2011 IEEE International Conference on Robotics and Automation
Shanghai International Conference Center
May 9-13, 2011, Shanghai, China

Motion Planning for Dynamic Folding of a Cloth with Two High-speed Robot Hands and Two High-speed Sliders

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Abstract— **The purpose of the work described in this paper is to achieve dynamic manipulation of a sheet-like flexible object. As one example, we examine dynamic folding of a cloth with two high-speed multifingered hands mounted on two sliders. First, dynamic folding by a human subject is analyzed in order to extract the necessary motions for realizing this task. Second, a model of a sheet-like flexible object is proposed by extending a linear flexible object model (algebraic equation) that takes advantage of high-speed robot motion. Third, motion planning of the robot system is performed by using the proposed model, and the simulation results are shown. Finally, an experiment was conducted with the robot motion obtained by the simulation.**

I. INTRODUCTION

Robotic manipulation is an important technique in robot engineering. Traditionally, various control schemes for manipulating rigid bodies have been proposed and analyzed. Although it has been expected that manipulation of flexible objects will be achieved, the manipulations required are extremely difficult to realize in practice. This is because many problems exist in carrying out the desired task, such as the manipulation strategy, motion planning, and modeling and control of the flexible object. Crucial reasons for the difficulties faced in flexible object manipulation include deformation of the flexible object during manipulation and estimating the deformation of the flexible object. One promising approach for solving these problems is the use of high-speed motion and high-speed sensory feedback. In particular, motion planning of a high-speed robot is enormously important.

Recently, there has been some research on rope knotting, which is one type of manipulation of a linear flexible object [1], [2]. Similar ideas have been extended to the manipulation of sheet-like flexible objects, such as cloth [3], origami paper [4], [5], and towels [6]. As described above, flexible objects have been manipulated typically in a static manner with slow motions of the robot, and high-speed manipulation of flexible objects has been infeasible thus far. In order to achieve high-speed, dynamic manipulation of a flexible object and to solve the above problems, we have demonstrated dynamic knotting manipulation of a flexible rope [7]. In that study, we proposed a simple rope deformation model (algebraic equation) that takes advantage of high-speed robot motion, and we confirmed the effectiveness of this model. We also proposed a motion planning method that derives the robot trajectory from the rope configuration.

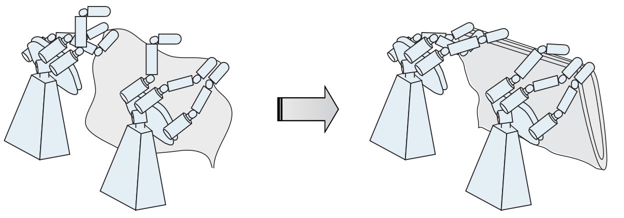

Fig. 1. Illustration of cloth folding task.

Here, as an extension of our previous work [7], we study dynamic manipulation of a sheet-like flexible object. More concretely, we consider dynamic folding of a cloth held in the air, as shown in Fig. 1. First, we extract the necessary motions in order to achieve the task by analyzing the dynamic folding performed by a human subject. Second, we construct a simple deformation model of the cloth by extending the previous rope model [7]. Third, we discuss a motion planning method based on the proposed model. The validity of the robot motion obtained by the motion planning method is confirmed by simulation. Finally, we show experimental results of dynamic cloth folding using two high-speed multifingered hands and two high-speed sliders.

In general, a cloth can be folded by placing it on a table. However, this method is not effective and is quite difficult to perform at high speed. Instead, we aimed to achieve high-speed folding of a cloth held in the air. Since this folding method dynamically uses the deformation of the cloth, we expect that it will be possible to carry out high-speed, dynamic manipulation of the cloth. Features of this research are expanding the types of target objects that can be manipulated and proposing a high-speed, dynamic manipulation strategy for cloth folding.

II. SYSTEM CONFIGURATION

The experimental system consists of two high-speed multifingered hands [8], two high-speed sliders, and a real-time control system, as shown in Fig. 2. The multifingered hands are mounted on the sliders.

Each hand has three fingers: a left thumb, an index finger, and a right thumb. Each finger is divided into a top link and a root link. The index finger has two degrees of freedom (2-DOF), and the other fingers have 3-DOF. The joints of the hands can be closed at a speed of 180 deg$/0.1$ s. In addition, each hand has two wrist joints. The wrist part has

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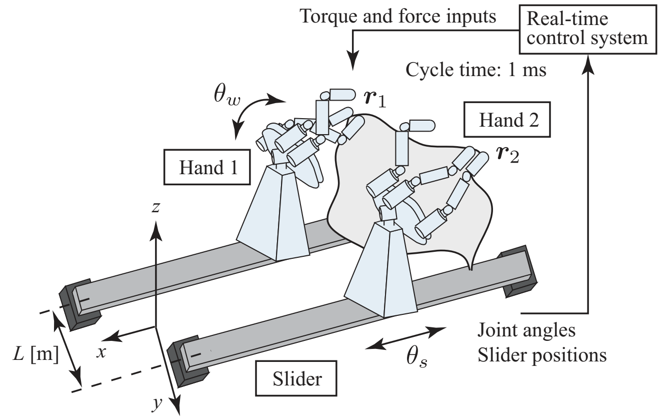


Fig. 2. Overview of robot system.

a differential rotation mechanism and moves about two axes (a bending-extension axis and a rotation axis).

Each slider has a 1-DOF translational motion mechanism. The maximum speed of the sliders is about 2 m/s.

The experimental system can be controlled at a sampling time of 1 ms by the real-time control system. The joint angles of the hand and the positions of the sliders can be controlled within 1 ms by a PD system.

III. ANALYSIS OF DYNAMIC FOLDING

In order to extract the motions required for performing dynamic folding of the cloth, we first analyze the dynamic folding performed by a human subject. Fig. 3(a) shows the initial condition. Fig. 3(b) shows the subject pulling the cloth toward his body using shoulder and elbow motions. Fig. 3(c) shows that when the shoulder and elbow motions stop, the free end (the far point from the grasped position) of the cloth is folded on itself by an inertial force. Fig. 3(d) shows the subject grasping the free end of the cloth and the final state where dynamic folding is completed. As a result, dynamic folding can be carried out by deforming the cloth to fold it and then grasping the deformed cloth.

The key elements to achieve this dynamic folding of the cloth are: appropriate deformation to fold the cloth and grasping the end of the deformed cloth. It is necessary to design a motion planning method in order to execute these elements. In this paper, we focus on the motion planning for the first element, that is, appropriate deformation to fold the cloth. A method for the second element, that is, grasping the end of the deformed cloth, will be discussed in the future. Folding of the cloth by the inertial force is not analyzed in this paper.

We do not use human motion for this task. The robot motion can be obtained by a motion planning method based on a deformation model of the cloth, as described in Section IV.

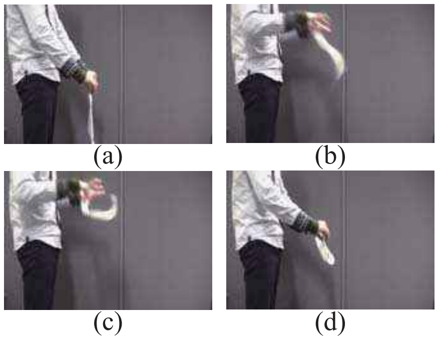


Fig. 3. Dynamic cloth folding by human subject.

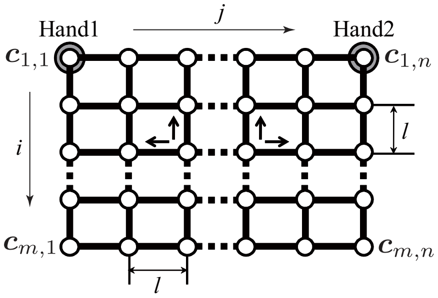


Fig. 4. Model of cloth.

IV. DEFORMATION MODEL AND SIMULATION

In this section, we explain a cloth deformation model that takes advantage of high-speed motion. In a previous paper [7], we demonstrated that the deformation model of a linear flexible object can be approximated by an algebraic equation by taking advantage of the high-speed motion of the robot. In this paper, we propose a deformation model of a sheet-like flexible object using the high-speed motion. As a physical model, we consider a multi-link model of the cloth and the grasping positions shown in Fig. 4. Then, we suggest a motion planning method based on the proposed model. Finally, we show the simulation results of the dynamic folding using the robot motion obtained with this motion planning method.

A. Kinematics of High-speed Hand and High-speed Slider

Here we consider the kinematics of the high-speed multifingered hands and the high-speed slider system in order to derive the grasp positions of the cloth. Using the fingers of the hands to grasp the cloth, the grasp positions do not depend on the finger positions. Thus, the degrees of freedom of the fingers can be neglected. As a result, the robot system has 4-DOF, including the bending-extension motions of the hands and the translational motions of the sliders. The 2-DOF variables for each hand side are assumed to be described by $\boldsymbol{\theta} = [\theta_w \ \ \theta_s]^T \in \boldsymbol{R}^2$. The grasp positions of the cloth by the experimental system are defined as $\boldsymbol{r} \in \boldsymbol{R}^3$. In general, using a Denabit–Hartenberg description, the following equations hold:

$$\begin{aligned} \boldsymbol{r}_1(t) &= \boldsymbol{f}(\boldsymbol{\theta}_1(t)) \\ \boldsymbol{r}_2(t) &= \boldsymbol{f}(\boldsymbol{\theta}_2(t)) + [0 \ \ L \ \ 0]^T, \end{aligned} \tag{1}$$

where the subscripts $(1, 2)$ are numbers representing the hands (hand 1 and hand 2). In this paper, the details of the kinematics are omitted.

B. Deformation Model of Cloth

Conventional mathematical models of flexible objects can be described by partial differential equations as the physical model of a distributed parameter system and matrix differential equations as the physical model of a multi-link system. These models have many parameters, which makes parameter estimation very difficult. As a result, the modeling is also very difficult. Furthermore, since these models are complicated, it is extremely hard to analyze these models and to design a control scheme.

We have previously demonstrated the possibility that the deformation model of a flexible rope can be approximated by an algebraic equation by taking advantage of high-speed robot motion [7]. In the present paper, the deformation model of the cloth will be described based on those results. In the modeling of the cloth under high-speed robot motion, the following assumptions are made:

1) The physical model of the cloth is approximated by a two-dimensional (2-D) multi-link model.
2) The behavior of the part of the cloth located "near" the grasp position of the robot does depend on the robot motion.
3) The distance between two joint coordinates of the cloth is not variable.
4) There exists a time delay between the robot motion and the cloth deformation.
5) The twist of the links in the cloth is not taken into account.

The second assumption means that the cloth deformation is determined by the grasp position of the robot system. The third assumption is the constraint that the link distance in the multi-link model does not change. The fourth assumption means that even if the robot moves, the part of the cloth located far from the grasp position does not deform during the time delay.

Considering the above assumptions and extending the model of the linear flexible object [7], the deformation model of the cloth can be algebraically represented as follows:

$$\boldsymbol{c}(t) = \boldsymbol{h}(\boldsymbol{r}_1(t), \boldsymbol{r}_2(t), d_i, d_j, t), \tag{2}$$

where

$$\boldsymbol{c} = \begin{bmatrix} \boldsymbol{c}_{1,1} & \cdots & \boldsymbol{c}_{1,j} & \cdots & \boldsymbol{c}_{1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{c}_{i,1} & \cdots & \boldsymbol{c}_{i,j} & \cdots & \boldsymbol{c}_{i,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{c}_{m,1} & \cdots & \boldsymbol{c}_{m,j} & \cdots & \boldsymbol{c}_{m,n} \end{bmatrix} \in \boldsymbol{R}^{3m \times n},$$

$m \times n$ is the number of joints, 3 indicates $[x \quad y \quad z]^T$ coordinates, $\boldsymbol{c}_{i,j}$ is the joint coordinate of element (i,j), $\boldsymbol{c}_{1,1}$ and $\boldsymbol{c}_{1,n}$ are the same as the grasp positions of the robot hands 1 and 2, respectively, and d_i and d_j represent longitudinal and transverse time delays between the robot motion and the cloth deformation, respectively. At the (i,j) joint, the time delay is given by $d_{i,j} = \lambda \times l \times (i, j-1)$ (λ is a normalized time delay, and l is the link distance).

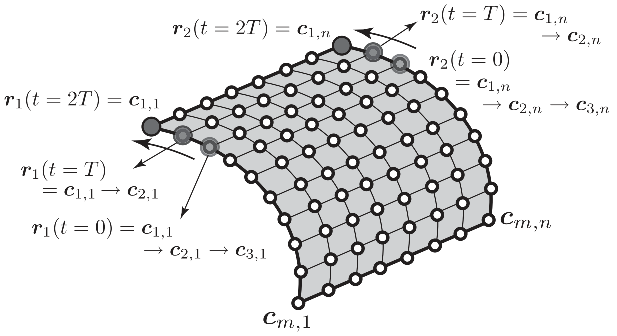


Fig. 5. Algebraic equation model.

Taking the time delays into account and introducing the model assumptions, the joint coordinates of the cloth can be given by the following equation:

$$\begin{aligned} \boldsymbol{c}_{i,j}(t) &= \boldsymbol{r}_1\left(t - (d_i + d_j)\right) \quad j \leq n/2 \\ \boldsymbol{c}_{i,j}(t) &= \boldsymbol{r}_2\left(t - (d_i + d_j)\right) \quad j > n/2. \end{aligned} \tag{3}$$

Fig. 5 illustrates the proposed model (algebraic equation). Since the proposed model does not include an inertia term, Coriolis and centrifugal force terms, or a spring term, we do not need to estimate the dynamic model parameters; only the normalized time delay λ has to be estimated. The value of λ may be dependent on the characteristics of the cloth. The advantage of the proposed model is that the number of model parameters is smaller than in typical models. Therefore, the proposed model itself is robust. Moreover, since the cloth model can be algebraically calculated, the simulation time becomes much shorter. In particular, the most critical advantage is that the motion planning can be obtained by an algebraic calculation from the configuration of the flexible object.

In the calculation sequence, the joint coordinates of the cloth can be calculated, while changing each joint number in the longitudinal direction and the lateral direction from the hand grasp position. In this study, since there are two hand grasp positions, the calculation of the joint coordinates is performed from the grasp positions of hands 1 and 2. Then, the central joint coordinate of the cloth is finally calculated.

C. Correction of Link Distance

Describing the cloth deformation using Eqn. (3), there exists a case where the distance between two joint coordinates cannot be kept constant. Therefore, in order to satisfy assumption 3), the joint coordinates of the cloth need to be converted as follows:

The joint coordinates to be converted are defined as $\boldsymbol{c}_{i,j} = [x_{i,j} \quad y_{i,j} \quad z_{i,j}]$, and the prior joint coordinates (that is, nearer to the position grasped by the robot) are $\boldsymbol{c}_{i-1,j} = [x_{i-1,j} \;\; y_{i-1,j} \;\; z_{i-1,j}]$. The distance between these two joint coordinates can be described by

$$D = \parallel \boldsymbol{c}_{i,j} - \boldsymbol{c}_{i-1,j} \parallel . \tag{4}$$

In the case where D is not equal to l (l is the link distance), the joint coordinates $\boldsymbol{c}_{i,j}$ are corrected in terms of polar

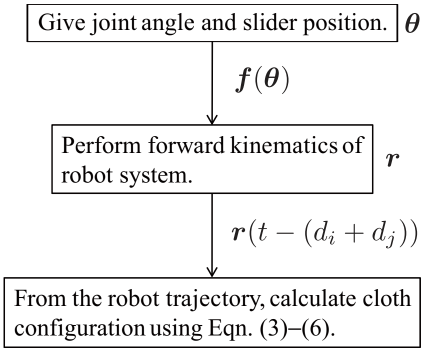


Fig. 6. Simulation flow of forward problem.

coordinates as follows:

$$\begin{aligned} x_{i,j} &= l\sin\theta\cos\phi + x_{i-1,j} \\ y_{i,j} &= l\sin\theta\sin\phi + y_{i-1,j}, \\ z_{i,j} &= l\cos\theta + z_{i-1,j} \end{aligned} \tag{5}$$

where

$$\begin{aligned} \theta &= \cos^{-1}\left(\frac{z_{i,j}-z_{i-1,j}}{D}\right) \\ \phi &= \cos^{-1}\left(\frac{x_{i,j}-x_{i-1,j}}{\sqrt{(x_{i,j}-x_{i-1,j})^2+(y_{i,j}-y_{i-1,j})^2}}\right). \end{aligned} \tag{6}$$

In the same way, the joint coordinates ($\boldsymbol{c}_{i,j} = [x_{i,j} \;\; y_{i,j} \;\; z_{i,j}]$) should be also corrected for the neighboring joint coordinates ($\boldsymbol{c}_{i,j-1} = [x_{i,j-1} \;\; y_{i,j-1} \;\; z_{i,j-1}]$). Then, a weighting factor can be calculated from each correction value of the longitudinal and lateral directions. Finally, joint coordinates for the longitudinal and lateral directions are corrected.

Fig. 6 shows a simulation flow of the forward problem using the cloth deformation model. First, the joint angle of the wrist θ_w and the slider position θ_s are given. Then, the grasp positions $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ of the cloth are calculated from the forward kinematics. Finally, the cloth deformation $\boldsymbol{c}$ is derived with Eqns. (3)–(6).

D. Validity of model

Here we discuss the validity of the proposed model (algebraic equation, Eqn. (3)). Fig. 7 shows images of the simulation results and the experimental results as viewed from the side. In this experiment, the motions of the bending-extension wrist joint θ_w and the translation of the slider θ_s are properly given. As can be seen from Fig. 7, the cloth deformations in the simulation and the experiment are the almost same, confirming the validity of the proposed model.

E. Inverse problem

This section explains the inverse problem of deriving the joint angle of the wrist and the slider position from the cloth configuration. It is difficult and complex to give all control points (the cloth configuration) of a curved surface on a 3-D plane. Thus, let us consider the cloth configuration on the x-z plane (side view) in order to simplify the inverse problem. Here, we assume that the cloth deformations at both hands are the same. Namely, the grasp position of each hand is the same ($\boldsymbol{r} = \boldsymbol{r}_1 = \boldsymbol{r}_2$). Fig. 8 shows the simulation flow of the inverse problem.

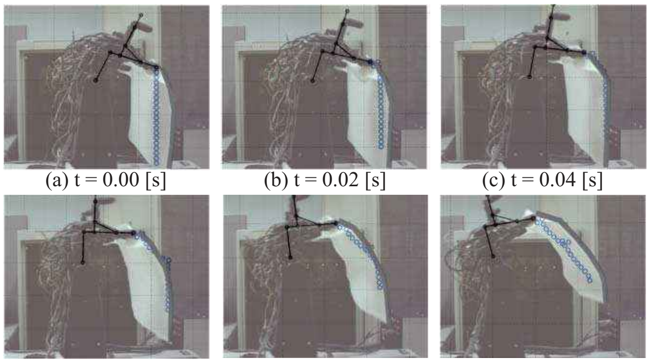

(a) t = 0.00 [s] (b) t = 0.02 [s] (c) t = 0.04 [s]
(d) t = 0.06 [s] (e) t = 0.08 [s] (f) t = 0.10 [s]

Fig. 7. Validity of model.

First, we give the number of links of the multi-link system of the cloth. The desired cloth configuration $\boldsymbol{c'}$ is graphically set in a 2-D plane by a human operator. There is a case where the link distance between the two joint coordinates for the given cloth configuration is not equal to l. Therefore, the cloth configuration is corrected using polar coordinates (Eqns. (4)–(6)).

Second, the cloth configuration $\boldsymbol{c}$ is converted so as to match the robot system kinematics to avoid problems such as singular points.

Third, the trajectory of the grasp position $\boldsymbol{r}$ of the robot system is calculated from the converted cloth configuration $\boldsymbol{c}$. From the assumption that the cloth deformation depends on the high-speed robot motion, the trajectory $\boldsymbol{r}$ of the robot system can be obtained, to track the given coordinates of each joint of the cloth. Namely, we have the following equations:

$$\boldsymbol{r}(t=0) = \boldsymbol{c}_N, \quad \boldsymbol{r}(t=T) = \boldsymbol{c}_1, \tag{7}$$

where N $(= 20)$ is the number of joints, and T $(= 0.4$ s$)$ is the motion time. The trajectory is determined so as to linearly move from the N-th link to the first link during the motion time T. Here, the trajectory is calculated so as to compensate for the effects of gravity.

Finally, the joint variables $\boldsymbol{\theta}$ of the robot system can be obtained by solving the inverse kinematics.

F. Simulation of Dynamic Folding

In this section, we show the simulation results of the dynamic folding. In this simulation, the size of the cloth is 0.4 m × 0.4 m. Since the number of joints (m, n) of the multi-link model is $(20, 20)$, the link distance l is 0.02 m.

In order to achieve dynamic folding of the cloth, motion planning of the robot system is extremely important. In particular, obtaining the desired cloth configuration is a key element to the success of this task. Motion planning is required to realize this.

In this simulation, the 2-D cloth configuration $\boldsymbol{c}$ is given as shown in Fig. 9. In Fig. 9, the blue circles are the given cloth configuration on the x-z plane. Then, the joint variables $\boldsymbol{\theta}$ of the robot system can be calculated by solving

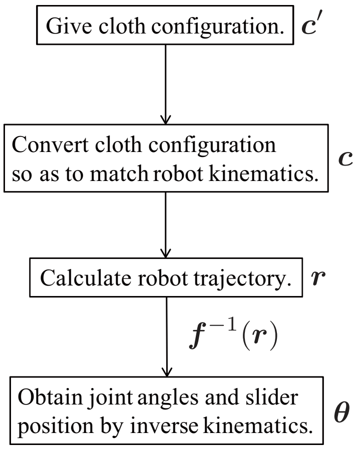
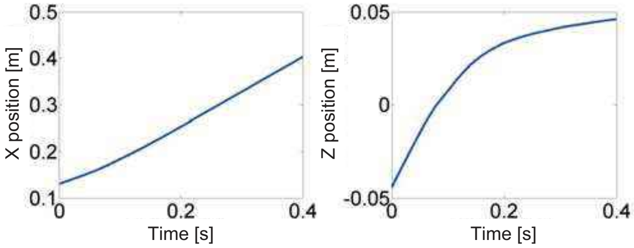
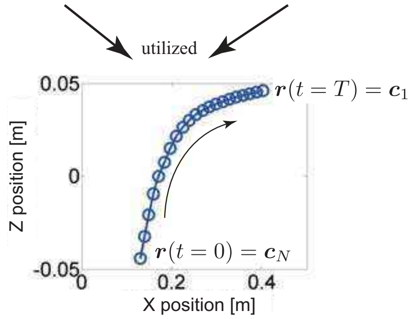


Fig. 8. Simulation flow of inverse problem.



(a) X position (b) Z position

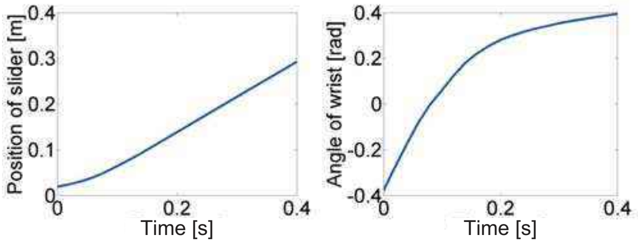


(c) X-Z position

Fig. 9. Cloth configuration.

the inverse kinematics. Fig. 10 shows the results of the trajectories of the wrist joint angle θ_w of the hand and the slider position θ_s. Fig. 11 shows the simulation results of the dynamic folding using the robot motion obtained in the inverse problem. In Fig. 11, the black circles and the blue circles depict the positions of both hands and the cloth coordinates, respectively. It can be seen from Fig. 11 that the cloth deforms depending on the hand and slider actions. The folding aspect of the cloth can be confirmed. Therefore, the validity of the cloth deformation model can be verified. The robot motion obtained by the simulation can be used in the experiment.

By using the proposed model, the trajectory of the robot system can be algebraically obtained when an arbitrary cloth configuration is given. In this simulation, folding of the cloth by the inertial force is not considered. Also, folding can be simulated by calculating the parabolic motion of each joint.



(a) Trajectory of slider (b) Trajectory of wrist of hand

Fig. 10. Result of motion planning.

In the parabolic motion calculation, the velocity of each joint at the moment that the robot stops is considered as the initial condition.

Finally, the grasp timing of the deformed cloth can be estimated from the simulation results. In the next section, we show experimental results using the robot trajectory obtained by the simulation.

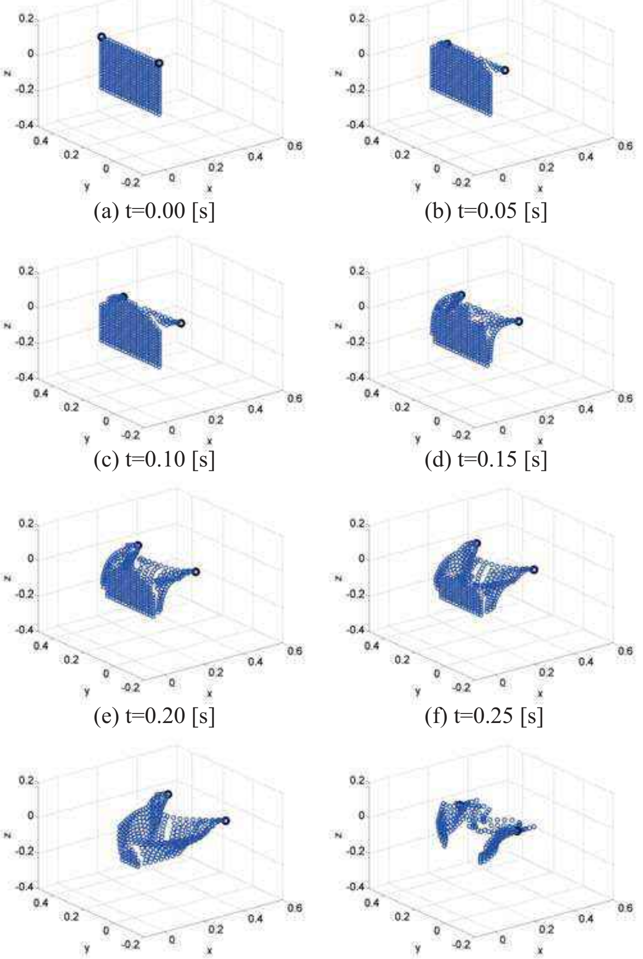
V. EXPERIMENT

In Fig. 12, the dynamic folding is shown as a continuous sequence of pictures taken at intervals of 57 ms. In this experiment, the robot motion obtained by the simulation is implemented. In addition, the grasp timing of the deformed cloth can be estimated from the simulation result. The size of the cloth is 0.4 m × 0.4 m. This video is available online [9].

Fig. 12(a) represents the initial condition of the experiment, where the two hands grasp the cloth. Fig. 12(b)–(e) show pulling of the cloth toward the grasp positions using the hand and slider motions. Fig. 12(f)–(g) show that when the hand and slider motions stop, the free end (the far point from the grasp position) of the cloth is folded by an inertial force. Fig. 12(h) shows grasping of the free end of the cloth. As can be seen from this result, dynamic folding of the cloth can be achieved by two high-speed multifingered hands and two high-speed sliders. Moreover, since the action time of the dynamic folding performed by the robot system is 0.4 s, high-speed folding can be carried out.

Although the success rate of cloth deformation is almost 100 %, the success rate of cloth grasping is only about 30 %. Thus, the success rate of the dynamic folding on the whole is about 30 %. The reason is that the cloth deformations at the left and right sides of the cloth are slightly different despite the same trajectories of the two hands and the two sliders. In order to improve the success rate of the grasping, high-speed visual feedback control should be introduced. However, since the desired cloth deformation can be obtained, the effectiveness of the proposed deformation model of the cloth and the motion planning method, as well as the validity of the simulation results, can be confirmed.

In the case where a typical (low-speed) robot is used, the proposed model cannot be applied, and the appropriate deformation of the cloth cannot be achieved. As a consequence, dynamic folding cannot be realized. Thus, a different folding strategy is required.



(a) t=0.00 [s] (b) t=0.05 [s] (c) t=0.10 [s] (d) t=0.15 [s] (e) t=0.20 [s] (f) t=0.25 [s] (g) t=0.30 [s] (h) t=0.40 [s]

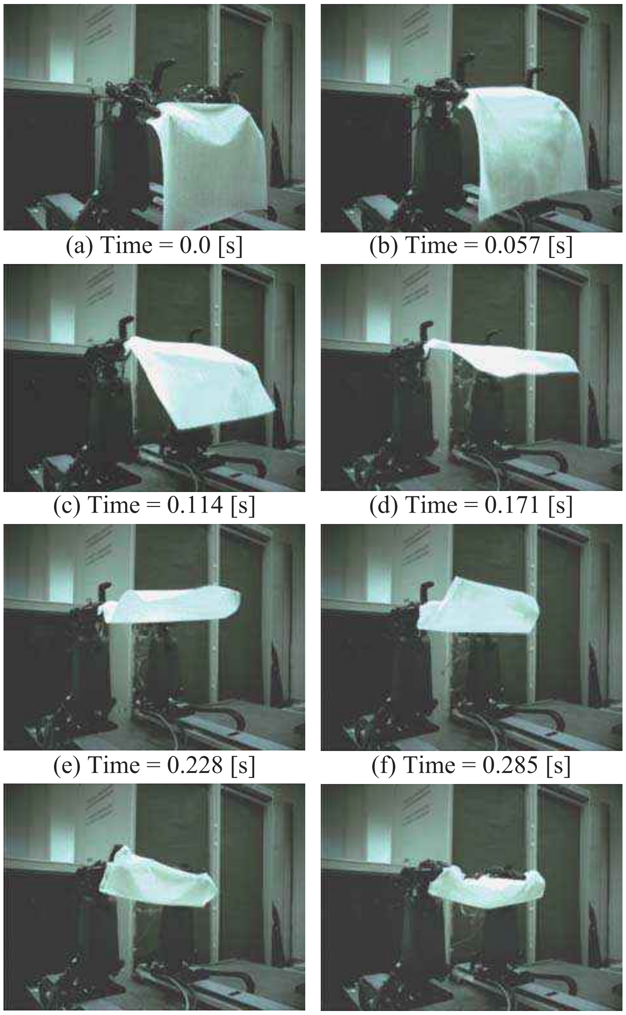
Fig. 11. Simulation result of dynamic folding.

VI. CONCLUSIONS

In this study, we aimed at dynamic manipulation of a sheet-like flexible object. As one example of the dynamic manipulation, we performed dynamic folding of a cloth with two high-speed multifingered hands and two high-speed sliders.

In the previous paper, we proposed a simple model (algebraic equation) of a linear flexible object by taking advantage of high-speed robot motion. In this study, we extended the model of the linear flexible object to the model of the sheet-like flexible object using the high-speed robot motion. Then, we proposed a motion planning method based on the proposed model of the sheet-like flexible object. We validated the robot trajectory obtained from the motion planning method using simulation results. Finally, we implemented the robot trajectory in the experimental system and demonstrated dynamic folding experimentally. We also confirmed the effectiveness of the proposed model and the motion planning method.

In future work, we will introduce high-speed visual feedback in order to improve the success rate.



(a) Time = 0.0 [s] (b) Time = 0.057 [s] (c) Time = 0.114 [s] (d) Time = 0.171 [s] (e) Time = 0.228 [s] (f) Time = 0.285 [s] (g) Time = 0.342 [s] (h) Time = 0.4 [s]

Fig. 12. Dynamic folding by robot system.

REFERENCES

[1] H. Inoue and M. Inaba, Hand-eye Coordination in Rope Handling, Robotics Research: The First International Symposium, MIT Press, pp. 163–174, 1984.

[2] H. Wakamatsu, E. Arai and S. Hirai, Knotting/Unknotting Manipulation of Deformable Linear Objects, Int. Journal of Robotics Research, Vol. 25, No. 4, pp. 371–395, 2006.

[3] M. Shibata, T. Ohta and S. Hirai, Fabric Unfolding Using Pinching Slip Motion, Journal of Robotics Society Japan, Vol. 27, No. 9, pp. 1029-1036, 2009. (in Japanese)

[4] K. Tanaka, Y. Kamotani and Y. Yokokohji, Origami Folding by a Robotic Hand, Proc. IEEE/RSJ Int. Conf. on Intelligent Robot Systems, pp. 2540–2547, 2007.

[5] D. J. Balkcom and M. T. Mason, Robotic origami folding, Int. Journal of Robotics Research, Vol. 27, No. 5, pp. 613–627, 2008.

[6] J. M. Shepard, M. C. Towner, J. Lei and P. Abbeel, Cloth Grasp Point Detection based on Multiple-View Geometric Cues with Application to Robotic Towel Folding, Proc. IEEE Int. Conf. on Robotics and Automation, pp. 2308–2315, 2010.

[7] Y. Yamakawa, A. Namiki and M. Ishikawa, Motion Planning for Dynamic Knotting of a Flexible Rope by a High-speed Robot Arm, Proc. IEEE/RSJ Int. Conf. on Intelligent Robot Systems, pp. 49–54, 2010.

[8] A. Namiki, Y. Imai, M. Ishikawa, and M. Kaneko, Development of a High-speed Multifingered Hand System and Its Application to Catching, Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp. 2666–2671, 2003.

[9] http://www.k2.t.u-tokyo.ac.jp/fusion/DynamicFolding/