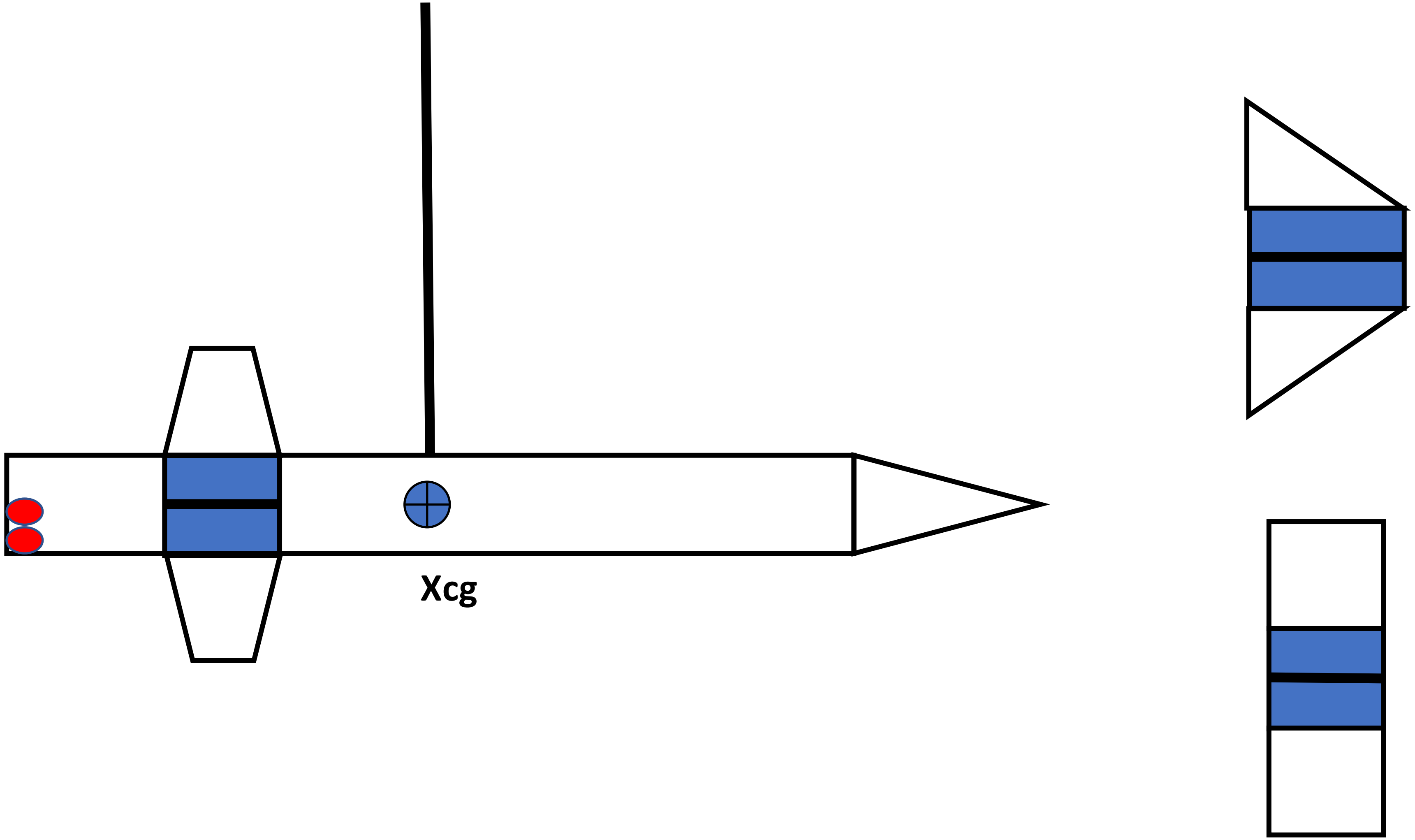
Exp. 1 – Stability as a Function of Fin Location

Since the fins move forward, small weights need to be added at the back to keep the rocket balanced.



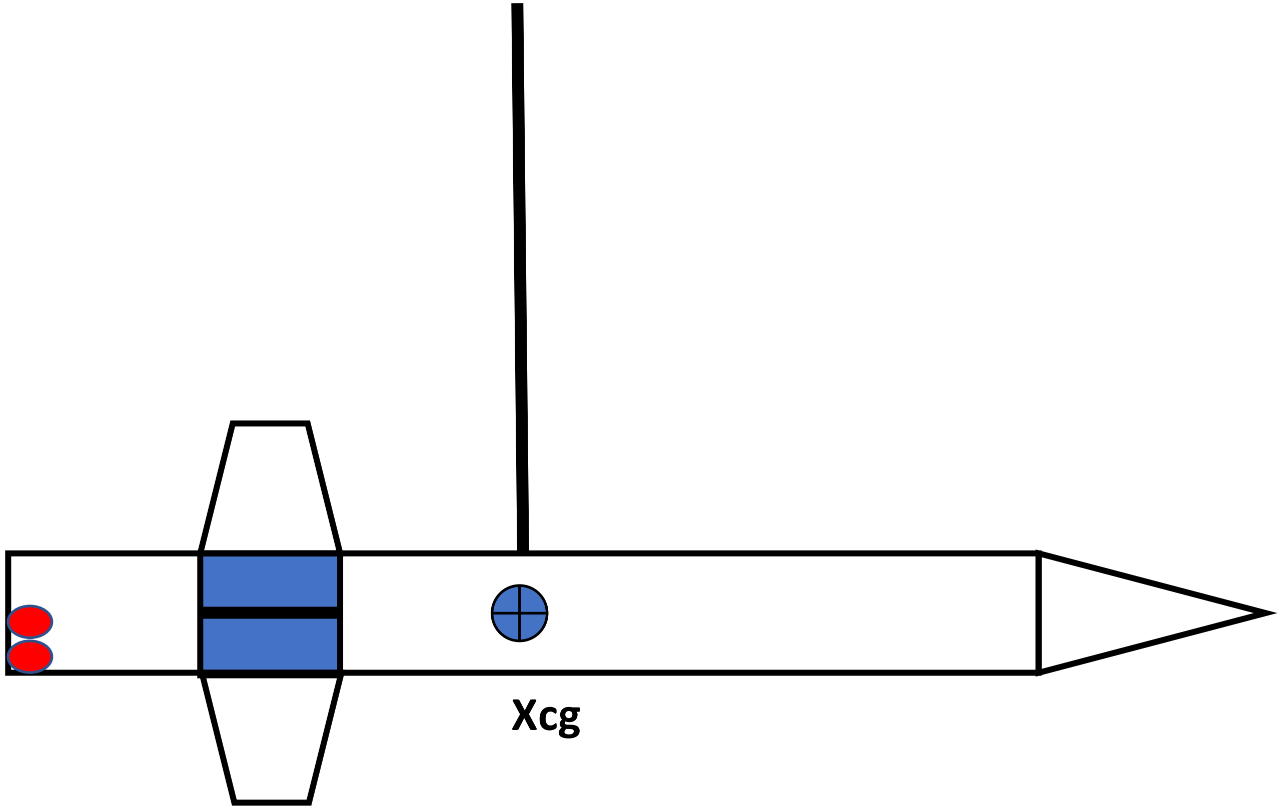
Exp. 1 – Stability as a Function of Fin Location

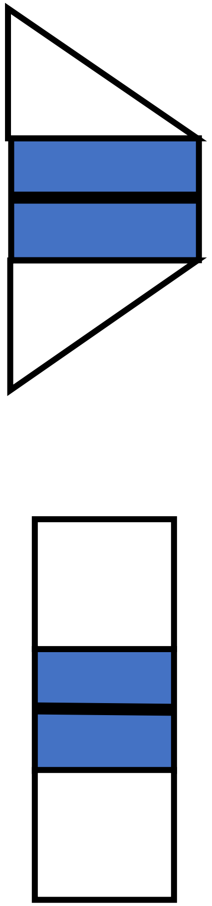
Since the fins move forward, small weights need to be added at the back to keep the rocket balanced.



Exp. 1 – Stability as a Function of Fin Location

Since the fins move forward, small weights need to be added at the back to keep the rocket balanced.





Repeat the test for the 2nd and 3rd fin sets.

Exp. 2 – Stability as Function of CG Location

- Scientific Question
  - How does the location of the Center of Pressure affect the stability of a rocket?

- Equipment and Materials
  - Experimental rocket with movable balance string
  - String

Exp. 2 – Stability as Function of CG Location

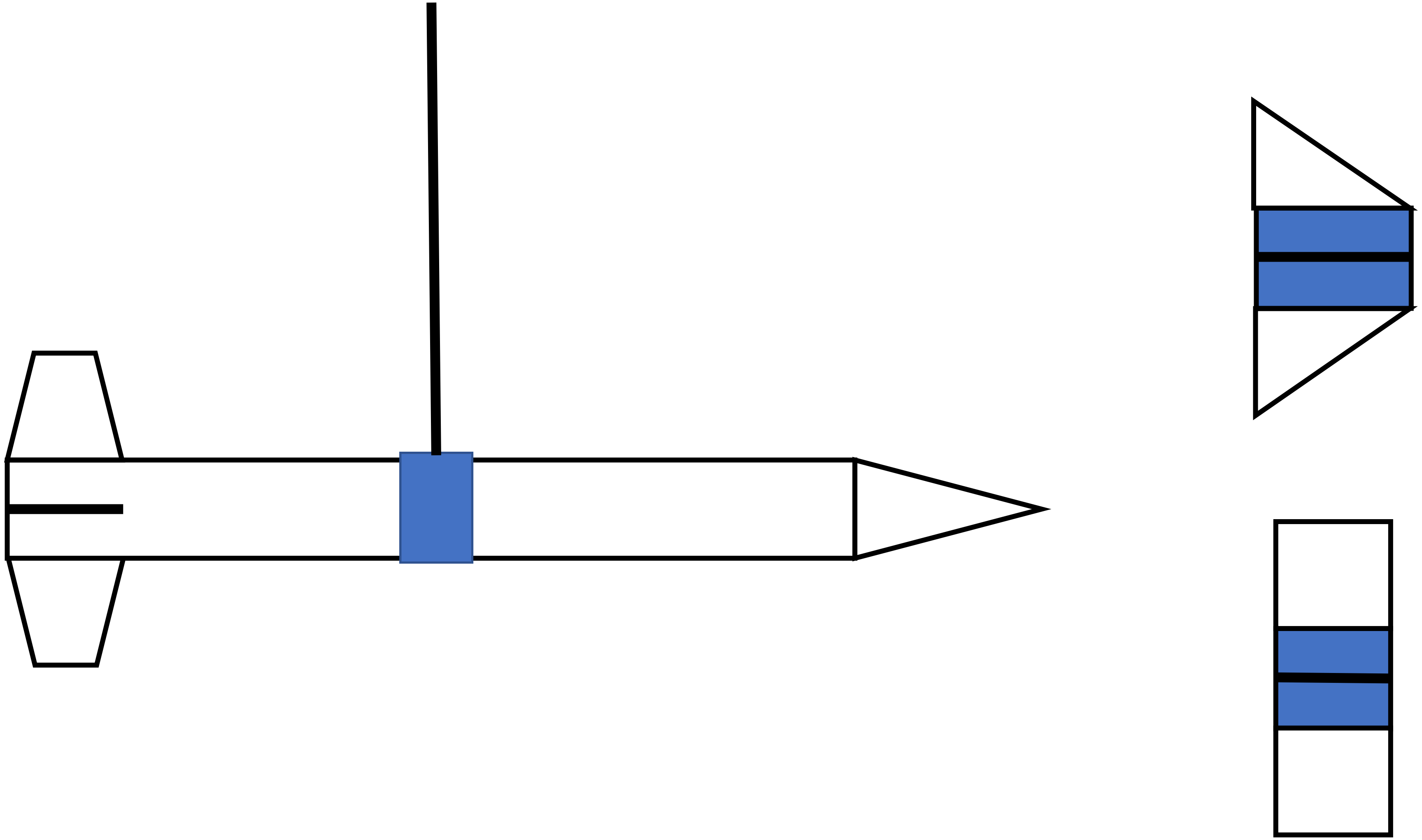
- Approach
  1. Build the test rocket with replaceable fin sets (use those from Exp. 1)
  2. Place the fin set 1 at the rear of the rocket
  3. Hang the rocket from its Center of Gravity
  4. Perform a spin test to see if the rocket is stable
  5. Move the string collar 1” aft and rebalance the rocket
  6. Perform a spin test to see check the rocket’s stability
  7. Repeat steps 4-5 until the rocket become unstable
  8. Change the fin set and repeat the experiment

- Data Analysis
  - Tabulate the CP locations with respect to the fin shape
  - Assess which shape of fin provides the best stability

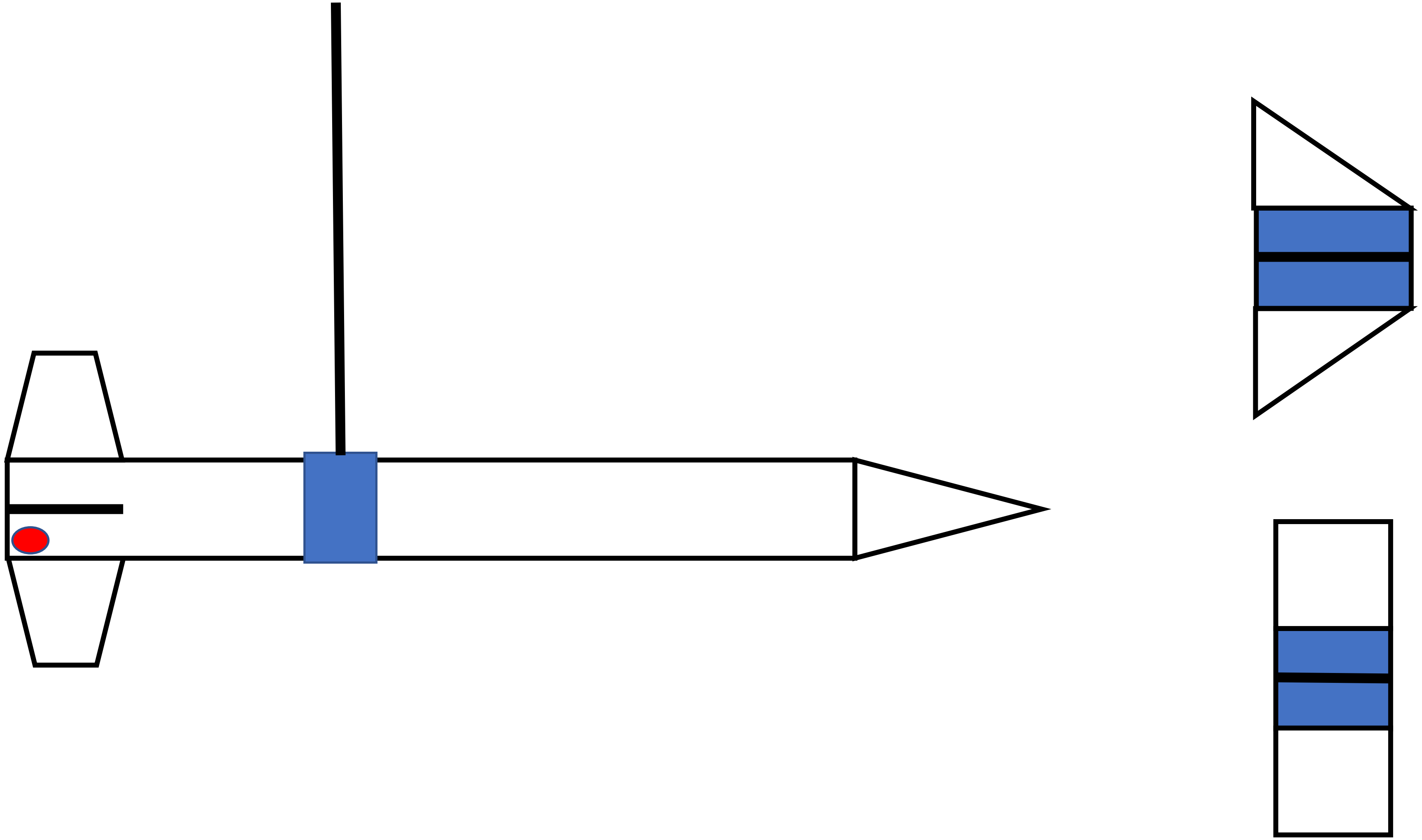
Exp. 2 – Stability as Function of CG Location

- Drawing Conclusions
  - Which fin set provides the best stability

Exp. 2 – Stability as Function of CG Location



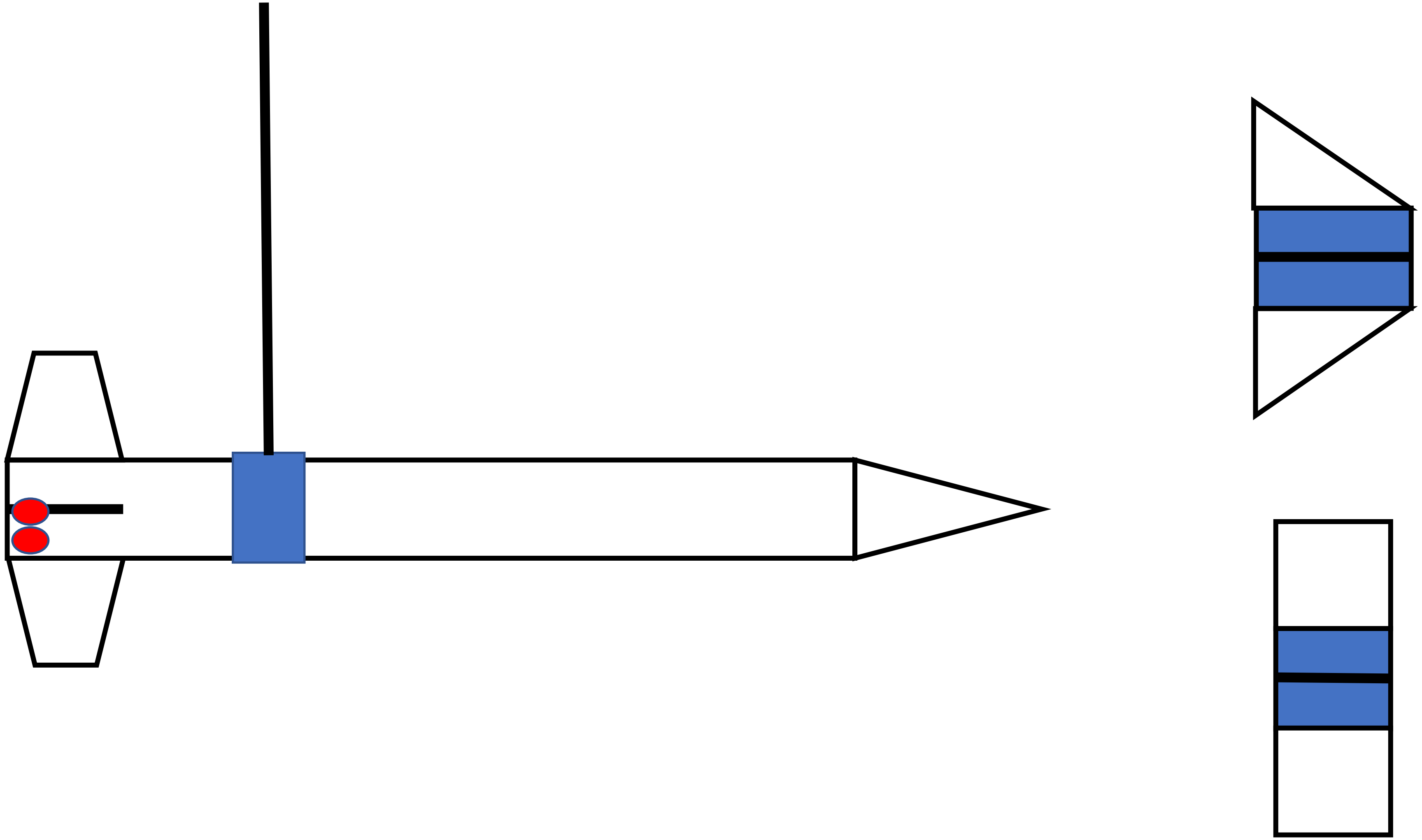
Exp. 2 – Stability as Function of CG Location



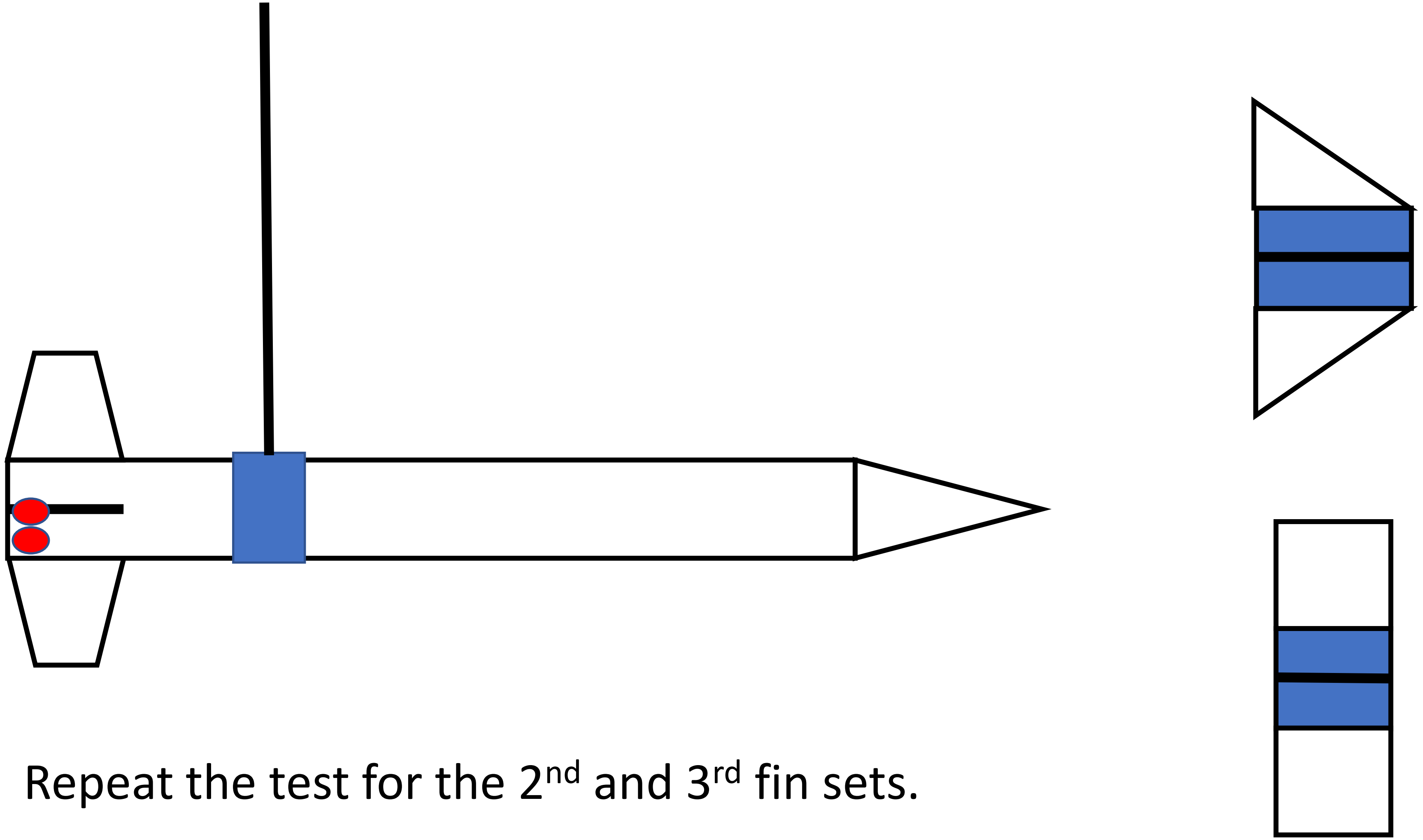
Add weights to the back of the rocket to shift the balancing point backwards.

Exp. 2 – Stability as Function of CG Location

Add weights to the back of the rocket to shift the balancing point backwards.



Exp. 2 – Stability as Function of CG Location



Add weights to the back of the rocket to shift the balancing point backwards.

Repeat the test for the 2nd and 3rd fin sets.